Slide 1 / 255

Slide 2 / 255

NEW JERSEY CENTER
FOR TEACHING & LEARNING

Geometry

Circles

2015-10-23

www.njctl.org

Slide 3 / 255

Table of Contents

Click on a topic to go to that section

Slide 4 / 255

Throughout this unit, the Standards for Mathematical Practice are used.

MP1: Making sense of problems & persevere in solving them.
MP2: Reason abstractly & quantitatively.
MP3: Construct viable arguments and critique the reasoning of others.
MP4: Model with mathematics.
MP5: Use appropriate tools strategically.
MP6: Attend to precision.
MP7: Look for & make use of structure.
MP8: Look for & express regularity in repeated reasoning.

Additional questions are included on the slides using the "Math Practice" Pull-tabs (e.g. a blank one is shown to the right on this slide) with a reference to the standards used.

If questions already exist on a slide, then the specific MPs that the questions address are listed in the Pull-tab.

Slide 4 (Answer) / 255

Throughout this unit, the St
are used.

MP1: Making sense of
MP2: Reason abstract
MP3: Construct viable
others.
MP4: Model with ma
MP5: Use appropriate
MP6: Attend to preci
MP7: Look for & make
MP8: Look for & expre

Math Practice

Additional questions a
Practice" Pull-tabs (e.g.
this slide) with a reference

If questions already exist on a slide, then the specific MPs that the questions address are listed in the Pull-tab.

Slide 5 / 255

Parts of a Circle

Return to the table of contents

Slide 6 / 255

Circles are a type of Figure

A figure lies in a plane and is contained by a boundary.

Euclid defined figures in this way:

Definition 13: *A boundary is that which is an extremity of anything.*

Definition 14: *A figure is that which is contained by any boundary or boundaries.*

A boundary divides a plane into those parts that are within the boundary and those parts that are outside it. That which is within the boundary is the "figure."

Slide 7 / 255

Circles

Euclid defined a circle and its center in this way:

Definition 15: *A circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure equal one another.*

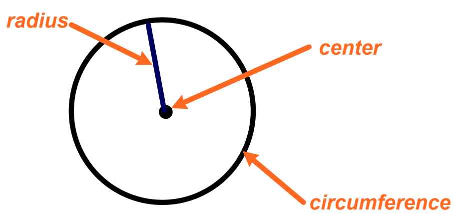
Definition 16: *And the point is called the center of the circle.*

This states that all the radii (plural of radius) drawn from the center of a circle are of equal length, which is a very important aspect of circles and their radii.

Slide 8 / 255

Circles and Their Parts

Another way of saying this is that a circle is made up of all the points that are an equal distance from the center of the circle.



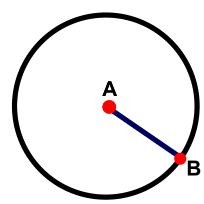
Slide 9 / 255

Circles and Their Parts

The symbol for a circle is ⊙ and is named by a capital letter placed by the center of the circle.

The below circle is named: *Circle* A or ⊙A

$\overline{\quad}$ is a *radius* of ⊙A



A *radius* (plural, *radii*) is a line segment drawn from the center of the circle to any point on the circle.

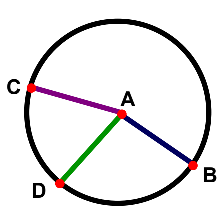
Slide 10 / 255

Radii

A *radius* (plural, *radii*) is a line segment drawn from the center of the circle to any point on its circumference.

It follows from the definition of a circle that all the radii of a circle are congruent since they must all have equal length.

An unlimited number of radii can be drawn in a circle.



That all radii of a circle are congruent will be important to solving problems.

In this drawing, we know that line segments AC, AD and AB are all congruent.

Slide 11 / 255

Diameters

Definition 17: *A diameter of the circle is any straight line drawn through the center and terminated in both directions by the circumference of the circle, and such a straight line also bisects the circle.*

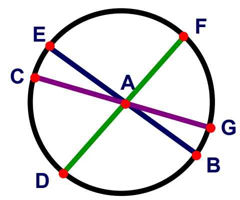
Since the diameter passes through the center of the circle and extends to the circumference on either side, it is twice the length of a radius of that circle.

Slide 12 / 255

Diameters

There are an unlimited number of diameters which can be drawn within a circle.

They are all the same length, so they are all congruent.



That all diameters of a circle are congruent will be important to solving problems.

In this drawing, we know that line segments BE, CG and DF are all congruent.

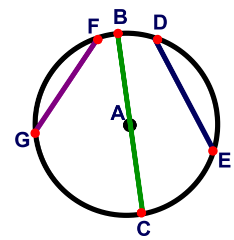
Slide 13 / 255

Chords

A *chord* is a line segment whose endpoints lie on the circumference of the circle.

So, a diameter is a special case of a chord.

Why is a radius not a chord?



All the line segments in this drawing are chords.

Slide 13 (Answer) / 255

Chords

A chor... e on the circur...

So, a...

Wh...

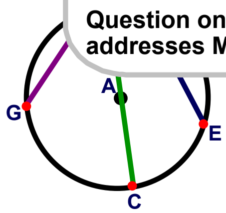
Answer

A radius starts at the center and has one endpoint on the circle.

A chord has 2 endpoints on the circle, w/ the center not being one of the endpoints.

Question on this slide addresses MP7.

...ments in this drawing

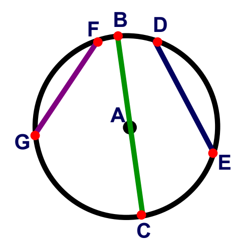


Slide 14 / 255

Chords

There are an unlimited number of chords which can be drawn in a circle.

Chords are not necessarily the same length, so are not necessarily congruent.



Chords can be of any length up to a maximum.

What is the longest chord that can be drawn in a circle?

Slide 14 (Answer) / 255

Chords

There are an un... n be drawn in a ci...

Chords are n... ot necessarily c...

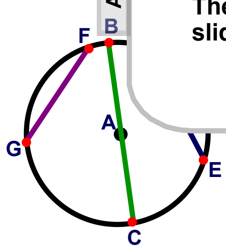
Answer

$\overline{BC}$, the diameter, is the longest chord.

The question on this slide addresses MP7.

...ngth up to a

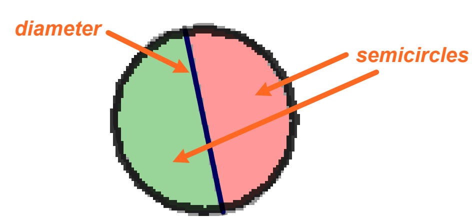
...chord that can be drawn in a circle?



Slide 15 / 255

Semicircles

Definition 18: *A semicircle is the figure contained by the diameter and the circumference cut off by it. And the center of the semicircle is the same as that of the circle.*



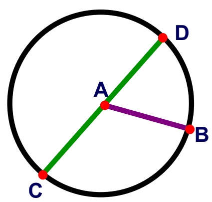
Slide 16 / 255

Diameters and Radii

The measure of the diameter, *d*, is twice the measure of the radius, *r*.

In this case, CD = 2 AB

In general, d = 2r or r = 1/2 d



Example: In the diagram to the left, AB = 5. Determine AC & DC.

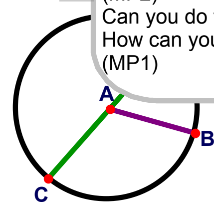
Slide 16 (Answer) / 255

Diameters and Radii

The measure of the diameter, *d*, is twice the measure of the radius, *r*.

In this case, CD = 2 AB

In general, d = 2r or r = 1/2 d



Example: In the diagram to the left, AB = 5. Determine AC & DC.

Answer

AC = 5
DC = 10

Additional Q's to address MP Standards:
What information are you given? (MP1)
What is the problem asking? (MP1)
What do you think the answers will be? (MP2)
Can you do this mentally? (MP1 & MP5)
How can you check your answers? (MP1)

Slide 17 / 255

1 A diameter of a circle is the longest chord of the circle.

○ True

○ False

Slide 17 (Answer) / 255

1 A diameter of a circle is the longest chord of the circle.

○ True

○ False

Answer

True

Slide 18 / 255

2 A radius of a circle is a chord of a circle.

○ True

○ False

Slide 18 (Answer) / 255

2 A radius of a circle is a chord of a circle.

○ True

○ False

Answer

False

Slide 19 / 255

3 The length of the diameter of a circle is equal to twice the length of its radius.

- True
- False

Slide 19 (Answer) / 255

3 The length of the diameter of a circle is equal to twice the length of its radius.

- True
- False

Answer

True

Slide 20 / 255

4 If the radius of a circle measures 3.8 meters, whatis the measure of the diameter?

Slide 20 (Answer) / 255

4 If the radius of a circle measures 3.8 meters, whatis the measure of the diameter?

Answer

7.6 m

Slide 21 / 255

5 How many diameters can be drawn in a circle?

- A 1
- B 2
- C 4
- D infinitely many

Slide 21 (Answer) / 255

5 How many diameters can be drawn in a circle?

- A 1
- B 2
- C 4
- D infinitely many

Answer

D

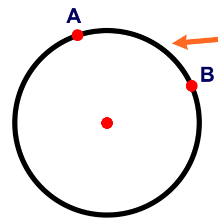
Slide 22 / 255

Central Angles & Arcs

Return to the table of contents

Slide 23 / 255

Arcs

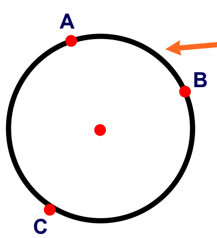


Arc AB or AB

Arc AB is the path between points A and B on the circumference of the circle.

Slide 24 / 255

Arcs



Arc AB or AB

Of course, you may have wondered why we went around the circle the way we did (in blue).

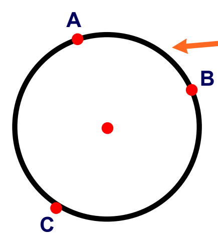
This shorter path (blue) is called the minor arc. We use 2 letters to denote minor arcs

The longer path (green) is called the major arc. If we want to refer to a major arc, we will add another point along that path and include it in the name.

For instance, we would name the green arc, Arc ACB to distinguish it from Arc AB.

Slide 25 / 255

Arcs

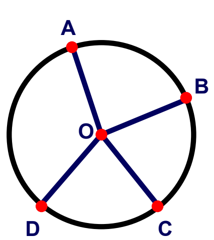


Arc AB or AB

Now let's discuss some ways we can use arcs, and measure them.

Slide 26 / 255

Central Angles



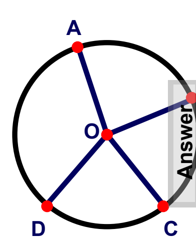
A central angle of a circle is any angle which has vertices consisting of the center of the circle and two points on the circumference.

How many different central angles can you find in this diagram?

Name them.

Slide 26 (Answer) / 255

Central Angles

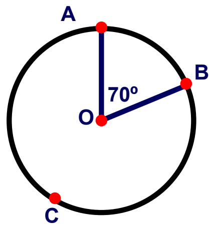


6 angles:
∠AOB
∠AOC
∠BOC
∠BOD
∠COD
∠AOD

Question on this slide addresses MP1.

Slide 27 / 255

Central Angles



The measure of a central angle is equal to the measure of the arc it intercepts.

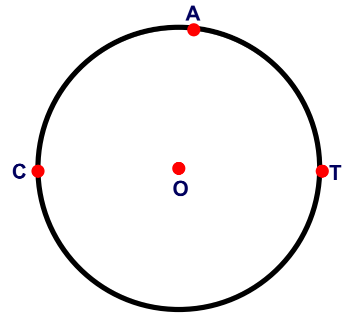
So, the measure of the angle AOB is the same as the measure of Arc AB, also denoted by mAB.

In this case, the mAB is 70°, since that is the m∠AOB.

That also means the mACB is 290°, since a full trip around the circle is 360°.

Slide 28 / 255

Adjacent Arcs

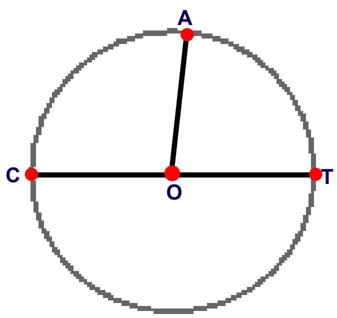


Adjacent arcs: two arcs of the same circle are adjacent if they have a common endpoint.

In this case, Arc CA and Arc AT are adjacent since they share the endpoint A.

Slide 29 / 255

Adjacent Arcs



From the Angle Addition Postulate we know that

m∠COT = m∠COA + m∠AOT

It follows then that the measures of adjacent arcs can be added to find the measure of the arc formed by the adjacent arcs.

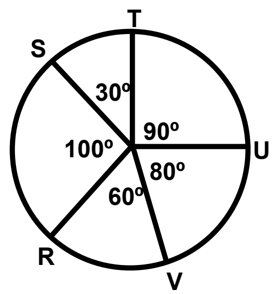
In this case that:

mCAT = mCA + mAT

which is the *Arc Addition Postulate*

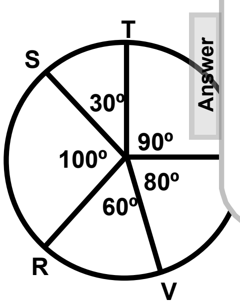
Slide 30 / 255

6 Find the measure of Arc RU.



Slide 30 (Answer) / 255

6 Find the measure of Arc RU.

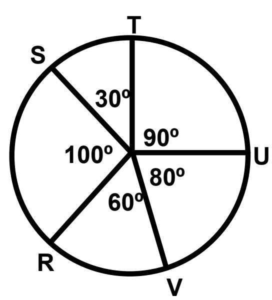


Answer

$m\overset{\frown}{RU} = m\overset{\frown}{RV} + m\overset{\frown}{VU}$
$= 60^0 + 80^0$
$= 140^0$

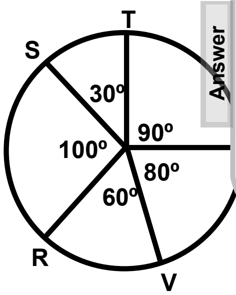
Slide 31 / 255

7 Find the measure of Arc RT.



Slide 31 (Answer) / 255

7 Find the measure of Arc RT.

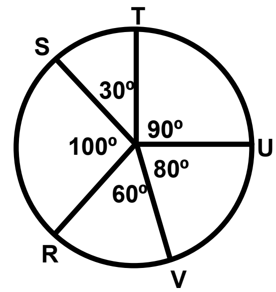


Answer

$m\overset{\frown}{RT} = m\overset{\frown}{RS} + m\overset{\frown}{ST}$
$= 100^0 + 30^0$
$= 130^0$

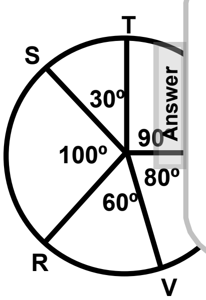
Slide 32 / 255

8 Find the measure of Arc RVT.



Slide 32 (Answer) / 255

8 Find the measure of Arc RVT.

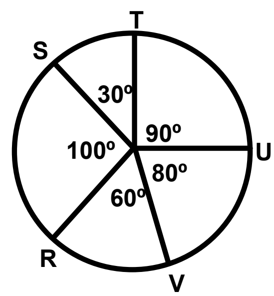


Answer

$m\overset{\frown}{RVT} = m\overset{\frown}{RV} + m\overset{\frown}{VU} + m\overset{\frown}{UT}$
$= 60^0 + 80^0 + 90$
$= 230^0$

