## APPENDIX LIST OF SYMBOLS

| $a$ | A calibration constant; for example, it is the service-employment multiplier or populationserving ratio (number of service jobs generated from one household or resident) |
| :---: | :---: |
| $\tilde{a}$ | Intercept regression-coefficient as a random variable |
| $\tilde{a}^{*}$ | Specific value of $\tilde{a}$ corresponding to a sample of data points |
| $a^{\prime}$ | Acceleration of a vehicle; also a constant parameter, such as unit cost of commuting (cost per unit-of-distance travelled), or the exponent of the development opportunity $W_{j}$ at destination $j$ |
| $a_{i}$ | Calibration parameter corresponding to the utility increase in zone $i$, where utility is some measure of composite accessibility to the zone; also the population-serving ratio at zone $i$ |
| $a_{t}$ | Estimation-error or noise term for a series of data ( $t=1,2, \ldots$ ) usually in a 'normalized' time-series, or after the data have been differenced to a stationary series; the estimated error or noise in Kalman filtering; also referred to as innovations when it is white noise |
| $a_{i}^{\prime}$ | Physical area of geographic sub-unit $i$ or the demand-generating potential of $i$ |
| $a_{t}^{\prime}$ | Measurement error in a Kalman-filter time-series, representing the difference between observed and measured data |
| $a_{D}$ | Error term in a demand econometric-equation |
| $a_{S}$ | Error term in a supply econometric-equation |
| $a^{W}$ | Weighted labor-force-participation-rate, where the weights are the percentages of regional population in each zone |
| $a^{p}$ | The $p$ th-sector employment-growth-rate in the entire study-area |
| $a_{i j}$ | Parallel to its single-dimension analogue, $a_{i j}$ is an error- or noise-term in the spatial context; it has a zero mean and a constant variance; also stands for the entries in the $\overline{\mathbf{A}}$ matrix |
| $a_{i}^{u}$ | Convex combination of the population-serving ratios, with normalized accessibilities to zone $i$ as weights |
| $a_{j}^{p}$ | Employment multiplier considering the population-serving ratio, i.e., $\left(1+a_{j}\right)$-segregated both by economic-sector $p$ and by zone $j$ here |
| $a^{k l}$ | Calibration parameter in a predictor-prey equation-set showing the interaction between the $k$ th and $l$ th species |
| $a_{k j}^{\prime}$ | The $k$ th output (benefit) measures due to decision-making-unit $j$ considering both nonspatial and spatial attributes (see also $\overline{\mathbf{A}}=a_{i t i f}$ ) |
| $a_{i j}^{p q}(k)$ | Impact of the $p$ th-state-variable-in-zone- $i$-at-time- $k$ on the $q$ th-state-variable-in-zone- $j$-at-time- $k+1$ |
| $a^{\prime \prime}$ | Threshold for a high-pass noise-filter |
| $\mathbf{a}=\left(\vdash a_{i} \rightarrow\right)^{T}$ | Vector of calibration coefficients in the second stage of 2-stage least-squares, consisting of $q$ entries; also stands for the vector of the error (noise) terms in a spatial-temporal forecasting-model |
| $\mathbf{a}^{\prime}$ | Vector whose $i$ th element is the ratio of the-household-income to the gross-output-in-the$i$ th-industrial-sector |
| ã | Interim error-vector or noise-term in a more efficient calibration-procedure for STARMA |
| $\mathbf{a}_{i j}=\left(\leftarrow a_{i j}^{k} \rightarrow\right)^{T}$ | Each entry of the $\overline{\mathbf{A}}=\left\|a_{v \mid}\right\|$ payoff-matrix is replaced by a vector in a linear program, mainly to facilitate a multicriteria, two-person, zero-sum, non-cooperative game; here $k$ is the index for a criterion |
| $\alpha$ | Calibration constant, or step size in "hill-climbing" algorithms; also the tail of a distribution |
| $\alpha^{\prime}$ | Angle between two criterion-functions in multicriteria linear-programming; also a calibration constant |
| $\alpha{ }^{\prime \prime}$ | Resulting problem-type after the original problem has been polynomially reduced |
| $\alpha_{t}$ | Random-shock or white-noise input-time-series in a transfer-function model |
| $\alpha_{j i}^{q p}$ | Exponent in a Cobb-Douglas production-function corresponding to the factor input $x_{j i}^{q p}$ |
| A | Accessibility expenditure for a household (part of locational expenditure); also the area |
| $A(\cdot)$ | Area of - |
| $A_{i}$ | Weighted labor-force participation-rate, with accessibility from zone $i$ as weights |
| $A_{j}$ | Gross acreage of subarea $j$ |
| $A_{j}^{\prime}$ | Useable gross-acreage of subarea $j$ |
| $A_{t}$ | Error term in a 'raw-data' time-series |
| $\bar{A}_{j}$ | Developable acreage in subarea $j$ |
| $A^{B}$ | Basic land-use ( $A_{j}^{B}$ is basic land-use in zone $j$ ) |


| $A^{R}$ | Retail land ( $A_{j}^{R}$ is retail land in zone $j$ ) |
| :---: | :---: |
| $A^{U}$ | Unusable land ( $A_{j}^{U}$ is unusable land in zone $j$ ) |
| $\underline{A}$ | Set of arcs in a network |
| $\hat{A}_{j}{ }^{k}$ | Net acreage in subarea $j$ devoted to the $k$ th land-use |
| $\mathbf{A}=\left(-A_{t} \rightarrow\right)^{T}$ | Vector of disturbance or error terms in econometric or spatial time-series models, consisting of $n$ observations; in 2-stage least-squares, it consists of $q$ entries, where $q$ is the number of endogenous variables |
| A | As a matrix (instead of a vector), A stands for node-arc incidence-matrix in network-flow programming |
| $\mathbf{A}^{\prime}=\left[A_{i j}^{\prime}\right]$ | An $n \times n$ square matrix; for a compartmental model, it is the rate-of-change matrix; and for the matrix of secondary (retail)-employment it is the distribution-rate by zone, where $n=n^{\prime}$. |
| $\mathbf{A}_{0}(t)$ | Vector showing rate-of-change with the "outside world" over time |
| $\mathbf{A}^{\prime \prime}=[(i, j)]$ | Contiguity matrix with nonzero arc-entries where $i$ is incident upon $j$ |
| $\hat{\mathbf{A}}$ | An $n \times n$ matrix, which converts value-added output vector by industrial sectors to the same vector measured in labor-force base |
| $\mathbf{A}_{j}$ | Vector of socioeconomic variables at location $j$, representing such activities as population and employment |
| $\mathbf{A}(j)$ | Column vector in the network-simplex tableau for arc $j$ |
| $\overline{\mathbf{A}}={ }^{\text {a }} a_{i j}$ | Coefficient matrix of linear-programming constraints, where $a_{i j}$ expresses the incidence relationship between row $i$ and column $j$; an example is the $k$ th output measures due to decision-making-unit $j, a_{k j}$, in a data-envelopment analysis. |
| $\mathbf{A}_{B}$ | Basis of a linear program |
| $\mathbf{A}_{N}$ | Nonbasic part of the tableau in a linear program |
| $\mathbf{A}^{1}$ | The complicated set of constraints in a mixed integer-program |
| $\mathbf{A}^{2}$ | The straightforward set of constraints in a mixed integer-program |
| $b$ | Generally a constant parameter, denoting a growth rate, intercept or slope in a linear equation, or the positive exponent of a spatial cost-function etc. |
| $\tilde{b}$ | 'Slope' regression-coefficient as a random variable |
| $\tilde{b}^{*}$ | Specific value of $\tilde{b}$ for a sample of data points |
| $b^{U}$ | Household budget |
| $b_{j}$ | The fixed cost of siting a depot at node $j$ |
| $b^{\prime}$ | Travel-cost elasticity for activity $j$ |
| $b^{k}(m)$ | A scale factor used to adjust the $k$ th zonal-retail-employment from one loop of the Lowry model $m$ to another $m+1$, where $m=1,2, \ldots$ |
| $b_{k i}, b_{i k}$ | Slack-flow capacity on slack arc $(k, i)$ or $(i, k)$; also the benefit variable in data-envelopment analysis, denoting the weight placed on the $k$ th benefit of the $i$ th alternative |
| $b_{k j i}$ | Benefit variable used in the combined data-envelopment-analysis-and-location model, showing the relative importance of assigning the $k$ th benefit to the demand-facility pair $i j$ |
| $\mathbf{b}=\left(-b_{i} \rightarrow\right)^{T}$ | Vector of estimated parameters in ordinary least-squares regression or other calibration procedures, consisting of $k+1$ parameters (including the'intercept'); also the right-hand-side of a linear or mixed integer program |
| $\mathbf{b}^{\prime}=\left(\leftarrow b_{i}^{\prime} \rightarrow\right)^{T}$ | A given vector of the right-hand-side of a mathematical program; also the fixed externalflows in a network-flow program |
| $\overline{\text { b }}$ | Updated right-hand-side of a linear program during a simplex procedure; also the birth rates in a cohort-survival analysis |
| $\mathbf{b}^{1}$ | The portion of the right-hand-side corresponding to the complicated set of constraints in a mixed-integer-program |
| $\mathbf{b}^{2}$ | The portion of the right-hand-side corresponding to the straightforward set of constraints in a mixed-integer-program |
| $\beta$ | A calibration constant, such as the positive exponent of a spatial cost-function or the roundtrip factor in stochastic facility-location. (This same constant $\beta$ is also referred to as $b$ ) |
| $\beta^{\prime}$ | A calibration constant |
| $\beta_{i}$ | Current level of inventory at location $i$ |
| $\beta_{t}$ | Prewhitened output time-series in a transfer-function model |
| $B$ | An arbitrarily large integer; also the backshift operator in a time series |
| $B^{\prime}$ | Bifurcation set of control variables |
| $B^{\prime \prime}$ | Blue-collar employment |
| $B_{k}$ | Percentage reflectance in band $k$ of a satellite sensor |


