NAME _____

ES 442 Homework #3

(**Spring 2016** – Due <u>February 22, 2016</u>)
Print out homework and do work on the printed pages.

Textbook: B. P. Lathi & Zhi Ding, ***Modern Digital and Analog Communication Systems***, 4th edition, Oxford University Press, New york, 2009.

Problem 1 (Problem 4.2-3 of Lathi & Ding) (35 points)

You are asked to design a double-sideband-suppressed carrier (DSB-SC) modulator to generate a modulated signal $km(t) \cdot \cos(\omega_c t + \theta)$, where $m(t)$ is a signal band-limited to B Hertz and k is a constant. The figure below shows a DSB-SC modulator available commercially. The carrier generator is not $\cos(\omega_c t)$, but rather $\cos^3(\omega_c t)$. That is, the cube of $\cos(\omega_c t)$. Explain if you will still be able to use $\cos^3(\omega_c t)$ to generate the output of $km(t) \cdot \cos(\omega_c t + \theta)$ as required from this modulator. You are free to choose any filter you wish (that is, any bandwidth).

(a) (b)
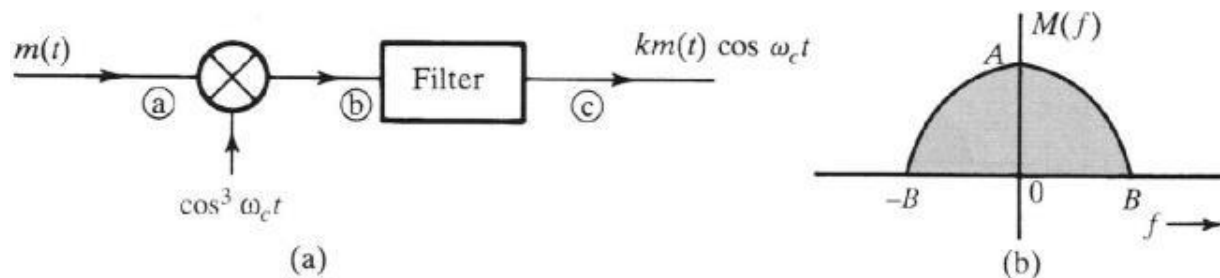
(a) If $\cos^3(\omega_c t)$ does work, describe the filter you would need to generate an output of $km(t) \cdot \cos(\omega_c t + \theta)$?

(b) Determine the signal spectra at nodes labeled "b" and "c," and indicate the frequency bands occupied by at the spectra at both nodes.

(c) What is the minimum usable value of ω_c possible?

(d) Would this modulator scheme work if the carrier output were changed to $\sin^3(\omega_c t)$ instead? Explain clearly the reason for your answer.

(e) Will the same modulator scheme work if the carrier generator output were $\cos^n(\omega_c t)$, for any integer $n \geq 2$, instead? If yes, why does it?

Problem 2 Audio Scrambler (Problem 4.2-8 Lathi & Ding) (20 points)

The system shown below is used for scrambling audio signals (although it is not a very sophisticated scrambler). The output $y(t)$ is the scrambled version of the input $m(t)$.

$M(f)$

-15 kHz 15 kHz

f kHz

$m(t)$

$2 \cos 30{,}000\pi t$

Low-pass filter
0–15 kHz

$y(t)$
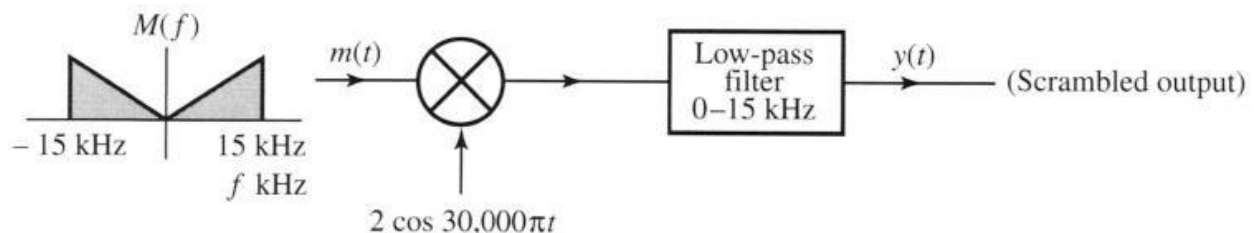
(Scrambled output)

(a) Sketch the spectrum of the scrambled output signal $y(t)$ on the graph below.
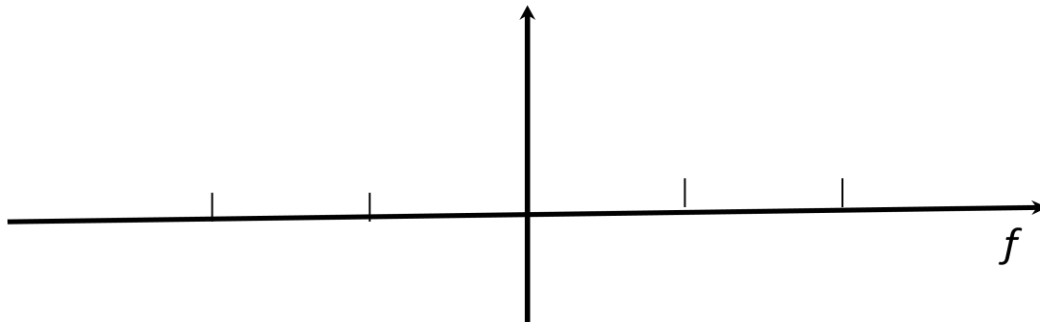
(b) Suggest a method for descrambling $y(t)$ to recover $m(t)$.