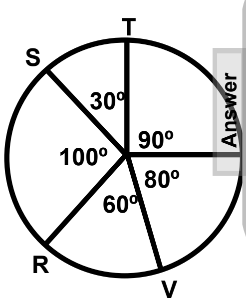
Slide 33 / 255

9 Find the measure of Arc UST.



Slide 33 (Answer) / 255

9 Find the measure of Arc UST.



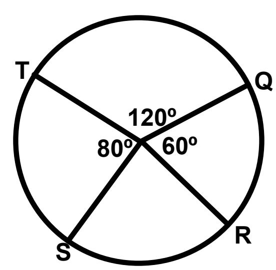
Answer

$m\overset{\frown}{UST} = 360 - m\overset{\frown}{TU}$
$= 360^0 - 90$
$= 270^0$

Slide 34 / 255

10 Which type of arc is Arc TQR?

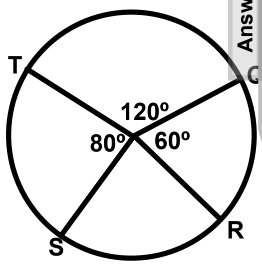
- A Minor Arc
- B Major Arc
- C Semicircle
- D None of these



Slide 34 (Answer) / 255

10 Which type of arc is Arc TQR?

- A Minor Arc
- B Major Arc
- C Semicircle
- D None of these



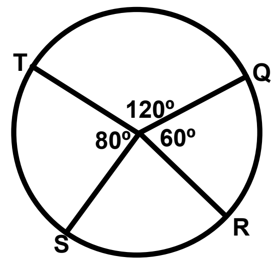
Answer

C

Slide 35 / 255

11 Which type of arc is Arc QRT?

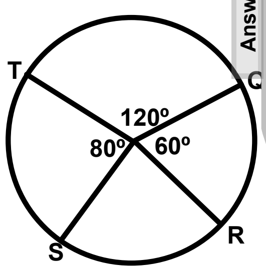
- A Minor Arc
- B Major Arc
- C Semicircle
- D None of these



Slide 35 (Answer) / 255

11 Which type of arc is Arc QRT?

- A Minor Arc
- B Major Arc
- C Semicircle
- D None of these



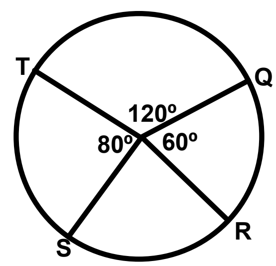
Answer

B

Slide 36 / 255

12 Which type of arc is Arc QS?

- ❑ Minor Arc
- ❑ Major Arc
- ❑ Semicircle
- ❑ None of these



Slide 36 (Answer) / 255

12 Which type of arc is Arc QS?

- ❑ Minor Arc
- ❑ Major Arc
- ❑ Semicircle
- ❑ None of these

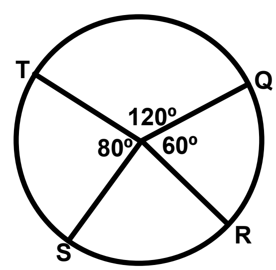
Answer

A

Slide 37 / 255

13 Which type of arc is Arc TS?

- ❑ Minor Arc
- ❑ Major Arc
- ❑ Semicircle
- ❑ None of these



Slide 37 (Answer) / 255

13 Which type of arc is Arc TS?

- ❑ Minor Arc
- ❑ Major Arc
- ❑ Semicircle
- ❑ None of these

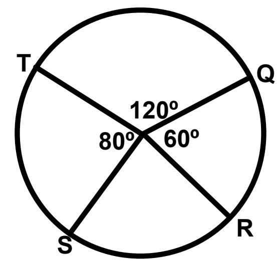
Answer

A

Slide 38 / 255

14 Which type of arc is Arc RST?

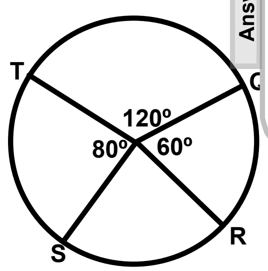
- ○ A Minor Arc
- ○ B Major Arc
- ○ C Semicircle
- ○ D None of these



Slide 38 (Answer) / 255

14 Which type of arc is Arc RST?

- ○ A Minor Arc
- ○ B Major Arc
- ○ C Semicircle
- ○ D None of these



Answer

C

Slide 39 / 255

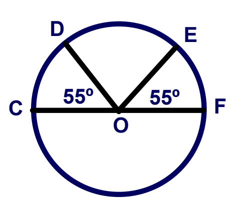
Congruent Circles and Arcs

- All circles are similar since they all have the identical shape
- Circles which have the same radius are congruent, since they will overlap at every point if their centers are lined up.

Slide 40 / 255

Congruent Circles and Arcs

- Arcs are similar if they have the same measure
- Arcs are congruent if they have the same measure and are either part of the same circle or of congruent circles.

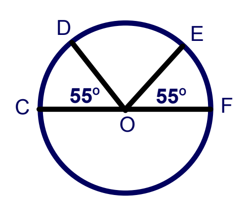


Arc CD and Arc EF are congruent because they are in the same circle and have the same measure.

Slide 41 / 255

Congruent Circles and Arcs

Since the measure of an arc is equal to that of the central angle which intercepts it, arcs within the same circle which are intercepted by central angles of the same measure, are congruent.

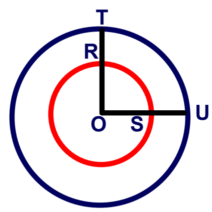


Arc CD and Arc EF are congruent because they are in the same circle and have the same measure.

Slide 42 / 255

Congruent Circles and Arcs

- Arcs are similar if they have the same measure
- Arcs are congruent if they have the same measure and are either part of the same circle or of congruent circles.



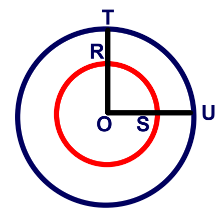
Arc RS and Arc TU are similar since they have the same measure.

But they are not congruent because they are arcs of circles that are not congruent.

Slide 43 / 255

Congruent Circles and Arcs

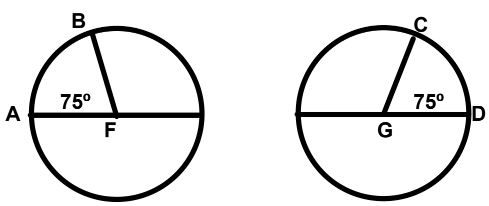
The two circles below are also called **concentric circles**, since they share the same center, but have different radii lengths. Concentric circles are similar, but not congruent.



Slide 44 / 255

15 Both Circle F and Circle G have a radius of 2.0 m. Are Arc AB and Arc CD similar?

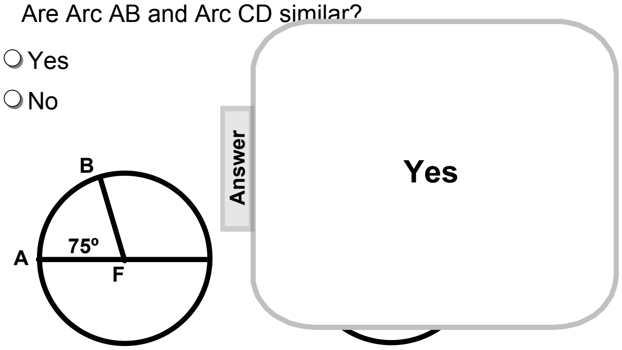
- ○ Yes
- ○ No



Slide 44 (Answer) / 255

15 Both Circle F and Circle G have a radius of 2.0 m. Are Arc AB and Arc CD similar?

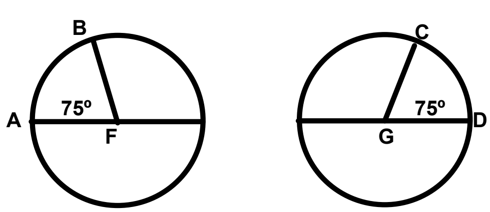
- ○ Yes
- ○ No



Slide 45 / 255

16 Both Circle F and Circle G have a radius of 2.0 m. Are Arc AB and Arc CD congruent?

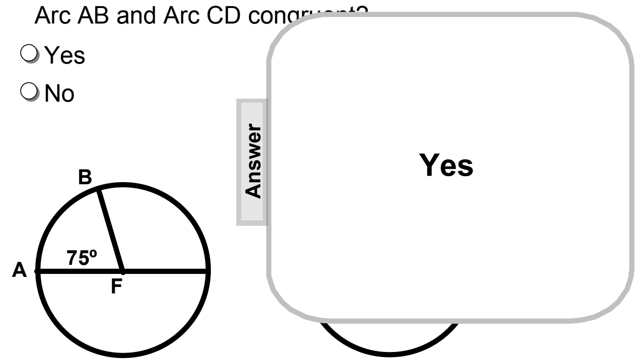
- ○ Yes
- ○ No



Slide 45 (Answer) / 255

16 Both Circle F and Circle G have a radius of 2.0 m. Are Arc AB and Arc CD congr…

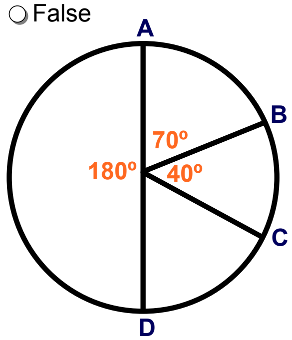
- ○ Yes
- ○ No



Slide 46 / 255

17

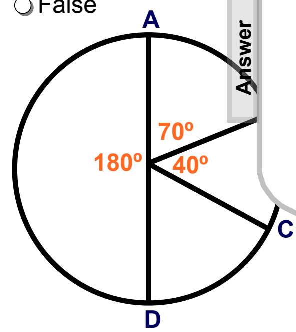
- True
- False



Slide 46 (Answer) / 255

17

- True
- False



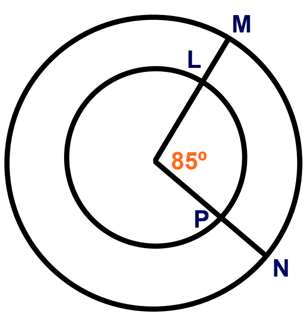
Answer

True

Slide 47 / 255

18

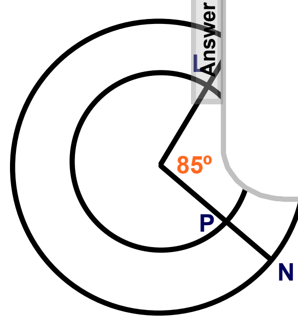
- True
- False



Slide 47 (Answer) / 255

18

- True
- False



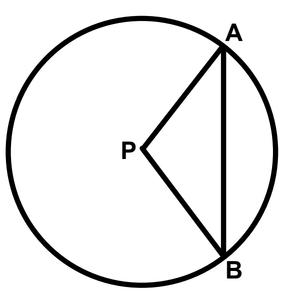
Answer

False

Slide 48 / 255

19 Circle P has a radius of 3 and AB has a measure of 90°. What is the length of $\overline{AB}$?

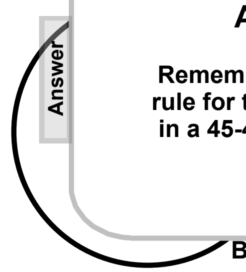
- A $3\sqrt{2}$
- B $3\sqrt{3}$
- C 6
- D 9



Slide 48 (Answer) / 255

19 Circle P has a radius of 3 and AB has a measure of 90°. What is the l

- A $3\sqrt{2}$
- B $3\sqrt{3}$
- C 6
- D 9



Answer

A

Remember the rule for the hyp. in a 45-45-90 Δ.

20 Two concentric circles always have congruent radii.

- True
- False

20 Two concentric circles always have congruent radii.

- True
- False

Answer

False

21 If two circles have the same center, they are congruent.

- True
- False

21 If two circles have the same center, they are congruent.

- True
- False

Answer

False

22 Tanny cuts a pie into 6 congruent pieces. What isthe measure of the central angle of each piece?

22 Tanny cuts a pie into 6 congruent pieces. What isthe measure of the central

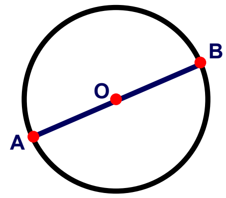
Answer

360 ÷ 6 = 60°

Arc Length & Radians

Return to the table of contents

π

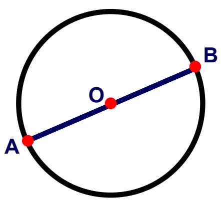


Before we extend our thinking of central angles and arc measures to arc lengths, it's worth reflecting on the number π, which will be central to our work.

This number was a devastating discovery to Greek mathematicians.

In fact, the reason that *The Elements* was written without relying on numbers, was because numbers were considered unreliable to the Greeks after π was discovered.

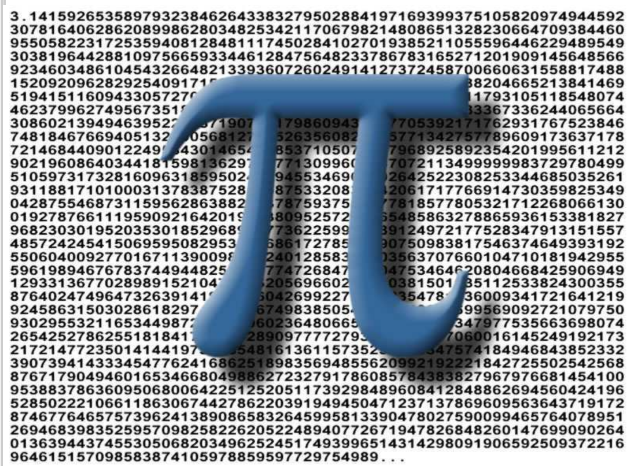
π



Until then, Pythagoras, and his followers believed that "All was Number."

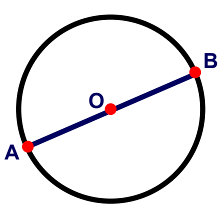
But when they sought to find the number that is ratio of the circumference to the diameter of a circle, they found that there wasn't one.

The closer they looked, the more impossible it became to find a number solution to the simple expression of C/d: the circumference divided by the diameter of a circle.



http://bobchoat.files.wordpress.com/2013/06/pi-day004.jpg

π



π is an example of an irrational number. A number that is not the ratio of two integers. So, no matter how far you take it, it keeps going without settling down.

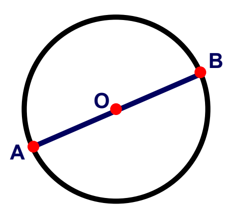
We are comfortable with irrational numbers now, but the Greeks weren't.

Now we know that there are many more irrational numbers than rational numbers.

Rational numbers are like islands in a sea of irrational numbers.

But, we are more familiar with those islands as that's where we grew up.

π



In mathematics, it's best to just leave answers with the symbol π.

In science, engineering and other fields which need a rational answer, and where π shows up a lot, the value of π is just estimated with the number of digits necessary for the problem.

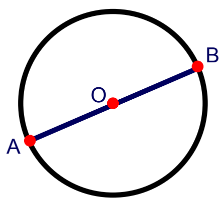
For most problems, 3.14 is close enough.

For others, you might use 3.14159...but you will rarely need more than that.

