APPENDIX LIST OF SYMBOLS

а	A calibration constant; for example, it is the service-employment multiplier or population- serving ratio (number of service jobs generated from one household or resident)
ã	Intercept regression-coefficient as a random variable
ã*	Specific value of \tilde{a} corresponding to a sample of data points
<i>a'</i>	Acceleration of a vehicle; also a constant parameter, such as unit cost of commuting (cost per unit-of-distance travelled), or the exponent of the development opportunity W_j at destination <i>j</i>
a_i	Calibration parameter corresponding to the utility increase in zone <i>i</i> , where utility is some measure of composite accessibility to the zone; also the population-serving ratio at zone <i>i</i>
a _t	Estimation-error or noise term for a series of data ($t = 1, 2,$) usually in a 'normalized' time-series, or after the data have been differenced to a stationary series; the estimated error or noise in Kalman filtering; also referred to as innovations when it is white noise Physical area of generarbia when it is at the domand generating potential of <i>i</i> .
$a_i a_t'$	Measurement error in a Kalman-filter time-series, representing the difference between observed and measured data
a_D	Error term in a demand econometric-equation
a_{S_W}	Error term in a supply econometric-equation
<i>a</i> "	weighted labor-force-participation-rate, where the weights are the percentages of regional population in each zone
a^{r}	Parallel to its single dimension analogue <i>a</i> is an error or poise term in the spatial
u _{ij}	context; it has a zero mean and a constant variance; also stands for the entries in the $\bar{\mathbf{A}}$ matrix
a_i^u	Convex combination of the population-serving ratios, with normalized accessibilities to zone <i>i</i> as weights
a_j^p	Employment multiplier considering the population-serving ratio, i.e., $(1+a_j)$ —segregated both by economic-sector <i>p</i> and by zone <i>i</i> here
a^{kl}	Calibration parameter in a predictor-prey equation-set showing the interaction between the <i>k</i> th and <i>l</i> th species
a'_{kj}	The <i>k</i> th output (benefit) measures due to decision-making-unit <i>j</i> considering both non-spatial and spatial attributes (see also $\overline{\mathbf{A}} = [\mathbf{a}_{ij}]$)
$a_{ij}^{pq}(k)$	Impact of the <i>p</i> th-state-variable-in-zone- <i>i</i> -at-time- <i>k</i> on the <i>q</i> th-state-variable-in-zone- <i>j</i> -at-time- $k+1$
<i>a</i> "	Threshold for a high-pass noise-filter
$\mathbf{a} = (\leftarrow a_i \rightarrow)^T$	Vector of calibration coefficients in the second stage of 2-stage least-squares, consisting of q entries; also stands for the vector of the error (noise) terms in a spatial-temporal forecasting-model
a'	Vector whose <i>i</i> th element is the ratio of the-household-income to the gross-output-in-the- <i>i</i> th-industrial-sector
$\tilde{\mathbf{a}}_{ij} = (-a_{ij}^k)^T$	Interim error-vector or noise-term in a more efficient calibration-procedure for STARMA Each entry of the $\bar{\mathbf{A}} = {}_{\mathbf{i}} a_{\mathbf{i}\mathbf{j}}$ payoff-matrix is replaced by a vector in a linear program, mainly to facilitate a multicriteria, two-person, zero-sum, non-cooperative game; here k is the index for a criterion
α α'	Calibration constant, or step size in "hill-climbing" algorithms; also the tail of a distribution Angle between two criterion-functions in multicriteria linear-programming; also a calibration constant
α " α_t	Resulting problem-type after the original problem has been polynomially reduced Random-shock or white-noise input-time-series in a transfer-function model
α_{ii}^{qp}	Exponent in a Cobb–Douglas production-function corresponding to the factor input x_{ii}^{qp}
A	Accessibility expenditure for a household (part of locational expenditure); also the area
$A(\cdot)$	Area of ·
A_i	Weighted labor-force participation-rate, with accessibility from zone <i>i</i> as weights
A_j	Gross acreage of subarea <i>j</i>
A_{i}	Error term in a 'raw-data' time-series
\overline{A} .	Developable acreage in subarea <i>j</i>
A^{B}	Basic land-use (A_i^B) is basic land-use in zone i)

A - 2	Appendix - Symbols
$\frac{A^{R}}{A^{U}}$	Retail land $(A_j^R$ is retail land in zone j) Unusable land $(A_j^U$ is unusable land in zone j) Set of arcs in a network
\hat{A}_{\cdot}^{k}	Net acreage in subarea <i>j</i> devoted to the <i>k</i> th land-use
$\mathbf{A} = (-A_t)^T$	Vector of disturbance or error terms in econometric or spatial time-series models, consisting of n observations; in 2-stage least-squares, it consists of q entries, where q is the number of endogenous variables
Α	As a matrix (instead of a vector), A stands for node-arc incidence-matrix in network-flow programming
$\mathbf{A}' = [A'_{ij}]$ $\mathbf{A}_{0}(t)$ $\mathbf{A}'' = [(i, i)]$	An $n \times n$ square matrix; for a compartmental model, it is the rate-of-change matrix; and for the matrix of secondary (retail)-employment it is the distribution-rate by zone, where $n = n'$. Vector showing rate-of-change with the "outside world" over time Continuity matrix with ponzero are entries where <i>i</i> is incident upon <i>i</i> .
$\mathbf{A} = [(i,j)]$	An $n \times n$ matrix, which converts value added output vector by industrial sectors to the same
\mathbf{A} \mathbf{A}_{j}	vector measured in labor-force base Vector of socioeconomic variables at location <i>j</i> , representing such activities as population and employment
$\mathbf{A}(j)$	Column vector in the network-simplex tableau for arc j
$\overline{\mathbf{A}} = \begin{bmatrix} a_{ii} \end{bmatrix}$	Coefficient matrix of linear-programming constraints, where a_{ij} expresses the incidence
	relationship between row <i>i</i> and column <i>j</i> ; an example is the <i>k</i> th output measures due to decision-making-unit <i>j</i> , a_{kj} , in a data-envelopment analysis. Basis of a linear program Nonbasic part of the tableau in a linear program The complicated set of constraints in a mixed integer-program The straightforward set of constraints in a mixed integer-program Generally a constant parameter denoting a growth rate intercept or slope in a linear
0	equation, or the positive exponent of a spatial cost-function etc.
ĥ	'Slope' regression-coefficient as a random variable
	Specific value of $\tilde{\boldsymbol{b}}$ for a sample of data points Household budget The fixed cost of siting a depot at node <i>j</i> Travel-cost elasticity for activity <i>j</i>
$\mathcal{O}(m)$	A scale factor used to adjust the kth zonal-retain-employment from one loop of the Lowry model <i>m</i> to another $m+1$, where $m = 1, 2,$ Slack-flow capacity on slack arc (<i>k i</i>) or (<i>i k</i>): also the benefit variable in data-envelopment
b_{kii}	analysis, denoting the weight placed on the k th benefit of the i th alternative Benefit variable used in the combined data-envelopment-analysis-and-location model,
$\mathbf{b} = (-b_i)^T$	showing the relative importance of assigning the <i>k</i> th benefit to the demand-facility pair ij . Vector of estimated parameters in ordinary least-squares regression or other calibration procedures, consisting of $k+1$ parameters (including the 'intercept'); also the right-hand-side of a linear or mixed integer program.
$\mathbf{b}' = (-b_i')^T$	A given vector of the right-hand-side of a mathematical program; also the fixed external- flows in a network-flow program
b	Updated right-hand-side of a linear program during a simplex procedure; also the birth rates in a cohort survival analysis
\mathbf{b}^1	The portion of the right-hand-side corresponding to the complicated set of constraints in a mixed-integer-program
b ²	The portion of the right-hand-side corresponding to the straightforward set of constraints in a mixed-integer-program
β	A calibration constant, such as the positive exponent of a spatial cost-function or the round- trip factor in stochastic facility-location. (This same constant β is also referred to as b)
β'	A calibration constant
β_i	Current level of inventory at location <i>i</i> Prewhitened output time-series in a transfer-function model
$\overset{P_t}{B}$	An arbitrarily large integer; also the backshift operator in a time series
B'	Bifurcation set of control variables
$\mathcal{B}_k^{\prime\prime}$	Blue-collar employment Percentage reflectance in band k of a satellite sensor

$B_L, B_R \\ B'_k \\ B^k \\ B^k$	Left and right boundaries of a firm's market area Number of times a facility is exposed to demands in period k Bound value for distance from a vertex, used to locate the intersecting point q_k or a candidate location for a center
$B_{\min}^{M}, B_{\max}^{M}$	Lower and upper bounds for the border-line length of a subregion
$\mathbf{B} = \begin{bmatrix} b_j \end{bmatrix}$ $\mathbf{B} = \begin{bmatrix} b_{ij} \end{bmatrix}$	Birth matrix with nonzero diagonal-elements showing the 'birth' rate within subarea <i>j</i> Arbitrary matrix in a tableau of network-with-side-constraint program, corresponding to the flow
$\mathbf{B'}=[\beta_{ij}]$	Calibration-coefficient matrix in the first stage of a 2-stage least-squares, which measures $q \times k$, where q is the number of endogenous variables and k the exogenous variables
$\tilde{\mathbf{B}} = \tilde{b}_{ij} $	Quasi-deterministic transition-matrix in a compartmental model
\mathbf{B}_i	Diagonal-block <i>i</i> of the inverse of a network node-arc incidence-matrix, expressed in terms of a spanning subgraph
$\mathbf{B}''=[b_{ij}']$	Fixed cyclic-permutation δ' expressed in terms of a matrix operation, where $b'_{t,t'0} = 1$ and all
	other elements $b'_{ii}=0$
Ē	Initial basis for a network-with-side-constraint model
С	Cost of operation, unit-cost, or a constant in general (e.g., c_i is the unit cost at location <i>i</i> ; c_{kl} is the "interaction cost" of moving materials between workstations <i>k</i> and <i>l</i> in an assembly line)
<i>c'</i>	Proportionality constant
c^k	Weight reflecting the relative importance of home-based retail-trips for purpose k
^r c ^s (x)	<i>r</i> th-stop coverage of state <i>s</i> by routing-variable x
$\mathbf{c} = (-c_j)$	Cost vector in the objective function of a linear program, which is also the gradient of the objective function; here c_i is the constant unit-cost
c'	Consumption-coefficient vector, whose <i>i</i> th element is the ratio of the purchased-value-of- the-commodity-from-the- <i>i</i> th-industrial-sector to the household income
\mathbf{c}_B	The part of the cost-vector c corresponding to the basic variables
\mathbf{c}_N	The part of the cost-vector c corresponding to the nonbasic variables
\mathbf{c}' $\mathbf{c}^{k+r}(k)$	Binary vector of <i>r</i> th-stage coverage-requirements in the decomposed recursive-program Binary vector of <i>r</i> th-stage coverage-requirements on each origin–destination pair in cycle $k \in \mathbf{C}(k) = [\leftarrow \mathbf{c}^{k+r}(k) \rightarrow]$
$\operatorname{conv}(\tilde{\boldsymbol{\varrho}}')$ C	Convex combination of discrete points $\mathbf{\tilde{g}}'$ in a feasible region of an integer program Generalized cost to include both time and monetary outlay, or unit composite-cost in general (e.g., C_i is the generalized cost of operation or the inventory-carrying cost at location <i>i</i> , C_{ij} is the composite transportation-cost from location <i>i</i> to <i>j</i> , C_{ij}^p is the composite transportation-cost from location <i>i</i> to <i>j</i> for compodity <i>n</i> etc.)
C'	Number of columns in a lattice, grid or a pixel image; also household expenditure on community amenities (which is part of non-locational expenditure)
C_0	Overhead of a firm
C_o	Operating cost
C_s	Capital cost
C_j	Equity factor in districting algorithms
C_X	Coefficient-of-variation of variable X, or $s_{\mathbf{X}}/\mathbf{X}$
$C_{XY} C(C_{ij})$	Cross-covariance between random variables X and Y Propensity, distribution, or accessibility function between <i>i</i> and <i>j</i> , assuming such forms as exponential function or power function of spatial cost C
$C[a](\mathbf{x})$	Performance of arc or path a as a function flow-vector x
$C'(\tau)$	Accessibility to work-opportunities as a function of time τ
$C^{k}(\tau)$	Accessibility to the kth non-work-opportunity as a function of time τ
$C_i(\cdot)$	The cost function (including land rent), or performance function, of firm <i>i</i> —expressed in
$C_{ij}(V_{ij})$	terms of the supply volume V_i^s or other arguments Transportation cost between origin-destination pair $i-j$ as a function of flow V_{ij} between
Ck,l	them Transportation aget between origin k and destinction l
$C^{mn}(r)$	Connectivity requirement between origin–destination pair $m-n$ via at most rth-stop itineraries
$\mathbf{C} = [C_{ij}]$	Arbitrary matrix in a tableau of network-with-side-constraint program, corresponding to the non-flow variables; also the covariance matrix

