PHY 399 ***Exam 2*** **Name:** ____________________________

1 [17 pts] Recall the definitions $\hat{a}_{\pm} = \frac{1}{\sqrt{2\hbar m\omega}}(\mp i\hat{P} + m\omega\hat{X})$ and the algebraic solution to the TISE for the harmonic oscillator $\psi_n = A_n(\hat{a}_+)^n\psi_0$, where A_n is a normalization constant.

(a) Write down an explicit differential equation that ψ_0 must satisfy. Box your answer.

(b) The solution to this differential equation is $\psi_0 = A_0 e^{-\frac{m\omega}{2\hbar}x^2}$. Find ψ_2, the second excited state of the harmonic oscillator. You may leave an underdetermined normalization constant A_2 in the solution...you don't have to find it.

$\psi_2 =$ ____________________________

(c) The allowed energies for a particle of mass m in a harmonic oscillator potential giving classical

angular frequency ω are $E_n =$ ______________________ (No work need be shown.)

2 [8 pts] Compute the commutator $[\hat{X}^2, \hat{P}]$ and simplify as much as possible.

$[\hat{X}^2, \hat{P}] =$ ________________

3 [13 pts] A <u>free particle</u> is in the state $\Psi(x,0) = \begin{cases} A & 0 < x < a \\ 0 & else \end{cases}$.

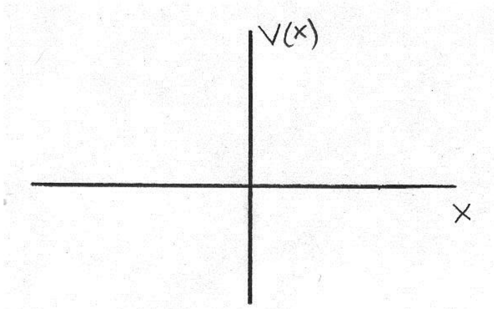
(a) Find A to normalize the state. (No work need be shown.) $A =$ ____________

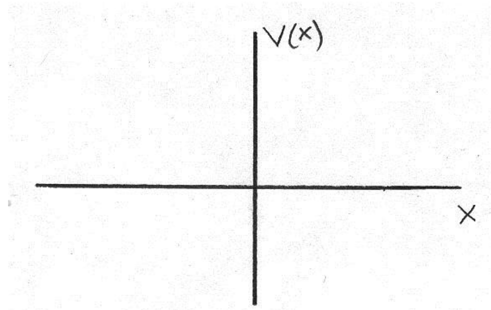
(b) Find $\Psi(x,t)$. Your final answer may contain one integral over a "dummy variable", the variables x and t, and the constants a, m and $\hbar$.

$\Psi(x,t) =$ ______________________________________

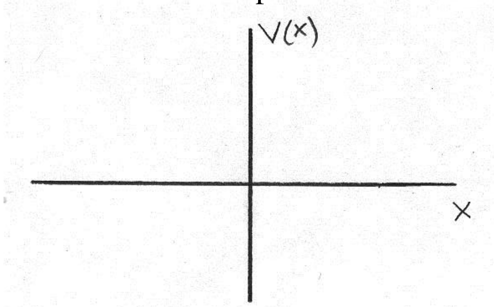
4 [9 pts] In the following sketches I will assume that the general trend at the edge of your graph continues out to infinity.

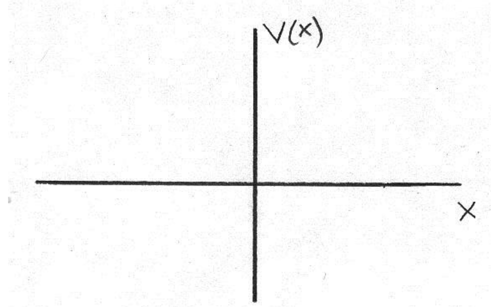
(a) Sketch two different potential functions that will only allow bound state solutions to the TISE.



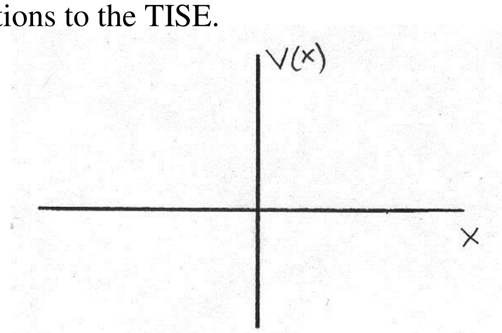


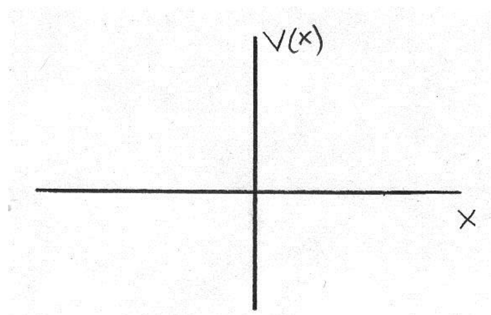
(b) Sketch two different potential functions that will only allow scattering state solutions to the TISE.