For this course, just leave your answers with the number π as part of your answer.

Slide 58 / 255

Arc Lengths



The relationship between the circumference of a circle to its diameter is

$C = \pi d$

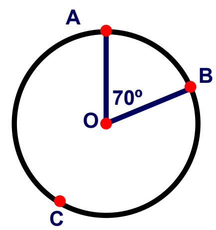
Since $d = 2r$, this is usually expressed as

$C = 2\pi r$

We know that a full trip around a circle is equal to 360°, so if we know the angle of an arc and the radius, we can determine the length of the arc.

Slide 59 / 255

Arc Lengths



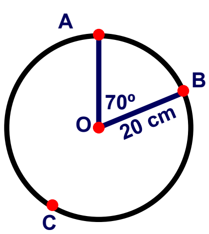
In the figure off to the left, we know that the measure of Arc AB is equal to that of the Central Angle...70°.

But, if we are also told that the radius of the circle is 20 cm, we can determine the length of Arc AB, also denoted as AB.

That's how far you'd have to travel along that arc to get from Point A to Point B.

Slide 60 / 255

Arc Lengths



We could do this by first figuring out the circumference using

$C = 2\pi r = 2\pi(20 \text{ cm}) = 40\pi \text{ cm}$

Then figuring what percentage of the circumference Arc AB is by the ratio of

$$\frac{AB}{C} = \frac{70°}{360°} = 0.1944$$

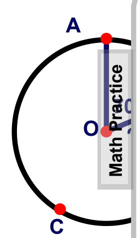
Since 360° is the entire circumference.

So, AB is

$(0.1944)(40\pi \text{ cm}) = 7.8\pi \text{ cm}$ (about 24 cm)

Slide 60 (Answer) / 255

Arc Lengths



out the

f the

atio of

rence.

Additional Q's to address MP Standards:

What connection do you see between calculating a percentage & the arc length of a circle? (MP4)

- Ans: arc length is a part of the circumference; part to whole, similar to percentages

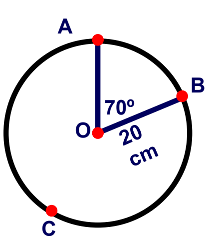
Can you find another method to calculate the arc length? (MP8)

- Ans: a proportion (shown on the next slide)

$(0.1944)(40\pi \text{ cm}) = 7.8\pi \text{ cm}$ (about 24 cm)

Slide 61 / 255

Arc Lengths



Alternatively, we could just set this up as a ratio and solve it in one step.

$$\frac{\text{Arc length}}{\text{Circumference}} = \frac{\text{Central angle}}{360°}$$

$$\frac{AB}{2\pi r} = \frac{70°}{360°}$$

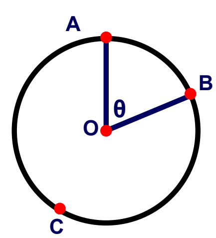
$$AB = \frac{70°}{360°}(2\pi r)$$

$$AB = \frac{70°}{360°}(2\pi)(20)$$

$$AB = 7.8\pi \text{ cm}$$

Slide 62 / 255

Arc Lengths



For any arc, you can find its length by multiplying the circumference of the circle ($2\pi r$) by the angle of arc divided by 360.

In this case, the measure of the arc is θ, since it is equal to the central angle.

Then,

$$\frac{AB}{2\pi r} = \frac{\theta}{360°}$$

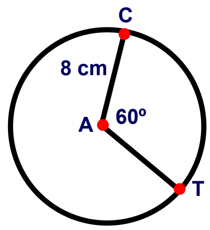
$$AB = \frac{\theta}{360°} 2\pi r$$

Slide 63 / 255

Example

In Circle A, the central angle is 60° and the radius is 8 cm.

Find the length of Arc CT

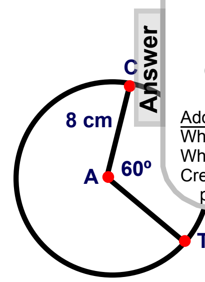


Slide 63 (Answer) / 255

Example

In Circle A, the central angle is 60° and the radius is 8 cm.

Find the length of Arc CT



$$CT = \frac{mCT}{360^\circ} \cdot 2\pi r$$

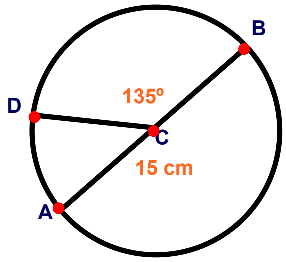
$$= \frac{60^\circ}{360^\circ} \cdot 2\pi(8)$$

$$CT = \frac{8\pi}{3} \approx 8.38 \text{ cm}$$

Additional Q's to address MP Standards:
What information are you given? (MP1)
What is the problem asking? (MP1)
Create an equation to represent the problem. (MP2)

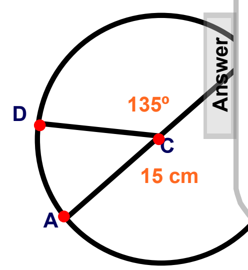
Slide 64 / 255

23 In circle C where $\overline{AB}$ is a diameter, find the length of Arc BD. The length of diameter $\overline{AB}$ is 15 cm.



Slide 64 (Answer) / 255

23 In circle C where $\overline{AB}$ is a diameter, find the length of Arc BD. The length of diameter $\overline{AB}$ is 15 cm.



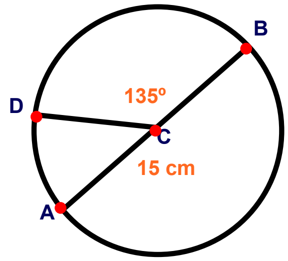
$$\frac{135^\circ}{360^\circ} = \frac{DB}{15\pi}$$

$$= 360DB$$

$$DB \approx 17.67 \text{ cm}$$

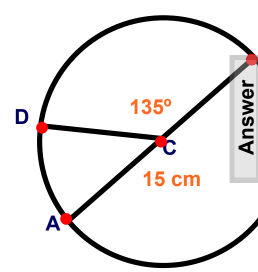
Slide 65 / 255

24 In circle C where $\overline{AB}$ is a diameter, find the length of Arc DAB. The length of $\overline{AB}$ is 15 cm.



Slide 65 (Answer) / 255

24 In circle C where $\overline{AB}$ is a diameter, find the length of Arc DAB. The length of $\overline{AB}$ is 15 cm.

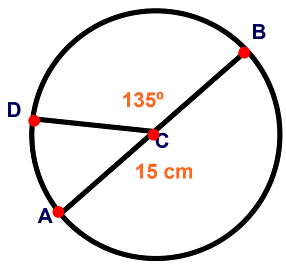


$$\frac{225^\circ}{360^\circ} = \frac{DAB}{15\pi}$$

$$DAB \approx 29.45 \text{ cm}$$

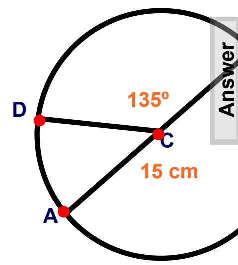
Slide 66 / 255

25 In circle C where $\overline{AB}$ is a diameter, find the length of Arc ADB. The length of $\overline{AB}$ is 15 cm.



Slide 66 (Answer) / 255

25 In circle C where $\overline{AB}$ is a diameter, find the length of Arc ADB. The length of $\overline{AB}$ is 15 cm.



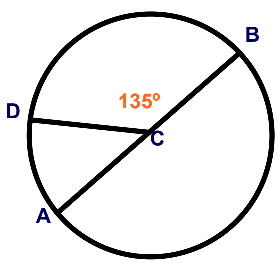
$$\frac{180°}{360°} = \frac{ADB}{15\pi}$$

ADB ≈ 23.56 cm

Slide 67 / 255

26 In circle C can it be assumed that $\overline{AB}$ is a diameter?

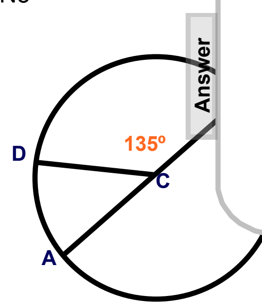
- ❍ Yes
- ❍ No



Slide 67 (Answer) / 255

26 In circle C can it be assumed that $\overline{AB}$ is a diameter?

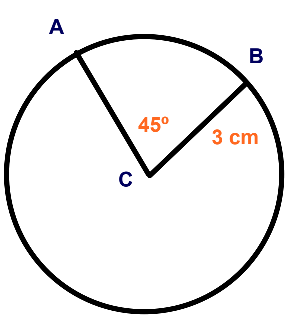
- ❍ Yes
- ❍ No



Yes

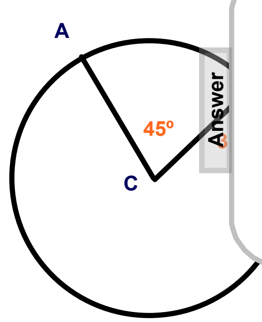
Slide 68 / 255

27 Find the length of AB



Slide 68 (Answer) / 255

27 Find the length of AB

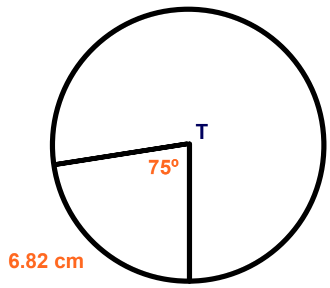


$$\frac{45°}{360°} = \frac{AB}{6\pi}$$

AB ≈ 2.36 cm

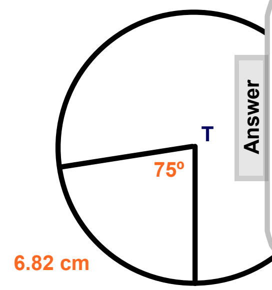
Slide 69 / 255

28 Find the circumference of circle T.



Slide 69 (Answer) / 255

28 Find the circumference of circle T.



Answer

$$\frac{75^\circ}{360^\circ} = \frac{6.82}{C}$$

$$C = 32.736cm$$

Slide 70 / 255

29 In circle T, Lines WY & XZ are diameters and have lengths of 6. The m∠XTY = 140º, what is YZ?

- A $\frac{2}{3}\pi$
- B 6
- C $\frac{4}{3}\pi$
- D 4

Hint:
click to reveal

Slide 70 (Answer) / 255

29 In circle T, Lines WY & XZ are diameters and have lengths of 6. The m∠XTY = 140º, what is YZ?

- A $\frac{2}{3}\pi$
- B 6
- C $\frac{4}{3}\pi$
- D 4

Answer

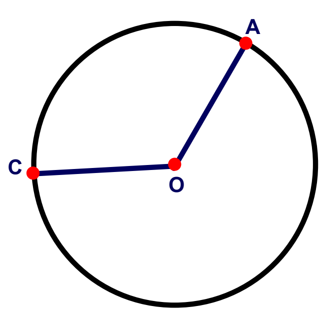
A

$$\frac{40^\circ}{360^\circ} = \frac{\overset{\frown}{YZ}}{6\pi}$$

Hint:
click to reveal

Slide 71 / 255

Radians



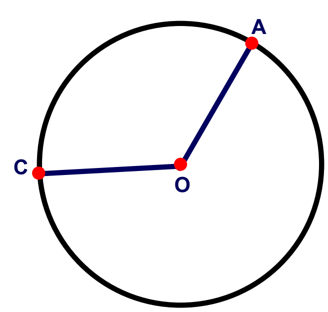
Another way of measuring angles, as an alternative to degrees, is radians.

Where degrees are arbitrary (Why are there 360 degrees in circle?), radians are very natural.

Radians are the ratio of an arc length divided by the radius of the arc.

Slide 72 / 255

Radians



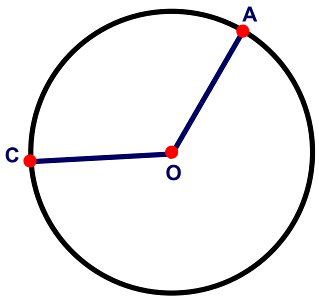
So, the radian measure of angle AOC is just the length of Arc AC divided by the length of radius (AO or CO).

The circumference of a circle is given by C = 2πr.

So, the radian measure of a full trip around a circle is 2πr/r = 2π.

Slide 73 / 255

Radians



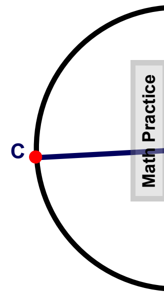
There are no units for radians, since the lengths cancel out, but you can write "rads" or "radians" just to indicate what you are doing.

Since there are no units, these angle measures are much easier to use when you study trigonometry, physics and calculus.

All scientific calculators allow you to use degrees or radians. Just make sure it is set to the correct one when you are entering angle measurements.

Slide 73 (Answer) / 255

Radians



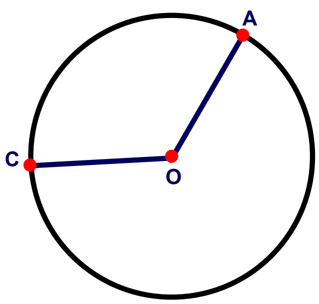
MP6: Attend to precision

Emphasize the correct unit of measurement (the use of "rads" or "radians") as the students answer the examples & SMART Response Q's

one when you are entering angle measurements.

Slide 74 / 255

Radians



Since a full trip around a circle is 360°, and is also 2π radians, they must be equal.

So, to convert from one to the other, just multiply by the appropriate conversion factor.

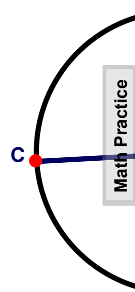
$$\frac{2\pi}{360°} = \frac{360°}{2\pi} = 1$$

Since 2π = 360°, each of these fractions is just equal to 1.

Multiplying anything by them doesn't change it's value, since multiplying by 1 has no effect.

Slide 74 (Answer) / 255

Radians

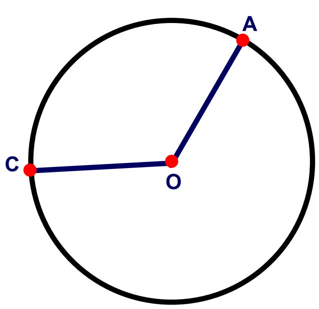


<u>Additional Q's to address MP Standards:</u>
What connection do you see between calculating a percentage & radian measure of a circle? (MP4)
- <u>Ans:</u> the number of radians is a part of 2π; part to whole, similar to percentages
Can you find another method to calculate the radian measurement? (MP8)
- <u>Ans:</u> a proportion (shown on the next slide)

doesn't change it's value, since multiplying by 1 has no effect.

Slide 75 / 255

Radians



Alternatively, you could write a proportion and use the cross-product property to find your missing measurement.

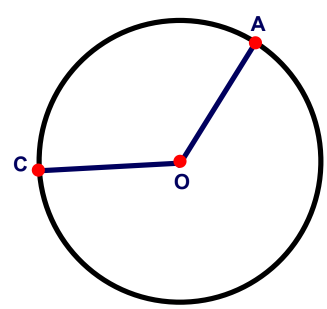
$$\frac{\text{degrees}}{360°} = \frac{\text{radians}}{2\pi}$$

Both methods will be shown in the next example. Use whichever method is easier for you.

Slide 76 / 255

Radians

$$\frac{2\pi}{360°} = \frac{360°}{2\pi} = 1$$



For instance, m∠AOC = 125°.