Problem 3 High-Q Resonant *RLC* Circuit (25 points)

One possible frequency selective circuit is an *LRC* resonator as schematically shown below. The resonant frequency is given by

$$f_{resonance} = \frac{1}{\sqrt{LC}}$$

The <u>Quality factor</u> of a resonant circuit (*Q*-factor for short) is defined as the resonant frequency divided by the half-power (or -3 dB) bandwidth; in symbols it is

$$Q = \frac{\text{resonance frequency}}{\text{half-power bandwidth}} = \frac{f_{resonance}}{B}$$

In this problem our focus is on passing an amplitude modulated signal through the *RLC* filter and how it can affect the modulation of the carrier.
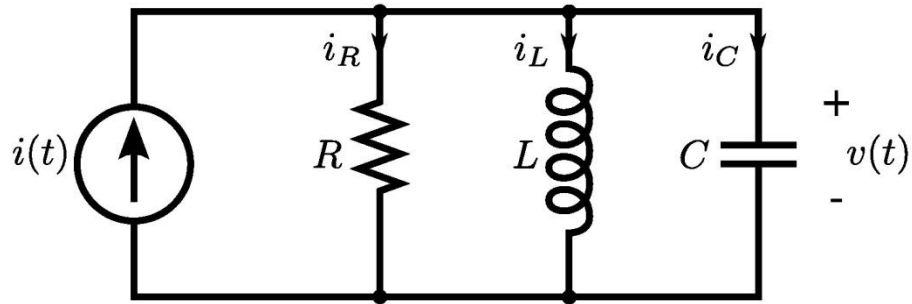
(a) Derive the transfer function $H(2\pi f)$ for this parallel RLC circuit. Assume the sinusoidal steady-state in deriving the transfer function.

(b) Derive the half-power bandwidth B (that is, the -3 dB bandwidth) for this circuit. Express B as a function of R, L and C.

(c) Starting with the definition of Q-factor at the beginning of the problem statement, derive an expression for Q as a function of the circuit parameters.

(d) A carrier wave of frequency of 1 MHz is modulated at 50% by a sinusoidal frequency of 5 kHz. [Note: A **modulation index of 50%** means the modulation signal amplitude range is 50% of the amplitude range of the carrier signal.] We assume that L and C have been chosen so that the circuit's resonance frequency is exactly 1 MHz. Resistor R is chosen so the **Q-factor is 200**. If a modulated signal with 50% modulation is passed through this RLC filter network, with the carrier frequency passed with no loss, how much is the 5 kHz modulation affected by the RLC circuit? Another way to ask this question, by how much is the modulation reduced at 5 kHz by the RLC circuit?

Problem 4 Free Space Propagation Loss (20 points)

Wireless communications depends upon the transmission of radio waves from transmitting antenna to receiving antenna. In ES 442 we do not have the time to discuss antenna operation in much detail, but this problem illustrates an important feature of wireless communication. Point-to-point wireless (such as line-of-sight microwave links) depends upon the free space propagation model. The free space power $P_r(d)$ received by the antenna separated by distance d from the transmitting antenna is given by the Friis free space equation.

$$P_r(d) = \frac{P_t G_r G_t \lambda^2}{(4\pi)^2 d^2 L} ,$$

where P_t is the transmitted power, G_t is the transmitter antenna gain, G_r is the receiver antenna gain, d is the distance between antennas in meters, L is the system loss not related to propagation losses ($L \geq 1$) and λ is the carrier wavelength in meters. The two antenna gain terms depend upon the effective apertures of the antennas – for this problem we will ignore that and assume G_t and G_r both equal unity. The wavelength depends upon frequency and the speed of electromagnetic waves (c = 300,000,000 meters per second); given by the equation

$$\lambda = \frac{c}{f} = \frac{2\pi c}{\omega_c} \ ,$$

where f is the frequency in Hertz, and ω_c is the carrier frequency in radians per second. The values for P_r and P_t must be expressed in appropriate units.

(a) All of this is background. If a transmitter produces 50 watts of radiated power at the antenna [P_t (dBW) = 17 dBW = 47 dBm], what is the received power in dBm at the receiver if the carrier frequency is 900 MHz (that is GSM cellular frequency) at d = 100 meters? Assume both antenna gains to be unity and let L = 1.

(b) Now let the distance be 10 kilometers; what is the received power in dBm?

(c) What conclusion do you draw as distance d increases? How many dB per decade as distance change?