(c) Sketch two different potential functions that will allow some bound state and some scattering state solutions to the TISE.





5 [6 pts] Which of the following collections of functions form a vector space under the usual sense of addition and scalar multiplication by complex numbers? Circle all letters which are vector spaces.

(a) The set of all functions whose magnitude never exceed 1.

(b) The set of all continuous functions.

(c) The set of all functions satisfying $f(5) = 0$.

(d) The set of all constant functions.

(e) The set of all functions which are normalized to 1.

(f) The set of all sinusoidal functions (that can be written as $A\sin(kx+\phi)$ for some constants A, k and ϕ).

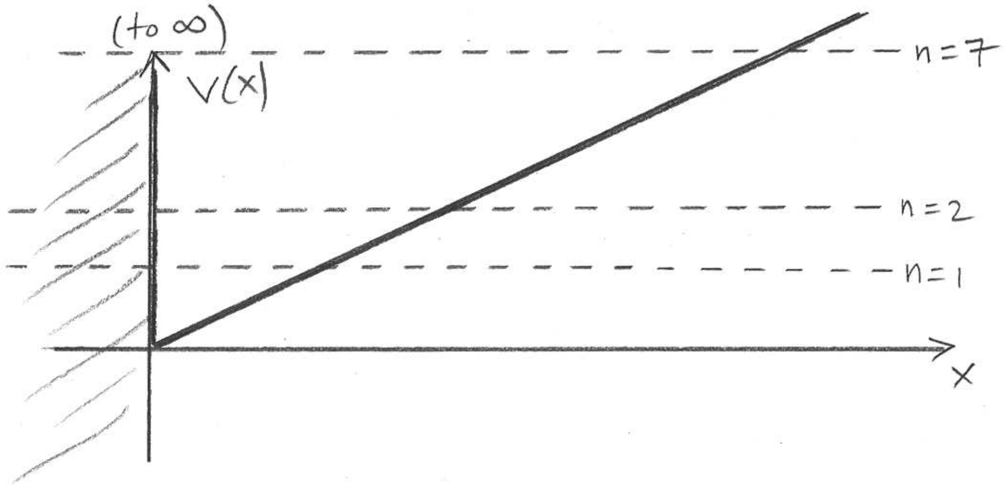
$\boxed{6}$ [17 pts] Suppose that you write down the TISE for a certain potential. After a change of variables $x \to y$ you obtain the differential equation $y\frac{d^2\psi}{dy^2} - \psi = 0$.

(a) Propose a series solution $\psi =$ ______________________

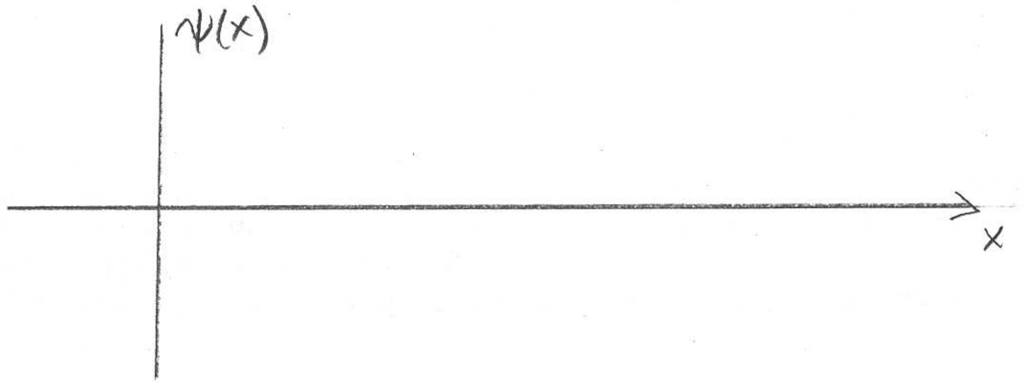
(b) Find a recursion relation for the coefficients in your series. Box your answer.

(c) If boundary conditions require that the coefficient of y^1 in your series is 1 (i.e. $a_1 = 1$), find the first three nonzero coefficients of your series. The y^1 term can count for one of the three. Box your answer.