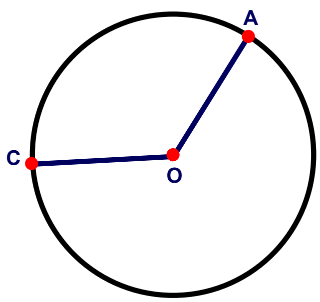
It is also equal to

$$m\angle AOC = (125°)\ \frac{2\pi}{360°}$$

m∠AOC = 0.69 π radians

Slide 77 / 255

Radians



If you are using the proportion and the cross-product property, you could set up & solve the problem with the work shown below.

$$\frac{\text{degrees}}{360°} = \frac{\text{radians}}{2\pi}$$

$$\frac{125°}{360°} = \frac{x}{2\pi}$$

$$250\pi = 360x$$

$$x = \frac{25\pi}{36} = 0.69\pi \text{ radians}$$

Slide 78 / 255

30 How many radians is 180 degrees?
(Round π to 3.14 in your answer.)

Slide 78 (Answer) / 255

30 How many radians is 180 degrees?
(Round π to 3.14 in your answer.)

Answer

The approximate answer is 3.14 The exact answer is π.

Slide 79 / 255

31 How many radians is 90 degrees?
(Round π to 3.14 in your answer.)

Slide 79 (Answer) / 255

31 How many radians is 90 degrees?
(Round π to 3.14 in your answer.)

Answer

The approximate answer is 1.57
The exact answer is π/2.

Slide 80 / 255

32 How many radians is 140°?
(Round π to 3.14 in your answer.)

Slide 80 (Answer) / 255

32 How many radians is 140°?
(Round π to 3.14 in your answer.)

Answer

The approximate answer is 0.81
The exact answer is 7π/9.

Slide 81 / 255

33 How many degrees is π radians?

Slide 81 (Answer) / 255

33 How many degrees is π radians?

Answer

180º

Slide 82 / 255

34 How many degrees is 2π radians?

Slide 82 (Answer) / 255

34 How many degrees is 2π radians?

Answer

360

Slide 83 / 255

35 How many degrees is 1.0 radian?

Slide 83 (Answer) / 255

35 How many degrees is 1.0 radian?

Answer

The approximate answer is 57.3
The exact answer is 180/π.

Slide 84 / 255

36 How many degrees is 1.6 radians?

Slide 84 (Answer) / 255

36 How many degrees is 1.6 radians?

Answer

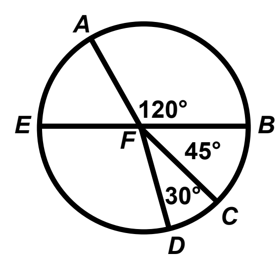
The approximate answer is 91.67
The exact answer is (1.6)(180)/π.

Slide 85 / 255

37 Circle F has a radius of 3.

What is the arc length for ∠AFB?

- A $\frac{\pi}{2}$
- B π
- 2π
- $\frac{3\pi}{4}$



PARCC Released Question - EOY - Question #3 - Non-Calculator

Slide 85 (Answer) / 255

37 Circle F has a radius of 3.

What is the arc length for ∠AFB?

- A $\frac{\pi}{2}$
- B π
- 2π
- $\frac{3\pi}{4}$

Answer

C

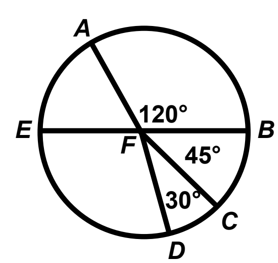
PARCC Released Question - EOY - Question #3 - Non-Calculator

Slide 86 / 255

38 Circle F has a radius of 3.

What is the arc length for ∠BFC?

- A $\frac{\pi}{2}$
- B π
- 2π
- $\frac{3\pi}{4}$



PARCC Released Question - EOY - Question #3 - Non-Calculator

Slide 86 (Answer) / 255

38 Circle F has a radius of 3.

What is the arc length

A $\frac{\pi}{2}$

B π

2π

$\frac{3\pi}{4}$

Answer

D

PARCC Released Question - EOY - Question #3 - Non-Calculator

Slide 87 / 255

39 Circle F has a radius of 3.

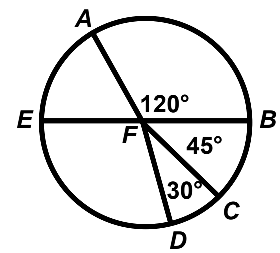
What is the arc length for ∠CFD?

A $\frac{\pi}{2}$

B π

2π

$\frac{3\pi}{4}$



PARCC Released Question - EOY - Question #3 - Non-Calculator

Slide 87 (Answer) / 255

39 Circle F has a radius of 3.

What is the arc leng

A $\frac{\pi}{2}$

B π

2π

$\frac{3\pi}{4}$

Answer

A

PARCC Released Question - EOY - Question #3 - Non-Calculator

Slide 88 / 255

40 Circle F has a radius of 3.

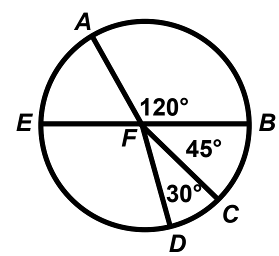
What is the arc length for ∠AFE?

A $\frac{\pi}{2}$

B π

2π

$\frac{3\pi}{4}$



PARCC Released Question - EOY - Question #3 - Non-Calculator

Slide 88 (Answer) / 255

40 Circle F has a radius of 3.

What is the arc lengt

A $\frac{\pi}{2}$

B π

2π

$\frac{3\pi}{4}$

Answer

B

PARCC Released Question - EOY - Question #3 - Non-Calculator

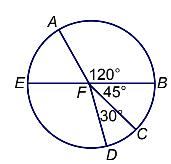
Slide 89 / 255

Question 3/7 **Topic: Angles, Arcs and Arc Lengths**

The circle with center F is divided into sectors. In circle F, $\overline{EB}$ is a diameter. The radius of circle F is 3 units.

Drag and drop each arc length to its subtended central angle.

$\frac{\pi}{2}$ π 2π $\frac{3\pi}{4}$



Subtended Central Angle	Arc Length
∠AFB	
∠BFC	
∠CFD	
∠AFE	

PARCC Released Question - EOY

Slide 89 (Answer) / 255

Question 3/7 **Topic: Angles, Arcs and Arc Lengths**

The circle with center F is divided into sectors. In circle F, $\overline{EB}$ is a diameter. The radius of circle F is 3 units.

Drag and drop each arc length t

$\frac{\pi}{2}$ | π | 2π

Answer

$arc\ length(s) = \frac{n°}{360°} \times 2\pi r$

$radius = 3\ units$

$n = Subtended\ Central\ Angle\ given$

$arc\ length\angle AFB = \frac{120°}{360°} \times (2)(3)\pi = 2\pi$

$arc\ length\angle BFC = \frac{45°}{360°} \times (2)(3)\pi = \frac{3\pi}{4}$

$arc\ length\angle CFD = \frac{30°}{360°} \times (2)(3)\pi = \frac{\pi}{2}$

$\angle AFE = 180° - 120° = 60°$

$arc\ length\angle AFE = \frac{60°}{360°} \times (2)(3)\pi = \pi$

Subtended Central Angle	Arc Length
$\angle AFB$	2π
$\angle BFC$	$\frac{3\pi}{4}$
$\angle CFD$	$\frac{\pi}{2}$
$\angle AFE$	π

$\angle AFE$

PARCC Released Question - EOY

Slide 90 / 255

Chords, Inscribed Angles & Triangles

Return to the table of contents

Slide 91 / 255

Click on the link below and complete the labs before the Chords lesson.

Lab - Properties of Chords

Slide 91 (Answer) / 255

Click on the link bel labs before th

Lab - Prope

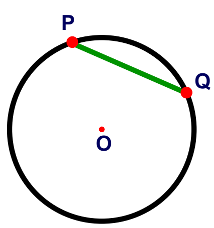
Math Practice

This lab addresses MP3 & MP5

Slide 92 / 255

The Arc of the Chord

Definition: When a chord and a minor arc have the same endpoints, we call the arc *The Arc of the Chord.*



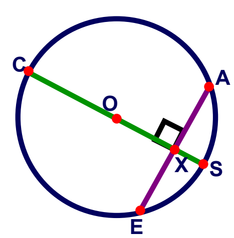
PQ is the arc of $\overline{PQ}$

Recall the definition of a chord: a line segment with endpoints on the circle.

Slide 93 / 255

Chord Bisector Theorem

If a diameter or radius of a circle is perpendicular to a chord, then the diameter or radius bisects the chord and its arc.

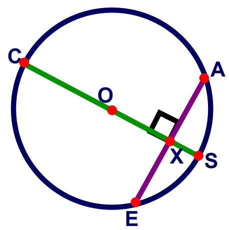


$\overline{CS}$ is a diameter of the circle and is perpendicular to chord $\overline{AE}$

Therefore, $\overline{XE} \cong \overline{XA}$ & AS ≅ ES

Slide 94 / 255

Proof of Chord Bisector Theorem

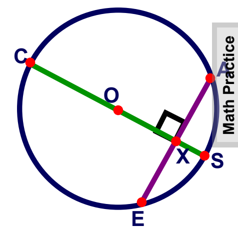


Given: $\overline{CS}$ is a diameter which is perpendicular to $\overline{AE}$

Prove: $\overline{CS}$ bisects $\overline{AE}$ and its minor arc AE

Slide 94 (Answer) / 255

Proof of Chord Bisector Theorem

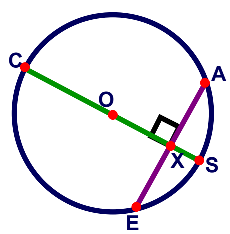


This example accomplishes MP3.

s

Slide 95 / 255

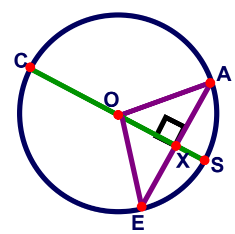
Proof of Chord Bisector Theorem



As a first step, let's draw radii from the Center O to the points A and E to construct ΔAOE.

Slide 96 / 255

Proof of Chord Bisector Theorem



Now, we can use the triangles which have been formed in our proof .

Given: $\overline{CS}$ is a diameter which is perpendicular to $\overline{AE}$

Prove: $\overline{CS}$ bisects $\overline{AE}$ and its minor arc AE

Slide 97 / 255

Proof of Chord Bisector Theorem

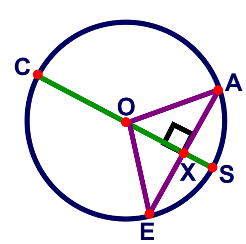
Use the proof below for the next six questions.

	Statement	Reason
1	$\overline{CS}$ is a diameter which is perpendicular to $\overline{AE}$	Given
2	?	Right angles are formed by perpendicular lines
3	$\overline{OE} \cong \overline{OA}$	?
4	?	Reflexive Property of ≅
5	?	Hypotenuse - Leg Theorem
6	$\overline{XE} \cong \overline{XA}$ and $\angle XOE \cong \angle XOA$	?
7	SE ≅ SA	Arcs intercepted by ≅ central ∠s of a circle are ≅
8	$\overline{CS}$ bisects $\overline{AE}$ and Minor Arc AE	?

Slide 98 / 255

41 Fill in the statement for step #2.

- ○ $\angle OXA$ and $\angle OXE$ are complementary
- ○ $\angle OXA$ and $\angle OXE$ are right angles
- ○ $\overline{OX} \cong \overline{OX}$
- ○ D
- ○ E



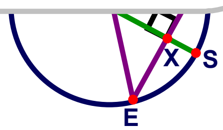
Slide 98 (Answer) / 255

41 Fill in the statement fo

- ∠OXA and ∠OXE are
- ∠OXA and ∠OXE are
- $\overline{OX} \cong \overline{OX}$
- D
- E

Answer

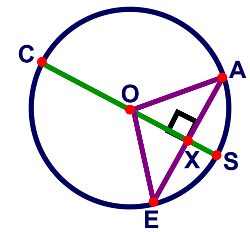
B



Slide 99 / 255

42 Fill in the reason for step #3.

- Radii of circles are ≅
- Definition of supplementary
- Corresponding parts of ≅ Δs are ≅
- D
- E



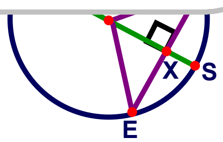
Slide 99 (Answer) / 255

42 Fill in the reason for s

- Radii of circles are ≅
- Definition of supple
- Corresponding parts
- D
- E

Answer

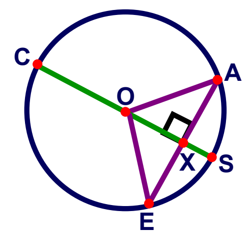
A



Slide 100 / 255

43 Fill in the statement for step #4.

- ∠OXA and ∠OXE are complementary
- ∠OXA and ∠OXE are right angles
- $\overline{OX} \cong \overline{OX}$
- D
- E



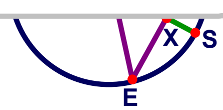
Slide 100 (Answer) / 255

43 Fill in the statement for

- ∠OXA and ∠OXE are
- ∠OXA and ∠OXE are
- $\overline{OX} \cong \overline{OX}$
- D
- E

Answer

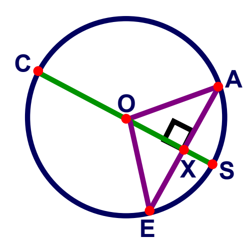
C



Slide 101 / 255

44 Fill in the statement for step #5.

- ∠OXA and ∠OXE are complementary
- ∠OXA and ∠OXE are right angles
- $\overline{OX} \cong \overline{OX}$
- D
- E



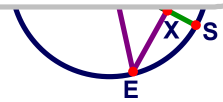
Slide 101 (Answer) / 255

44 Fill in the statement fo

- ∠OXA and ∠OXE are
- ∠OXA and ∠OXE are
- $\overline{OX} \cong \overline{OX}$
- D
- E

Answer

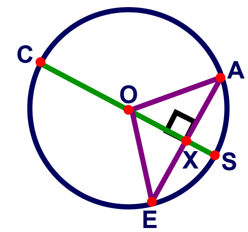
E



Slide 102 / 255

45 Fill in the reason for step #6.

- Radii of circles are ≅
- Definition of supplementary
- Corresponding parts of ≅ Δs are ≅
- D
- E



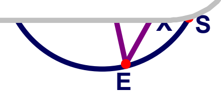
Slide 102 (Answer) / 255

45 Fill in the reason for ste

- Radii of circles are ≅
- Definition of supplem
- Corresponding parts
- D
- E

Answer

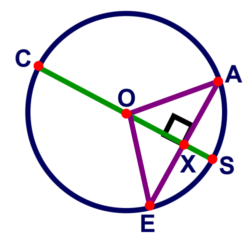
C



Slide 103 / 255

46 Fill in the reason for step #8.

- Radii of circles are ≅
- Definition of supplementary
- Corresponding parts of ≅ Δs are ≅
- D
- E



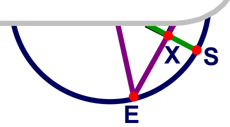
Slide 103 (Answer) / 255

46 Fill in the reason for st

- Radii of circles are ≅
- Definition of supplem
- Corresponding parts
- D
- E

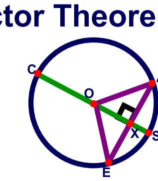
Answer

E



Slide 104 / 255

Proof of Chord Bisector Theorem

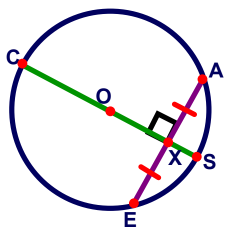


	Statement	Reason
1	$\overline{CS}$ is a diameter which is perpendicular to $\overline{AE}$	Given
2	∠OXA and ∠OXE are right angles	Right angles are formed by perpendicular lines
3	$\overline{OE} \cong \overline{OA}$	Radii of a circle are ≅
4	$\overline{OX} \cong \overline{OX}$	Reflexive Property of ≅
5	ΔOXA ≅ ΔOXE	Hypotenuse - Leg Theorem
6	$\overline{XE} \cong \overline{XA}$ and ∠XOE ≅ ∠XOA	Corresponding parts of ≅ Δs are ≅
7	SE ≅ SA	Arcs intercepted by ≅ central ∠s of a circle are ≅
8	$\overline{CS}$ bisects $\overline{AE}$ and Minor Arc AE	Definition of bisector

Slide 105 / 255

Converse of Chord Bisector Theorem

In a circle, if one chord is a perpendicular bisector of another chord, then the first chord is a diameter.



