DEFINITION. Any DE of the form

$$a(t)\frac{d^2x}{dt^2} + b(t)\frac{dx}{dt} + c(t)x = g(t)$$

where $a(t)$, $b(t)$, $c(t)$, and $g(t)$ are continuous on some interval I, $a(t) \neq 0$ on I, is a second-order linear DE.

DEFINITION (9.1). Any DE of the form

$$(*) \quad a\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = 0$$

where a, b, and c are constants, $a \neq 0$, is a second-order homogeneous linear DE with constant coefficients.

Goal: To solve $(*)$ for all possible values of a, b, and c.

EXAMPLE.

(1) $x'' - x' = 0$.

Let $y = x'$. Then the DE becomes

$$y' - y = 0,$$

a first-order linear DE with a specific, not general, solution

$$y = e^t.$$

Then

$$x' = e^t \Longrightarrow$$
$$x = \int e^t \, dt = e^t$$

is a solution.