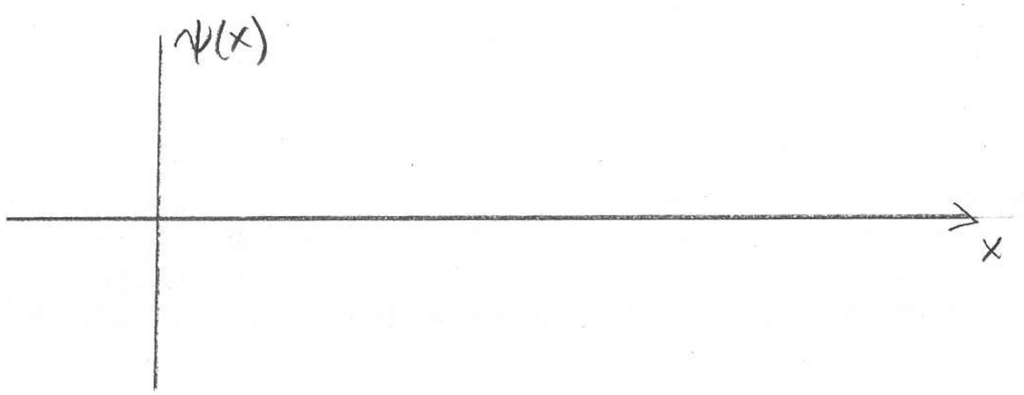
7 [13 pts] Consider a potential $V(x) = \begin{cases} \infty & x < 0 \\ mx & x \geq 0 \end{cases}$ shown in the sketch. The solutions to the TISE are called Airy functions. Sketch the Airy functions corresponding to the ground state (n = 1) and the states corresponding to n = 2 and n = 7, with energies shown by the dashed lines. Make sure that the x values on your sketch line up with the x values directly above on the potential sketch.



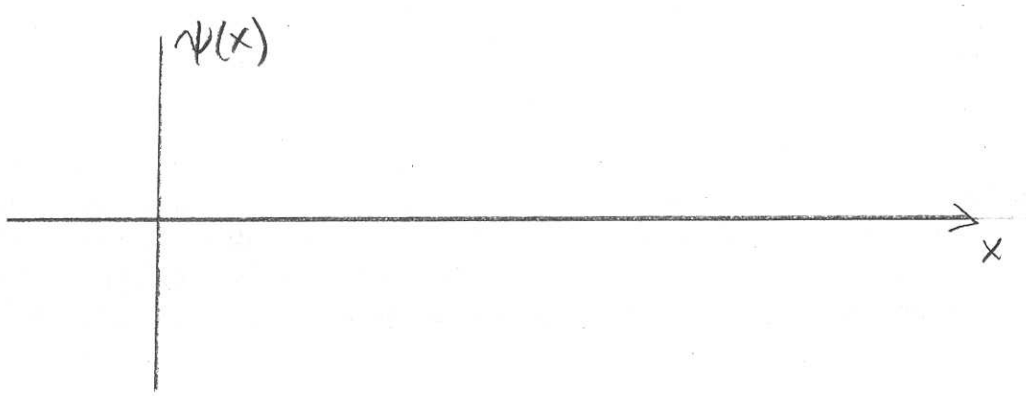
n = 1:



n = 2:



n = 5:



8 [17 pts] Consider the finite square well, $V(x)=\begin{cases}-V_0 & -a<x<a\\ 0 & else\end{cases}$, where V_0 is a positive constant.

The general solution to the TISE for the bound states of this potential is

$$\psi(x)=\begin{cases}Ae^{\kappa x}+Be^{-\kappa x} & x\le -a\\ C\sin(\ell x)+D\cos(\ell x) & -a<x<a\\ Fe^{\kappa x}+Ge^{-\kappa x} & x\ge a\end{cases},\ \text{where}\ \ell=\frac{\sqrt{2m(E+V_0)}}{\hbar}\ \text{and}\ \kappa=\frac{\sqrt{-2mE}}{\hbar}.$$

(a) There are six undetermined constants in this solution, so there should be six conditions to fix these values. Suppose we are looking for the <u>odd function bound states</u>. If you immediately know the values for any of the constants in this case, state the value of the constant and the reason you know this. For the remaining constants give enough equation(s) or other conditions that will determine their values. Your equations may contain the constants κ and ℓ, which will be determined by choosing one of the allowed values of E. You should, of course, have six values or equations/conditions.

Known Value or Condition	Justification

(b) Write down an equation that determines the allowed energies for the odd solutions to the finite well. It should involve E, of course, and may involve the other constants in the problem: m, V_0, $\hbar$ and a. You don't have to solve this equation for the allowed E values. Box your answer.