A BILEVEL PROGRAMMING FRAMEWORK FOR ENTERPRISE-WIDE SUPPLY CHAIN PLANNING PROBLEMS UNDER UNCERTAINTY

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Abstract

Enterprise-wide supply chain planning problems inherently exhibit multi-level decision network structures, where for example, one level may correspond to a local plant control/scheduling/planning problem and another level to a plant-wide planning/distribution network problem. Such multi-level decision network structures can be mathematically represented using multi-level mathematical programming principles. In this paper, we address bilevel decision-making problems under uncertainty in the context of enterprise-wide supply chain optimization with one level corresponding to a plant-specific planning problem and the other to a distribution network problem. We first describe how such problems can be modelled as bilevel programming problems and then present an effective solution strategy based on parametric programming techniques.

Keywords

supply chain planning, uncertainty, bilevel programming, parametric programming.

Introduction

Supply chains typically involve multiple enterprise-wide activities, from the procurement of the raw materials, through a series of process operations, to the distribution of end-products to customers. It is not surprising that their design and operation issues pose a number of important theoretical, technical and practical challenges, which have started to receive increasing attention in academia and industry (see representative publications in Table 1). However little attention has been given to actual supply chain principles, particularly (i) hierarchical decision structures from local, independent to global, centralized objectives, which are often conflicting each other, and (ii) incomplete data and information to significant uncertainty involved in characteristics at the various levels of the hierarchy *i.g.* demand forecasts, raw material availabilities, etc.

In order to bridge the gap between the industrial practices and the lack of corresponding research, we propose an approach that directly captures their multilevel and uncertainty aspects based on bilevel optimization principles. The solutions of the resulting stochastic bilevel programming problems are obtained by proposing an effective solution strategy based on parametric programming techniques.

Supply chain planning - a bilevel optimization model

Bilevel programming problems refer to hierarchical optimization problems (leader's or outer problems) that are constrained by another optimization problem (follower's problem or inner problem). It is often used to describe situations involving several indifferent groups which are inter-connected in a hierarchical structure (see some of representative references on bilevel programming in Table 2). Each group may correspond to an individual or an agency, often with a corresponding independent

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objective. The two problems are inter-connected: the outer problem sets parameters influencing the inner problem; the outer problem, in turn, is affected by the outcome of the inner problem. Bilevel programming problems are challenging since even they typically involve non-convexities for linear models and attention has been given to only deterministic ones (see, for example, Visweswaran et al., 1996).

Table 1. Recent research on Supply chain planning problem

Researcher	Solution Method	Uncertainty	Key issue
Bose and Pekny (2000)	Simulation Optimization	Demand	Model predictive control (MPC)
Zhou et al.(2000)	Optimization (multi-objective)	no.	Refinery example
Gupta and Maranas (2000)	Optimization	Demand	Stochastic programming
Flores et al.(2000)	Simulation	Demand	MPC
Gjerdrum et al.(2001)	Optimization (MILP)	no.	Profit distribution Game-theory
Perea-Lopez et al.(2001)	Simulation	Demand	Decentralized Control
Papageorgiou et al.(2001)	Optimization (MILP)	no	Tax, scale-up cost Phamaceutical

Table 2. Representative applications of the bilevel programming

Area	Reference
Economy	Cassidy et al. (1971) Hobbs and Nelson (1992)
Civil Eng.	Clegg et al. (2000) Boyce and Mattsson (1999) Chiou (1999), Migdalas (1995)
Environ. Eng.	Amouzegar and Moshirvaziri (1999)
Finance	Bard et al. (2000)
Chem. Eng.	Clark and Westerberg (1983, 1990) Grossmann and Floudas (1987) Brengel and Seiderm (1992) Visweswaran et al. (1998) Floudas et al. (2001)

In view of multiple enterprise activities in actual supply chains, their planning problems can be naturally posed as bilevel optimisation models. Consider the following manufacturing supply chain that consists of a production part involving two plants, *PL1, PL2* and a distribution part, involving an inventory warehouse, *WH* for two products *A* and *B*, as shown in Figure 1. Based on the mathematical notation in Table 3, its individual production and distribution problem can be mathematically modelled separately as follows:

Table 3. Notation

Indices	
I	Product (1,…,*NM*)
L	Plant (1,…,*NL*)
Variables	
Y_{li}	Production amount of product I at plant l(ton)
X_i	Inventory holding amount of product I (ton)
Parameter	
DM_i	Demand of product i(ton)
pc_{li}, pd_i, CRS_{li}, $CRSB$, IRS_{li}, $IRSB_l$, dc_i, dd_{li}, $b2$, $INVRS_i$, $INVB$: *cost parameter*	

