Performance Based Learning and Assessment Task

Discover Radians!

I. ASSESSSMENT TASK OVERVIEW & PURPOSE:

Students will use a unit circle and string to measure the length of the radius and then measure how many radii are in the arc of the circle defined by the central angle.

II. UNIT AUTHOR:

Thomas Abernethy, Jr., Lee-Davis High School, Hanover County Public Schools
Jerry Dawson, Glen Allen High School, Henrico County Public School

III. COURSE:

Geometry

IV. CONTENT STRAND:

Geometry, Measurement. SOL G.8: The student will solve real-world problems involving right triangles by using right triangle trigonometry.

V. OBJECTIVES:

Students will: 1)define the meaning of radians, 2)identify the degree measure of an arc of a central angle in the unit circle, 3)identify the radian measure of an arc of a central angle in the unit circle, 4) identify the difference between degree measure and radian measure.

VI. REFERENCE/RESOURCE MATERIALS:

Students will use: 1)a 12-inch diameter unit circle cut out of poster board, 2)string, 3)a straight-edge inch and metric measuring tool, 4)pencil, 5) scissors, 6)nail or Philips head screwdriver, 7)protractor, 8)TI-83 or higher graphing calculator, 9)high school Geometry textbook and Trigonometry textbook

VII. PRIMARY ASSESSMENT STRATEGIES:

Students and teachers will use an assessment rubric

VIII. EVALUATION CRITERIA:

Students will perform a self assessment using the same criteria the teacher will use to perform the final assessment.

IX. INSTRUCTIONAL TIME:

One ninety-minute block to prepare the unit circle, collect data, and complete the data chart; One sixty-minute block for students to report their findings and elaborate in their conclusions.

Discover Radians !

Strand
Geometry, Measurement.
SOL Standard G.8: The student will solve real-world problems involving right triangles by using right triangle trigonometry.

Mathematical Objective(s)
Students will use a unit circle and string to measure the length of the radius and then measure how many radii are in the arc of the circle defined by the central angle. Students will discover the meaning of the term "radians". Students will construct a large 12-inch diameter circle using a thumb tack, string, and a pencil. Students will construct a trigonometric unit circle using a compass and Geometry and Trigonometry textbooks as reference sources for the angle and radian measures of the unit circle. Students will identify by algebraic proof that "pi" is the number of diameters of any circle that fit into the circumference of the circle. Students will then identify what radians are by measuring the length of the unit circle radius with string, then using that piece of string to measure how many radii there are in the arc of that central angle. Students will eventually identify how many radii fit into the circumference of the circle, and relate that to the distance of 2π radians around the circle. The specific goal of this activity is for students to identify the actual meaning of "radians". The skills of accurate measuring, division, algebraic deduction, and understanding unit circle measures are being reinforced in this activity.

Related SOL

SOL G.8: The student will solve real-world problems involving right triangles by using right triangle trigonometry.

NCTM Standards List all applicable NCTM standards related to each task/activity. Example:
- Establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others
- Use trigonometric relationships to determine lengths and angle measures
- Make decisions about units and scales that are appropriate for problem situations involving measurement
- Use unit analysis to check measurement computations
- Analyze precision, accuracy, and approximate error in measurement situations
- Build new mathematical knowledge through problem solving
- Make and investigate mathematical conjectures
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others
- Use the language of mathematics to express mathematical ideas precisely

Materials/Resources
- Graphing calculators
- Straight edge inch and metric rulers, string
- Poster board, nail or Philips head screwdriver, string, pencil, scissors
- Geometry and Trigonometry textbooks

Assumption of Prior Knowledge

Students should come with the ability to use a straight edge measuring tool in inches and centimeters, and the ability to round off centimeter measures to decimals. Students should understand what a ratio is and how to accurately record a ratio's numerator and denominator, be aware of what a trigonometric unit circle is, understand what quadrants are and accurately identify them, what a diameter is, what a radius is, what a central angle is, what an arc of a central angle is, what the formula for circumference of a circle is, that an angle can be measured in degrees or radians, what degrees are, and what the numerical representation of "pi" is, and how to place a calculator in degree and radian mode. Students will use the terms quadrant I, II, III, and IV, diameter, radius, "pi", central angle, and arc during this activity. Student may have difficulty expressing what a degree is, which is essential to their understanding of the comparison between degrees and radians.

Introduction: Setting Up the Mathematical Task

In this activity you will investigate the relationship between degrees and radians of a circle, and clearly define what radians are. The activity will take one 90-minute class period and one-60-minute class period. You will construct a circle, turn that circle into a trigonometric unit circle, and use string to discover what radians are. You will identify the difference between degrees of a circle and radians of a circle. You will be able to explain to others what degrees are, what radians are, how you discovered what radians are, what different units of measure such as "60 degrees" and "$\pi/6$ really mean, and ultimately what 2π means, in a 5-minute presentation to the class after your activity. During the presentation you will also describe what the challenges of the task were, and what you learned from the activity that you were not aware of before.