A - 4	Appendix - Symbols
$\mathbf{C} = [\mathbf{c}^1, \dots, \mathbf{c}^q]^T$ $\mathbf{C}(\cdot)$	A $q \times n$ matrix of cost coefficients in a multicriteria linear-program, where each criterion <i>j</i> has a cost and a gradient vector \mathbf{c}^{j} State-connectivity function linking to past decisions and connectivity requirements in a recursive program
C '	Diagonal matrix converting the gross-output vector to value-added vector
Ĉ	Matrix of estimated coefficients in stage 1 of 2-stage least-squares, measuring $q \times k$
$ar{m{\mathcal{C}}}{\gamma} \gamma^{\prime} \gamma^{\prime} \gamma^{\prime} \gamma^{pq}_{j}$	Number of cell columns in a grid region or in a raster image Unit price at the market, Lagrange multiplier, and a calibration constant in general Capacity-utilization rate, bounded between zero and unity Dual variable associated with the input–output coefficients in an entropy-maximization model
$\mathbf{\gamma}' = [q_j']$	Matrix of subareal growth-rates along its diagonal
Ÿ	Economic-base multiplier over a time-increment Δt , combining the activity-rate f and the population-serving-ratio a ; $\overline{\mathbf{y}}_{g}$ (with the subscript) would include the locational attributes as captured in work- and nonwork-accessibilities t_{g} and u_{g} .
$\gamma_i(p,s)$	General 'strain' or the savings from including new-demand i via a triangular-inequality- style route-replacement between points p and s
Г	The gross economic-multiplier deriving the total employment from the initial basic- employment
Г	Vector of economic-multipliers deriving the total employment in the study area from the initial basic-employment, including c_j , f and a
$\mathbf{\Gamma}_t$	Observation matrix in Kalman filter; when multiplied against the observed time-series,
$\Gamma(W,p)$	Optimization results from a facility-location model where p facilities are relocated to respond to a maximum demand of W
$\mathbf{\Gamma}(k) = [-\gamma_i(k)]$ Vector o	f payoff-function consisting of q entries, where $q \le \mathbf{\hat{\mu}}$
d d'	Distance or spatial separation; also a proxy for a particular spatial order
a d"	A decision in a Markovian decision-process
d_i	Distance from location i (notice this is not necessarily Euclidean distance); or deviation from a standard or ideal in dimension i ; also the capacity of arc i or the weights in a transfer function
d^k	Minimum threshold of retail-employment by trade-class k ; d^R is the threshold for the case when there is only one trade class Multidimensional decision boundary in a Bayesian classifier
$d(B) = d_0 + d_1 B + d_2 B^2 + \dots$	Transfer function in a multivariate time-series, consisting of weights d_0 , d_1 , d_2 , etc. and backshift operators B
$d_{ij} \\ d_{ijk} \\ d_{ii}^h$	Euclidean distance or the spatial-cost in general between locations i and j Euclidean distance or the spatial cost between locations i and j in state k Distance or travel time between nodes i and j by salesman or vehicle h
d^{i^j} d^{ij}	Time a salesman or vehicle visits node <i>i</i> in a tour or a route Distance or time between locations <i>i</i> and <i>j</i> , starting with arrival at <i>i</i> and terminating at arrival at <i>i</i> (notice this is not necessarily the Euclidean distance)
$d(\mathbf{i},\mathbf{j}) \\ d(\mathbf{x}_i,\mathbf{x}_{i+1}) \\ \mathbf{d}, \mathbf{d}'$	Planar Euclidean distance between two Cartesian coordinate points i and j Spatial separation between consecutive stops \mathbf{x}_i , \mathbf{x}_{i+1} Vector of arc capacities in network-flow programming
$\mathbf{d}^{j} = (-d^{k}_{i})$	Extreme direction along the <i>j</i> th axis in a linear program Direction of steepest ascent in the <i>k</i> th step of a hill-climbing optimization-algorithm, as characterized by <i>n</i> components of the vector
δ δ(<i>i</i>)	Change in a quantity (e.g., δx is the increase or decrease in quantity x); δ_{ij} is the distance savings in directly going from <i>i</i> to <i>j</i> , instead of through an intermediate point k. The steady state decision whenever the state is <i>i</i> in a Markovian decision process.
ð	Policy in a Markovian decision-process
š′	Improved stationary-policy in the policy-iteration procedure of a Markovian-decision-
δ* δ', δ"	process Optimal policy in a Markovian-decision-process Fixed cyclic-permutation

δ _i	Binary decision-variable to be switched on, conditional upon another decision-variable
	being engaged; also a calibration constant; or a nonnegative real-number denoting the number of legs in a subtour-breaking constraint
$\delta\Omega$	Boundary of the bounded-domain Ω
$\delta^{\dagger}(t)$	Set of nodes reachable from <i>i</i>
δ ⁻ (i)	Set of nodes incident upon <i>i</i>
δ _{ii}	Route-distance savings by including demands <i>i</i> and <i>j</i> in a single, rather than separate tours,
	in accordance with the Clarke–Wright heuristic
δ ▲	Vector of estimated-parameters in nonlinear regression
$\delta = (1 \delta \cdot 1)^T$	Least-squares estimate of δ , usually obtained as a conditional estimate Orthonormal base of the transition rate space when the system is in compartment i
$\mathbf{o}_{j} = (\leftarrow o_{ji} \rightarrow)$	Distance or time of specified length
D D'	Data, population density, or a measure of crowding
<i>D</i> ″	Dual polyhedron of a linear program; or a subset of nodes/vertices
D_{ab}	Shortest distance from demand or customer a to demand b along a path, or along a tour from denot a to demand b
D(i)	Decision set in a Markovian decision-process
D(a,b)	Shortest distance along a vehicle route from terminal a to terminal b
D_i $D_i(V_i^d)$	Demand at location <i>l</i> showing price against flow-quantity in other words, price paid at
	demand quantity V_l^d
D'_i	Cumulative distance (along a path) to demand <i>i</i> from all facility candidate-sites
D^{H}	Upper-bound distance
$D^L_{-\mu}$	Lower-bound distance
D_j^H D = [d] Death	Maximum allowable household-density in zone j
$\mathbf{D} = [a_{j}]$ Death \mathbf{D}'	Calibration-coefficient matrix in the first stage of 2-stage least-squares, measuring $q \times q$,
_	where q is the number of endogenous variables
$\overline{\boldsymbol{D}} = [D_{ab}]$	$ I \times I $ matrix of shortest cumulative-distances along a path from vertex <i>a</i> to vertex <i>b</i>
$\bar{\boldsymbol{D}}' = [D_{qk}]$	$ I \times m$ matrix of distances from vertex q to arc k
$\Delta_i^{I} = \langle a \rangle = \langle $	I he difference between two utility measures <i>i</i> and <i>j</i> Gradient of a function over <i>n</i> variables
$\mathcal{O}(\mathbf{x}) = (\mathcal{O}(\mathbf{y}_{i}, \mathbf{y}_{i}) = (\mathcal{O}_{\mathbf{x}}, \mathcal{O}_{\mathbf{y}}, \mathcal{O}_{\mathbf{x}}, \dots)$ $\mathcal{O}(\mathbf{x}) = (\mathcal{O}(\mathbf{y}_{i}, \mathbf{y}_{i}) = (\mathcal{O}(\mathbf{y}_{i}, \mathbf{y}_{i}) = (\mathcal{O}(\mathbf{y}_{i}, \mathbf{y}_{i}) = (\mathcal{O}(\mathbf{y}_{i}, \mathbf{y}_{i}) = (\mathcal{O}(\mathbf{y}_{i}) = (\mathcal{O}(\mathbf{y}) = (\mathcal{O}(\mathbf{y}) = (\mathcal{O}(\mathbf{y}) = (\mathcal{O}(y$	The exponent value of 2.7183; also a calibration constant
e'	Number of exogenous variables left in the econometric model after estimation
e'' e.	Number of endogenous variables left in the econometric model after estimation Index to denote the <i>i</i> th type of industrial employment: also the <i>i</i> th arc in a network
e,	Arc <i>j</i> associated with node/vertex <i>i</i>
$\mathbf{e}^{i(j)}$	Unitary column-vector for arc j with unitary entry in the ith row
e	A very small number or a random perturbation
c_k	curve-fitting a distance function
E	Normally-distributed error-vector with zero mean; when it has a constant variance, it could
	be a vector of random perturbations in the forecast using a transfer function, due to white
Ε	Total employment
$\overline{E'}$	Number of exogenous variables
E''_{F^B}	Number of endogenous variables Basic employment (F^B is basic employment in zone i)
E^{R}	Service employment (L_j) is basic employment in Zone j
E^k	Retail employment by trade-class k (E_j^k is retail employment by trade class k in zone j)
E(t)	Relative smoothed-errors in adaptive-response-rate exponential-smoothing
E_j	Employment in the <i>j</i> th zone as projected from an areawide growth-rate for each sector
E_{ijk} $E(i, i, h, h_{-})$	Expected number of demands i in period k at location j Net change in travel-distance from an exchange of demands i and i between tours h and
L(1,12,11],112)	h_2