| $B_{L}, B_{R}$ | Left and right boundaries of a firm's market area |
| :--- | :--- |
| $B_{k}^{\prime}$ | Number of times a facility is exposed to demands in period $k$ |
| $B^{k}$ | Bound value for distance from a vertex, used to locate the intersecting point $q_{k}$ or a |
|  | candidate location for a center |


| $\mathbf{C}=\left[\mathbf{c}^{1}, \ldots, \mathbf{c}^{q}\right]^{T}$ | A $q \times n$ matrix of cost coefficients in a multicriteria linear-program, where each criterion $j$ has a cost and a gradient vector $\mathbf{c}^{j}$ |
| :---: | :---: |
| C( $\cdot$ ) | State-connectivity function linking to past decisions and connectivity requirements in a recursive program |
| $\mathbf{C}^{\prime}$ | Diagonal matrix converting the gross-output vector to value-added vector |
| $\hat{\mathbf{C}}$ | Matrix of estimated coefficients in stage 1 of 2-stage least-squares, measuring $q \times k$ |
| $\bar{C}$ | Number of cell columns in a grid region or in a raster image |
| $\gamma$ | Unit price at the market, Lagrange multiplier, and a calibration constant in general |
| $\gamma^{\prime}$ | Capacity-utilization rate, bounded between zero and unity |
| $\gamma_{j}^{p q}$ | Dual variable associated with the input-output coefficients in an entropy-maximization model |
| $\boldsymbol{\gamma}^{\prime}=\left[q_{j}^{\prime}\right]$ | Matrix of subareal growth-rates along its diagonal |
| $\bar{\gamma}$ | Economic-base multiplier over a time-increment $\Delta t$, combining the activity-rate $f$ and the population-serving-ratio $a ; \bar{\gamma}_{i j}$ (with the subscript) would include the locational attributes as captured in work- and nonwork-accessibilities $t_{i j}$ and $u_{i j}$ |
| $\gamma_{i}(p, s)$ | General 'strain' or the savings from including new-demand $i$ via a triangular-inequalitystyle route-replacement between points $p$ and $s$ |
| $\Gamma$ | The gross economic-multiplier deriving the total employment from the initial basicemployment |
| $\Gamma$ | Vector of economic-multipliers deriving the total employment in the study area from the initial basic-employment, including $c_{j}$, $f$ and $a$ |
| $\Gamma_{t}$ | Observation matrix in Kalman filter; when multiplied against the observed time-series, specifies what is actually observable |
| $\Gamma(W, p)$ | Optimization results from a facility-location model where $p$ facilities are relocated to respond to a maximum demand of $W$ |
| $\boldsymbol{\Gamma}(k)=\left[\gamma_{i}(k) \rightarrow\right]$ Vector of payoff-function consisting of $q$ entries, where $q \leq \dot{\eta} \mu$ |  |
| $d$ | Distance or spatial separation; also a proxy for a particular spatial order |
| $d^{\prime}$ | Amount of differencing to induce stationarity in a time-series |
| $d^{\prime \prime}$ | A decision in a Markovian decision-process |
| $d_{i}$ | Distance from location $i$ (notice this is not necessarily Euclidean distance); or deviation from a standard or ideal in dimension $i$; also the capacity of arc $i$ or the weights in a transfer function |
| $d^{k}$ | Minimum threshold of retail-employment by trade-class $k ; d^{R}$ is the threshold for the case when there is only one trade class |
| $d_{j}(\mathbf{x})$ | Multidimensional decision-boundary in a Bayesian classifier |
| $d(B)=d_{0}+d_{1} B+d_{2} B^{2}+\ldots$ | Transfer function in a multivariate time-series, consisting of weights $d_{0}, d_{1}, d_{2}$, etc. and backshift operators $B$ |
| $d_{i j}$ | Euclidean distance or the spatial-cost in general between locations $i$ and $j$ |
| $d_{i j k}$ | Euclidean distance or the spatial cost between locations $i$ and $j$ in state $k$ |
| $d_{i j}^{h}$ | Distance or travel time between nodes $i$ and $j$ by salesman or vehicle $h$ |
| $d^{i}$ | Time a salesman or vehicle visits node $i$ in a tour or a route |
| $d^{i j}$ | Distance or time between locations $i$ and $j$, starting with arrival at $i$ and terminating at arrival at $j$ (notice this is not necessarily the Euclidean distance) |
| $d(\mathbf{i}, \mathbf{j}$ | Planar Euclidean distance between two Cartesian coordinate points i and $\mathbf{j}$ |
| $d\left(\mathbf{x}_{i}, \mathbf{x}_{i+1}\right)$ | Spatial separation between consecutive stops $\mathbf{x}_{i}, \mathbf{x}_{i+1}$ |
| d, d' | Vector of arc capacities in network-flow programming |
| $\mathbf{d}^{j}$ | Extreme direction along the $j$ th axis in a linear program |
| $\mathbf{d}^{k}=\left(\leftarrow d_{i}^{k} \rightarrow\right)$ | Direction of steepest ascent in the $k$ th step of a hill-climbing optimization-algorithm, as characterized by $n$ components of the vector |
| $\delta$ | Change in a quantity (e.g., $\delta x$ is the increase or decrease in quantity $x$ ); $\delta_{i j}$ is the distance savings in directly going from $i$ to $j$, instead of through an intermediate point $k$ |
| $\delta(i)$ | The steady-state decision whenever the state is $i$ in a Markovian-decision-process |
| $\tilde{\delta}$ | Policy in a Markovian decision-process |
| $\tilde{\delta}^{\prime}$ | Improved stationary-policy in the policy-iteration procedure of a Markovian-decisionprocess |
| $\delta^{*}$ | Optimal policy in a Markovian-decision-process |
| $\delta^{\prime}, \delta^{\prime \prime}$ | Fixed cyclic-permutation |


| $\delta_{i}$ | Binary decision-variable to be switched on, conditional upon another decision-variable <br> being engaged; also a calibration constant; or a nonnegative real-number denoting the |
| :--- | :--- |
|  | number of legs in a subtour-breaking constraint |
| $\delta \Omega$ | Boundary of the bounded-domain $\Omega$ |


| $E^{\prime}\left(i_{1}, i_{2}, h_{1}, h_{2}\right)$ | Modified generalized-savings-measure from an exchange of demands $i_{1}$ and $i_{2}$ between tours $h_{1}$ and $h_{2}$ |
| :---: | :---: |
| E | Row vector of employment-levels, made up of individual zonal employment $E_{i}$ |
| $f$ | Average household-size in terms of the number of employed residents per household, or reciprocal of the labor-force participation-rate (also called the activity rate) |
| $f(\cdot)$ | Regular function of the argument (e.g., the criterion function in dynamic programming) |
| $f\left(\mathbf{x}_{q}, \mathbf{x}-\mathbf{x}_{q}\right)$ | A functional for which the directional derivative is being considered, approaching point $\mathbf{x}_{q}$ from point $\mathbf{x}$ |
| $f^{\prime}$ | Functional-attribute score, including spatial separation |
| $f^{\prime}(t)$ | Cumulative demand at time-period $t$ |
| $f_{i}$ | Demand-for-service frequency at location $i$; also the natural growth-rate of population in subarea $i$ (the activity rate) |
| $f^{W}$ | Weighted activity-rate, where the weights are the percentages of regional population at each zone |
| $f_{i k}$ | Demand-for-service frequency at location $i$ in state $k$ |
| $f_{i k}^{\prime}$ | Number of demands $k$ serviced by facility $i$ |
| $f_{i}^{t}$ | Convex combination of activity-rate $f_{i}$, where the weights are the normalized accessibilities into zone $i$ |
| $f_{j}^{(t)}$ | Speed-of-adjustment function for the $j$ th zone and $l$ th activity |
| $f_{r}^{m n}$ | $r$ th-stop demand between origin-destination $m-n$ |
| $f(x)=d f d x$ | Derivative of function $f$ over variable $x$ |
| f | Partial-flow pattern in the decomposed RISE algorithm |
| $F$ | Set of candidate or new facilities to be sited, or an objective functional |
| $F(f(x))=F\left(u^{\prime}\right)$ | Fourier transform of function $f(x)$ in frequency $u^{\prime}$ |
| $F^{\prime}(\mathbf{z})$ | Production function with input rates $\mathbf{z}=(\leftarrow z \rightarrow)^{T}$ |
| $F^{\prime}(\cdot)$ | Regional-growth-rate function |
| $F_{k}$ | Fibonacci numbers; also the weighted activity-rate, with work-accessibilities from zone $k$ as the weights |
| $F_{X}$ | Derivative of function $F$ with respect to variable $X$ |
| $\dot{F}=\nabla F$ | Gradient of the function $F$ being maximized |
| $F_{i}^{\prime}$ | Unsatisfied demand or remaining service-capacity at each demand-node $i$ to entertain additional vehicle-deliveries |
| $F_{i j}$ | Accessibility factor between locations $i$ and $j$, expressed as an inverse function of travel cost |
| $F_{i k}$ | Probability that a demand from $i$ is of type $k$ |
| $\mathbf{F}=\left[F_{i j}\right]$ | Square matrix of population-distribution rate by zone, measuring $n^{\prime} \times n^{\prime}$ |
| $\mathbf{F}^{\prime}(\mathbf{x})=\left(\vdash F_{i}(\mathbf{x}) \rightarrow\right)$ | A vector of functions whose interactions $\partial F_{i}^{\prime}(\mathbf{x}) \partial x_{j} \neq \partial F_{j}^{\prime}(\mathbf{x}) \partial x_{i}$ are asymmetric, where $\mathbf{x}=\left(\leftarrow x_{i} \rightarrow\right)^{T}$ for $i=1, \ldots, n$ |
| $g$ | A scale factor; when serialized against argument $m$ for example, $g(m)$, it is used to adjust zonal population from one loop of the Lowry model $m$ to another $m+1$, where $m=1,2, \ldots$ |
| $g(\cdot)$ | A special function of • , such as the state equation; the relocation-cost function in stochastic facility-location; or the expected-master-travelling-salesman-tour length in probabilistic travelling-salesman-problem |
| $g_{k}$ | Generalized unit-cost at facility $k$ or for vehicle $k$ |
| $g_{i}^{\prime \prime}$ | Load to be picked up at node/vertex $i$ |
| $g_{i}^{\prime \prime}$ | Spatial 'drift' of activities toward location $i$, in accordance with a profit/benefit motive or some gravitational potential-function |
| $g_{i j}$ | Short-hand notation for nonwork accessibility between $i$ and $j$ |
| g | Vector of coefficients associated with the discrete-variables $\mathbf{y}$; when used as a function, it is the subgradient |
| $\mathbf{g}(j)=\left(\leftarrow g_{h(j)}^{\rightarrow}\right)^{T}$ | Vector of input measures for a decision-making unit $j$ |
| G | Number of salespersons in a travelling-salesman problem, or the number of vehicle-tours out of a depot |
| $G^{\prime}$ | Maximum fleet-size available at a depot; or share of the population which are immigrants |
| $G(\cdot)$ | Multiple-travelling-salesmen expected-tour-length-function involving $k$ salespersons |
| $G(\xi)$ | Generating function for the probability distribution $P_{0}, P_{1}, P_{2}, \ldots, P_{n}$ where $\xi$ takes on values of $0,1,2, \ldots, n$ |
| $G(\xi, t)$ | Generating function for the probability distribution $P\left(\mathbf{X}_{0}^{*}, \mathbf{X}^{*}, t\right)$; where $\mathbf{X}_{0}^{*}$ is the initialcondition vector, $\mathbf{X}^{*}=\left[X_{1}^{*}(t), X_{2}^{*}(t), \ldots, X_{\mathrm{n}}^{*}(t)\right]^{T}$, and where the $n$-dimensional-vector $\xi$ takes |