Student Exploration

Small Group Work

Preparing the Circle:

Students will work in groups of two. Exploration is as follows:

Take a piece of poster board. In the center of the poster board set a 12 inch ruler. At the 6 inch mark place the pointed edge of a nail (or Philips head screwdriver) with one loop-end of a piece of non-stretchable string under it. Loop the other end of the string around the bottom edge of a pencil as low as possible without it slipping off of the pencil point, 6 inches from the tip of the nail. Press down on the nail to anchor it into the poster board. Draw a circle with the pencil moving around the nail 6 inches out, keeping consistent string tension. Cut out the resulting circle with scissors right on the pencil edge.

Preparing the Unit Circle:

The teacher should use one or two models of previous student work to give students a benchmark of what their unit circle should look like. If there is no previous work, the teacher should prepare a unit circle to look like what the students should produce, and place in the front of the room during the activity for students to use as a visual model. Then the students should begin as follows:

Use a straight edge and draw a diameter. Take a protractor and measure off a 30 degree angle using the nail hole created by pressing down into the poster board as the center of the circle. Use the straight edge to draw a radius to the outer edge of the circle, forming a 30 degree angle with the diameter. Measure off another angle, this time 45 degrees, using the nail hole (indentation) as circle center. Construct another radius to the outer edge of the circle. Now measure a 60 degree angle using the nail indentation as circle center, and construct a radius forming a 60 degree angle with the diameter. Finally, construct a 90 degree angle with the diameter, and draw a radius that connects with the center of the diameter. Continue this for quadrants II, III, and IV until the entire unit circle has been created. Use a Geometry or Trigonometry textbook to find a unit circle labeled with appropriate radian measures. Neatly label each circle edge endpoint of each radius with degree measure and radian measure.

Radians discovery:
Cut a piece of string to exactly the length of the radius of the circle. Do not stretch the string. Preserve the accuracy of the edges of the string to prevent fraying during the entire activity. Measure the length of the string in centimeters. Now place the string on the outer edge of the circle around (beside) the arc created by the first angle, 30 degrees. Do not stretch the string, just lay it on the outer edge of the 30 degree arc. Mark the string or place your finger nail on the edge to mark the arc length. Now straighten the string and use the centimeter straight edge to measure the length of the string. This is arc length. Identify the ratio of arc length to radius length, reduce if necessary, and place that ratio in the proper column on the data sheet. Using the TI-83 (or above) graphing calculator, convert that to decimal form, rounding to hundredths, and place that figure in the next column. Next to that, in the next column, place the "pi" ratio of the radian measure of that arc from the unit circle, then convert that value to decimal form to the nearest hundredths and place that value in the next column. Compare the two decimal values.

Follow the same procedure for each angle in the unit circle, performing measurements and recording values in the data chart. Note: when an angle begins requiring the string to be placed end to end to measure arcs that are longer than the string length, be careful to maintain accuracy when calculating string length in centimeters. (By the time you get to 180 degrees, you will be measuring a little over 3 string lengths.)

After completing this procedure for the entire unit circle and completing the data chart, record what you have found. How does the decimal value of the ratio of arc length to radius length compare to the decimal ratio of "pi" values of each point on the unit circle? How would you define the term "radians"? How would your data be different if the circle had a different diameter? Would your conclusions be different? Would your definition of "radians" be different? How has this activity solidified your understanding of what "pi" actually is? Is "pi" a value that was invented by ancient mathematicians? Is "pi" a discovery of mathematical truth?
Record your conclusions.

Teacher activity:
During the activity the teacher will circulate and provide coaching and assistance, monitoring how each group is proceeding toward completion of the unit circle itself and then toward data accumulation. The

recording of conclusions by students should be monitored as well to ensure complete sentences and thoroughness.

Small Group Work:
During the following class students will present their unit circles and their 5-minute report of their findings, what was challenging for them in the activity, and what they became aware of that they did not know before the activity.

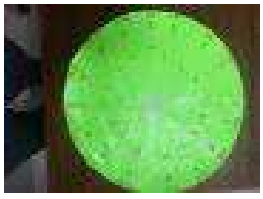
Whole Class Sharing/Discussion
After each small group has made their presentations, the teacher will give the class a one minute Think/Pair/Share time of reflection with their group partners or for groups to interact with one another, and then call on volunteers to share how the activity could be altered or improved in the future.