A - 6	Appendix - Symbols
$E'(i_1, i_2, h_1, h_2)$	Modified generalized-savings-measure from an exchange of demands i_1 and i_2 between
E f	tours h_1 and h_2 Row vector of employment-levels, made up of individual zonal employment E_i Average household-size in terms of the number of employed residents per household, or
$f(\cdot) \\ f(\mathbf{x}_q, \mathbf{x} - \mathbf{x}_q)$	Regular function of the argument (e.g., the criterion function in dynamic programming) A functional for which the directional derivative is being considered, approaching point \mathbf{x}_q
$f'_{f'(t)}$	Functional-attribute score, including spatial separation
f_i	Demand-for-service frequency at location <i>i</i> ; also the natural growth-rate of population in subarea <i>i</i> (the activity rate)
f^{W}	Weighted activity-rate, where the weights are the percentages of regional population at each zone
f_{ik} f_{ik}	Demand-for-service frequency at location <i>i</i> in state <i>k</i> Number of demands <i>k</i> serviced by facility <i>i</i>
Ji	into zone i
$ f_j^{(l)}(\cdot) $ $ f_r^{mn} $	Speed-of-adjustment function for the <i>j</i> th zone and <i>l</i> th activity <i>r</i> th-stop demand between origin–destination $m-n$
<i>f</i> (x)= df _/ dx f	Derivative of function f over variable x Partial-flow pattern in the decomposed RISE algorithm
F	Set of candidate or new facilities to be sited, or an objective functional
F(f(x)) = F(u')	Fourier transform of function $f(x)$ in frequency u'
$F(\mathbf{z})$ $F'(\cdot)$	Regional-growth-rate function
F_k	Fibonacci numbers; also the weighted activity-rate, with work-accessibilities from zone k as the weights
F_X	Derivative of function F with respect to variable X
$\vec{F} = \nabla F$	Gradient of the function F being maximized
Γ_i	additional vehicle-deliveries
F_{ij}	Accessibility factor between locations <i>i</i> and <i>j</i> , expressed as an inverse function of travel cost
F_{ik} F - [F]	Probability that a demand from i is of type k Square matrix of nonulation distribution rate by zone, measuring $n' \times n'$
$\mathbf{F} = [\mathbf{F}_{ij}]$ $\mathbf{F}'(\mathbf{x}) = (\mathbf{F}(\mathbf{x})) \rightarrow \mathbf{F}$	A vector of functions whose interactions $a\mathbf{E}'(\mathbf{r}) a\mathbf{r} \neq a\mathbf{E}'(\mathbf{r}) a\mathbf{r}$ are asymmetric, where $\mathbf{x} = (\mathbf{r} \neq \mathbf{r})^T$ for
$\mathbf{I}(\mathbf{A}) (\mathbf{I}_{i}(\mathbf{A}))$	i = 1,, n
g	A scale factor; when serialized against argument m for example, $g(m)$, it is used to adjust
$g(\cdot)$	zonal population from one loop of the Lowry model <i>m</i> to another $m+1$, where $m = 1, 2,$ A special function of \cdot , such as the state equation; the relocation-cost function in stochastic facility location: or the expected master travelling salesman tour length in probabilistic
	travelling-salesman-problem
g_k	Generalized unit-cost at facility k or for vehicle k
g'_i	Load to be picked up at node/vertex i
g_i	some gravitational potential-function
g_{ii}	Short-hand notation for nonwork accessibility between <i>i</i> and <i>j</i>
g	Vector of coefficients associated with the discrete-variables y; when used as a function, it
$\mathbf{\sigma}(i) = (\mathbf{\sigma}_{i}, \mathbf{\sigma}_{i})^{T}$	is the subgradient Vector of input measures for a decision-making unit <i>i</i>
G	Number of salespersons in a travelling-salesman problem, or the number of vehicle-tours out of a depot
G'	Maximum fleet-size available at a depot; or share of the population which are immigrants
$G(\cdot)$	Multiple-travelling-salesmen expected-tour-length-function involving k salespersons Concreting function for the probability distribution P_{1} , P_{2} , P_{3} , P_{4} ,
σ(ς)	values of 0, 1, 2,, n
G(ξ , t)	Generating function for the probability distribution $P(\mathbf{X}_0^*, \mathbf{X}^*, t)$; where \mathbf{X}_0^* is the initial- condition vector, $\mathbf{X}^* = [X_1^*(t), X_2^*(t), \dots, X_n^*(t)]^T$, and where the <i>n</i> -dimensional-vector $\boldsymbol{\xi}$ takes

	on values of $\boldsymbol{\xi}^{X^*} = (\boldsymbol{\xi}_1^T, \boldsymbol{\xi}_2^T, \dots, \boldsymbol{\xi}_n^T)^T$, for $ \boldsymbol{\xi}_i < 1$. Thus for the stationary, irreducible Markov-
	process it assumes the form $P(X_{\circ}^{*}) + r^{*} P[X_{\circ}^{*}] + r^{*} P[X_{\circ}^{*}] + r^{*} P[X_{\circ}^{*}]$
G	Class or group <i>i</i> : also a generalized spatial-statistic for point <i>i</i> .
$G_i(n,s)$	Generalized savings-measure from including demand node i between demand points p and
$\mathbf{G}_{i}(\mathbf{p},\mathbf{s})$	s in a location-routing heuristic
G'(ns)	Modified generalized savings-measure from including node <i>i</i> between points <i>n</i> and <i>s</i> after
$O_i(p,s)$	considering different denot-based tours
$G^*(h'')$	Net change in cost from displacing demand i from tour h to h"
$G_{i}^{**}(h'')$	Net change in cost from displacing demand i from tour h to h" considering different fleets
G^{ij}	Transaction of goods and services between the <i>i</i> th and <i>i</i> th industrial sectors
G_{ii}	General location-pair spatial-statistic
G_{ii}^{pq}	Monetary transaction between the <i>q</i> th industrial sector in zone <i>j</i> and the <i>p</i> th economic-
.,	sector in zone i in an input-output model; with shorthand notation being G_i^{pq} for
	consumption and G_{i}^{q} for production respectively, considering only the nonzero elements
$\mathbf{G} = [G_{ii}]$	The growth matrix showing the growth springing off from group/location <i>i</i> to group/
	location <i>j</i> (within a period of time); also a basic-feasible-solution to a simplex-on-a-graph
$\mathbf{G}(\cdot)$	Vector return-function in a recursive program
$\mathbf{G}' = [g_{hj}]$	Input matrix containing the h th input for decision-making-unit j
$\zeta_j^{(l)}(\cdot)$	Economic surplus- or deficit-function at zone j of the <i>l</i> th type
h	Index for a variable; generally to show a fleet type, a category of inputs (costs) in data-
	envelopment analysis, or the iteration number in a recursive program
h'	Minimum fleet size
$h'(\cdot)$	State-transition function in dynamic programming
n	Calibration parameter in a dynamic version of a spatial-location model; an example is the
h	United of a subsection k
n_k	neight of a subregion k A rate or collibration constant in a deterministic compartmental model: for example, the
n_{ij}	A rate- of canon autom-constant in a deterministic comparimental-induct, for example, the interaction between regions i and i in a multiple region predictor prevenue automote or a
	short-hand notation for work-accessibility
$\mathbf{h}(i) = (-h_{i})^T$	Vector of output-measures for target decision-making-unit <i>i</i>
H H H	Housing expenditure for a household (part of locational expenditure)
$H(\cdot)$	The Hamiltonian function in terms of the state equation, the costate or adjoint variable, and
()	the figure-of-merit at the present: it also stands for a general function
H'	An upper limit of discrete index h
$H'(\cdot)$	Regional growth-rate function
H"	Set of vehicles in a fleet, or the set of vehicle types in the fleet
H''	Cardinality of set H", or the number of members in the set; here it is the fleet size
H^i	Transaction of goods and services to the <i>i</i> th household-sector
H_i	Set of potential tours in which demand <i>i</i> can be included
H _p	Cost of one dispatch on route p
H_r^{o}	Imports to region r
H_i	Hazard a hode l is exposed to
Π_{ij} $\Pi'(.)$	Hazard a link (i,j) is exposed to Eleve rate function from compartment <i>i</i> to compartment <i>i</i>
$H_{ij}(\cdot)$	Monetary transaction between the household sector in zone i and the nth economic sector
11 _{ij}	in zone i in an input-output model
n	Elasticity of demand
ין ח	$100(1 - \alpha/2)$ percentile of the standard normal-distribution
θ	A parameter in general: for example it can show decline in demand per unit-of-spatial-
-	separation: θ is the rate-of-decline (or diffusion rate) of inflows into i
θ_{i}	Coefficient of the <i>t</i> th term in a moving-average time-series
$\theta(B)$	The backshift operation of a moving-average model
θ_{ii}	Proportion of activities (or trips) from origin-location <i>i</i> that end up in destination-location
.,	j based strictly on accessibility alone
Θ_{ii}	A short-hand notation for the spatial-interaction term, indicating the proportion of activities
-	(or trips) from origin-location i that end up in destination j —based on both accessibility
	and the attractiveness at the destination; i.e., the normalized accessibility-function between
	<i>i</i> and <i>j</i>
$\boldsymbol{\Theta}_{\boldsymbol{k}} = [\theta_{ijk}]$	A kth-order spatial-matrix of moving-average coefficients
$\Theta(B) = \left[\theta_{ii}(B) \right]$	A spatial matrix of moving-average operators
() L ij(=)]	

A - 8	Appendix - Symbols
i, j $i(k)$ $j(k)$ i I I	Indices for nodes/vertices; <i>i</i> normally stands for a demand node and <i>j</i> a facility node; or they can just be any counter Beginning node of arc k Terminating node of arc k Cartesian coordinates of a demand <i>i</i> Set of nodes/vertices in a network
$I(a)$ $ I/I_{k}$ I_{N} I_{D} I' I'_{k} I''_{k} $I''_{p_{k}}$	The spatial-statistic Moran s- <i>I</i> for a particular spatial-order as defined by the distance- parameter <i>d</i> Cardinality of set <i>I</i> , or the number of members in the set Profit or income for facility <i>k</i> Set of unlabelled nodes Dual objective-function in recursive program Household or aggregate income Aggregate income at time <i>t</i> Set of potential demands for exchange, with an existing demand on the tour <i>h</i> Subset of potential demand nodes within the set <i>I</i> , where demands are non-zero Any subset of nodes in the <i>k</i> th-stop route p_k
$I(i)$ $I(0)$ $I(t)$ $I_{i\kappa}$	Set of nodes/vertices which are input markets Set of nodes/vertices which are output markets 0–1 indicator-sequence reflecting the absence and presence of an intervention, overlaying the transfer-function on top of the time-series A binary variable assuming unity if the combination of facilities κ provides a satisfactory service to demand <i>i</i>
I_{Rx}	Total expected-mutual-information between the facility pattern in the region R and the demand spatial-pattern (when $x=I$), or between the facility pattern and an individual demand (when $x=i_k$); i.e., how probable the facility pattern is consistent with what is known about the demand pattern I or individual demand i_k
$I[\mathbf{X}(k), \mathbf{I}(k)]$	xth-stage payoff of objective-function of a recursive program, defined in terms of decision- variables X and constraint parameters Γ
$I(\mathbf{P};\mathbf{Q})$	Information that allows updating a prior probability-distribution Q to probability P
$r_{I}'(\cdot)$	Net-benefit function in a decomposed recursive-program
\mathbf{l} $i^*(k)$	Identity matrix Ontimal facility location in state k
j (k)	Cartesian coordinates of a facility <i>j</i>
Ĵ	Subset of nodes/vertices in a network, generally the candidate sites for facility location
J_q	Set of candidate production sites
<i>J</i> <i>I</i> '	A particular control-point in the bifurcation set
J"	The double values that the state variable assumes, corresponding to the control variable J'
	in the bifurcation set
J(i)	Set of Voronoi polygons adjacent to the <i>i</i> th polygon
	Basis k of a multicriteria linear-program Index to show category k (e.g. 7^k is the kth activity); it marks a node, the commodity the
<i>κ</i>	tree in a forest, or just serves as a counter
$k(\cdot)$	Equation for the control variable over time, expressed in terms of the state, the costate or
<i>k</i> _i	Calibration or scaling constant for zone i in a doubly-constrained gravity model; the
k	Moran's-1 or General Spatial statistic; alternatively, it is the propensity to save (invest) row vector consisting of $0 + 1 = 1$ entries marking an orthonormal base of the transition-
K	rate space
Κ	A discrete or continuous constant, or the upper limit of running index k
K(t)	Capital-stock investment over time
K_i, K'_i	I rip-production and -attraction rate at zone <i>i</i> respectively
K _j ^r	A scaling constant; it ensures that the inter-sectorial and inter-zonal flows sum up to the
.	non-labor input to the input-output table for sector- <i>p</i> and zone- <i>j</i>
К,	Combination of these commons for it is the formation of t
К v'	Combination of three or more facilities that perform a certain function
к к ^h	Cost of operating vehicle h
	cost of operating vehicle n