on values of $\xi^{\mathbf{X}^{*}} \equiv\left(\xi_{1}^{Z_{1}^{*}}, \xi_{2}^{\xi_{2}^{*}}, \ldots, \xi_{\xi_{n}^{*}}^{Z_{1}}\right)^{T}$, for $\left|\xi_{j}\right|<1$. Thus for the stationary, irreducible Markovprocess, it assumes the form $P\left(X_{0}^{*}\right)+\xi_{1}^{x_{1}^{*}} P\left[X_{1}^{*}\right]+\xi_{2}^{*_{2}^{*}} P\left[X_{2}^{*}\right]+\ldots+\xi_{n}^{*} P\left[X_{n}^{*}\right]$
$G_{i j}$
$G_{i}^{\prime \prime}(p, s)$
$G_{i}^{\prime}(p, s)$
$G_{i( }^{*}\left(h^{\prime \prime}\right)$
$G_{i=}^{* *}\left(h^{\prime \prime}\right)$
$G^{i j}$
$G_{i j}$
$G_{i j}^{p q}$

Class or group $i$; also a generalized spatial-statistic for point $i$
Generalized savings-measure from including demand node $i$ between demand points $p$ and $s$ in a location-routing heuristic
Modified generalized-savings-measure from including node $i$ between points $p$ and $s$, after considering different depot-based tours
Net change in cost from displacing demand $i$ from tour $h$ to $h^{\prime \prime}$
Net change in cost from displacing demand $i$ from tour $h$ to $h^{\prime \prime}$ considering different fleets Transaction of goods and services between the $i$ th and $j$ th industrial sectors
General location-pair spatial-statistic
Monetary transaction between the $q$ th industrial sector in zone $j$ and the $p$ th economicsector in zone $i$ in an input-output model; with shorthand notation being $G_{j}^{p q}$ for consumption and $G_{i j}^{q}$ for production respectively, considering only the nonzero elements
$\mathbf{G}=\left[G_{i j}\right] \quad$ The growth matrix showing the growth springing off from group/location $i$ to group/ location $j$ (within a period of time); also a basic-feasible-solution to a simplex-on-a-graph
G(•)
$\mathbf{G}^{\prime}=\left[g_{h j}\right]$
$\zeta_{j}^{(j)}(\cdot)$
$h$
$h^{\prime}$
$h^{\prime}(\cdot)$
Vector return-function in a recursive program
Input matrix containing the $h$ th input for decision-making-unit $j$
Economic surplus- or deficit-function at zone $j$ of the $l$ th type
Index for a variable; generally to show a fleet type, a category of inputs (costs) in dataenvelopment analysis, or the iteration number in a recursive program
Minimum fleet size
State-transition function in dynamic programming
$h^{\prime \prime} \quad$ Calibration parameter in a dynamic version of a spatial-location model; an example is the time-scale parameter to convert activity to a rate-of-change
Height of a subregion $k$
A rate- or calibration-constant in a deterministic compartmental-model; for example, the interaction between regions $i$ and $j$ in a multiple-region predictor-prey equation-set, or a short-hand notation for work-accessibility
$\mathbf{h}(j)=\left(\leftarrow h_{k(j)} \rightarrow\right)^{T} \quad$ Vector of output-measures for target decision-making-unit $j$.
Housing expenditure for a household (part of locational expenditure)
The Hamiltonian function in terms of the state equation, the costate or adjoint variable, and the figure-of-merit at the present; it also stands for a general function
$H^{\prime} \quad$ An upper limit of discrete index $h$
$H^{\prime}(\cdot)$
Regional growth-rate function
Set of vehicles in a fleet, or the set of vehicle types in the fleet
Cardinality of set $H^{\prime \prime}$, or the number of members in the set; here it is the fleet size
Transaction of goods and services to the $i$ th household-sector
Set of potential tours in which demand $i$ can be included
Cost of one dispatch on route $p$
Imports to region $r$
Hazard a node $i$ is exposed to
Hazard a link $(i, j)$ is exposed to
Flow-rate function from compartment $i$ to compartment $j$
Monetary transaction between the household sector in zone $i$ and the $p$ th economic-sector in zone $j$ in an input-output model

## Elasticity of demand

$100(1-\alpha / 2)$ percentile of the standard normal-distribution
A parameter in general; for example, it can show decline in demand per unit-of-spatialseparation; $\theta_{i}$ is the rate-of-decline (or diffusion rate) of inflows into $i$
Coefficient of the $t$ th term in a moving-average time-series
The backshift operation of a moving-average model
Proportion of activities (or trips) from origin-location $i$ that end up in destination-location $j$ based strictly on accessibility alone
$\Theta_{i j} \quad$ A short-hand notation for the spatial-interaction term, indicating the proportion of activities (or trips) from origin-location $i$ that end up in destination $j$-based on both accessibility and the attractiveness at the destination; i.e., the normalized accessibility-function between $i$ and $j$
$\boldsymbol{\Theta}_{k}=\left[\theta_{i j k}\right] \quad$ A $k$ th-order spatial-matrix of moving-average coefficients
$\boldsymbol{\Theta}(B)=\left[\theta_{i j}(B)\right]$

| A-8 | Appendix - Symbols |
| :---: | :---: |
| $i, j$ | Indices for nodes/vertices; $i$ normally stands for a demand node and $j$ a facility node; or they can just be any counter |
| $i(k)$ | Beginning node of arc $k$ |
| $j(k)$ | Terminating node of arc $k$ |
| i | Cartesian coordinates of a demand $i$ |
| I | Set of nodes/vertices in a network |
| $I(d)$ | The spatial-statistic Moran's-I for a particular spatial-order as defined by the distanceparameter $d$ |
| \|I/ | Cardinality of set $I$, or the number of members in the set |
| $I_{k}$ | Profit or income for facility $k$ |
| $I_{N}$ | Set of unlabelled nodes |
| $I_{D}$ | Dual objective-function in recursive program |
| $I^{\prime}$ | Household or aggregate income |
| $I_{t}^{\prime}$ | Aggregate income at time $t$ |
| $I_{h}^{\prime}$ | Set of potential demands for exchange, with an existing demand on the tour $h$ |
| $I^{\prime \prime}$ | Subset of potential demand nodes within the set $I$, where demands are non-zero |
| $I_{p_{k}}$ | Any subset of nodes in the $k$ th-stop route $p_{k}$ |
| $I(i)$ | Set of nodes/vertices which are input markets |
| $I(0)$ | Set of nodes/vertices which are output markets |
| $I(t)$ | $0-1$ indicator-sequence reflecting the absence and presence of an intervention, overlaying the transfer-function on top of the time-series |
| $I_{\text {iк }}$ | A binary variable assuming unity if the combination of facilities $\kappa$ provides a satisfactory service to demand $i$ |
| $I_{R x}$ | Total expected-mutual-information between the facility pattern in the region $R$ and the demand spatial-pattern (when $x=I$ ), or between the facility pattern and an individual demand (when $x=i_{k}$ ); i.e., how probable the facility pattern is consistent with what is known about the demand pattern $I$ or individual demand $i_{k}$ |
| $I[\mathbf{X}(k), \Gamma(k)]$ | $k t$ th-stage payoff or objective-function of a recursive program, defined in terms of decisionvariables $\mathbf{X}$ and constraint parameters $\mathbf{\Gamma}$ |
| $I(\mathbf{P} ; \mathbf{Q})$ | Information that allows updating a prior probability-distribution $\mathbf{Q}$ to probability $\mathbf{P}$ |
| ${ }^{7}{ }^{s}(\cdot)$ | Net-benefit function in a decomposed recursive-program |
| I | Identity matrix |
| $j^{*}(k)$ | Optimal facility location in state $k$ |
| j | Cartesian coordinates of a facility $j$ |
| $J$ | Subset of nodes/vertices in a network, generally the candidate sites for facility location |
| $J_{q}$ | Set of candidate production sites |
| $\|J\|$ | Cardinality of set $J$, or the number of members in the set |
| $J^{\prime}$ | A particular control-point in the bifurcation set |
| $J^{\prime \prime}$ | The double values that the state variable assumes, corresponding to the control variable $J^{\prime}$ in the bifurcation set |
| $J(i)$ | Set of Voronoi polygons adjacent to the $i$ th polygon |
| $\mathbf{J}_{k}$ | Basis $k$ of a multicriteria linear-program |
| $k$ | Index to show category $k$ (e.g., $Z^{k}$ is the k th activity); it marks a node, the commodity, the tree in a forest, or just serves as a counter |
| $k(\cdot)$ | Equation for the control variable over time, expressed in terms of the state, the costate or adjoint variables |
| $k_{i}$ | Calibration or scaling constant for zone $i$ in a doubly-constrained gravity model; the Moran's-I or General Spatial statistic; alternatively, it is the propensity to save (invest) |
| k | row vector consisting of $0,+1,-1$ entries marking an orthonormal base of the transitionrate space |
| K | A discrete or continuous constant, or the upper limit of running index $k$ |
| $K(t)$ | Capital-stock investment over time |
| $K_{i}, K_{i}^{\prime}$ | Trip-production and -attraction rate at zone $i$ respectively |
| $\bar{K}_{j}^{p}$ | A scaling constant; it ensures that the inter-sectorial and inter-zonal flows sum up to the non-labor input to the input-output table for sector- $p$ and zone- $j$ |
| $\dot{K}_{r}$ | Instantaneous rate-of-capital-accumulation in region $r$ |
| к | Combination of three or more facilities that perform a certain function |
| $\kappa^{\prime}$ | The complement of the set к |
| $\kappa^{h}$ | Cost of operating vehicle $h$ |