Assessment List and Benchmarks

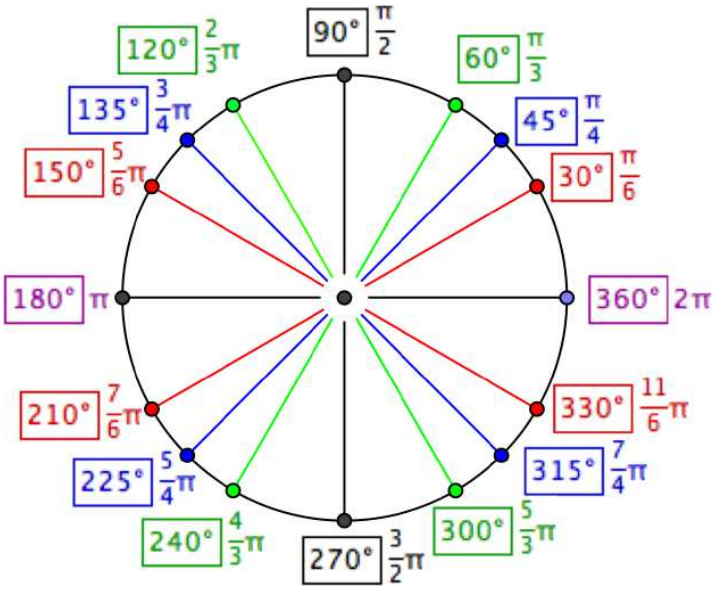
The Student Assignment which will be assessed is a data chart with questions to be answered on the back, or second page. See the "Unit Circle Data Chart" at the end of this document.

The tools for self-assessment and teacher assessment are the same, shown next. See "Self Assessment" and "Assessment" at the end of this document.

Below is a sample of student work from an Algebra 3 Trigonometry class where students made a model of a unit circle. The units are not very readable; however, this model is to display the general idea of cutting a circle out of poster board and labeling it with the angles and radians. The picture was taken with a mobile phone while the green unit circle was resting on the top of a dark oak teacher desk.



Below is another representation of the unit circle presenting a clearer picture of the information students need to show, only the students will have all their labeling inside the outer ring of the circle instead of outside the ring as this model has them.



Unit Circle Data Chart (Centimeters) Name_____Date_____Block____

Discover Radians! Activity

Central Angle Measure (degrees)(1)	Radius Length (same for all) (2)	Arc Length (3)	$\frac{ArcLength}{RadiusLength}$ (Ratio) (4)	$\frac{ArcLength}{RadiusLength}$ (Decimal) (5)	"Pi" Ratios (6)	"Pi" Ratios (Decimal) (7)

Over-→

Conclusions:

1. How do $\frac{ArcLength}{RadiusLength}$ (decimal form) and "Pi" Ratios (decimal form) compare (the two shaded columns)?

2. What have you learned about the term "radians"?

3. How do radians compare with degrees?

4. What was your prior knowledge of radians?

5. How would your data be different if the circle had a different diameter? Explain.

6. If the circle had a different diameter, would your conclusions be different? Would your definition of "radians" be different?

7. How has this activity solidified your understanding of what "pi" actually is? Is "pi" a value that was invented by ancient mathematicians?

Is "pi" a discovery of mathematical truth?

8. List any other discoveries or impressions you have gained from this activity. How might it be amended, how might it be improved?

Discover Radians! Self Assessment Name_____Grade: /95 points = _____

	5 pts: 100% completed	4 pts: 90% completed	3 pts: 80% completed	2 pts: 70% completed	1 pt: 60% completed	0 pts: not completed
Unit Circle 12" diameter						
Lines on Unit Circle						
Angles Marked on Unit Circle						
Trigonometric Radians on Unit Circle						
Data Chart Completed Column 1						
Data Chart Completed Column 2						
Data Chart Completed Column 3						
Data Chart Completed Column 4						
Data Chart Completed Column 5						
Data Chart Completed Column 6						
Data Chart Completed Column 7						
Data Chart Completed Column 1						
Question 1 Completed						
Question 2 Completed						
Question 3 Completed						
Question 4 Completed						
Question 5 Completed						
Question 6 Completed						
Question 7 Completed						
Totals:						

Discover Radians! Assessment Name_____Grade: /95 points = _____

	5 pts: 100% completed	4 pts: 90% completed	3 pts: 80% completed	2 pts: 70% completed	1 pt: 60% completed	0 pts: not completed
Unit Circle 12" diameter						
Lines on Unit Circle						
Angles Marked on Unit Circle						
Trigonometric Radians on Unit Circle						
Data Chart Completed Column 1						
Data Chart Completed Column 2						
Data Chart Completed Column 3						
Data Chart Completed Column 4						
Data Chart Completed Column 5						
Data Chart Completed Column 6						
Data Chart Completed Column 7						
Data Chart Completed Column 1						
Question 1 Completed						
Question 2 Completed						
Question 3 Completed						
Question 4 Completed						
Question 5 Completed						
Question 6 Completed						
Question 7 Completed						
Totals:						