κ_i^h	Marginal cost of serving demand-node <i>i</i>
K	Combination of three or more facilities
l(T)	Total cost of spanning-tree T, which is sum of the arc costs
<i>l'</i>	Discount rate (e.g., on the number of commuting trips, or traditionally in the time stream
7:	of cost or benefits)
l^{ι}	Lower bound of a specified time window for a salesman or vehicle to visit node/vertex i
l_j	Calibration constant for zone <i>j</i> in a doubly-constrained gravity model
l_k	Spatial order of the <i>k</i> th autoregressive-term in a spatial time-series
<i>l</i> ^{<i>m</i>}	Ordered set of neighboring points (p,s) representing candidate tour h^n
$l_{h''/i'}$ Imn()	Ordered set of neighboring points (p,s) in tour h^* after removing demand t^*
$l^{m}(r)$	Length of an P -stop route originating in m and terminating in n
'l '(x)	Route-length vector at stage r and in state s of a decomposed recursive-program, expressed
I	as a function of the decision variable x
L	hydrein fry souset of demand houes vertices, where a demand instance may be characterized in a hydre subject L of the network hydres (variase L the
	symbol also denotes twice the boundary length of district
	Cardinality of set L or the number of members in the set
L ~	Leads a state of the formation of the induction in the set
	Length of the perimeter of a subarea
	I ne length of a queue, including the entity being served
$L(\cdot)$	Lagrangian of maximum-likelinood function
	A solution constant in a biversite is the termination point for the trip
	A candidation constant in a bivariate predictor-prey difference-equation-set
L_q	Regional labor-input-factor
L_r	Spatial-lag operator on the value of spatial unit i where l refers to the lth contiguity-class
\mathbf{L} \mathbf{x}_{i}	such as the <i>l</i> th-order neighbors: alternatively we can write $L^{(0)}x$ as a matrix operation to
	compute the weighted sum of the neighboring values of i contained in vector \mathbf{x} or $(\mathbf{w}^{(0)})^T \mathbf{x}$
	In general, $L^{(0)}(\cdot)$ stands for spatial-lag operator of the <i>l</i> th-order, with the <i>l</i> th-order operator
	reproducing the observation itself, or $L^{(0)}(\cdot) = \cdot$
$L_{\tau}(\cdot)$	Length of a master travelling-salesman-tour, constructed out of the set of nodes/vertices ·
L_{ii}	Error (in terms of a "loss measure") when a Bayesian classifier mis-assigns a multi-attribute
Ŋ	observation $\mathbf{x} = (x_1, x_2,)^T$ to group <i>i</i> when it actually belongs to group <i>i</i> ; usually $L_{ii} = 0$
	if there is no error and $L_{ii} = 1$ if there is a misclassification
$L_i(\mathbf{x})$	Average misclassification error (in terms of a "loss measure") when assigning
	multi-attribute observation $\mathbf{x} = (x_1, x_2, \dots)^{t}$ to group j; a couple of computational
	transformations of this measure are $L'_j(\mathbf{x})$ and $L''_j(\mathbf{x})$
$\mathbf{L} = \left(\mathbf{x}_{L}(\boldsymbol{g}_{1}^{\prime}), \mathbf{x}_{L}(\boldsymbol{g}_{2}^{\prime}), \ldots\right)^{T}$	Matrix containing the left eigenvectors \mathbf{x}_{L}
λ	Dual variable or Lagrange multiplier, with a specific (not necessarily feasible) solution $\bar{\lambda}$
	and the optimal solution λ^*
λ'_i	A normalized weight, where $r_{\lambda} \chi = 1$ unless noted otherwise
λ"	Arrival rate for a queuing process
$\boldsymbol{\lambda^{k}} = (\leftarrow \mathcal{\lambda}_{i}^{k} \rightarrow)^{T}$	The k th solution-vector in a Lagrange-relaxation procedure
λ/*	Dual optimal-solution to the linear-program subproblem at the last iteration within Benders'
	decomposition
$\Lambda(\mathbf{J}_k)$	The weight cone for multicriteria linear-program, showing the λ' -weight combinations that
	characterize a particular solution \mathbf{J}_k among the nondominated set of solutions
<i>m</i> , <i>n</i>	Indices for dimension or for a node/vertex
<i>m'</i>	A calibration constant in a bivariate predictor–prey difference-equation-set
m	A critical bifurcation-value in a bivariate predictor-prey difference-equation-set
m^{1}	A collection of entities of characteristic 1; e.g., the number of complicated constraints in
÷	a Lagrangian-relaxation problem
m'	A collection of entities of characteristic <i>j</i> ; e.g., the number of high-frequency direction
	finders in a bundle located at station j
m_k	Spatial-order of the ktn moving-average term in a spatial time-series
m_r	ventue-neet requirement at depot r , of the number of deployed ventues at depot r .
m_i, m_i m(k)	Median for a median-filter using a $k \times k$ mask
m(n)	Groups of demand nodes to be served by route $1, 2, \dots, k'$ with $m \perp m \perp \dots \perp m \neq 1$
$m_1, m_2, \ldots, m_{k'}$ m'(a)	Maximum shortest-distance from point a
m(q)	maximum shorest-distance nom point q

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m'_{ii}	Binary variable that is "switched on" when demand <i>i</i> is allocated to facility <i>j</i> in a combined
	data-envelopment-analysis/location model; also the benefit valuation for such $i-j$ pair
M	Area specification for a districting model
M_i	Maximum inventory carried at node <i>i</i>
$M_{\rm max}$	Maximum number of nodes in a vehicle route
ĨĂ, ĨĂ	A couple of matchings in a spanning-tree/perfect-matching heuristic for the travelling- salesman-problem
M(t)	Absolute smoothed-error (used in conjunction with relative smoothed-error) for adaptive-
	response-rate exponential-smoothing over time
$M(\Xi)$	Maximum of the weighted distances from the center candidates to each of the demands in
	the candidate facility-locations Ξ
M'	Non-locational expenditure such as food, clothing, education, savings etc.
M''	A very large number or weight
M_{ii}	Minor of a square matrix
M(W,p)	Simulation results of a facility-location model where p facilities are relocated to respond to
	a maximum load of W
$\mathbf{M} = [m_{ii}]$	Migration matrix showing the migration rate between locations <i>i</i> and <i>j</i>
μ	Mean of a probability distribution
μ'	Service rate of a queuing process; also the number of intermediate stops in the longest
	vehicle-route
μ_i	Positive weights placed upon an extreme direction \mathbf{d}^{\prime} in a linear program
μ_i, μ_i	Mean of observations in group <i>i</i> in both scalar and vector form
u ^(j)	Scaling constant of the error ϵ associated the value v being measured, resulting in $v^{(j)} + u^{(j)} \epsilon^{(j)}$
V	A collection of integer numbers
V.	Route shape parameter (serialized by i) used in location-routing heuristics, assuming values
- 1	such as 1 or 2
V_{t}	Noise series in a transfer-unction multivariate time-series
μ^{p}	Dual variable associated with the control total of areawide-transportation-cost constraint
,	in an entropy-maximization model
Ξ	Collection of candidate facility-locations
$\Xi(X)$	Collection of all candidate facility-locations in the decision space X
$\Xi(\mathbf{y})$	Collection of candidate facility-locations which are open (i.e., for those locations where $y \neq y$)
$\Xi(z)$	Collection of candidate facility-locations in the Z space, whose distance bounds are within
	z units
ξ	As used in the Minkowski's distance-function, it is the proportion by which factor inputs
	have to be reduced to reach the efficient point on the production frontier
<i>n'</i>	The number of units in a spatial entity (e.g., the number of zones in a region, the number
	of subareas in a study area, or the total number of pixels in an image)
n_s	Number of sides in a subareal polygon (e.g., in a Dirichlet tesselation)
n(a,b)	Number of stops between origin-terminal <i>a</i> and destination-terminal <i>b</i>
N	Population or number of households (e.g., N_i is the population at location i)
N_j	Number of pattern vectors from class G_j , or the number of nodes or pixel vectors belonging
	to class j
N'(large)	A large number
Ν	Total working population in the study area
N^p	Population working in economic-sector <i>p</i>
N_j^c	Capacity for residential development in zone j
N'_i	Set of spatial units (including facilities) within a distance S from demand i
N_{ij}	Binary decision-variables in a districting model, serving as a 'pointer' across a district
	boundary separating a geographic sub-unit i and one that is not j ; it is unitarily value if
N	subunit j is acquired and i is not
IN N(1)	Kow vector of zonal population N_i
IN(K)	The nonbasic column associated with variable k in a linear-programming tableau
O_i	Export snare of region i
$O(l^{*})$	worst-case ktn-polynomial computational-complexity for input-data-length l
$O_i = (O'(D))$	Export from the <i>l</i> th region O right the parameters of $1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 $
$O(\Gamma) = \{\leftarrow O_i(\Gamma) \rightarrow \}$	orientation of the arc in the path sequence r_1 and r_1 entries, depending on the
O^i	Export from the <i>i</i> th industrial sector measured in dollars
O^i	Export from the <i>i</i> th industrial sector in subarea <i>i</i> managered in dollars
O_j	Export from the <i>t</i> th industrial sector in subarea <i>J</i> , measured in dollars

$\mathbf{O} = (0 \leftarrow O^i \rightarrow)^T$	Export vector in an aspatial input–output model, showing the convention that the first sector (the household sector) has no exports
$\mathbf{O} = (0 \leftarrow O_j^i \rightarrow)^T$	Export vector in a spatial input-output model, where i is the economic sector and j is the subarea
p	An integer number for the number of facilities, the number of services provided, the index for the <i>p</i> th vehicle route, the parameter in the l_p -metric, or the differencing parameter in a
<i>n</i> ′	time-series Number of facilities in a subset of the <i>n</i> facilities (i.e. $n' < n$)
P D _f	Price of fuel
p_{g}	Price of the good
p_k	Price of a commodity k , with p standing for a vector of commodity prices
p'_{i}	Probability of adopting strategy <i>i</i> in a two-person game
$p_{ik}^{o(\cdot)}$	Empirical probability that demand k patronizes facility i; or the probability of transitioning from state i to state k
\hat{p}_{ik}	Estimated value of p_{ik}
p_{i}	Empirical probability that a demand patronizes facility <i>i</i>
$p_{\cdot k}$	Empirical probability that a demand k is being served
$p_{j}^{\prime q}$	<i>q</i> th factor-of-production input-prices at subarea <i>j</i>
p_k	Number of facilities of the <i>k</i> th type (as used in a multi-product facility-location formulation)
p(t)	Capacity expansion at time t
р р.,.	Conditional probability that event-type i occurs at geographic-region i at time-of-day k
Р 1 <i>jk</i> р .а.	Prediction of p_{iik} based both on the hypothesized intervention model and historical data
ž yk Žia	Analytical prediction of the <i>relative</i> probabilities p_{iik} , for field implementation as a transfer
- yx	function
${ ilde p}_{ijk}$	Relative probabilities after intervention probabilities have been implemented, using the
	transfer function p_{ijk}
₽ _{ijk}	Deseasonalized relative-probabilities after intervention probabilities have been implemented
$\mathbf{p} = (-p^{(j)})$	Perron vector whose components are positive and sum to unity
$\mathbf{p}_{i}(t) = (-p_{ij}(t))^{T}$	Vector of transitioning probabilities from state <i>i</i> to state <i>j</i> (where $j = 1,, n$)
$\dot{\mathbf{p}}_{i}(t) = (-\dot{p}_{i}(t) -)^{T}$	Time-derivative vector of probabilities transitioning transitioning from state <i>i</i> to state <i>j</i>
. ,	(where $j = 1,, n$)
P	A path; also a set of vehicle routes generated for a network
P'	Potential surface for destination choice, whose derivative dP'/dC_{ij} is often operationalized by the trip distribution function
P	Dual space of the linear-programming relaxation problem
P(p)	Probability that p servers are occupied (busy)
$P(\cdot)$	Probability of an event ·
P_i	Nearest location for demand or customer <i>i</i> ; also the probability that the system is in state
$P_{i}(t)$	Probability that the system is in state <i>i</i> at time t
$P_k', P_{(k)}$	Steady-state probability of being in state k
$P_{id''}$	Steady-state probability that decision d'' is reached while in state <i>i</i>
P_{ij}	Binary decision-variables in a districting model, serving as a 'pointer' across a district
	<i>i</i> is acquired and <i>i</i> is not
P_{iik}	<i>Joint</i> probability of event-type <i>i</i> occurring in area <i>i</i> at time <i>k</i> , given that an event-type <i>i</i>
ijn.	occurred at time k
₽ _{ijk}	Analytical predictions of p_{ijk} aggregated monthly, based on the hypothesized
Dmn	Intervention-model Set of vehicle routes covering origin destination pair <i>m</i> , <i>n</i> via <i>k</i> stop itineraries
P ^{mn}	Set of vehicle routes covering origin–destination pair <i>m–n</i> via connect itineraries
\overline{P}	Scale of a facility as represented by its capacity capital outlay etc.
- P	Lower and upper bound of the supply at location <i>l</i>
$\frac{1}{p}$	Aggregate production-function with capital as input
$\mathbb{P}(\bullet)$	Logical predicate over the argument •
$P_i(p)$	Steady-state saturation-probability of all <i>p</i> service-units (in stochastic facility-location)