| $\kappa_{i}^{h}$ | Marginal cost of serving demand-node $i$ |
| :---: | :---: |
| K | Combination of three or more facilities |
| $l(T)$ | Total cost of spanning-tree $T$, which is sum of the arc costs |
| $l^{\prime}$ | Discount rate (e.g., on the number of commuting trips, or traditionally in the time stream of cost or benefits) |
| $l^{i}$ | Lower bound of a specified time window for a salesman or vehicle to visit node/vertex $i$ |
| $l_{j}$ | Calibration constant for zone $j$ in a doubly-constrained gravity model |
| $l$ | Spatial order of the $k$ th autoregressive-term in a spatial time-series |
| $l^{h^{\prime \prime}}$ | Ordered set of neighboring points ( $p, s$ ) representing candidate tour $h^{\prime \prime}$ |
| $l_{h^{\prime \prime \prime}}$ | Ordered set of neighboring points ( $p, s$ ) in tour $h^{\prime \prime}$ after removing demand $i^{\prime}$ |
| $l^{m n}(r)$ | Length of an $r$-stop route originating in $m$ and terminating in $n$ |
| ${ }^{r^{5}(\mathbf{x}}$ ) | Route-length vector at stage $r$ and in state $s$ of a decomposed recursive-program, expressed as a function of the decision variable $\mathbf{x}$ |
| $L$ | Nonempty subset of demand nodes/vertices, where a demand instance may be characterized by having actual demands realized in a node subset $L$ of the network nodes/vertices $I$; the symbol also denotes twice the boundary length of a district |
| $\|L\|$ | Cardinality of set $L$, or the number of members in the set |
| $\tilde{L}$ | Length of the perimeter of a subarea |
| $\bar{L}$ | The length of a queue, including the entity being served |
| $L(\cdot)$ | Lagrangian or maximum-likelihood function |
| $L^{\prime}$ | Probability that the location visited is the termination point for the trip |
| $L^{\prime \prime}$ | A calibration constant in a bivariate predictor-prey difference-equation-set |
| $L_{q}$ | Queue length (excluding the one being served) |
| $L_{r}$ | Regional labor-input-factor |
| $L^{(t)} x_{i}$ | Spatial-lag operator on the value of spatial unit $i$, where $l$ refers to the $l$ th contiguity-class such as the $l$ th-order neighbors; alternatively, we can write $L^{(l)} x_{i}$ as a matrix operation to compute the weighted sum of the neighboring values of $i$ contained in vector $\mathbf{x}$, or $\left(\mathbf{w}^{(l)}\right)^{T} \mathbf{x}$. In general, $L^{(l)}(\cdot)$ stands for spatial-lag operator of the $l$ th-order, with the 0 th-order operator reproducing the observation itself, or $L^{(0)}(\cdot)=$. |
| $L_{T}(\cdot)$ | Length of a master travelling-salesman-tour, constructed out of the set of nodes/vertices - |
| $L_{i j}$ | Error (in terms of a "loss measure") when a Bayesian classifier mis-assigns a multi-attribute observation $\mathbf{x}=\left(x_{1}, x_{2}, \ldots\right)^{T}$ to group $j$ when it actually belongs to group $i$; usually $L_{i j}=0$ if there is no error and $L_{i j}=1$ if there is a misclassification |
| $L_{j}(\mathbf{x})$ | Average misclassification error (in terms of a "loss measure") when assigning multi-attribute observation $\mathbf{x}=\left(x_{1}, x_{2}, \ldots\right)^{T}$ to group $j$; a couple of computational transformations of this measure are $L_{j}^{\prime}(\mathbf{x})$ and $L_{j}^{\prime \prime}(\mathbf{x})$ |
| $\mathbf{L}=\left[\mathbf{x}_{I}\left(q_{1}^{\prime}\right), \mathbf{x}_{I}\left(\underline{q}_{2}^{\prime}\right),\left.\ldots\right\|^{\top}\right.$ | Matrix containing the left eigenvectors $\mathbf{x}_{L}$ |
| $\lambda$ | Dual variable or Lagrange multiplier, with a specific (not necessarily feasible) solution $\bar{\lambda}$ and the optimal solution $\lambda^{*}$ |
| $\lambda_{i}^{\prime}$ | A normalized weight, where $\Sigma_{\Sigma_{i}} \lambda_{l}^{\prime}=1$ unless noted otherwise |
| $\lambda^{\prime \prime}$ | Arrival rate for a queuing process |
| $\lambda^{k}=\left(\leftarrow \lambda_{i}^{k} \rightarrow\right)^{T}$ | The $k t$ th solution-vector in a Lagrange-relaxation procedure |
| $\lambda^{* *}$ | Dual optimal-solution to the linear-program subproblem at the last iteration within Benders' decomposition |
| $\Lambda\left(\mathbf{J}_{k}\right)$ | The weight cone for multicriteria linear-program, showing the $\lambda^{\prime}$-weight combinations that characterize a particular solution $\mathbf{J}_{k}$ among the nondominated set of solutions |
| $m, n$ | Indices for dimension or for a node/vertex |
| $m^{\prime}$ | A calibration constant in a bivariate predictor-prey difference-equation-set |
| $m^{*}$ | A critical bifurcation-value in a bivariate predictor-prey difference-equation-set |
| $m^{1}$ | A collection of entities of characteristic 1; e.g., the number of complicated constraints in a Lagrangian-relaxation problem |
| $m^{j}$ | A collection of entities of characteristic $j$; e.g., the number of high-frequency direction finders in a bundle located at station $j$ |
| $m_{k}$ | Spatial-order of the $k$ th moving-average term in a spatial time-series |
| $m_{r}$ | Vehicle-fleet requirement at depot $r$, or the number of deployed vehicles at depot $r$ |
| $m_{i}^{\prime}, m_{i}^{\prime \prime}$ | In- and out-movement rate to and from subarea $i$ |
| $m(k)$ | Median for a median-filter using a $k \times k$ mask |
| $\begin{aligned} & m_{1}, m_{2}, \ldots, m_{k^{\prime}} \\ & m^{\prime}(a) \end{aligned}$ | Groups of demand nodes to be served by route $1,2, \ldots, k^{\prime}$, with $m_{1}+m_{2}+\ldots+m_{k^{\prime}} \leq\|I\|$ Maximum shortest-distance from point $q$ |