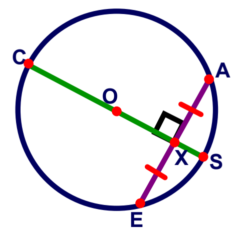
The chord CS is the perpendicular bisector of the chord AE.

Therefore, $\overline{CS}$ is a diameter of the circle and passes through the center of the circle.

Slide 106 / 255

Proof of Converse of Chord Bisector Theorem

In a circle, if one chord is a perpendicular bisector of another chord, then the first chord is a diameter.



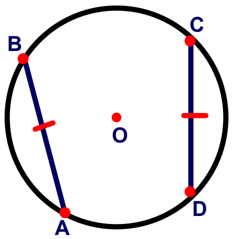
This proof is very much like that of the original theorem of which this is the converse.

Construct ΔOAX and ΔOEX and by proving them congruent, you show that $\overline{OA}$ and $\overline{OE}$ are radii, which means that chord $\overline{CS}$ passes through the center, and is a diameter.

Slide 107 / 255

Arcs and Chords Theorem

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.



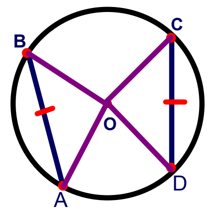
AB ≅ CD iff $\overline{AB} \cong \overline{CD}$

*iff stands for "if and only if"

Slide 108 / 255

Proof of Arcs and Chords Theorem

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.



This follows from the fact that the measure of an arc is equal to that of the central angle which intercepts it.

Since all the radii, $\overline{BO}$, $\overline{AO}$, $\overline{CO}$ and $\overline{DO}$ are congruent and the sides $\overline{BA}$ and $\overline{CD}$ are congruent, the triangles ABO and DOC are congruent by Side-Side-Side.

That means that the central angles are congruent which means that the arcs are congruent since arcs intercepted by congruent central angles are congruent.

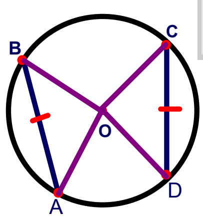
Slide 108 (Answer) / 255

Proof of Arcs and Chords Theorem

In the same circle, or in congruent circles, two minor arcs are congruent if and only if thei... t.

Math Practice

This example accomplishes MP3.

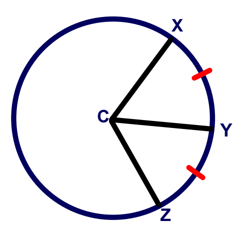


are congruent which means that the arcs are congruent since arcs intercepted by congruent central angles are congruent.

Slide 109 / 255

BISECTING ARCS

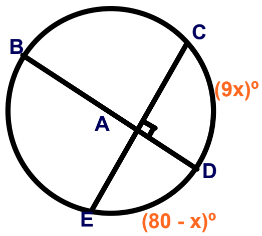
If XY ≅ YZ, then point *Y* and any line segment, or ray, that contains *Y*, bisects XYZ



This just follows from the definition of a bisector as dividing something into two pieces of equal measure.

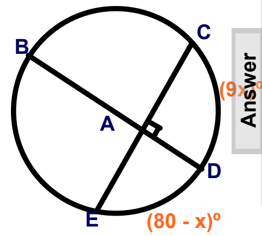
Slide 110 / 255

EXAMPLE



Slide 110 (Answer) / 255

EXAMPLE



9x = 80 - x
10x = 80
x = 8

CD = 9(8) = 72º
DE = 80 - 8 = 72º
CE = 72º + 72º = 144º

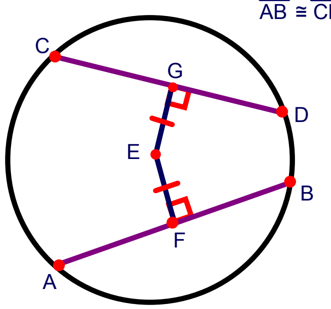
This example addresses MP2.

Slide 111 / 255

Theorem

In the same, or congruent, circles, two chords are congruent if and only if they are equidistant from the center.

$\overline{AB} \cong \overline{CD}$ iff $\overline{EF} \cong \overline{EG}$



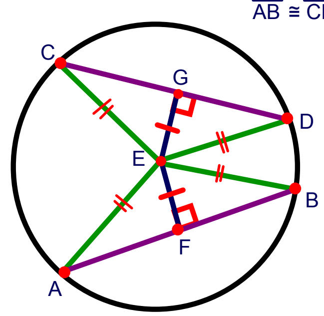
Proving this just requires creating Δ AEB and ΔCED and proving that they are congruent, which means that their altitudes are equal, which is their distance from the center of the circle.

Slide 112 / 255

Theorem

In the same, or congruent, circles, two chords are congruent if and only if they are equidistant from the center.

$\overline{AB} \cong \overline{CD}$ iff $\overline{EF} \cong \overline{EG}$

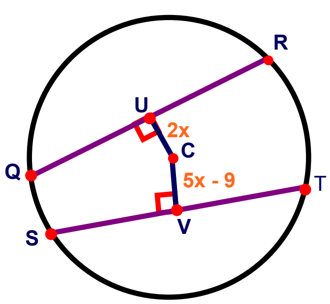


Here Δ AEB and ΔCED and their congruent sides are shown.

Slide 113 / 255

Example

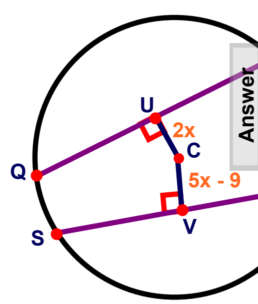
Given circle C, *QR* = *ST* = 16.
Find *CU*.



Slide 113 (Answer) / 255

Example

Given circle



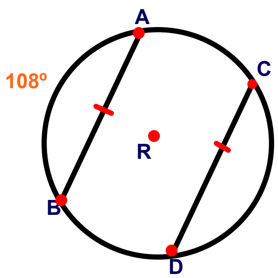
2x = 5x - 9
9 = 3x
3 = x

***CU* = 2(3) = 6**

This example addresses MP 2

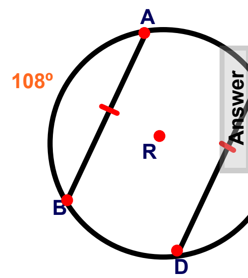
Slide 114 / 255

47 In circle *R*, $\overline{AB} \cong \overline{CD}$ and mAB = 108°. Find mCD.



Slide 114 (Answer) / 255

47 In circle *R*, $\overline{AB} \cong \overline{CD}$ and mAB = 108°. Find mCD.



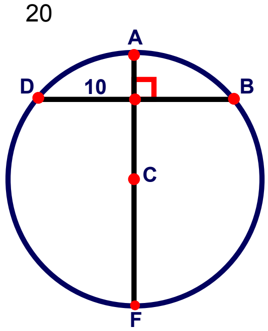
Answer

108º

Slide 115 / 255

48 Given circle *C* below, the length of $\overline{BD}$ is:

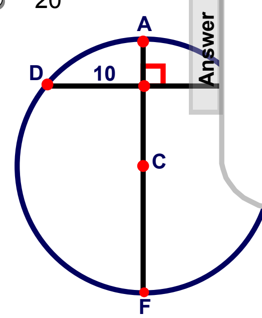
- ❍ 5
- ❍ 10
- ❍ 15
- ❍ 20



Slide 115 (Answer) / 255

48 Given circle *C* below, the length of $\overline{BD}$ is:

- ❍ 5
- ❍ 10
- ❍ 15
- ❍ 20



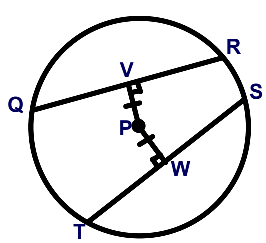
Answer

D

Slide 116 / 255

49

- ❍ A 1
- ❍ B 7
- ❍ C 20
- ❍ D 8



Slide 116 (Answer) / 255

49

- ❍ A 1
- ❍ B 7
- ❍ C 20
- ❍ D 8

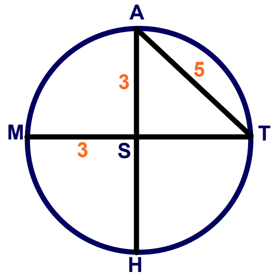
Answer

C

Slide 117 / 255

50 $\overline{AH}$ is a diameter of the circle.

- True
- False



Slide 117 (Answer) / 255

50 $\overline{AH}$ is a diameter of the circle.

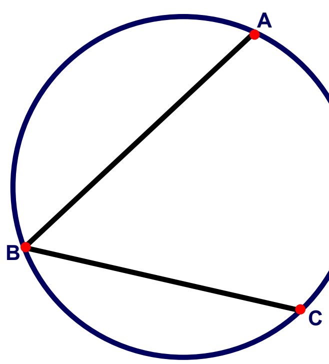
- True
- False

Answer

False

Slide 118 / 255

Inscribed Angles

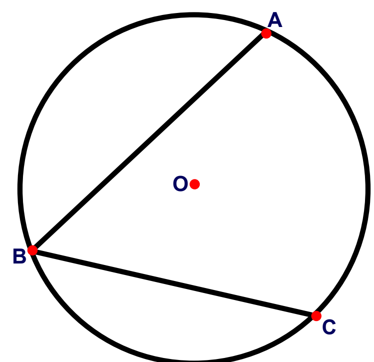


Inscribed angles are angles whose vertices lie on the circle and whose sides are chords of the circle.

Angle ABC is an inscribed angle and Arc AC is its intercepted arc.

Slide 119 / 255

Inscribed Angles



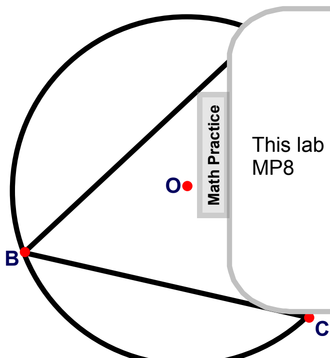
The minor arc that lies in the interior of the inscribed angle and has endpoints which are vertices of the angle is called the intercepted arc.

Arc AC is the intercepted arc of inscribed angle ABC.

Lab - Inscribed Angles

Slide 119 (Answer) / 255

Inscribed Angles



es in
cribed
oints
the

pted
e

Math Practice

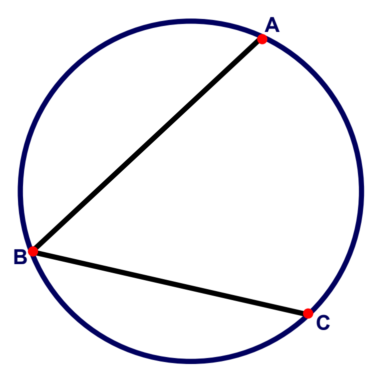
This lab addresses MP3, MP7 & MP8

Lab - Inscribed Angles

Slide 120 / 255

Inscribed Angle Theorem

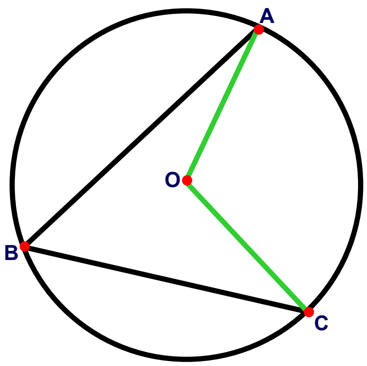
The measure of an inscribed angle is equal to half the measure of the intercepted arc or central angle



To prove this, we need to find the measure of $\angle ABC$ relative to the measure of the intercepted Arc AC

Slide 121 / 255

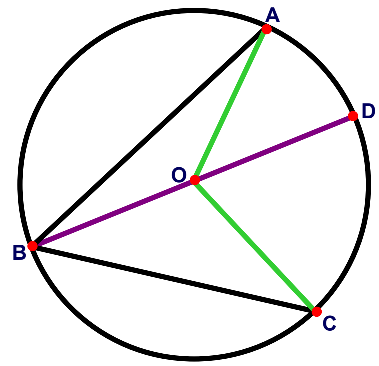
Inscribed Angle Theorem



The measure of ∠AOC, shown with green lines, is the same as the measure of the minor arc AC, shown in blue.