A - 12	Appendix - Symbols
$\mathbf{P} = (\leftarrow P_i \rightarrow) \text{ or } (\leftarrow V_{ij} \rightarrow)$	Updated probability-distribution for each of the <i>n</i> ' subareas or $ I $ nodes written in a vector form; also be the updated travel-vector between <i>i</i> and <i>j</i> , V_{11} , V_{12} ,, V_{ij} ,, $V_{ I J }$, measuring $ I \cdot J $ long
$\mathbf{P}(t) = (-P_i(t)) \text{Vector}$	or of the state probabilities $P_i(t)$; also the square matrix of transition probabilities over time
$\dot{\mathbf{P}}(t) = (+\dot{P}(t) \rightarrow)$	Time-derivative vector of state probabilities $P_i(t)$
$\mathbf{P}' = [\mathbf{x}_1, \dots, \mathbf{x}_n]$	Matrix containing independent eigenvectors $\mathbf{x}(q'_i), i = 1,, n$.
$\mathbf{P}_{t-1,t}$	Variance–covariance matrix for the difference between the observed and estimated Kalman- filter time-series-vector (or the estimation-error vector)
π_i	Dual variable in a network; such as the shadow price at node <i>i</i> , or a real number showing
	the amount of load carried on board a vehicle at node/vertex i
$\pi^{(j)}$	Probability that an individual reviews his/her choice of the <i>j</i> th compartment in a compartmental model
$\pi_{ij}(\cdot)$	Probability a given individual moves from compartment <i>i</i> to compartment <i>j</i> —as a function of, say, the state variable and time
π_i^j	Dual variable associated with the <i>i</i> th column of the spanning-tree $(j=1)$ or non-spanning-tree $(j=2)$ part of the basis (in a network-with-side-constraint tableau)
$\pi(\bullet)$	Permutation operator on the argument •
$\pi(j \mid i, d'')$	The probability of transitioning from state i to state j during one period of the Markov process, given a decision d'' has been made
Π	<i>n</i> -dimensional transition-rate space
^r π ^s (x,y)	Vector gross-return-function of decisions \mathbf{x} and \mathbf{y} (in a decomposition implementation of recursive-program)
$\mathbf{\Pi}(\cdot)$	Vector of gross return-functions of decisions in a recursive program
$\mathbf{\Pi}_{0}(t) = (-\pi_{i0}(t))^{T}$	Vector of transition rates with the "outside world" over time
$\mathbf{\overline{II}} = [\pi_{kl}]$	Transition-probability matrix in a Markov chain or compartmental model, with each entry denoting the given probability of transitioning from state k to state l ; also the matrix of transition rates from state k to state l
Ĩ	Matrix of transition rates from state k to state l , considering both arrival and service in a gueue
9	Index to show a node number, center number, median number, number of substations, or the number of attributes, criteria, endogenous variables, eigenvalues, or differencing

- parameter in a time series Candidate location for a center k q_k
- Probability that an event-type i occurs at time k $q_{ik} q'$

 q'_i

 \bar{q}_j

 $\begin{array}{c} Q_i \\ \tilde{\mathcal{Q}}_i, \bar{\mathcal{Q}}_i \\ Q' \\ \tilde{\mathcal{Q}}' \\ Q'' \end{array}$

- Eigenvalue, with q'_{max} as the principal eigenvalue; also the growth rate of an area (with q'_j being the subareal growth-rate)
- Probability that strategy *i* is followed (in a two-person game); also the *i*th eigenvalue $q_i(\cdot)$ Inventory-cost functions at demand-node *i*; or simply the unit cost-of-time (a constant) from
 - demand-origin i Mean queuing delay
 - Total economic-activity in the study area, such as consumption in dollars or number of trips executed Ratio of two accessibility definitions from location *i*

 - Lower and upper bounds for the demand at location l
 - Total number of servers, or number of suppliers Set of discrete points in the feasible region of an integer program
 - Cost per rejected demand in a loss-system location-model
 - A matrix of economic-base multiplier over a time-increment Δt
- $\overline{\mathbf{Q}} = [\overline{\mathbf{Y}}_{ij}]$ $\mathbf{Q} = (-Q_i)$ or $[Q_{ij}]$ Prior-probability distribution for locating in each of the n' subareas (written in a vector form); or the vector of prior-travel between *i* and *j*, Q_{ii}
- Variance–covariance matrix of the white-noise vector $\boldsymbol{\alpha}_{t}$ Q_{t-1} The $\mathbf{X}^T \mathbf{X}$ data-matrix in the nonlinear regression of a STARMA model; where \mathbf{X} is not **O**'
- explicitly given, and has to be numerically estimated $Q'' = [q_j]$ Matrix with eigenvalues q_1, q_2, \dots along its diagonal
- Rent or mortgage, as part of locational expenditure (e.g., r^i is the rent for a unit of land i at a distance d_i from market, and **r** is the vector of rents among these land units) Pearson correlation-coefficient r_0
- r_k Satisficing-level of criterion k; also the autocorrelation of lag-k in a time-series

r'^k	Land-consumption rate per retail-employee of trade-class k
r'	An l_{p} -metric deviational-measure from a standard or an ideal
$r'(\mathbf{y}', \mathbf{x})$	Generalized-Leontief distance-measure, as a function of inputs x and outputs y'
$r(\cdot)$	Spatial-separation or response-time function of argument \cdot ; or the return function in
	dynamic programming
r_0'	Partial correlation coefficient
r'_k	Partial-correlation-coefficient of lag-k in a time-series
\overline{r}_{i}	The expected response-time of service-unit <i>j</i> , consisting of mean queuing-delay and mean-
,	travel-time to the demand
r _{ii}	Direct user-charge at facility <i>i</i> for user from origin <i>i</i>
r'(i,d'')	Reward expected at state i by making decision d'' (in a Markovian-decision-process)
$r_{XY}, r(X,Y)$	Sample (cross) correlation-coefficient between random-variables X and Y
$r_{\rm viv}$ $(X_{\rm b})$	Partial-correlation-coefficient between Y and X_k , given X_i, X_i, \ldots are in the equation already
$r'(\cdot)$	Euclidean distance between demand <i>i</i> and a facility
* ()	kth-order spatial-temporal-autocorrelation between the <i>/</i> th and <i>m</i> th neighbors of the subject
^{Im(n)}	aita
D	Site A closed radion in Euclidean 2 space: the set of a subradions $\{P, P, \dots, P\}$ or the
Λ	A closed region in Euclidean 2-space, the set of n subregions $\{X_1, X_2, \ldots, X_n\}$, of the multiple correlation coefficient
R(I)	The set of <i>n</i> subregions, each identified by its service, facility location $\mathbf{x} : \{B(\mathbf{x})\} = B(\mathbf{x})\}$
R(J)	The set of <i>n</i> sub-regions, call incontinued by its set vice-item ty location x_i : $\{n(x_1), \ldots, n(x_n)\}$. Total physical region made up of sub-regions $R = R$, these regions can be of higher
\mathbf{n}_T	dimensions than the Fuclidean 2 -space
R^n	Domain of continuous non-negative variables in Euclidean <i>n</i> -space
$ R(k^*) $	The area of the largest empty-circle with center at k^* located at any vertex of the bounded Voronoi-
II((iv)	ling ram
$ R(\mathbf{x}) $	The area of subregion $R(\mathbf{x})$: $ R(\mathbf{x}^*) $ is the area of the optimal <i>i</i> th Voronoi polygon, with its
	facility at \mathbf{x}_i^*
<i>R'</i>	In stochastic facility-location models, R' is the required time in dispatching a special
	reserve-service-unit from a neighboring jurisdiction
R^2	Coefficient-of-multiple-determination in regression
\overline{R}^2	Coefficient-of-multiple-determination after adjusted for the degree-of-freedom
$R^{2}_{$	Coefficient-of-multiple-determination between Y and X_1, X_2, \ldots, X_t
$\frac{D'(x')}{D'(x')}$	Set of input requirements \mathbf{x} to produce \mathbf{x}' in a production function
R''	The entire image or entire region
R(+ -) R(- +)	Finite redictor/nrediction-space used in spatial-temporal canonical-analysis
$R_{\rm c}$	Subregion <i>i</i> within the entire region R^m also the production in subregion <i>i</i>
R'	Normalizing constant in a spatial-interaction function or the denominator of the function
R^p_i	Production output of the pth industry in zone i
R	Number of row cells in a grid region, a raster image, or a lattice
R^i	Monetary output from the <i>i</i> th industrial-sector
R^i	Monetary output from the <i>i</i> th industrial-sector located in subarea <i>i</i>
R'(d)	Norm deviate of the generalized spatial-statistic (analogous to the two-tailed t-statistic)
\overline{R}^{p}	The observed value of non-labor input to the input-output table for sector- <i>n</i> and zone- <i>i</i>
$\mathbf{R} = (\nu \pi^e \leftarrow R^i \rightarrow)$	Output vector in an assatial input output model showing the production in each economic
$\mathbf{K} = (y_2 \times K^{-1})$	sector starting with output from the household (or labor) sector (measured in wages) and
	followed by the first second industrial sectors <i>i</i>
$\mathbf{R} = (\nu \tau^e \leftarrow R^i \rightarrow)$	Output vector in a spatial input-output model showing the subareal production in each
$(y_{2j} + R_j)$	economic-sector i starting with the subareal output from the household (or labor) sector
	(measured in wayes) and followed by the first second industrial-sectors by subarea i
$\mathbf{R}' = [\mathbf{x}_{p}(a') \ \mathbf{x}_{p}(a')]$	Matrix containing the right eigenvectors \mathbf{x}_{n}
\mathbf{R}''	Commodity-value-added output-vector
R	Variance-covariance matrix of the measurement error (or noise) in a Kalman-filter time-
£	series
ρ	Parameter or dual variable to account for the delivery-vehicle capacity
Ď(ĨB)	Spectral radius of matrix $\tilde{\mathbf{B}}$
$o' = \lambda'' \mu'$	Utilization rate of a server in a queuing system or ratio of the arrival rate λ " and service
r ·· / r	rate μ'
ρ"	Intensity of activity in a subarea