| $m_{j i}^{\prime}$ | Binary variable that is "switched on" when demand $i$ is allocated to facility $j$ in a combined data-envelopment-analysis/location model; also the benefit valuation for such $i-j$ pair |
| :---: | :---: |
| M | Area specification for a districting model |
| $M_{i}$ | Maximum inventory carried at node $i$ |
| $M_{\text {max }}$ | Maximum number of nodes in a vehicle route |
| $\tilde{M}, \tilde{M}^{\prime}$ | A couple of matchings in a spanning-tree/perfect-matching heuristic for the travelling-salesman-problem |
| $M(t)$ | Absolute smoothed-error (used in conjunction with relative smoothed-error) for adaptive-response-rate exponential-smoothing over time |
| $M(\Xi)$ | Maximum of the weighted distances from the center candidates to each of the demands in the candidate facility-locations $\Xi$ |
| $M^{\prime}$ | Non-locational expenditure such as food, clothing, education, savings etc. |
| $M^{\prime \prime}$ | A very large number or weight |
| $M_{i j}$ | Minor of a square matrix |
| $M(W, p)$ | Simulation results of a facility-location model where $p$ facilities are relocated to respond to a maximum load of $W$ |
| $\mathbf{M}=\left[m_{i j}\right]$ | Migration matrix showing the migration rate between locations $i$ and $j$ |
| $\mu$ | Mean of a probability distribution |
| $\mu^{\prime}$ | Service rate of a queuing process; also the number of intermediate stops in the longest vehicle-route |
| $\mu_{j}$ | Positive weights placed upon an extreme direction $\mathbf{d}^{j}$ in a linear program |
| $\mu_{i}, \mu_{i}$ | Mean of observations in group $i$ in both scalar and vector form |
| $\mu^{(j)}$ | Scaling constant of the error $\epsilon$ associated the value v being measured, resulting in $\nu^{(j)}+\mu^{(j)} \epsilon^{(j)}$ |
| $v$ | A collection of integer numbers |
| $v_{i}$ | Route shape parameter (serialized by $i$ ) used in location-routing heuristics, assuming values such as 1 or 2 |
| $v_{t}$ | Noise series in a transfer-unction multivariate time-series |
| $\mu^{p}$ | Dual variable associated with the control total of areawide-transportation-cost constraint in an entropy-maximization model |
| $\Xi$ | Collection of candidate facility-locations |
| $\Xi(X)$ | Collection of all candidate facility-locations in the decision space $X$ |
| $\Xi(\mathbf{y})$ | Collection of candidate facility-locations which are open (i.e., for those locations where $y \neq$ ) |
| $\Xi(z)$ | Collection of candidate facility-locations in the $Z$ space, whose distance bounds are within $z$ units |
| $\xi$ | As used in the Minkowski's distance-function, it is the proportion by which factor inputs have to be reduced to reach the efficient point on the production frontier |
| $n^{\prime}$ | The number of units in a spatial entity (e.g., the number of zones in a region, the number of subareas in a study area, or the total number of pixels in an image) |
| $n_{s}$ | Number of sides in a subareal polygon (e.g., in a Dirichlet tesselation) |
| $n(a, b)$ | Number of stops between origin-terminal $a$ and destination-terminal $b$ |
| $N$ | Population or number of households (e.g., $N_{i}$ is the population at location $i$ ) |
| $N_{j}$ | Number of pattern vectors from class $G_{j}$, or the number of nodes or pixel vectors belonging to class $j$ |
| $N^{\prime}$ (large) | A large number |
| $\bar{N}$ | Total working population in the study area |
| $N^{p}$ | Population working in economic-sector $p$ |
| $N_{j}^{c}$ | Capacity for residential development in zone $j$ |
| $N_{i}^{\prime}$ | Set of spatial units (including facilities) within a distance $S$ from demand $i$ |
| $N_{i j}$ | Binary decision-variables in a districting model, serving as a 'pointer' across a district boundary separating a geographic sub-unit $i$ and one that is not $j$; it is unitarilly value if subunit $j$ is acquired and $i$ is not |
| N | Row vector of zonal population $N_{i}$ |
| $\mathbf{N}(k)$ | The nonbasic column associated with variable $k$ in a linear-programming tableau |
| $o_{i}$ | Export share of region $i$ |
| $O\left(l^{k}\right)$ | Worst-case $k$ th-polynomial computational-complexity for input-data-length $l$ |
| $O_{i}$ | Export from the $i$ th region |
| $O^{\prime}(P)=\left\{-O_{i}^{\prime}(P) \rightarrow\right\}$ | Orientation sequence of a path $P$, consisting of +1 and -1 entries, depending on the orientation of the arc in the path sequence |
| $O^{i}$ | Export from the $i$ th industrial sector, measured in dollars |
| $O_{j}^{i}$ | Export from the $i$ th industrial sector in subarea $j$, measured in dollars |


| $\mathbf{O}=\left(0 \leftarrow O^{i} \rightarrow\right)^{T}$ | Export vector in an aspatial input-output model, showing the convention that the first sector (the household sector) has no exports |
| :---: | :---: |
| $\mathbf{O}=\left(0 \leftarrow O_{j}^{i} \rightarrow\right)^{T}$ | Export vector in a spatial input-output model, where $i$ is the economic sector and $j$ is the subarea |
| $p$ | An integer number for the number of facilities, the number of services provided, the index for the $p$ th vehicle route, the parameter in the $l_{p}$-metric, or the differencing parameter in a time-series |
| $p^{\prime}$ | Number of facilities in a subset of the $p$ facilities (i.e., $p^{\prime} \leq p$ ) |
| $p_{f}$ | Price of fuel |
| $p_{g}$ | Price of the good |
| $p_{k}$ | Price of a commodity $k$, with $\mathbf{p}$ standing for a vector of commodity prices |
| $p_{i}^{\prime}$ | Probability of adopting strategy $i$ in a two-person game |
| $p^{(j)}(\cdot)$ | Probability function of choosing alternative $j, j=1, \ldots, n$ |
| $p_{i k}$ | Empirical probability that demand $k$ patronizes facility $i$; or the probability of transitioning from state $i$ to state $k$ |
| $\hat{p}_{i k}$ | Estimated value of $p_{i k}$ |
| $p_{i}$. | Empirical probability that a demand patronizes facility $i$ |
| $p_{\cdot k}$ | Empirical probability that a demand $k$ is being served |
| $p_{j}^{\prime q}$ | $q$ th factor-of-production input-prices at subarea $j$ |
| $p_{k}^{\prime}$ | Number of facilities of the $k$ th type (as used in a multi-product facility-location formulation) |
| $\bar{p}(t)$ | Capacity expansion at time $t$ |
| $p^{\prime \prime}$ | Price of composite consumption-good |
| $p_{i j k}$ | Conditional probability that event-type $i$ occurs at geographic-region $j$ at time-of-day $k$ |
| $\hat{p}_{i j k}$ | Prediction of $p_{i j k}$ based both on the hypothesized intervention model and historical data |
| $\breve{p}_{i j k}$ | Analytical prediction of the relative probabilities $p_{i j k}$, for field implementation as a transfer function |
| $\tilde{p}_{l \mid k}$ | Relative probabilities after intervention probabilities have been implemented, using the transfer function $\breve{p}_{i j k}$ |
| $\vec{P}_{l j}$ | Deseasonalized relative-probabilities after intervention probabilities have been implemented |
| $\mathbf{p}=\left(\leftarrow p^{(j)} \rightarrow\right)$ | Perron vector whose components are positive and sum to unity |
| $\mathbf{p}_{\mathbf{i}}(t)=\left(-p_{j}(t) \rightarrow\right)^{T}$ | Vector of transitioning probabilities from state $i$ to state $j$ (where $j=1, \ldots, n$ ) |
| $\dot{\mathbf{p}}_{\mathbf{i}}(t)=\left(-\dot{p}_{i j}(t) \rightarrow\right)^{T}$ | Time-derivative vector of probabilities transitioning transitioning from state $i$ to state $j$ (where $j=1, \ldots, n$ ) |
| $P$ | A path; also a set of vehicle routes generated for a network |
| $P^{\prime}$ | Potential surface for destination choice, whose derivative $d P^{\prime} / d C_{i j}$ is often operationalized by the trip-distribution function |
| $P_{D}$ | Dual space of the linear-programming relaxation problem |
| $P(p)$ | Probability that $p$ servers are occupied (busy) |
| $P(\cdot)$ | Probability of an event - |
| $P_{i}$ | Nearest location for demand or customer $i$; also the probability that the system is in state i |
| $P_{i}(t)$ | Probability that the system is in state $i$ at time $t$ |
| $P_{k}^{\prime}, P_{(k)}$ | Steady-state probability of being in state $k$ |
| $P_{i d^{\prime \prime}}$ | Steady-state probability that decision $d^{\prime \prime}$ is reached while in state $i$ |
| $P_{i j}$ | Binary decision-variables in a districting model, serving as a 'pointer' across a district boundary separating a geographic sub-unit $i$ from one that is not $j$; it is unitarilly value if $i$ is acquired and $j$ is not |
| $P_{i j k}$ | Joint probability of event-type $i$ occurring in area $j$ at time $k$, given that an event-type $i$ occurred at time $k$ |
| $\breve{P}_{y j}$ | Analytical predictions of $p_{i j k}$ aggregated monthly, based on the hypothesized intervention-model |
| $P_{k}^{m n}$ | Set of vehicle routes covering origin-destination pair m-n via $k$-stop itineraries |
| $P_{c}^{n n}$ | Set of vehicle routes covering origin-destination pair $m-n$ via connect itineraries |
| $\bar{P}$ | Scale of a facility as represented by its capacity, capital outlay etc. |
| $\tilde{P}_{l} \bar{P}_{l}$ | Lower and upper bound of the supply at location $l$ |
| $\bar{P}^{\prime}$ | Aggregate production-function with capital as input |
| $\mathbb{P}(\bullet)$ | Logical predicate over the argument - |
| $P_{j}(p)$ | Steady-state saturation-probability of all $p$ service-units (in stochastic facility-location) |