Slide 122 / 255

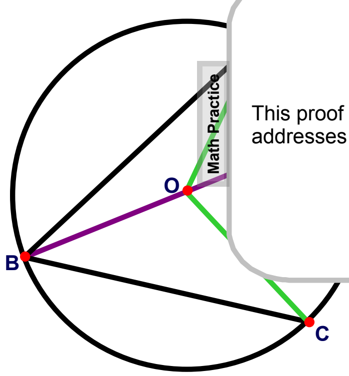
Inscribed Angle Theorem



To prove the theorem, we first draw the diameter $\overline{BD}$ which creates ΔAOB and ΔCOB

Slide 122 (Answer) / 255

Inscribed Angle Theorem



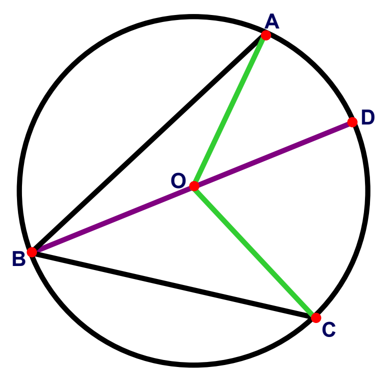
Math Practice

This proof (the next 9 slides) addresses MP1, MP2, MP3 & MP7

Slide 123 / 255

51 What are $\overline{OB}$, $\overline{OA}$, $\overline{OD}$ and $\overline{OC}$?

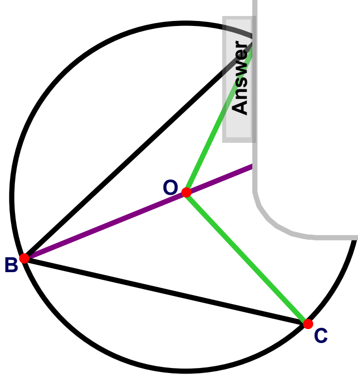
- A
- B
- C
- D



Slide 123 (Answer) / 255

51 What are $\overline{OB}$, $\overline{OA}$, $\overline{OD}$ and $\overline{OC}$?

- A
- B
- C
- D



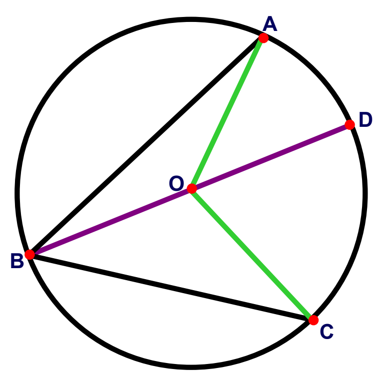
Answer

B

Slide 124 / 255

52 What is true of all radii of the same circle?

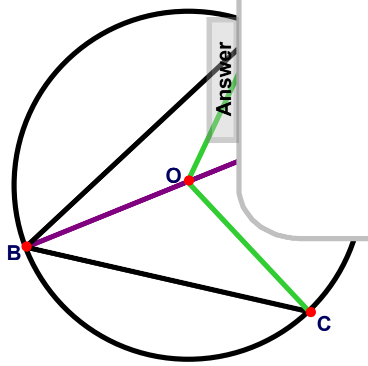
- A
- B
- C
- D



Slide 124 (Answer) / 255

52 What is true of all radii of the same circle?

- ○ A
- ○ B

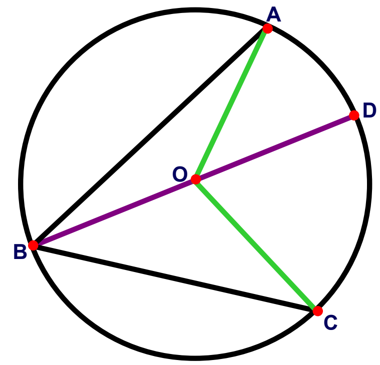


C

Slide 125 / 255

53 What types of triangles are ΔBOA and ΔBOC?

- ○ A
- ○ B
- ○ C
- ○ D

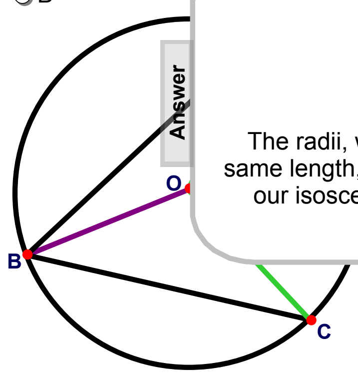


Why?

Slide 125 (Answer) / 255

53 What types of triangles are ΔBOA and ΔBOC?

- ○ A
- ○ B



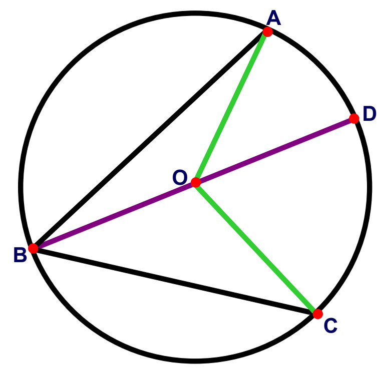
B

The radii, which are the same length, are the legs of our isosceles triangles.

Slide 126 / 255

54

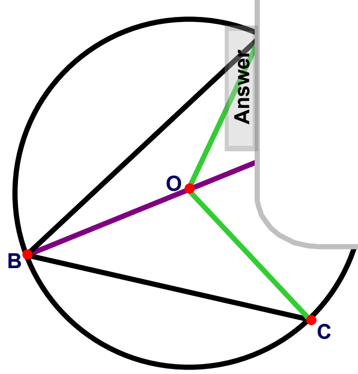
- ○ A
- ○ B
- ○ C
- ○ D



Slide 126 (Answer) / 255

54

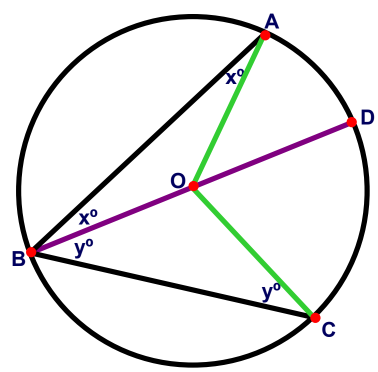
- ○ A
- ○ B
- ○ C



A

Slide 127 / 255

Inscribed Angle Theorem

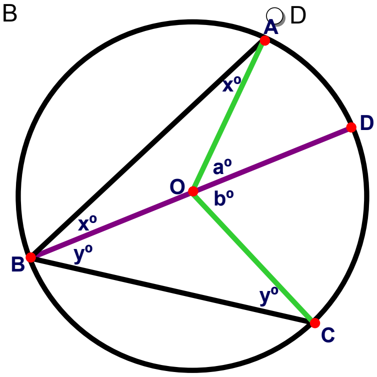


Equal angles are marked by x and y.

Slide 128 / 255

55 What types of angle are angles a and b of Δ BOA and ΔBOC?

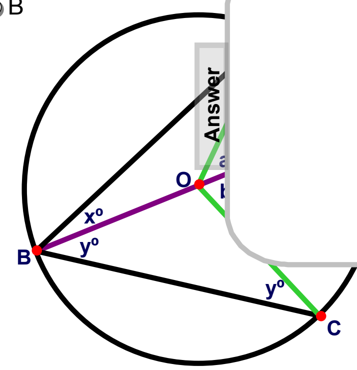
- ○ A
- ○ B
- ○ C
- ○ D



Slide 128 (Answer) / 255

55 What types of angle are angles a and b of Δ BOA and ΔBOC?

- ○ A
- ○ B



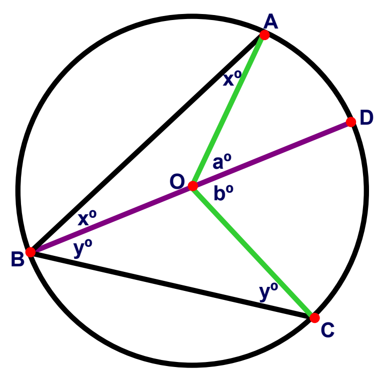
Answer

B

Slide 129 / 255

56 Which of the below is true? (Select all that apply.)

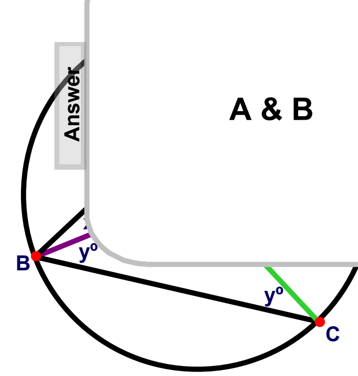
- ❑ a = 2x
- ❑ b = 2y
- ❑ a = b
- ❑ x = y



Slide 129 (Answer) / 255

56 Which of the below is true? (Select all that apply.)

- ❑ a = 2x
- ❑ b = 2y
- ❑ a = b
- ❑ x = y



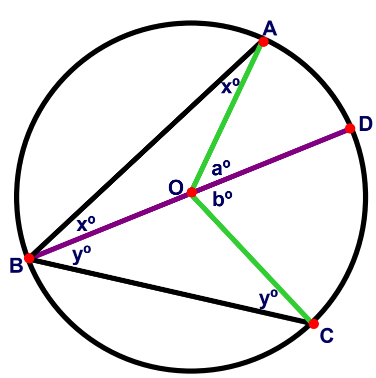
Answer

A & B

Slide 130 / 255

57 To what is the measure of the sum of a + b equal?

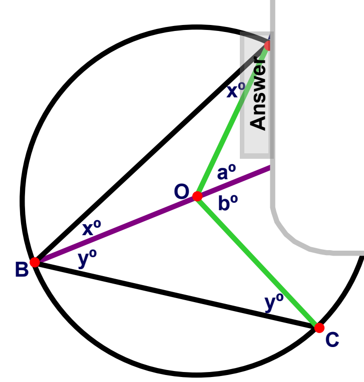
- ○ A a + b = arc AD
- ○ B a + b = arc DC
- ○ C a + b = arc AC
- ○ D a + b = x + y



Slide 130 (Answer) / 255

57 To what is the measure of the sum of a + b equal?

- ○ A a + b = arc AD
- ○ B a + b = arc DC
- ○ C a + b = arc AC

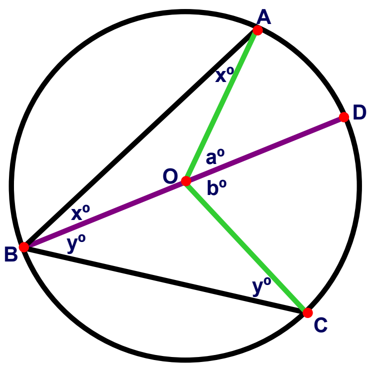


Answer

C

Slide 131 / 255

Inscribed Angle Theorem



So, a + b = m∠AOC

Also, a + b = 2x + 2y = 2(x+y)

And x + y = m∠ABC

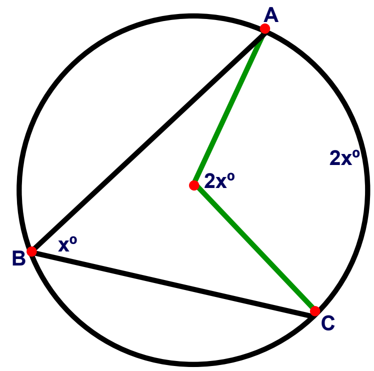
So, 2(m∠ABC) = m∠AOC

And m∠ABC = 1/2(m∠AOC)

The measure of the inscribed angle is equal to half the measure of the central angle and, therefore, half the measure of the intercepted arc.

Slide 132 / 255

Inscribed Angle Theorem



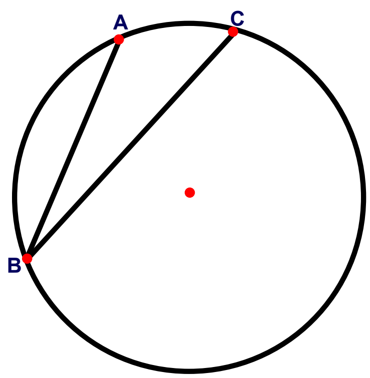
The measure of an inscribed angle, such as ∠ABC, is equal to half the measure of the intercepted arc, which is AC.

Also, it is equal to half the measure of the central angle intercepting that arc.

This was proven if the center is within the angle, but is true for all cases.

Slide 133 / 255

Inscribed Angle Theorem

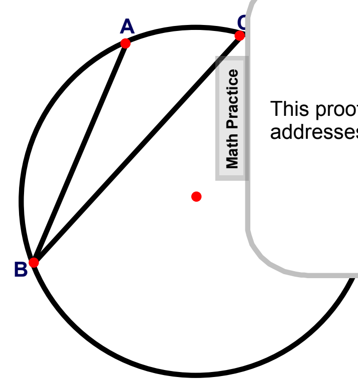


It's not essential to go through the proof of the other two cases, but if you have time, it's good practice.

First, if the inscribed angle does not include the center.

Slide 133 (Answer) / 255

Inscribed Angle Theorem

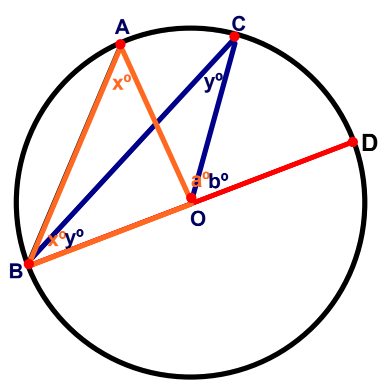


Math Practice

This proof (the next 2 slides) addresses MP1, MP2, MP3 & MP7

Slide 134 / 255

Inscribed Angle Theorem



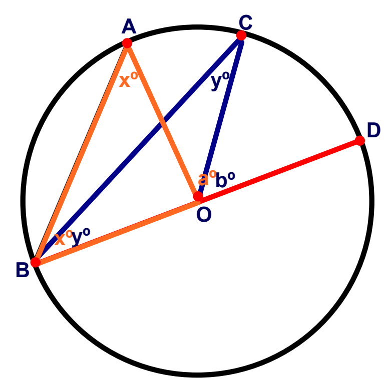
Since these triangles overlap so much one side of the inscribed angle, AB, and its associated isosceles triangle and external angle are shown in orange.

The other leg and its associated isosceles triangle and external angle are shown in dark blue.

The proof then follows the same form.

Slide 135 / 255

Inscribed Angle Theorem



a = 2x
b = 2y

m∠ABC = x - y

m∠AOC = a - b
m∠AOC = 2x - 2y = 2(x - y)

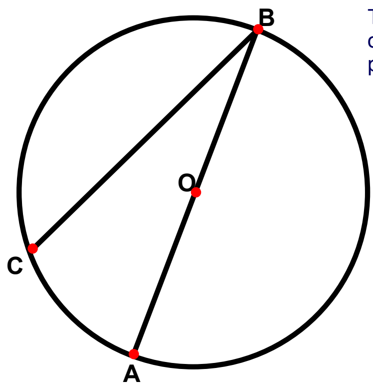
So,

m∠ABC = 1/2(m∠AOC)

The inscribed angle equals half the measure of the intercepted arc

Slide 136 / 255

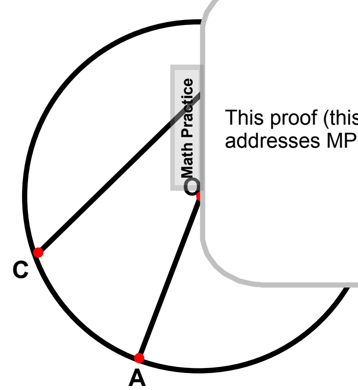
Inscribed Angle Theorem



The last case is if the a leg of the inscribed angle passes through the center.

Slide 136 (Answer) / 255

Inscribed Angle Theorem



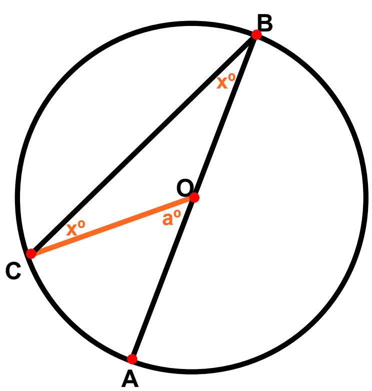
Math Practice

This proof (this and the next slide) addresses MP1, MP2, MP3 & MP7

e a leg
e
enter.

Slide 137 / 255

Inscribed Angle Theorem



m∠ABC = x

a = 2x

m∠AOC = a = 2x

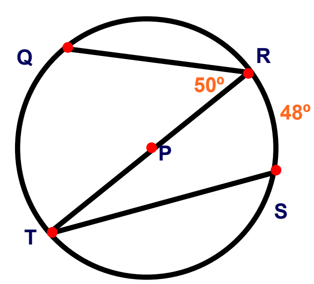
So, m∠ABC = 1/2(m∠AOC)

The inscribed angle equals half the measure of the intercepted arc

Slide 138 / 255

Example

Find m∠T, mTQ and mQR.

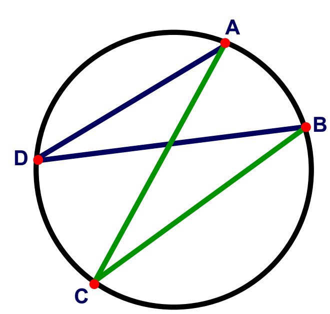


Slide 138 (Answer) / 255

Slide 139 / 255

Theorem

If two inscribed angles of a circle intercept the same arc, then the angles are congruent.