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ρ	Utilization-rate of a service-unit <i>j</i> in stochastic facility-location; also the import rate of
<i>p</i>	region j
ρ^{r}	Trade coefficient between regions <i>i</i> and <i>i</i>
$ ho^{pq}$	Technical coefficients showing the transactions between the <i>p</i> th and <i>q</i> th economic-sectors
d^{pq}	in an input-output model Technical coefficient at the receiving-sector zone- <i>i</i>
$ ho_{ij}^{pq}$	Technical coefficients showing the transactions between the <i>p</i> th economic-sector in zone
	<i>i</i> and the <i>q</i> th economic-sector in zone <i>j</i> in an input–output model
ρ	spatial-technical coefficients $[\rho_{ij}]$
ρ	Diagonal matrix of trade coefficients, $[\rho_{ii}]$
$\boldsymbol{\rho^{\prime}}=[ho_{h}^{j}]$	A matrix of economic-multipliers for the <i>j</i> th economic-sector, disaggregated by each zone- <i>h</i>
ρ _s , ρ _T	The consumption and production multi-sectorial components of the input/output-
	coefficient-matrix ρ , derived from row- and column-sum normalization of transaction flows respectively with $\rho_{\alpha} = \rho_{\alpha}$ the spatial multi-subareal version assumes $\sigma_{\alpha}^{R} = P_{\alpha}^{R}$ and $\sigma_{\alpha}^{R} = P_{\alpha}^{R}$
	$\mathbf{p}_{s} \mathbf{p}_{t} = \mathbf{p}_{t}$ and $\mathbf{p}_{s} \mathbf{p}_{t} = \mathbf{p}_{t}$ and $\mathbf{q}_{t} \mathbf{p}_{t}$
ô	Population cross-correlation between random-variables X and Y
ô ²	Relative size of the variance: $(1 - \delta^2)$ is the variance reduction
S	Source of a network
S_p	Autoregressive season-length in a seasonal time-series
S_q	Moving-average season-length in a seasonal time-series Prescribed frequency-of-visit at a node/vertex
<u>s</u>	Standard deviation of the random-variable X
$\hat{s(j)}$	Sum of vertex(node)-arc(link) distances for facility j (the smallest sum identifies the
c!(i)	general median)
s ())	median)
s^2	Sample variance, with s being the standard deviation
S_{ij}	Length of the border separating geographic sub-unit <i>i</i> from sub-unit <i>j</i> ; also surviving ratio
<i>s'</i>	Average size of a site; or the ratio between the demand potentials at sites i and j
<i>s</i> "	Slack node/vertex in a network
S	A set of alternatives (e.g., the set of solutions that satisfies a predetermined goal or standard the branch and bound search small incore programming relauction at a)
<i>S</i> (•)	Sum-of-squares surface constructed out of the parameters • in nonlinear regression
S'	Consumers' surplus (or net benefit) to a tripmaker in making a trip; alternatively it refers
C''	to a predetermined maximum-service-distance in discrete facility-location
S S	Set of demand vertices or nodes that would be covered by a center at a
$\tilde{S}_{i}(p',q')$	The increase (or savings) via a triangular-inequality-style inclusion (or exclusion) of
	demand <i>i</i> between the adjacent points (p',q')
$S_i(l_{h''}t')$	increase in travel-distance from serving demand t via tour n (after the former-demand t has been removed)
S^i	Marginal-cost function for path <i>i</i>
$S_l(V_l^S)$	Supply function showing price against flow quantity, in other words price charged at
C'	supply-quantity V_{i}^{a}
S_{kj}	The kth site-specific attribute of the <i>i</i> th facility (such as the acreage of a state park)
$\tilde{S}^{\prime k,l}$	Marginal-cost function between origin k and destination l
\mathbf{S}_{ij}	Vector of level-of-service variables between locations <i>i</i> and <i>j</i> , including such variables as travel time and travel cost
σ	standard deviation of a probability distribution
$\ddot{\sigma}^2$	Variance of a probability distribution (see also the sample-variance s^2)
σ'	Vendor score or simply a constant in a model
σ_i	Real number showing the 'odometer' reading of a vehicle at node/vertex <i>i</i>
σ _ŷ	randomness of the regression coefficients)
	randominess of the regression coefficients)

σ_M^2	'Tilting' effect, as measured in terms of the variance, on the regression line-when an
σ_y^2	additional data-point x' is added to the regression Total regression-based prediction- or estimation-error, as expressed in terms of the variance
σ^2	of the predicted- or estimated-values Y Total regression based prediction error, as expressed in terms of the variance of the
0 _Y	predicted values Y'
$\sigma^2_{M^*}$	Variance of a normally-distributed set of residuals, around the sample regression-line at $X = x^*$
σ_{ij}^{pq}	Calibration coefficient such as the subareal investment-coefficient or the marginal capital-
	output-ratio, quantifying the multiplier effect of investment among economic sectors and between subareas
σ_j^2	Variance (or second moment) of service-time at service-unit <i>j</i>
$\mathbf{\sigma}^{h} = \left(\leftarrow \mathcal{O}_{i}^{h} \rightarrow\right)$	Vector of dual-variables corresponding to the <i>i</i> th constraints defining the <i>h</i> th travelling-salesman-polytope
$\boldsymbol{\sigma} = [a_j]$	Zonal population-serving-ratios along the diagonal of an $n' \times n'$ matrix
$\boldsymbol{\Sigma} = [\operatorname{cov}(\epsilon_i \epsilon_j)]$	Error covariance-matrix Time dimension or simply a counter for a series of data ($\alpha = N(t)$ is the population at time
l	The dimension of simply a counter for a series of data (e.g., $N(t)$ is the population at time t , Δt is a time increment)
<i>t'</i>	Subareal share of transportation-accessibility-to-employment
t_b	Student-t statistic for calibration-parameter b t-statistic at $100(1-\alpha)$ % confidence-level and $n=2$ degrees-of-freedom
t_N	Sink node/vertex of a network
t''_{k}	Technical-attribute score Stan give in iteration k of a hill alimbing antimization algorithm
t^{-}	Dwell time at a terminal
t_j^0	Delivery- or dwell-time at node <i>j</i> by salesman or vehicle <i>h</i>
t_{ij}	Normalized work-accessibility-function between <i>i</i> and <i>j</i>
$\tilde{t}^{s}(\mathbf{x}, \mathbf{\Phi}, \mathbf{V})$	Cost of providing service at state s and stage r of a recursive-program
<u>t</u>	Random service-time on-scene \tilde{t}_i or off-scene \tilde{t}_j
t -/	Expected value of on-scene service-times to all demands <i>i</i>
$\frac{t'}{t}$	Katio between intra-nodal distances at <i>i</i> and <i>j</i> Average service-time for a service-unit stationed at denot <i>i</i> consisting of on-scene service-
'j	time at the demand t_i^1 and the off-scene service-time at the depot t_i^2
$\mathbf{t} = [t_{ij}]$	Matrix of normalized work-accessibilities, measuring $n' \times n'$
$\mathbf{t}^{\mathbf{k}} = \left[t^{\mathbf{k}}_{ij} \right]$	Matrix of travel-times between i and j Time duration (e.g., τ or $\tau(i, i)$ is the travel time from location i to i)
τ'	Calibration constant in a dynamicized input-output model
$ au_k$	A user-defined scalar in the subgradient optimization routine ranged (say) between 0 and
ĩ	Random variable for service-time in a queuing process: τ_{i} is the random service-time for
	demand <i>i</i> from depot <i>j</i>
$\overline{\tau}_{j}$	Expected one-way travel-time to a random demand from depot <i>j</i>
$\overline{\mathfrak{r}}_{j}'(k)$	Expected travel-time from j to all demands in state k
T	Transportation cost as part of locational expenditure; also quantifies other technological factors
T or $T(\cdot)$	A-priori travelling-salesman-tour as a function of ·
<i>T'</i>	Minimum spanning-tree of a graph
T''	Multi-graph, derived from the minimum spanning-tree by duplicating every arc of the graph; also an instance of the travelling salesman problem.
T_N	Alternate sink-node/vertex in a network for excess flows
T_{j}	Number-of-neighbors surrounding geographic sub-unit j
T'_i	Proportion of sales from subject location to demand at <i>i</i>
T_{ii}	i i i i i i i i i i
= //	Number of <i>i</i> th-group neighbors for a <i>j</i> th-group geographic sub-unit
\hat{T}_{ij}	Number of <i>i</i> th-group neighbors for a <i>j</i> th-group geographic sub-unit Current estimate on random-variable T_{ij}
r t _y T	Number of <i>i</i> th-group neighbors for a <i>j</i> th-group geographic sub-unit Current estimate on random-variable T_{ij} Diagonal matrix of zonal activities such as population
\hat{r}_{g} T T(·)	Number of <i>i</i> th-group neighbors for a <i>j</i> th-group geographic sub-unit Current estimate on random-variable T_{ij} Diagonal matrix of zonal activities such as population Vector of cost-functions in a recursive-program Basis for a simplex-on-a-graph represented graphically as a tree