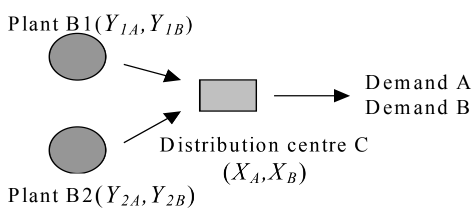

Figure 1 Process configuration of an illustrative supply chain

PRODUCTION MODEL

$$\min_{Y_{li}} Z_{PC} = \sum_{l=1}^{NL}\sum_{i=1}^{NM} pc_{li} Y_{li} + \sum_{i=1}^{NM} pd_i X_i \qquad (1)$$

$$s.t. \quad \sum_{l=1}^{NL}\sum_{i=1}^{NM} CRS_{li} Y_{li} \le CRSB \qquad (2)$$

$$\sum_{i=1}^{NM} IRS_{li} X_{li} \le IRSB_l \quad \forall l \qquad (3)$$

$$\sum_{l=1}^{NL} Y_{li} \ge X_i \quad \forall i \qquad (4)$$

where the objective function ***(1)*** is to minimize production / delivery costs; ***(2)*** denote that commonly used resources at each plants can be shared; ***(3)*** represent that allocations of some resources may be controlled by individual plant conditions; ***(4)*** indicate that the production should exceed the inventory warehouse levels.

DISTRIBUTION MODEL

$$\min_{X_i} Z_{DC} = \sum_{i=1}^{NM} dc_i X_i + \sum_{l=1}^{NL}\sum_{i=1}^{NM} dd_{li} Y_{li} \quad (5)$$

$$s.t. \quad \sum_{i=1}^{NM} INVRS_i X_i \le INVB, \quad (6)$$

$$X_i \ge DM_i \quad \forall i \quad (7)$$

where *(5)* represents the minimization of warehouse distribution costs; *(6)* are bounds for the inventory levels; *(7)* denote that inventory levels should meet demands.

Note that the decisions of the distribution part are generally based on those of the production part: for example, inventory policies are made using the outcome of production decisions. Similarly, decisions on the production part are affected by decisions of the distribution part: for example, production levels are decided from the given information regarding the inventory conditions. Therefore the overall supply chain planning model can be posed as the following bilevel optimization problem:

$$\begin{aligned}
&\min_{X_i} Z_{DC} = \sum_{i=1}^{NM} dc_i X_i + \sum_{l=1}^{NL}\sum_{i=1}^{NM} dd_{li} Y_{li}\\
&s.t. \quad \sum_{i=1}^{NM} INVRS_i X_i \le INVB,\\
&\quad X_i \ge DM_i \quad \forall i\\
&(Y_{li}, \forall l, i) \in \left\{ \begin{aligned}
&\min_{Y_{li}} Z_{PC}\\
&= \sum_{l=1}^{NL}\sum_{i=1}^{NM} pc_{li} Y_{li} + \sum_{i=1}^{NM} pd_i X_i\\
&s.t. \quad \sum_{l=1}^{NL}\sum_{i=1}^{NM} CRS_{li} Y_{li} \le CRSB\\
&\quad \sum_{i=1}^{NM} IRS_{li} X_{li} \le IRSB_l \quad \forall l\\
&\quad \sum_{l=1}^{NL} Y_{li} \ge X_i \quad \forall i
\end{aligned} \right\} \quad \textbf{(8)}
\end{aligned}$$

where the inner problem corresponds to the production optimization problem and the outer problem to the distribution problem. By denoting X_{li} as x, Y_i as y and by also including uncertainty (present in, *i.g.* demand forecast, equipment availability etc.) denoted as θ, *(8)* may be recast as the following bilevel programming problem under uncertainty:

$$\begin{aligned}
&\min_x F(x,y,\theta) = c_1^T x + d_1^T y + ct_1^T \theta\\
&s.t. \quad A_1 x + B_1 y \le b_1 + K_1 \theta \qquad \textbf{(9)}\\
&\min_y f(x,y,\theta) = c_2^T x + d_2^T y + ct_2^T \theta\\
&s.t. \quad A_2 x + B_2 y \le b_2 + K_2 \theta
\end{aligned}$$

where $x \in X \subseteq R,\ y \in Y \subseteq R, \theta \in \Theta \subseteq R,$ $b_1, b_2, c_1, c_2\ , d_1, d_2, ct_1, ct_2$ are constant vectors and $A_1, A_2, B_1, B_2, K_1, K_2$ are constant matrices.