| $\mathbf{P}=\left(\leftarrow P_{i} \rightarrow\right.$ ) or $\left(\leftarrow V_{i j} \rightarrow\right.$ ) | form; also be the updated travel-vector between $i$ and $j, V_{11}, V_{12}, \ldots, V_{i j}, \ldots, V_{\|I\|\|J\|}$, measuring $\|I\| \cdot\|J\|$ long |
| :---: | :---: |
| $\mathbf{P}(t)=\left(\leftarrow P_{i}(t) \rightarrow\right) \quad \mathrm{Ve}$ | Vector of the state probabilities $P_{i}(t)$; also the square matrix of transition probabilities over time |
|  | Time-derivative vector of state probabilities $P_{i}(t)$ |
| $\mathbf{P}^{\prime}=\left[\mathbf{x}_{1}, \ldots, \mathbf{x}_{n}\right]$ | Matrix containing independent eigenvectors $\mathbf{x}\left(q_{j}^{\prime}\right), j=1, \ldots, n$. <br> Variance-covariance matrix for the difference between the observed and estimated Kalmanfilter time-series-vector (or the estimation-error vector) |
| $\mathbf{P}_{t}$ |  |
| $\pi_{i}$ | Dual variable in a network; such as the shadow price at node $i$, or a real number showing the amount of load carried on board a vehicle at node/vertex $i$ |
| $\pi^{(j)}$ | Probability that an individual reviews his/her choice of the $j$ th compartment in a compartmental model |
| $\pi_{i j}(\cdot)$ | Probability a given individual moves from compartment $i$ to compartment $j$-as a function of, say, the state variable and time |
| $\pi i_{i}$ | Dual variable associated with the $i$ th column of the spanning-tree $(j=1)$ or non-spanningtree $(j=2)$ part of the basis (in a network-with-side-constraint tableau) |
| $\pi(\cdot)$ | Permutation operator on the argument - <br> The probability of transitioning from state $i$ to state $j$ during one period of the Markov process, given a decision $d^{\prime \prime}$ has been made |
| $\pi\left(j \mid i, d^{\prime \prime}\right)$ |  |
| $\pi$ | $n$-dimensional transition-rate space |
| ${ }^{\prime} \boldsymbol{\pi}^{s}(\mathbf{x}, \mathbf{y}$ | Vector gross-return-function of decisions $\mathbf{x}$ and $\mathbf{y}$ (in a decomposition implementation of recursive-program) |
| ■ ( $\cdot$ ) | Vector of gross return-functions of decisions in a recursive program |
| $\boldsymbol{\Pi}_{0}(t)=\left(\leftarrow \pi_{i 0}(t) \rightarrow\right)^{T}$ | Vector of transition rates with the "outside world" over time |
| $\Pi=\left[\pi_{k}\right]$ | Transition-probability matrix in a Markov chain or compartmental model, with each entry denoting the given probability of transitioning from state $k$ to state $l$; also the matrix of transition rates from state $k$ to state $l$ |
| пII | Matrix of transition rates from state $k$ to state $l$, considering both arrival and service in a queue |
| $q$ | Index to show a node number, center number, median number, number of substations, or the number of attributes, criteria, endogenous variables, eigenvalues, or differencing parameter in a time series |
| $q_{k}$ | Candidate location for a center $k$ |
| $q_{i k}$ | Probability that an event-type $i$ occurs at time $k$ <br> Eigenvalue, with $q_{\max }^{\prime}$ as the principal eigenvalue; also the growth rate of an area (with $q_{j}^{\prime}$ being the subareal growth-rate) |
| $q^{\prime}$ |  |
| $q_{i}^{\prime}$ | Probability that strategy $i$ is followed (in a two-person game); also the $i$ th eigenvalue Inventory-cost functions at demand-node $i$; or simply the unit cost-of-time (a constant) from demand-origin $i$ |
| $q_{i}(\cdot)$ |  |
| $q_{j}$ | Mean queuing delay <br> Total economic-activity in the study area, such as consumption in dollars or number of trips executed |
| $Q$ |  |
| $Q_{i}$ | Ratio of two accessibility definitions from location $i$ |
| $\hat{Q}_{1}, \bar{Q}_{1}$ | Lower and upper bounds for the demand at location $l$ |
| $Q^{\prime}$ | Total number of servers, or number of suppliers |
| $\tilde{Q}^{\prime}$ | Set of discrete points in the feasible region of an integer program |
| $Q^{\prime \prime}$ | Cost per rejected demand in a loss-system location-model |
| $\overline{\mathrm{Q}}=$ | A matrix of economic-base multiplier over a time-increment $\Delta t$ Prior-probability distribution for locating in each of the $n^{\prime}$ subareas (written in a vector form); or the vector of prior-travel between $i$ and $j, Q_{i j}$ |
| $\mathbf{Q}=\left(\stackrel{Q_{i} \rightarrow}{ }\right.$ ) |  |
| $\mathbf{Q}_{t}$ | Variance-covariance matrix of the white-noise vector $\boldsymbol{\alpha}_{t}$ |
| $\mathbf{Q}^{\prime}$ | The $\mathbf{X}^{T} \mathbf{X}$ data-matrix in the nonlinear regression of a STARMA model; where $\mathbf{X}$ is not explicitly given, and has to be numerically estimated |
| Q"= | Matrix with eigenvalues $q_{1}, q_{2}, \ldots$ along its diagonal |
|  | Rent or mortgage, as part of locational expenditure (e.g., $r^{i}$ is the rent for a unit of land $i$ at a distance $d_{i}$ from market, and $\mathbf{r}$ is the vector of rents among these land units) |
| $r_{0}$ | Pearson correlation-coefficient |
|  | Satisficing-level of criterion $k$; |



| $\rho_{j}$ | Utilization-rate of a service-unit $j$ in stochastic facility-location; also the import rate of region $j$ |
| :---: | :---: |
| $\rho^{p}$ | Productivity-in-the-pth-economic-sector per unit-of-labor |
| $\rho_{i j}$ | Trade coefficient between regions $i$ and $j$ |
| $\rho^{p q}$ | Technical coefficients showing the transactions between the $p$ th and $q$ th economic-sectors in an input-output model |
| $\rho_{j}^{p q}$ | Technical coefficient at the receiving-sector zone-j |
| $p_{i j}^{p q}$ | Technical coefficients showing the transactions between the $p$ th economic-sector in zone $i$ and the $q$ th economic-sector in zone $j$ in an input-output model |
| $\rho$ | Matrix of technical or input-output coefficients [ $\rho^{p q}$ ], trade coefficients [ $\rho_{i j}$ ], or combined spatial-technical coefficients [ $\rho_{i j}^{p}$ ] |
| $\hat{\boldsymbol{\rho}}$ | Diagonal matrix of trade coefficients, $\left[\rho_{i i}\right]$ |
| $\boldsymbol{\rho}^{\boldsymbol{j}}=\left[\rho_{h}^{j}\right]$ | A matrix of economic-multipliers for the $j$ th economic-sector, disaggregated by each zone-h |
| $\rho_{S}, \rho_{T}$ | The consumption and production multi-sectorial components of the input/output-coefficient-matrix $\rho$, derived from row- and column-sum normalization of transaction flows respectively, with $\rho_{s} \rho_{T}=\rho$; the spatial, multi-subareal version assumes $G_{j}^{p q} \mathcal{G}_{j}^{q q}=\rho_{j}^{p q}$ and $G_{i j}^{q} \mathcal{F}_{j}^{q}$ $=\rho_{i j}^{q}$ |
| $\hat{p}_{X Y}$ | Population cross-correlation between random-variables $X$ and $Y$ |
| $\hat{\rho}^{2}$ | Relative size of the variance; ( $1-\hat{\rho}^{2}$ ) is the variance reduction |
| $s$ | Source of a network |
| $s_{p}$ | Autoregressive season-length in a seasonal time-series |
| $s_{q}$ | Moving-average season-length in a seasonal time-series |
| $\underline{s}$ | Prescribed frequency-of-visit at a node/vertex |
| $s_{X}$ | Standard deviation of the random-variable $X$ |
| $s(j)$ | Sum of vertex(node)-arc(link) distances for facility $j$ (the smallest sum identifies the general median) |
| $s^{\prime}(j)$ | Sum of point-arc distances for facility $j$ (the smallest sum identifies the general absolute median) |
| $s^{2}$ | Sample variance, with $s$ being the standard deviation |
| $s_{i j}$ | Length of the border separating geographic sub-unit $i$ from sub-unit $j$; also surviving ratio of cohort-group $j$ from cohort-group $i$ |
| $s^{\prime}$ | Average size of a site; or the ratio between the demand potentials at sites $i$ and $j$ |
| $s^{\prime \prime}$ | Slack node/vertex in a network |
| $S$ | A set of alternatives (e.g., the set of solutions that satisfies a predetermined goal or standard, the branch-and-bound search-space in a linear-programming relaxation etc.) |
| $S(\bullet)$ | Sum-of-squares surface constructed out of the parameters • in nonlinear regression |
| $S^{\prime}$ | Consumers' surplus (or net benefit) to a tripmaker in making a trip; alternatively it refers to a predetermined maximum-service-distance in discrete facility-location |
| $S^{\prime \prime}$ | Another set of alternatives (for example, the set after introducing a new alternative) |
| $S_{k}$ | Set of demand vertices or nodes that would be covered by a center at $q_{k}$ |
| $S_{i}\left(p^{\prime}, q^{\prime}\right)$ | The increase (or savings) via a triangular-inequality-style inclusion (or exclusion) of demand $i$ between the adjacent points ( $p^{\prime}, q^{\prime}$ ) |
| $S_{t}\left(l_{n^{\prime \prime}} / i^{\prime}\right)$ | Increase in travel-distance from serving demand $i$ via tour $h^{\prime \prime}$ (after the former-demand $i^{\prime}$ has been removed) |
| $S^{i}$ | Marginal-cost function for path $i$ |
| $S_{l}\left(V_{l}^{S}\right)$ | Supply function showing price against flow quantity, in other words price charged at supply-quantity $V_{l}^{S}$ |
| $S_{k j}^{\prime}$ | Unit benefit of assigning the $k$ th activity (or activity from zone $k$ ) to zone $j$ |
| $S_{i k}$ | The $k$ th site-specific attribute of the $j$ th facility (such as the acreage of a state park) |
| $S^{k, l}$ | Marginal-cost function between origin $k$ and destination $l$ |
| $\mathbf{S}_{i j}$ | Vector of level-of-service variables between locations $i$ and $j$, including such variables as travel time and travel cost |
| $\sigma$ | Standard deviation of a probability distribution |
| $\sigma^{2}$ | Variance of a probability distribution (see also the sample-variance $s^{2}$ ) |
| $\sigma^{\prime}$ | Vendor score or simply a constant in a model |
| $\sigma_{i}$ | Real number showing the 'odometer' reading of a vehicle at node/vertex $i$ |
| $\sigma_{\text {\% }}^{2}$ | 'Tilting' effect, as measured in terms of the variance, on the regression line (due to the randomness of the regression coefficients) |