A - 16	Appendix - Symbols
u $u(t)$ u' u'' u'' $u_i(t)$ u^i	Accessibility-to-population, or a calibration parameter in general; for example, u_{ij} is the normalized nonwork-accessibility between <i>i</i> and <i>j</i> The set of infinite control-paths between the initial point $t=a$ and end point $t=b$ Frequency of a signal Ratio of the maximum travel-distances between nodes <i>i</i> and <i>j</i> Dual variables in a recursive-program for $t = 1, 2,$ Upper bound of a specified time-window when a salesman or a vehicle visits node/vertex <i>i</i>
\overline{u}_{ii}	Capacity on arc (i,j) in a network
$r_{u}(\mathbf{x}, \mathbf{y})$ $\mathbf{u} = [u_{ij}]$ U $U(h)$ U' $U(t)$ $\mathbf{U} = (-U_{i})$	Inference dual-variable to show the value (or shadow price) of relaxing an <i>r</i> th connectivity- requirement at state <i>s</i> Surplus variables in a linear program; also a subset of control-variables U Matrix of nonwork accessibilities, measuring $n' \times n'$ Utilities (e.g., U^* is the maximum amount of utility from a given income or budget) Route length or the range of a vehicle tour for vehicle type h ($h = 1, 2,$) Maximum route-length or range among a fleet of vehicles, $U' = \max_h [U(h)]$ Control variables over time <i>t</i> Vector of control-variables in control theory (slow variables), usually expressed as a
$\mathbf{U}_{v} = \begin{bmatrix} -\mathbf{u}^{k+r} \end{bmatrix} \mathbf{M}_{i}$ \mathbf{V}_{ij} $\mathbf{V}(k)$ \mathbf{V}'_{i} \mathbf{V}_{i} \mathbf{V}_{w} \mathbf{V}_{max}	function of t ; U_j also stands for just the j th canonical-variate Diagonal matrix of zonal activities such as employment atrix of inference dual-variables in a binary recursive-program Value or utility function, or simply the metric resulting from such a measurement The composite travel-cost, or the "utility function," between zones i and j , combining time, cost and other travel impedances into a single metric Average filter using the k th-order neighbors A given parameter (such as housing subsidy per household) Velocity of a service vehicle in stochastic facility-location Dual variable associated with node/vertex i Walking speed Maximum velocity of a vehicle
$ \begin{array}{l} v^{(j)}(\cdot) \text{ or } v^{j}(\cdot) \\ \overline{v}_{ij} \\ \mathbf{v}_{i} = (\leftarrow v^{j}_{i} \rightarrow) \end{array} $	Deterministic value-function for alternative j The reduced-cost for arc (i,j) in network-flow programming An eigenvector consisting of as many entries as the number of alternatives; this is
v V	equivalent to $\mathbf{x}_i = \mathbf{x}(q_i^i)$ Surplus variables in a linear program The amount of economic activities, traffic flow or patronage (e.g., V_i is the amount of activities or trips originating or terminating at location <i>i</i> , and V_{ij} is the exchange of economic activities or traffic movement between locations <i>i</i> and <i>j</i>); \mathcal{P} is the estimated value
$V(h)$ $V'(h)$ $V'(\cdot)$ V^{d} V'_{ij} V_{ijk} V_{ijk}^{k}	and V is the observed value. Capacity of vehicle-type h, where $h = 1, 2,$ Capacity remaining on each vehicle h Normalized vehicle-capacity Inverse demand-function, or the price schedule expressed as a function of a firm's (firms') total output; V_i^d is the excess demand at subarea i Set of vertices or nodes in a graph or network The <i>i</i> th canonical-variate Flow between origin-destination pair $i-j$; \tilde{v}_g is the lower bound and \bar{v}_g is the upper bound Probability that a demand <i>i</i> of type <i>k</i> is received by facility <i>j</i> Trips of type <i>k</i> from <i>i</i> to <i>j</i> Provident distance subarean is and <i>i</i> .
$ \begin{array}{c} \nu_{y} \\ \tilde{\nu}_{dq} \\ \overline{\nu}_{i}^{s} \\ \phi' \phi' $	 Amount supplied by all the firms other than q to demand-location i Output of firm i; also standing for the excess supply of a firm located in subarea i Calibration constant representing such parameters as the trip-generation rate or response rate of the system Polytope (feasible region) defined by the hth travelling-salesman-problem Probability distribution (e.g., probability that the surplus resulting from the trip to j has a value in the neighborhood of S')

Φ ϕ_k $\phi(B)$	Cumulative distribution (e.g., $\Phi(v) = [F(v)]^n$ is the cumulative distribution-function of the largest-utility v among n independent samples; $\Phi_{ij}(S')$ is the cumulative-distribution-function of the surplus accruing from the preferred (optimal) trip between location i and j) Coefficient of the <i>k</i> th-lag term in an autoregressive-time-series The backshift operation of an autoregressive model
φ(<i>D</i>) δ.	Partial-autocorrelation-coefficient for the kth-lag term in an autoregressive-time-series
т. Ф.,	Partial-autocorrelation-coefficient at temporal-lag k and spatial-lag l in an autoregressive
• 12	spatial-time-series
$\boldsymbol{\phi}(\cdot)$	Flow-vector function at stage s of a decomposed recursive-program
φ (x)	Demand density-function on Voronoi polygons
$\mathbf{\Phi}^{T} = (\leftarrow \phi_i \rightarrow)$	Vector of pertinent flows at stage r and state s of a decomposed recursive-program; these flows can be expressed in terms of the pertinent demand-vector f Vector of autoregressive coefficients in a conditional spatial-econometric model
$\mathbf{\Phi}_{k} = \left[\phi_{iik} \right]$	A spatial autoregressive-coefficient matrix of order-k
$\mathbf{\Phi}(B) = [\phi_{ii}(B)]$	A spatial autoregressive-operator matrix
$\mathbf{\Phi}^{k}(\cdot) = [-\mathbf{\Phi}^{k+r}] \text{Matrix}$	t of flow-vectors $\mathbf{\Phi}^{k+r}[\mathbf{x}^{k+r}(k) \rightarrow \mathbf{I}$
$\mathbf{\Phi}^{k+r}[\mathbf{x}^{k+r}(k)]$	Flow-vector at the k th cycle and r th stage, showing origin-destination-connectivity as a function of the iterative multi-stop routing-decisions
χ^2	Chi-square statistic
φ	Expected cost between stockout and storage in a newsboy problem
$\mathbf{\phi} = [f_j]$	Zonal activity-rates along the diagonal of the $n' \times n'$ matrix
Ψ <i>ttr</i> .	Weights used in time-series forecasting
$\Psi_{L}^{n}(l)$	Dynamic-program recursion-function for computing the shortest-route-length l
Ω	Dual variable corresponding to the terminal capacity constraint—a parameter to account
	for the given warehouse capacity; also regular vector space
Ω	A bounded domain including the boundary $\delta\Omega$
$ \Omega_q = \{\mathbf{x}_q\} $	A teasible region within the vector space Ω ; e.g., a set of constraints in a spatial-equilibrium model expressed in terms of the flow decision-variables x for each of the suppliers <i>a</i> .
$arOmega_{ii}$	Percentage-change-of-patronage at facility <i>j</i> from the demands that originate at <i>i</i>
$\Omega(B) = \sum_{i} \Omega_{i} B^{i}$	Backshift operator containing the dynamic multipliers <i>Q</i> in a set of dynamic simultaneous-
2	equations
$\underline{\mathbf{D}}_{t-1,t}$	I ransition matrix in a Kalman filter
ή _k	Connectivity requirement on the origin–destination pairs during the kin cycle
$\hat{\eta}_k(r)$	in the <i>k</i> th cycle: i.e. the number of constraint functions defining the local flow pattern in
	a recursive program for the RISE algorithm
W	A constant, or an aggregate weight-parameter, placed on a variable or an estimator-measure (such as Moran's- <i>I</i> , and its variance, plus the mean and expected variance of the general spatial-statistic)
W_k	A constant or a weight placed on entity or attribute k; when these weights are normalized
	and summed to unity, we write $\sum_{\mathbf{\lambda}} \mathbf{\lambda}'_{\mathbf{r}} = 1$
w^k	Weight reflecting the relative importance of workplace-based retail-trips for purpose k
\tilde{w}_1, \tilde{w}_2	Weight-parameters used in the formulas for the variance of Moran's- <i>I</i>
Wk	Width of a subregion k
w'	A white-noise series, consisting of a sequence of uncorrelated random-variables, each with zero mean and finite variance; engineers consider them as independent 'shocks' that are transformed by a "transfer function" to another time-series whose successive values are highly dependent.
$W_p^{\prime\prime}$	Frequency on route p Weight placed on the demand facility pair <i>i</i> i or the weight placed on are flow (<i>i</i> i).
w _{ij}	otherwise referred to as cost coefficients in the equivalent linear-program; also denotes the weight entry in a spatial-weight-matrix \mathbf{W} , with $0 \le w_{ii} \le 1$
$W_{ij}(d)$	Binary valuations of w_{ij} when an activity at j is within a distance d from i
W _{ijp}	Frequency on the (i,j) segment of route p
\mathbf{W}_{i}^{\prime}	weight contribution toward criterion i by alternative j

A - 18	Appendix - Symbols
$\mathbf{w}^{s}(\mathbf{\phi}, \mathbf{V})$ $\mathbf{w} = (-w_{i} \rightarrow)^{T}$ $-(0) = (-w_{i} \rightarrow)^{T}$	Vector of route-frequencies at stage r and state s of a decomposed recursive-program Eigenvector consisting of q entries—this is equivalent to \mathbf{v}_i and \mathbf{x}_i ; also the cost vector in a network-flow program
$\mathbf{W}^{\otimes} = (\leftarrow \mathbf{W}_{ij}^{\otimes} \rightarrow)^{*}$	weights associated with the <i>l</i> th-order neighbors—notice this is equivalent to the spatial operator $L^{(0)}(\bullet)$
W W _i	White-collar employment; also work load or demand placed on a service-unit Size of demand or activity at i , which is proxy for development opportunity at the zone; W ' is the vector of development-opportunities among all zones Delay time in gueue
W_T^{q} W(t) W'	Total time in system, including delay time in queue and the time being served Rate of investment in new capacity over time Revised size of demand or activity at <i>i</i>
W_{ij}^{i}	Service-effectiveness weight expressed as a function of the separation between demand i and facility j ; i.e., the further apart i and j are, the less effective it is for service to be rendered
$ar{W}_i^p$ W^h	Observed value of attractiveness or the opportunity of zone- <i>i</i> as a location for industry- <i>p</i>
W_i W_i^p	Observed zonal-shopping attractiveness or opportunity
$\mathbf{W} = [w_{ij}]$	A $q \times q$ pairwise-comparison weight-matrix used in the analytic hierarchy process; also denotes the weight matrix in spatial econometric-models, measuring $n \times n$
$\mathbf{W}' = [W_{gh}]$	An $n' \times n'$ activity derivation-and-allocation matrix of Lowry–Garin model, with each entry denoting a zone pair $g = h$
$\mathbf{W}'' = [w_j] \\ \mathbf{W}^j$	The diagonal matrix consisting of per-capita value-added productivity (wage rate) Activity derivation–allocation, transition or spatial-weight matrix for the <i>j</i> th activity in a
$\mathbf{W}^{(l)} = [w_{ij}^{(l)}]$	Lowry–Garin model Spatial weight-matrix for the <i>l</i> th-contiguity class; with the normalized spatial-weights sum to unity $\sum_{j} w_{ij}^{(l)} = 1$, and $\mathbf{W}^{(0)} = [w_{ij}^{(0)}] = \mathbf{I}$, or the 0 <i>th</i> -order neighbors being the subject entry itself.
\mathbf{W}_t	Gain matrix in Kalman filter, representing the net percentage of measurement-error or noise that is left after filtering
$(\mathbf{W}^{(l)}\mathbf{y})_{-\mathbf{y}_{l}}$	Preprocessing of data \mathbf{y} by removing the subject <i>i</i> th-entry, and then replace it with a value
~*	resulting from 'filtering' with a spatial-'mask' $\mathbf{W}^{(l)}$ of order <i>l</i>
$\frac{x}{x'}$	A particular observation for the random-variable X
x'_t	Actual, accurate data in a Kalman-filter time-series (to be differentiated from what is observable)
x_0, x_0', x_0', \dots	Decision boundary between pattern groups 1 and 2, 2 and 3, 3 and 4, etc.
x_{ij} x^{t}	Flow on path <i>i</i> in a network
X^p_{i}	Equilibrium economic-activity at each subarea i and sector p
\tilde{x}_i^P	Projected sales of product p in subarea i
X_{ijk}	Allocation of demand <i>i</i> to facility <i>j</i> in state <i>k</i>
x^{x} $x^{m,n}(C^{m,n})$	Demand-for-transportation between origin-destination pair $m-n$ as a function of the transportation and how them
x_p^{mn}	Binary link-allocation of demand between origin–destination pair $m-n$ to non-stop route or itinerary p
x_{jl}^{qp}	Input of commodity- q from subarea- j in the production of commodity- p in subarea- i
x ^{mn} x _{mip}	Binary allocation of demand $m-n$ on route p as indicated by the usage of segment (m,i) in
$\mathbf{x} = (-x_j)^T$	the itinerary Vector of decision-variables, or empirical readings (such as change-in-accessibility for all the activities <i>j</i>)
\mathbf{X}_q $\mathbf{X}_q = (\mathbf{x} \cdot \mathbf{x} \cdot \mathbf{y})^T$	An interior point in the feasible-region Ω_q
$\mathbf{x}_t = (x_1, x_2, \dots)^T$ $\mathbf{x}_t' = (x_1', x_2', \dots)^T$	Actual readings over time in a Kalman-filter time-series
\mathbf{X}_L \mathbf{X}_R	The left eigenvector of a square matrix The right eigenvector of a square matrix