∠ADB ≅ ∠ACB

This follows directly from our prior proofs showing that the measure of an inscribed angle is equal to half that of the intercepted arc.

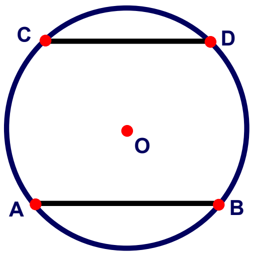
By the transitive property of equality, if two inscribed angles intercept the same are they must be equal.

In this case, m∠ADB = m∠ACB since the both intercept Arc AB.

Slide 140 / 255

Theorem

In a circle, parallel chords intercept congruent arcs.



In circle *O*, if Chord AB is parallel to Chord CD then Arc CA is equal to Arc DB.

Note that this does NOT mean that Arc CD is equal to Arc AB.

Slide 141 / 255

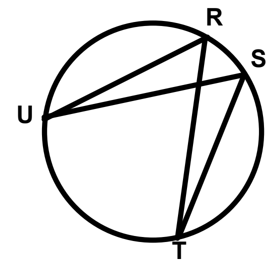
58 Given the figure below, which pairs of angles are congruent?

○ A $\angle R \cong \angle S$
$\angle U \cong \angle T$

○ B $\angle R \cong \angle U$
$\angle S \cong \angle T$

○ C $\angle R \cong \angle T$
$\angle U \cong \angle S$

○ D $\angle R \cong \angle T$
$\angle R \cong \angle S$



Slide 141 (Answer) / 255

58 Given the figure below, which pairs of angles are congruent?

○ A $\angle R \cong \angle S$
$\angle U \cong \angle T$

○ B $\angle R \cong \angle U$
$\angle S \cong \angle T$

○ C $\angle R \cong \angle T$
$\angle U \cong \angle S$

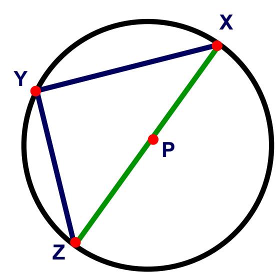
○ D $\angle R \cong \angle T$
$\angle R \cong \angle S$

Answer

A

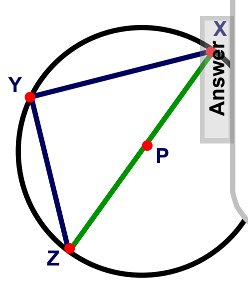
Slide 142 / 255

59 Find $m\angle Y$



Slide 142 (Answer) / 255

59 Find $m\angle Y$



Answer

90º

Slide 143 / 255

60 In a circle, two parallel chords on opposite sides of the center have arcs which measure 100º and 120º.

Find the measure of one of the arcs included between the chords.

Slide 143 (Answer) / 255

60 In a circle, two parallel chords on opposite sides of the center have arcs which

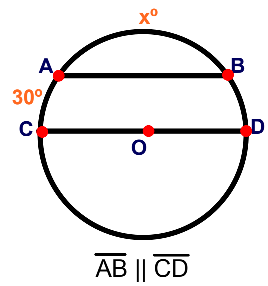
Find the measure of the chords.

Answer

70º

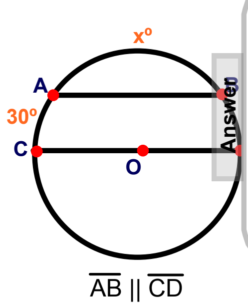
Slide 144 / 255

61 Given circle *O*, find the value of x.



Slide 144 (Answer) / 255

61 Given circle *O*, find the value of x.

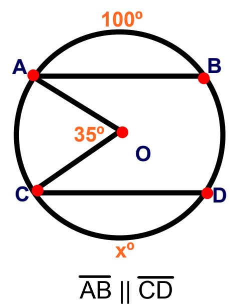


Answer

120º

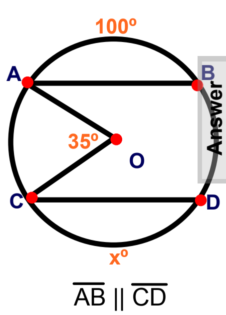
Slide 145 / 255

62 Given circle *O*, find the value of x.



Slide 145 (Answer) / 255

62 Given circle *O*, find the value of x.



Answer

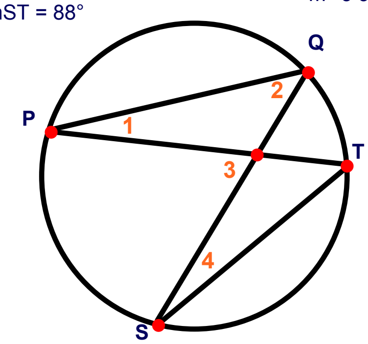
190º

Slide 146 / 255

Try This

In this circle:
mPQ = 52°
mQS = 112°
mST = 88°

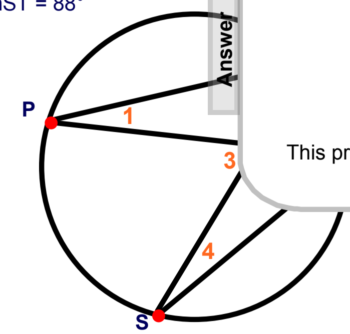
Find
m∠1, m∠2,
m∠3 & m∠4



Slide 146 (Answer) / 255

Try This

In this circle:
mPQ = 52°
mQS = 112°
mST = 88°



Answer

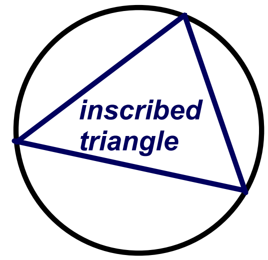
m∠1 = 12°
m∠2 = 98°
m∠3 = 110°
m∠4 = 12°

This problem addresses MP2

Slide 147 / 255

Inscribed Triangles

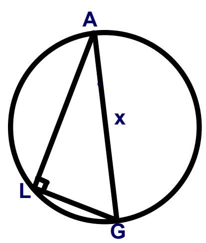
A triangle is inscribed if all its vertices lie on a circle.



Slide 148 / 255

Corollary to Inscribed Angle Theorem

If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle.



m∠ALG = 90°

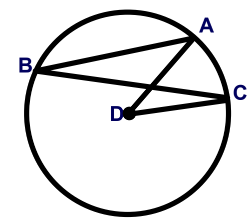
$\overline{AG}$ is a diameter of the circle if ΔALG is a right Δ, and *∠ALG* is its right angle, follows directly from the inscribed angle theorem.

Since ∠ALG has a measure of 90°, it must intercept an arc whose measure is 180°, this is half the circle, so $\overline{AG}$ must be a diameter

Slide 149 / 255

63 In the diagram, ∠ADC is a central angle and m∠ADC = 60º. What is m∠ABC?

- A 15º
- B 30º
- C 60º
- D 120º



Slide 149 (Answer) / 255

63 In the diagram, ∠ADC is a central angle and m∠ADC = 60º. What is m∠ABC?

- A 15º
- B 30º
- C 60º
- D 120º

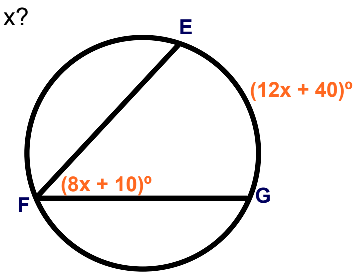
Answer

B

Slide 150 / 255

64 What is the value of x?

- A 5
- B 10
- C 13
- D 15



Slide 150 (Answer) / 255

64 What is the value of x?

- A 5
- B 10
- C 13
- D 15

Answer

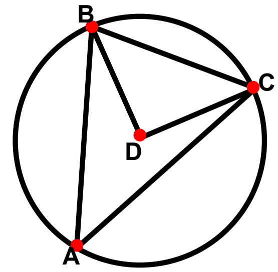
A

Slide 151 / 255

Question 1/25 **Topic: Inscribed Angles**

The figure shows ΔABC inscribed in circle D.

65 If m ∠CBD = 44°, find m∠BAC.



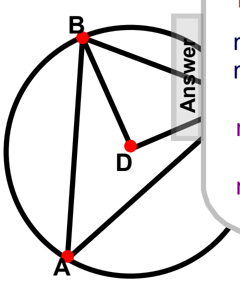
PARCC Released Question - EOY - Calculator

Slide 151 (Answer) / 255

Question 1/25 **Topic: Inscribed Angles**

The figure shows ΔABC inscribed in circle D.

65 If m ∠CBD = 44°, find m∠BAC.

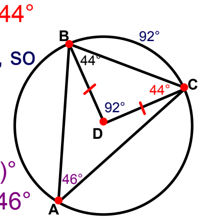


Answer

∠DBC ≅ ∠DCB, so
m∠DCB = 44°

m∠D = 92°, so
mBC = 92°

m∠BAC =
1/2(92)°
m∠BAC = 46°

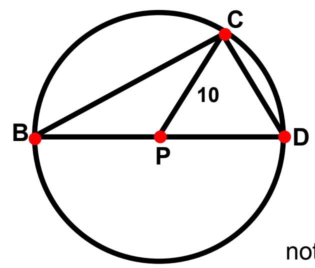


PARCC Released Question - EOY - Calculator

Slide 152 / 255

Question 17/25 **Topic: Inscribed Angles**

66



not to scale

PARCC Released Question - EOY - Calculator

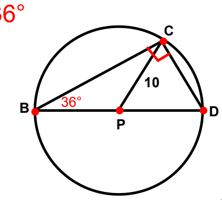
Slide 152 (Answer) / 255

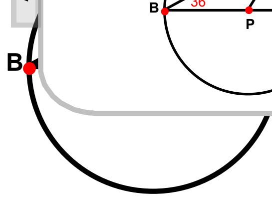
Question 17/25 **Topic: Inscribed Angles**

66

Answer

x + 54 = 90
x = 36°





not to scale

PARCC Released Question - EOY - Calculator

Slide 153 / 255

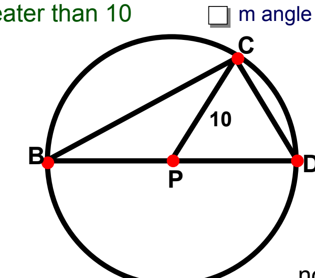
Question 17/25 **Topic: Inscribed Angles**

67 **PART B**

The length of CD is __________ because ________.

- ☐ 10
- ☐ less than 10
- ☐ greater than 10

- ☐ ΔCPD is equilateral
- ☐ m angle CPD < 60°
- ☐ m angle CPD > 60°



not to scale

PARCC Released Question - EOY

Question 17/25 **Topic: Inscribed Angles**

67 **PART B**

The length of CD is __________ because _______.

- 10
- ΔCPD is equilateral
- less th... ...0°
- greater ...

Answer

Part B:

The length of $\overline{CD}$ is c. greater than 10

because f. m∠CPD > 60°.

not to scale

PARCC Released Question - EOY

Question 22/25 **Topic: Inscribed Angles**

Point B is the center of a circle, and $\overline{AC}$ is a diameter of the circle. Point D is a point on the circle different from A and C.

68 **PART A**

The statement AD > CD is:

- Always True
- Sometimes True
- Never True

PARCC Released Question - EOY - Calculator

Question 22/25 **Topic: Inscribed Angles**

Point B is the center of a circle, and $\overline{AC}$ is a diameter of the circle. Point D is a point on ... ...nd C.

68 **PART A**

The statement AD > C...

- Always True
- Sometimes True
- Never True

Answer

B

PARCC Released Question - EOY - Calculator

Question 22/25 **Topic: Inscribed Angles**

Point B is the center of a circle, and $\overline{AC}$ is a diameter of the circle. Point D is a point on the circle different from A and C.

69 **PART A**

The statement m∠CBD = 1/2 (m∠CAD) is:

- Always True
- Sometimes True
- Never True

PARCC Released Question - EOY - Calculator

Question 22/25 **Topic: Inscribed Angles**

Point B is the center of a circle, and $\overline{AC}$ is a diameter of the circle. Point D is a point ... ...d C.

69 **PART A**

The statement m∠C...

- Always True
- Sometimes True
- Never True

Answer

C

PARCC Released Question - EOY - Calculator

Question 22/25 **Topic: Inscribed Angles**

Point B is the center of a circle, and $\overline{AC}$ is a diameter of the circle. Point D is a point on the circle different from A and C.

70 **PART A**

The statement that m∠CBD = 90º is:

- Always True
- Sometimes True
- Never True

PARCC Released Question - EOY - Calculator

Slide 156 (Answer) / 255

Question 22/25 **Topic: Inscribed Angles**

Point B is the center of a circle, and $\overline{AC}$ is a diameter of the circle. Point D is a point ... d C.

70 **PART A**

The statement that m

- ❍ Always True
- ❍ Sometimes True
- ❍ Never True

Answer

B

PARCC Released Question - EOY - Calculator

Slide 157 / 255

Question 22/25 **Topic: Inscribed Angles**

Point B is the center of a circle, and $\overline{AC}$ is a diameter of the circle. Point D is a point on the circle different from A and C.

71 **PART A**

The statement that $m\angle ABD = 2(m\angle CBD)$ is:

- ❍ Always True
- ❍ Sometimes True
- ❍ Never True

PARCC Released Question - EOY - Calculator

Slide 157 (Answer) / 255

Question 22/25 **Topic: Inscribed Angles**

Point B is the center of a circle, and $\overline{AC}$ is a diameter of the circle. Point D is a point ... d C.

71 **PART A**

The statement that m

- ❍ Always True
- ❍ Sometimes True
- ❍ Never True

Answer

B

PARCC Released Question - EOY - Calculator

Slide 158 / 255

Question 22/25 **Topic: Inscribed Angles**

Point B is the center of a circle, and $\overline{AC}$ is a diameter of the circle. Point D is a point on the circle different from A and C.

72 **PART B**

If $m\angle BDA = 20°$, what is $m\angle CBD$?

- ❍ 20°
- ❍ 40°
- ❍ 70°
- ❍D 140°

PARCC Released Question - EOY - Calculator

Slide 158 (Answer) / 255

Question 22/25 **Topic: Inscribed Angles**

Point B is the center of a circle, and $\overline{AC}$ is a diameter of the circle. Point D is a point on ... d C.

72 **PART B**

If $m\angle BDA = 20°$, what

- ❍ 20°
- ❍ 40°
- ❍ 70°
- ❍D 140°

Answer

B 40°

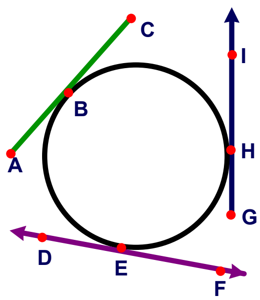
PARCC Released Question - EOY - Calculator

Slide 159 / 255

Tangents & Secants

Return to the table of contents

Tangents



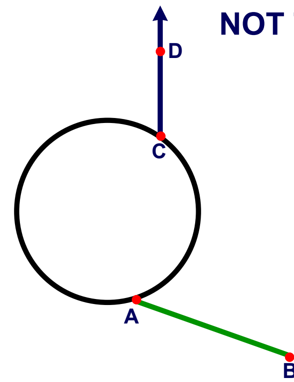
A tangent is a line, ray or segment which touches a circle at just one point.

All three types of tangent lines are shown on this drawing.

The point where the line touches the circle is called the "point of tangency."

In this case, those points are B, E and H.