Parametric programming-based solution methodology

There is little research on methodology for stochastic bilevel programming problems like *(9)* to the best of our knowledge. We therefore propose a novel solution methodology involving the following three steps:

Step 1

Formulate the inner optimisation problem as a multi-parametric linear programming (mp-LP) problem by regarding the variables of the outer problem and the uncertain parameters as parameters:

$$\begin{aligned}
&\min_y d_2^T y + (c_2^T \quad ct_2^T)\begin{pmatrix} x \\ \theta \end{pmatrix}\\
&s.t. \quad B_2 y \le b_2 + (-A_2 \quad K_2)\begin{pmatrix} x \\ \theta \end{pmatrix} \qquad \textbf{(10)}\\
&\quad x^L \le x \le x^U\\
&\quad \theta^L \le \theta \le \theta^U
\end{aligned}$$

Step 2.

Solve problem *(10)* using multi-parametric LP algorithms (refer to Dua, 2000 and POP software). The corresponding parametric solutions are of the following form:

$$y = \begin{cases}
\zeta_1(x,\theta) = l_1 + m_1 x + n_1 \theta & if\ H_1 x \le h_1 + I_1 \theta,\\
\zeta_2(x,\theta) = l_2 + m_2 x + n_2 \theta & if\ H_2 x \le h_2 + I_2 \theta,\\
\vdots & \vdots\\
\zeta_k(x,\theta) = l_k + m_k x + n_k \theta & if\ H_k x \le h_k + I_k \theta,
\end{cases} \qquad \textbf{(11)}$$

where k denotes the number of the computed parametric solutions, l_k is a constant parameter, H_k and I_k are constant matrices and h_k is a constant vector.

Step 3.

Using the parametric expression in *(11)*, the outer problem is then transformed into a family of single parametric optimization problems. By solving those single problems, all local optimal solutions of the original problem are obtained and the global optimum may be determined consequently.

A typical solution for the illustrative example is shown in Table 4, where uncertainty in demands is incorporated as θ_A, θ_B. The proposed methodology is novel because it provides a complete set of optimal planning strategies of individual supply chain elements as a function of uncertain parameters and other design variables which are decided in advance hierarchically.

Conclusion

This paper has proposed a bilevel programming framework to address industrial supply chain planning problems under uncertainty. The solutions of the resulting problem are computed using a novel methodology based on parametric optimization.

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Table 4. A typical Result of the illustrative example

#	Critical region	Optimal operation plan: Production	Optimal operation plan: Distribution
1	$\begin{cases} -1.5X_A - X_B \le -250 \\ 1.143X_A + X_B \le 228.571 \\ 1.143\theta_A + \theta_B \le 228.571 \end{cases}$	$\begin{cases} Y_{1A} = -0.5X_B + 100 \\ Y_{1B} = X_B \\ Y_{2A} = X_A + 0.5X_B - 100 \\ Y_{2B} = 0 \end{cases}$	$\begin{cases} X_A = -0.875\theta_B + 200 \\ X_B = \theta_B \end{cases}$
2	$\begin{cases} -1.5X_A - X_B \le -250 \\ 1.143X_A + X_B \le 228.571 \\ -1.143\theta_A - \theta_B \le -228.571 \\ 1.182\theta_A + \theta_B \le 250 \\ 1.294\theta_A + \theta_B \le 267.647 \end{cases}$	$\begin{cases} Y_{1A} = -0.5X_B + 100 \\ Y_{1B} = X_B \\ Y_{2A} = X_A + 0.5X_B - 100 \\ Y_{2B} = 0 \end{cases}$	$\begin{cases} X_A = \theta_A \\ X_B = \theta_B \end{cases}$
3	$\begin{cases} -1.5X_A - X_B \le -250 \\ 1.182X_A + X_B \le 250 \\ -1.143X_A - X_B \le -228.571 \\ 1.143\theta_A + \theta_B \le 228.571 \\ -1.5\theta_A - \theta_B \le -250 \end{cases}$	$\begin{cases} Y_{1A} = \frac{4}{3}X_A + \frac{2}{3}X_B - 166.667 \\ Y_{1B} = -\frac{8}{3}X_A - \frac{4}{3}X_B + 533.333 \\ Y_{2A} = -\frac{1}{3}X_A - \frac{2}{3}X_B + 166.667 \\ Y_{2B} = \frac{8}{3}X_A + \frac{7}{3}X_B - 533.333 \end{cases}$	$\begin{cases} X_A = \theta_A \\ X_B = \theta_B \end{cases}$