 \mathbf{X}^{i}

 \mathbf{x}^k

 $\mathbf{x}_{\delta}^{0} \mathbf{x}_{t}^{0} \mathbf{x}_{t}^{i_{k}}$

 $\mathbf{x}^{*}(\mathbf{x}_{s}^{0})$ $\mathbf{x}^{*}(\mathbf{x}_{t}^{0})$ $\mathbf{\hat{x}}_{t}^{\prime}=(\mathbf{\hat{x}}_{1},\mathbf{\hat{x}}_{2},\dots)^{T}$

The <i>i</i> th discrete-point proposal in a branch-and-bound tree, corresponding to a constraint in the Lagrangian-dual linear-program
The <i>k</i> th basic-solution in a linear-program, or the <i>k</i> th set of decision-variables (e.g., solution alternative in a branch-and-bound tree, the location of the <i>k</i> th-facility or the
routing decision-variables for the <i>k</i> th-vehicle)
Coordinates for the origin of a trip
Coordinates for the destination of a trip
The <i>i</i> th discrete-point proposal in a branch-and-bound tree during the <i>k</i> th-step of the subgradient-optimization procedure of Lagrangian relaxation
Nearest public-transportation terminal for a trip starting at origin \mathbf{x}_s^0
Nearest public-transportation terminal for a trip terminating at destination \mathbf{x}_t^0
Estimated-values of the observations in a Kalman-filter time-series
The kth iterative multi-stop-routing decision-variables for the <i>r</i> th-stage

$\mathbf{x}^{k+r}(k)$	The kth iterative multi-stop-routing decision-variables for the rth-stage
$\overline{\mathbf{x}}^{k+r}(k)$	Realized values for the kth iterative multi-stop routing-decision-variables in the rth-stage
X	The decision-variable X; or the decision or alternative space in multi-criteria
	decision-making
X'	The state-space in Markovian-decision-processes
\overline{X}	Average of the independent random-variable X in a regression model
X^p	Control-total of areawide-transportation-cost for commodity p
X(t)	Random-variable for the state at time t
X_k	Random-variable for the state at stage or time k
$X_i(\cdot)$	Accessibility from origin <i>i</i> to all destinations as a function of such parameter as travel cost
X'_{lj}	Activity- <i>l</i> 's accessibility to zone- <i>j</i>
X_{ij}	Observed patronage of facility <i>j</i> by demand from location- <i>i</i>
X_i^k	Amount of activity k in zone i
[←X ₁ →	
$\mathbf{X} = [\mathbf{X}_{i}] = [X_{i}]$	Exogenous- or independent-variable $n \times (k+1)$ matrix in ordinary-least-squares regression,
[.]	

 $\mathbf{X}(t) = (-X_i(t))^T$ vector of state-variables in control theory (fast variables), expressed as a function of t in terms of the individual state variables $X_i(t)$ for states i = 1, ..., n

marvia	Lat state variables $A_i(t)$ for states $t = 1, \dots, n$
$\mathbf{X}(0) = \mathbf{X}_0 = (-X_i(0))^T$	Initial condition of the state-vector at time 0 for states $i = 1,, n$
$\mathbf{X}_{\max}^{*}(t)$	The most-likely state
$\mathbf{X}(k) = [-\mathbf{x}^{k+r}(k)]$	Matrix of binary-decision-variables in a recursive-program during the kth-cycle and the rth-
	stage
$\mathbf{X}_{l}(\Delta) = (-X_{lj}(\Delta))$	Activity-l's accessibility to individual-zone-j expressed as change in the regional-share-in-
	accessibility
$\mathbf{X}'' = (-X''_i \rightarrow)$	Stationary states in system of interacting differential-equations
$\mathbf{X}^{j} = [X_{gh}^{j}]$	A matrix of accessibilities between zones g and h for activity j
<i>y</i>	Wage rate for a household or total wages across the labor-force
<i>y</i> *	Sample observation or the optimal-value of y
<i>v</i> ′	Regression-based prediction corresponding to a given x'
V _n	The <i>p</i> th-component of the y '-vector in a network-tableau
V_t	Ordinate of an observed-data-point in the series $t = 1, 2,$
\hat{y}_t	Estimated ordinate of an observed-data-point in the series $t = 1, 2,$
v^q	Household-wage expenditure on the <i>q</i> th industrial-sector
V_{ik}	Binary-decision-variable to assign facility to node <i>i</i> in state- k
V _{iik}	Binary-decision-variable to indicate that node/vertex- <i>i</i> is served by facility- <i>i</i> in state- <i>k</i>
V_{k}^{mn}	Binary indicator that there is kth-stop service between origin-destination pair $m-n$
$\mathcal{Y}_{u(k),v(l)}$	Binary decision-variable to indicate moving a facility from node/vertex u to v as the state
$()^T$	
$\mathbf{y} = (-y_j - y_j)^{T}$	vector of integer-variables in a mathematical-program, or simply a point within the regular
	vector-space
\mathbf{y}_q	A point other than \mathbf{x}_q within the feasible-region Ω_q
$\mathbf{y}' = (-y_i')'$	A vector of criterion-measures consisting of attributes <i>i</i> , also the updated or 'refreshed'
	column in a network-flow-tableau during the simplex-iterations
У″	Interim solution in Benders' decomposition
$\mathbf{y}(k)$	The updated (or 'refreshed') kth column in a network-tableau

A - 20	Appendix - Symbols
$\mathbf{y}^{j} = (\leftarrow y_{i}^{j} \rightarrow)^{T}$ Y	A vector of criterion-measures for alternative j , or the j th group of y_i -variables (e.g., the delivery commitment of vehicle j toward demand i) The decision-variable Y , or random-variable notation of the explanatory or dependent variable in ordinary-least-squares regression; also the regional income
r Y'	Mean of the random-variable <i>Y</i> Outcome or criterion space of multi-criteria decision-making; also the prediction random-variable in regression
Y'' Y_{ij}	The combinatorial space of the discrete-variables y_i A spatial-variable defined by the coordinates i and j —a variable that is related to its neighbors in both axes of this coordinate system; this cross-product is the covariance between the observations at i and i
$\mathbf{Y} = (-y_i \rightarrow)^T$ $\mathbf{Y}^{ij}(k) = [-y_i^{ij} \rightarrow]$	Explanatory- or dependent-variable vector in ordinary-least-squares regression, consisting of <i>n</i> observations; \hat{r} denotes the estimated-values of random-variable <i>Y</i> Binary parameters of each constraint-function in recursive programming (<i>p'</i> in total),
	where <i>i</i> is the state-index and <i>j</i> the stage-index; $\mathbf{Y}(k) = \begin{bmatrix} 1 \\ \mathbf{y}_{k}^{k+r} \end{bmatrix}$
$\mathbf{Y}(\cdot)$	State-connectivity linkage-function of past decisions and available vehicle-capacity in a recursive-program
Y' z	Labor-force-value-added output-vector Objective-function of an optimization-problem; also used to denote the activity-generation rate
z' z(i)	A bound on z Objective-function value of the <i>i</i> th alternative
Z_c Z_i	Largest demand-facility assignment-distance Amount of product or services sold at demand-point i ; or a transformed observation from the raw-data Z_i
Z_t $Z_{\rm IP}$	Stationary time-series with zero mean An integer-programming objective-function that is to be estimated by Lagrangian- relaxation
\hat{z}_t	Stationary time-series with non-zero mean; also the estimated-value in an adaptive time-
z'_{0} z'_{0} z'_{0i} z'_{j} z'_{ij} z'_{i}	series Binary variable to denote the location of a facility at j ; z_j is used after y_j when there is more than one type of facility to be located Amount-of-output produced at supply-facility or plant j Amount-of-output produced at plant j and sent to output-market i The optimal benefit of opening facility- j in a generalized p -median-problem (as defined in a subproblem of Lagrangian-relaxation solution) `Trunk' traffic from supply-source i to distribution-center j Amount of input- i used by plant- j
z ^{<i>a</i>} ,	Employment by the <i>e</i> _i th-type industry
z_i^{i} z_y^{i} z_L^{i} z_U^{i} z' z_{ij}	Supply-of-labor by household in zone- <i>i</i> to zone- <i>j</i> for employment by the e_i th-type industry Lower-bound of objective-function-value at iteration- <i>i</i> Upper-bound of objective-function-value at iteration- <i>i</i> Lower or upper bound of objective-function-value Binary indicator-variable to show whether a multiattribute observation $\mathbf{x} = (x_1, x_2,)^T$ for a pixel of color <i>j</i> has been properly classified into group <i>i</i> ; $z_{ij}=1$ when it is properly classified into group <i>i</i> (or $i=j$) and $z_{ij}=0$ when it is improperly classified ($i\neq j$). In vector notation for two groups <i>i</i> and <i>i</i> , we write $\mathbf{z} = (z_1, z_2)^T = (1, 0)^T$ and the random variable
$ z_{ij}' z_{LD} \\ z_{LP} \\ z_{LR} \\ z_{i} \\ z $	corresponding to $\mathbf{z}_i = (z_{ii}, z_{ij})^T$ is $\tilde{\mathbf{z}}_i = (\tilde{\mathbf{z}}_{ij}, \tilde{\mathbf{z}}_{ij})^T$. Impedance between zones <i>i</i> and <i>j</i> Objective-function-value of a Lagrangian-dual Objective-function-value for a linear-program relaxation Objective-function-value for a Lagrangian-relaxation problem Goods in storage at location- <i>i</i> vector of \mathbf{Z} values induced for stationary and with mean set to zero; also stands for

\mathbf{Z}_{j}	Vector-of-pixels z for group j in a Bayesian classifier
Ζ	Activity level (where the activity can be population, employment, grey values, or any
7(i)	Expect value of the decision made at state <i>i</i>
Z'(i)	Expected-value of the improved-decision made at state- <i>i</i> according to Howard's policy- iteration
Z_i	Objective-function value or activity level at location- <i>j</i>
\vec{Z}_t	Raw-data time-series before inducing stationarity
Z'_t	Actual, accurate daa in a Kalman-filter time series (to be differentiated from what is observable)
$\dot{Z}_{t}, \ddot{Z}_{t}$	First and second differencing of time-series Z_t
Z' Z'' Z _{ij}	Preference structure in multi-criteria decision-making Deviation-measures from the efficient-contour of unity in the Minkowski distance-function Value of spatial-data at grid-point $i-j$; often simplified to read Z_j to stand for the spatial- data value at location i
Z_j^l	Value of the <i>j</i> th spatial-data at spatial-lag <i>l</i>
Z_{+}^{n}	n-dimensional Euclidean-space of positive discrete-values
$\mathbf{Z} = (\leftarrow Z_i \rightarrow)$	Vector of exogenous-variables Z_i of such activities as population and employment in each zone- <i>i</i> ; Z_0 is the initial-values of Z
$\mathbf{Z}(t)$	Density or relative-frequency of the state-vector $\mathbf{X}(t)$; in other words, the normalized state-vector
$\mathbf{Z}_{i}=(\leftarrow Z_{i},\rightarrow)$	The <i>i</i> th-activity assigned to zone- <i>i</i>
\mathbf{Z}^{i}	Vector of the <i>total</i> -population/employment activity-levels at time-period (iteration) i , with \mathbb{Z}^0 as the given final-period <i>basic-activities</i> (from which other activities are generated)