| $\sigma_{M}^{2}$ | 'Tilting' effect, as measured in terms of the variance, on the regression line-when an additional data-point $x$ ' is added to the regression |
| :---: | :---: |
| $\sigma_{Y}^{2}$ | Total regression-based prediction- or estimation-error, as expressed in terms of the variance of the predicted- or estimated-values $Y$ |
| $\sigma_{Y}^{2}$ | Total regression-based prediction-error, as expressed in terms of the variance of the predicted values $Y^{\prime}$ |
| $\sigma_{M^{*}}^{2}$ | Variance of a normally-distributed set of residuals, around the sample regression-line at $X$ $=x^{*}$ |
| $0_{i j}{ }^{p}$ | Calibration coefficient such as the subareal investment-coefficient or the marginal capital-output-ratio, quantifying the multiplier effect of investment among economic sectors and between subareas |
| $\sigma_{j}^{2}$ | Variance (or second moment) of service-time at service-unit $j$ |
| $\boldsymbol{\sigma}^{h}=\left(\leftarrow \sigma_{i}^{h} \rightarrow\right)$ | Vector of dual-variables corresponding to the $i$ th constraints defining the $h$ th travelling-salesman-polytope |
| $\boldsymbol{\sigma}=\left[a_{j}\right]$ | Zonal population-serving-ratios along the diagonal of an $n^{\prime} \times n^{\prime}$ matrix |
| $\boldsymbol{\Sigma}=\left[\operatorname{cov}\left(\epsilon_{i} \epsilon_{j}\right)\right]$ | Error covariance-matrix |
| $t$ | Time dimension or simply a counter for a series of data (e.g., $N(t)$ is the population at time $t, \Delta t$ is a time increment) |
| $t^{\prime}$ | Subareal share of transportation-accessibility-to-employment |
| $t_{b}$ | Student- $t$ statistic for calibration-parameter $b$ |
| $t_{\alpha / 2, n-2}$ | $t$-statistic at $100(1-\alpha) \%$ confidence-level and $n-2$ degrees-of-freedom |
| $t_{N}$ | Sink node/vertex of a network |
| $t^{\prime \prime}$ | Technical-attribute score |
| $t^{k}$ | Step size in iteration- $k$ of a hill-climbing optimization-algorithm |
| $t_{0}$ | Dwell time at a terminal |
| $t_{j}$ | Delivery- or dwell-time at node $j$ by salesman or vehicle $h$ |
| $t_{i j}$ | Normalized work-accessibility-function between $i$ and $j$ |
| ${ }_{t}{ }^{s}(\mathbf{x}, \mathbf{\Phi}, \mathbf{V})$ | Cost of providing service at state $s$ and stage $r$ of a recursive-program |
| $t$ | Random service-time on-scene $\tilde{t}_{t}$ or off-scene $\tilde{t}_{j}$ |
| $t$ | Expected value of on-scene service-times to all demands $i$ |
| $\bar{t}$ | Ratio between intra-nodal distances at $i$ and $j$ |
| $\bar{t}_{j}$ | Average service-time for a service-unit stationed at depot $j$, consisting of on-scene servicetime at the demand $t_{i}^{1}$ and the off-scene service-time at the depot $t_{j}^{2}$ |
| $\mathbf{t}=\left[t_{i j}\right]$ | Matrix of normalized work-accessibilities, measuring $n^{\prime} \times n^{\prime}$ |
| $\mathbf{t}^{k}=\left[\tau_{i j}^{k}\right]$ | Matrix of travel-times between $i$ and $j$ |
| $\tau$ | Time duration (e.g., $\tau_{i j}$ or $\tau(i, j)$ is the travel time from location $i$ to $j$ ) |
| $\tau^{\prime}$ | Calibration constant in a dynamicized input-output model |
| $\tau_{k}$ | A user-defined scalar in the subgradient optimization routine ranged (say) between 0 and 2 |
| $\tilde{\boldsymbol{\tau}}$ | Random variable for service-time in a queuing process; $t_{j \mid i}$ is the random service-time for demand $i$ from depot $j$ |
| $\overline{\tau_{j}}$ | Expected one-way travel-time to a random demand from depot $j$ |
| $\bar{\tau}_{j}^{\prime}(k)$ | Expected travel-time from $j$ to all demands in state $k$ |
| $T$ | Transportation cost as part of locational expenditure; also quantifies other technological factors |
| $T$. or $T(\cdot)$ | A-priori travelling-salesman-tour as a function of - |
| $T^{\prime}$ | Minimum spanning-tree of a graph |
| $T^{\prime \prime}$ | Multi-graph, derived from the minimum spanning-tree by duplicating every arc of the graph; also an instance of the travelling-salesman problem |
| $T_{N}$ | Alternate sink-node/vertex in a network for excess flows |
| $T_{j}$ | Number-of-neighbors surrounding geographic sub-unit $j$ |
| $T_{i}^{\prime}$ | Proportion of sales from subject location to demand at $i$ |
| $T_{i}^{\prime \prime}$ | Electrical-flow capacity of a substation $i$ |
| $T_{i j}$ | Number of $i$ th-group neighbors for a $j$ th-group geographic sub-unit |
| $\hat{T}_{i j}$ | Current estimate on random-variable $T_{i j}$ |
| T | Diagonal matrix of zonal activities such as population |
| T( $\cdot$ ) | Vector of cost-functions in a recursive-program |
| $\boldsymbol{T}_{B}$ | Basis for a simplex-on-a-graph, represented graphically as a tree |




| $\begin{aligned} & { }^{r} \mathbf{w}^{s}(\boldsymbol{\phi}, \mathbf{V}) \\ & \mathbf{w}=\left(-w_{i} \rightarrow\right)^{T} \end{aligned}$ |
| :---: |
| $\mathbf{w}^{(l)}=\left(\leftarrow \mathrm{W}_{i j}^{(l)} \rightarrow\right)^{T}$ |
| W |
| $W_{i}$ |
| $W_{q}$ |
| $W_{T}$ |
| $W(t)$ |
| $W_{i}^{\prime}$ |
| $W_{i j}$ |
| $\bar{W}_{i}^{p}$ |
| $W_{i}^{h}$ |
| $W_{i}^{p}$ |
| $\mathbf{W}=\left[w_{i j}\right]$ |
| $\mathbf{W}^{\prime}=\left[W_{g h}\right]$ |
| $\mathbf{W}^{\prime \prime}=\left[w_{j}\right]$ |
|  |
| $\mathbf{W}^{(t)}=\left[w_{i j}{ }^{(\nu)}\right]$ |
| $\mathbf{W}_{t}$ |
| $\left(\mathbf{W}^{\boldsymbol{\theta}} \mathbf{y}\right)_{\boldsymbol{y}_{t}}$ |
| $x^{*}$ |
| $x^{\prime}$ |
| $x_{t}^{\prime}$ |
| $x_{0}, x_{0}^{\prime}, x_{0}^{\prime \prime}, \ldots$ |
| $x_{i j}$ |
| $\chi^{p}$ |
| $\tilde{x}_{i}^{p}$ |
| $x_{i j k}$ |
| $x^{n n}$ |
| $x^{m, n}\left(C^{m, n}\right)$ |
| $x_{p}^{m n}$ |
| $x_{j i}^{q p}$ |
| $x_{\text {mip }}^{m p}$ |
| $\mathbf{x}=\left(-x_{j} \rightarrow\right)^{T}$ |
| $\mathbf{x}_{q}$ |
| $\begin{aligned} & \mathbf{x}_{t}^{q}=\left(x_{1}, x_{2}, \ldots\right)^{T} \\ & \mathbf{x}_{t}^{\prime}=\left(x_{1}^{\prime}, x_{2}^{\prime}, \ldots\right)^{T} \end{aligned}$ |
| $\mathbf{x}_{L}$ |
| $\mathbf{x}_{R}$ |

${ }^{r} \mathbf{w}^{s}(\boldsymbol{\phi}, \mathbf{V})$
$\mathbf{w}=\left(\leftarrow w_{i} \rightarrow\right)^{T}$
$\mathbf{w}^{(l)}=\left(\leftarrow \mathrm{W}_{i j}^{(l)} \rightarrow\right)^{T}$
$\mathbf{W}_{t}$
$\left(\mathbf{W}^{\oplus} \mathbf{y}_{\boldsymbol{y}_{t}}\right.$
$x^{*}$
$x^{\prime}$
$x^{\prime}$
$x_{0}, x_{0}^{\prime}, x_{0}^{\prime \prime}, \ldots$
$x_{i j}$
$x^{i}$
$x_{i}^{p}$
$\tilde{x}_{i}^{p}$
$x_{i j k}$
$x^{n n}$
$x^{m, n}\left(C^{m, n}\right)$
$x_{p}^{m n}$
$x_{j i}^{q p}$
$x_{m i p}^{m n}$
$\mathbf{x}=\left(\leftarrow x_{j} \rightarrow\right)^{T}$
$\mathbf{x}_{q}$
$\mathbf{x}_{t}=\left(x_{1}, x_{2}, \ldots\right)^{T}$
$\mathbf{x}_{t}^{\prime}=\left(x_{1}^{\prime}, x_{2}^{\prime}, \ldots\right)^{T}$
$\mathbf{x}_{R}$

Vector of route-frequencies at stage $r$ and state $s$ of a decomposed recursive-program Eigenvector consisting of $q$ entries-this is equivalent to $\mathbf{v}_{i}$ and $\mathbf{x}_{i}$; also the cost vector in a network-flow program
The vector of spatial-weights associated with the lth contiguity-class; an example is the weights associated with the $l$ th-order neighbors-notice this is equivalent to the spatial operator $L^{(l)}(\bullet)$
White-collar employment; also work load or demand placed on a service-unit
Size of demand or activity at $i$, which is proxy for development opportunity at the zone; $\mathbf{W}{ }^{\prime}$ is the vector of development-opportunities among all zones
Delay time in queue
Total time in system, including delay time in queue and the time being served
Rate of investment in new capacity over time
Revised size of demand or activity at $i$
Service-effectiveness weight expressed as a function of the separation between demand $i$ and facility $j$; i.e., the further apart $i$ and $j$ are, the less effective it is for service to be rendered
Observed value of attractiveness or the opportunity of zone- $i$ as a location for industry- $p$ Observed zonal-residence attractiveness or opportunity
Observed zonal-shopping attractiveness or opportunity
A $q \times q$ pairwise-comparison weight-matrix used in the analytic hierarchy process; also denotes the weight matrix in spatial econometric-models, measuring $n \times n$
An $n^{\prime} \times n^{\prime}$ activity derivation-and-allocation matrix of Lowry-Garin model, with each entry denoting a zone pair $g-h$
The diagonal matrix consisting of per-capita value-added productivity (wage rate)
Activity derivation-allocation, transition or spatial-weight matrix for the $j$ th activity in a Lowry-Garin model
Spatial weight-matrix for the $l$ th-contiguity class; with the normalized spatial-weights sum to unity $\sum_{j} w_{i j}^{(l)}=1$, and $\mathbf{W}^{(0)}=\left[w_{i j}^{(0)}\right]=\mathbf{I}$, or the 0 th -order neighbors being the subject entry itself.
Gain matrix in Kalman filter, representing the net percentage of measurement-error or noise that is left after filtering
Preprocessing of data $\mathbf{y}$ by removing the subject $i$ th-entry, and then replace it with a value resulting from 'filtering' with a spatial-'mask' $\mathbf{W}^{(t)}$ of order $l$
Sample observation or the optimal value of $x$
A particular observation for the random-variable $X$
Actual, accurate data in a Kalman-filter time-series (to be differentiated from what is observable)
Decision boundary between pattern groups 1 and 2, 2 and 3, 3 and 4, etc.
Allocation of demand $i$ to facility $j$; or flow from $i$ to $j$
Flow on path $i$ in a network
Equilibrium economic-activity at each subarea $i$ and sector $p$
Projected sales of product $p$ in subarea $i$
Allocation of demand $i$ to facility $j$ in state $k$
Lost calls between origin-destination pair $m-n$
Demand-for-transportation between origin-destination pair $m-n$ as a function of the transportation cost between them
Binary link-allocation of demand between origin-destination pair $m-n$ to non-stop route or itinerary $p$
Input of commodity- $q$ from subarea- $j$ in the production of commodity- $p$ in subarea- $i$
Binary allocation of demand $m-n$ on route $p$ as indicated by the usage of segment $(m, i)$ in the itinerary
Vector of decision-variables, or empirical readings (such as change-in-accessibility for all the activities $j$ )
An interior point in the feasible-region $\Omega_{q}$
Observed readings in a time-series
Actual readings over time in a Kalman-filter time-series
The left eigenvector of a square matrix
The right eigenvector of a square matrix

| $\mathbf{x}^{i}$ | The $i$ th discrete-point proposal in a branch-and-bound tree, corresponding to a constraint |
| :--- | :--- |
|  | in the Lagrangian-dual linear-program |
| $\mathbf{x}^{k}$ | The $k$ th basic-solution in a linear-program, or the $k$ th set of decision-variables (e.g., |


| $\mathbf{y}^{j}=\left(\leftarrow y_{i}^{j} \rightarrow\right)^{T}$ | A vector of criterion-measures for alternative $j$, or the $j$ th group of $y_{i}$-variables (e.g., the delivery commitment of vehicle $j$ toward demand $i$ ) |
| :---: | :---: |
| Y | The decision-variable $Y$, or random-variable notation of the explanatory or dependent variable in ordinary-least-squares regression; also the regional income |
| $\bar{Y}$ | Mean of the random-variable $Y$ |
| $Y^{\prime}$ | Outcome or criterion space of multi-criteria decision-making; also the prediction randomvariable in regression |
| $Y^{\prime \prime}$ | The combinatorial space of the discrete-variables $y_{i}$ |
| $Y_{i j}$ | A spatial-variable defined by the coordinates $i$ and $j$-a variable that is related to its neighbors in both axes of this coordinate system; this cross-product is the covariance between the observations at $i$ and $j$ |
| $\mathbf{Y}=\left(\leftarrow y_{i} \rightarrow\right)^{T}$ | Explanatory- or dependent-variable vector in ordinary-least-squares regression, consisting of $n$ observations; $\hat{Y}$ denotes the estimated-values of random-variable $Y$ |
| $\mathbf{Y}^{i j}(k)=\left[-y_{l}^{i j} \rightarrow\right]$ | Binary parameters of each constraint-function in recursive programming ( $p^{\prime}$ in total), where $i$ is the state-index and $j$ the stage-index; $\mathbf{Y}(k)=\left[\begin{array}{c}1 \\ \mathbf{Y}^{\mathbf{c}, k+r} \\ 1\end{array}\right]$ |
| $\mathbf{Y}(\cdot)$ | State-connectivity linkage-function of past decisions and available vehicle-capacity in a recursive-program |
| $\mathbf{Y}^{\prime}$ | Labor-force-value-added output-vector |
| $z$ | Objective-function of an optimization-problem; also used to denote the activity-generation rate |
| $z^{\prime}$ | A bound on $z$ |
| $z(j)$ | Objective-function value of the $j$ th alternative |
| $z_{c}$ | Largest demand-facility assignment-distance |
| $z_{i}$ | Amount of product or services sold at demand-point $i$; or a transformed observation from the raw-data $Z_{i}$ |
| $z_{t}$ | Stationary time-series with zero mean |
| $z_{\text {IP }}$ | An integer-programming objective-function that is to be estimated by Lagrangianrelaxation |
| $z_{t}$ | Stationary time-series with non-zero mean; also the estimated-value in an adaptive timeseries |
| $z_{j}^{\prime}$ | Binary variable to denote the location of a facility at $j ; z_{j}$ is used after $y_{j}$ when there is more than one type of facility to be located |
| $z_{0}^{j}$ | Amount-of-output produced at supply-facility or plant $j$ |
| $z_{0, i}^{j}$ | Amount-of-output produced at plant $j$ and sent to output-market $i$ |
| $z_{j}^{\prime}$ | The optimal benefit of opening facility- $j$ in a generalized $p$-median-problem (as defined in a subproblem of Lagrangian-relaxation solution) |
| $z_{i j}$ | 'Trunk' traffic from supply-source $i$ to distribution-center $j$ |
| $z_{i}^{j}$ | Amount of input-i used by plant- $j$ |
| $z^{e_{i}}$ | Employment by the $e_{i}$ th-type industry |
| $z_{i}^{e}$ | Number-of-households in zone-i employed by industry |
| $z_{i j}{ }^{e_{1}}$ | Supply-of-labor by household in zone- $i$ to zone-j for employment by the $e_{i}$ th-type industry |
| $z_{L}^{i}$ | Lower-bound of objective-function-value at iteration-i |
| $z_{U}^{i}$ | Upper-bound of objective-function-value at iteration-i |
| $z^{\prime}$ | Lower or upper bound of objective-function-value |
| $z_{i j}$ | Binary indicator-variable to show whether a multiattribute observation $\mathbf{x}=\left(x_{1}, x_{2}, \ldots\right)^{T}$ for a pixel of color $j$ has been properly classified into group $i ; z_{i j}=1$ when it is properly classified into group $i$ (or $i=j$ ) and $z_{i j}=0$ when it is improperly classified $(i \neq j$ ). In vector notation for two groups $i$ and $j$, we write $\mathbf{z}_{\mathrm{i}}=\left(z_{i i}, z_{i j}\right)^{T}=(1,0)^{T}$; and the random variable corresponding to $\mathbf{z}_{i}=\left(z_{i i}, z_{i j}\right)^{T}$ is $\tilde{\mathbf{z}}_{i}=\left(\tilde{z}_{\mathbb{\pi}} \tilde{z}_{i j}\right)^{T}$. |
| $z_{i j}{ }^{\prime}$ | Impedance between zones $i$ and $j$ |
| $z_{\text {LD }}$ | Objective-function-value of a Lagrangian-dual |
| $z_{\text {LP }}$ | Objective-function-value for a linear-program relaxation |
| $z_{\text {LR }}$ | Objective-function-value for a Lagrangian-relaxation problem |
| $\grave{z}_{i}$ | Goods in storage at location-i |
| Z | vector of $\mathbf{Z}$ values induced for stationary and with mean set to zero; also stands for endogenous variables in an econometric model |


| $\mathbf{z}_{j}$ | Vector-of-pixels $\mathbf{z}$ for group $j$ in a Bayesian classifier |
| :---: | :---: |
| $Z$ | Activity level (where the activity can be population, employment, grey values, or any economic or non-economic activity) |
| Z(i) | Expected-value of the decision made at state-i |
| $Z^{\prime}(i)$ | Expected-value of the improved-decision made at state- $i$ according to Howard's policyiteration |
| $Z_{j}$ | Objective-function value or activity level at location-j |
| $Z_{t}$ | Raw-data time-series before inducing stationarity |
| $Z_{t}^{\prime}$ | Actual, accurate daa in a Kalman-filter time series (to be differentiated from what is observable) |
| $\dot{Z}_{t}, \ddot{Z}_{t}$ | First and second differencing of time-series $Z_{t}$ |
| $Z^{\prime}$ | Preference structure in multi-criteria decision-making |
| Z" | Deviation-measures from the efficient-contour of unity in the Minkowski distance-function |
| $Z_{i j}$ | Value of spatial-data at grid-point $i-j$; often simplified to read $Z_{j}$ to stand for the spatialdata value at location- $j$ |
| $Z_{j}^{l}$ | Value of the $j$ th spatial-data at spatial-lag $l$ |
| $Z_{+}^{n}$ | $n$-dimensional Euclidean-space of positive discrete-values |
| $\mathbf{Z}=\left(\vdash Z_{i} \rightarrow\right.$ ) | Vector of exogenous-variables $Z_{i}$ of such activities as population and employment in each zone- $i ; \mathbf{Z}_{0}$ is the initial-values of $\mathbf{Z}$ |
| $\mathbf{Z}(t)$ | Density or relative-frequency of the state-vector $\mathbf{X}(t)$; in other words, the normalized statevector |
| $\mathbf{Z}_{j i}=\left(-Z_{j i} \rightarrow\right.$ ) | The $j$ th-activity assigned to zone- $i$ |
| $\mathbf{Z}^{i}$ | Vector of the total-population/employment activity-levels at time-period (iteration) $i$, with $\mathbf{Z}^{0}$ as the given final-period basic-activities (from which other activities are generated) |