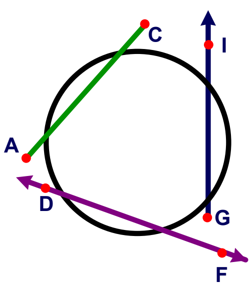
NOT Tangents



Note that for a ray or segment to be a tangent, it must not touch the circle in more than one point even if it were extended.

The segment and ray shown to the left areNOT tangents because if they were extended they would touch the circle in more than one point.

Secants



A secant is a line, ray or segment which touches a circle at two points.

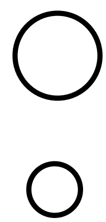
All three types of secant lines are shown on this drawing.

Intersections of Circles

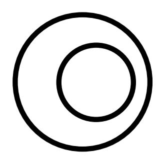
Coplanar circles can intersect at zero, one, or two points.

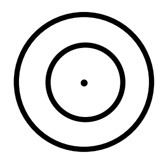
Below are shown three ways in which a pair of circles can have no points of contact.

Circles which are side-by-side may not intersect.



Circles within one another may not intersect.



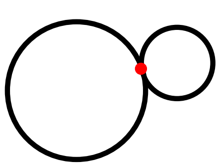


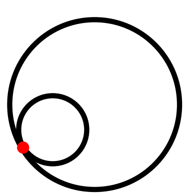
Remember that:
Circles within one another and have a common center are "concentric."

Intersections of Circles

Tangent Circles intersect at one point.

The two types of tangent circles are shown below.

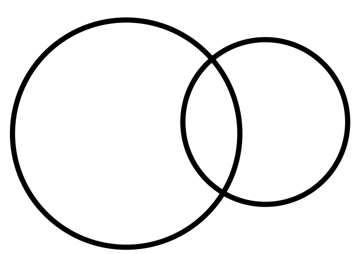




Intersections of Circles

These circles intersect at two points.

Can two distinct circles intersect at three points?



Slide 165 (Answer) / 255

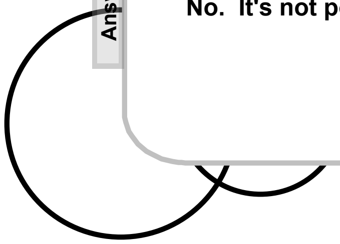
Intersections of Circles

These circles intersect at t

Can two distinct circles

Answer

No. It's not possible.

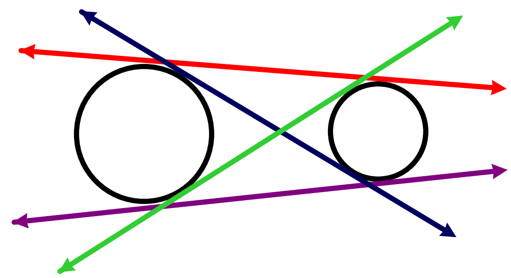


Slide 166 / 255

Common Tangents

Two circles can have between zero and four common tangents.

Two completely separate circles have four common tangents.

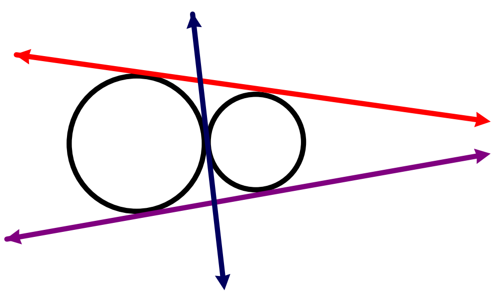


Slide 167 / 255

Common Tangents

Two circles can have between zero and four common tangents.

Two externally tangent circles have three common tangents.

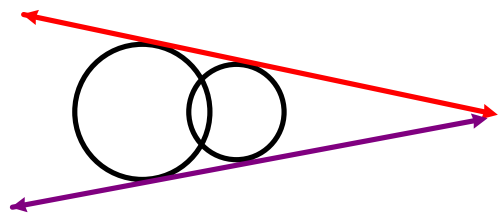


Slide 168 / 255

Common Tangents

Two circles can have between zero and four common tangents.

Two overlapping circles have two common tangents.

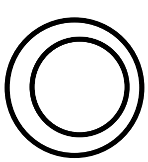


Slide 169 / 255

Common Tangents

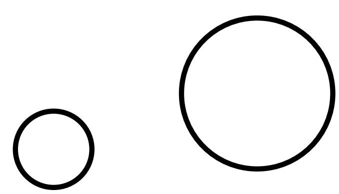
Two circles can have between zero and four common tangents.

Two non-tangent circles within one another have no common tangents.



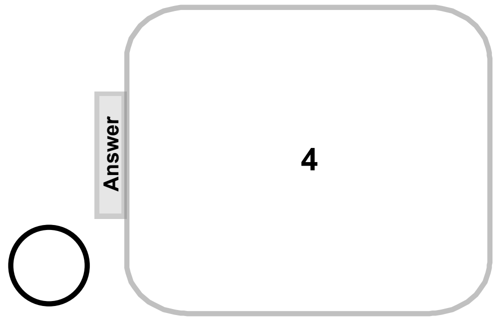
Slide 170 / 255

73 How many common tangent lines do the circleshave?



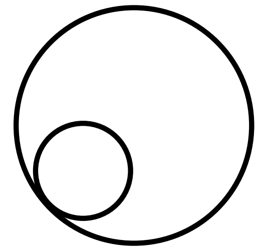
Slide 170 (Answer) / 255

73 How many common tangent lines do the circleshave?



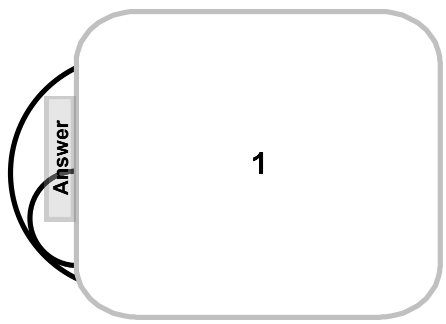
Slide 171 / 255

74 How many common tangent lines do the circleshave?



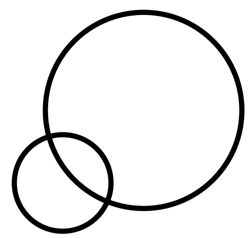
Slide 171 (Answer) / 255

74 How many common tangent lines do the circleshave?



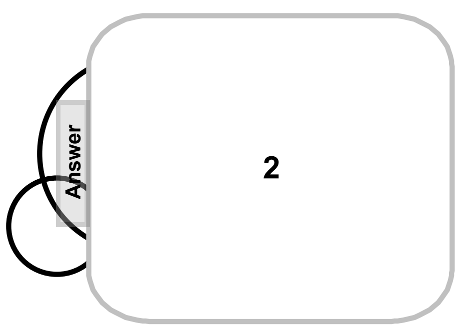
Slide 172 / 255

75 How many common tangent lines do the circleshave?



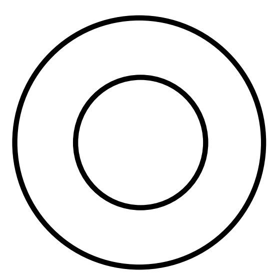
Slide 172 (Answer) / 255

75 How many common tangent lines do the circleshave?



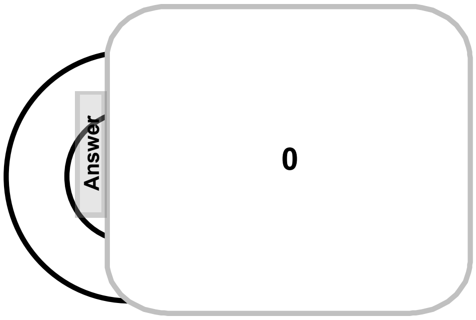
Slide 173 / 255

76 How many common tangent lines do the circleshave?



Slide 173 (Answer) / 255

76 How many common tangent lines do the circles have?

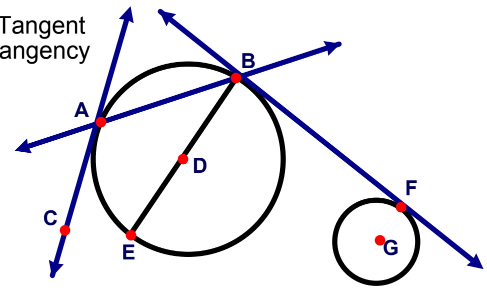


Slide 174 / 255

77 Which term best describes Line BE?

- A Radius
- B Diameter
- C Chord
- D Secant

Tangent
Common Tangent
Point of Tangency
Center

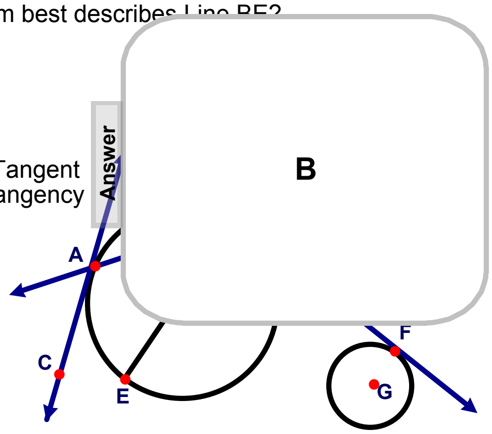


Slide 174 (Answer) / 255

77 Which term best describes Line BE?

- A Radius
- B Diameter
- C Chord
- D Secant

Tangent
Common Tangent
Point of Tangency
Center

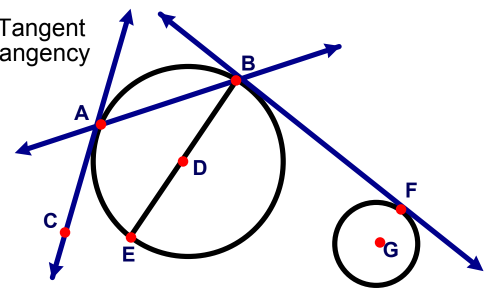


Slide 175 / 255

78 Which term best describes Line DE?

- A Radius
- B Diameter
- C Chord
- D Secant

Tangent
Common Tangent
Point of Tangency
Center

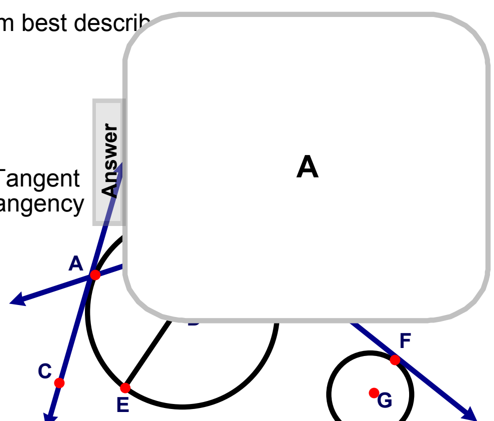


Slide 175 (Answer) / 255

78 Which term best describ

- A Radius
- B Diameter
- C Chord
- D Secant

Tangent
Common Tangent
Point of Tangency
Center

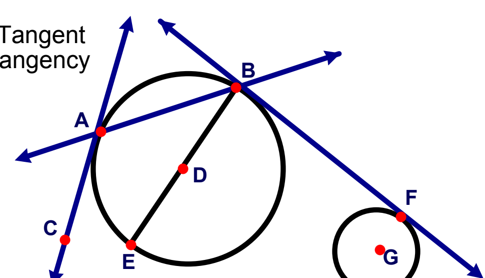


Slide 176 / 255

79 Which term best describes Line AB?

- A Radius
- B Diameter
- C Chord
- D Secant

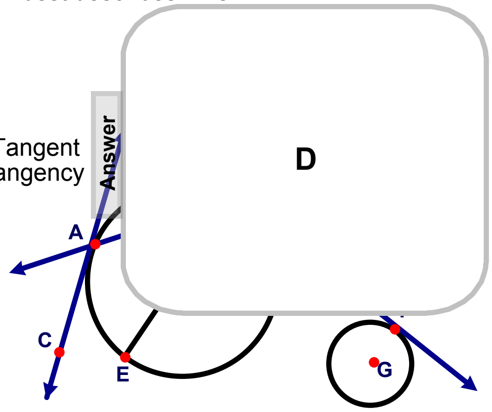
Tangent
Common Tangent
Point of Tangency
Center



Slide 176 (Answer) / 255

79 Which term best describes Line AB?

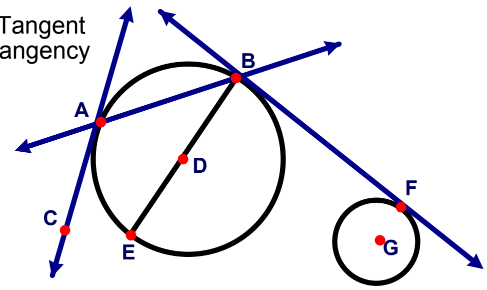
- A Radius
- B Diameter
- C Chord
- D Secant
- Tangent
- Common Tangent
- Point of Tangency
- Center



Slide 177 / 255

80 Which term best describes Points D and G?

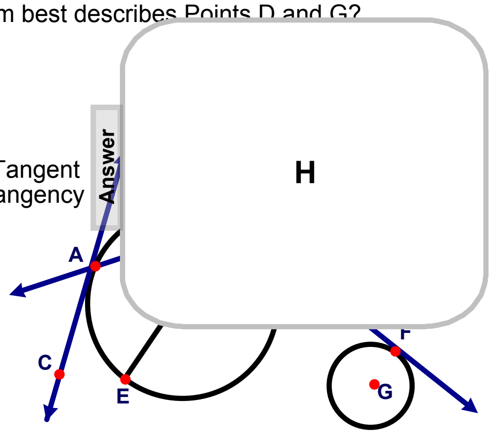
- A Radius
- B Diameter
- C Chord
- D Secant
- Tangent
- Common Tangent
- Point of Tangency
- Center



Slide 177 (Answer) / 255

80 Which term best describes Points D and G?

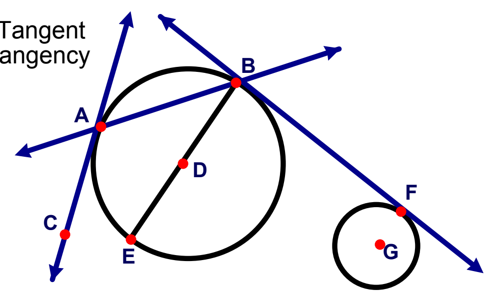
- A Radius
- B Diameter
- C Chord
- D Secant
- Tangent
- Common Tangent
- Point of Tangency
- Center



Slide 178 / 255

81 Which term best describes Line BF?

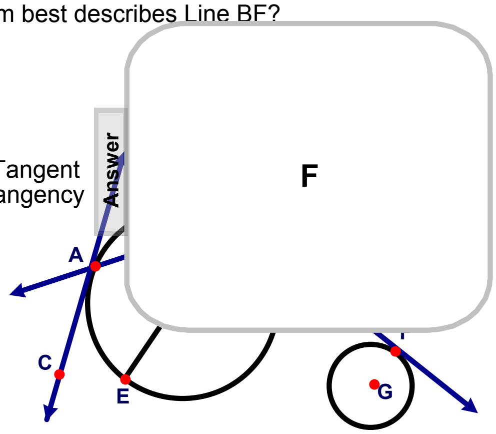
- A Radius
- B Diameter
- C Chord
- D Secant
- Tangent
- Common Tangent
- Point of Tangency
- Center



Slide 178 (Answer) / 255

81 Which term best describes Line BF?

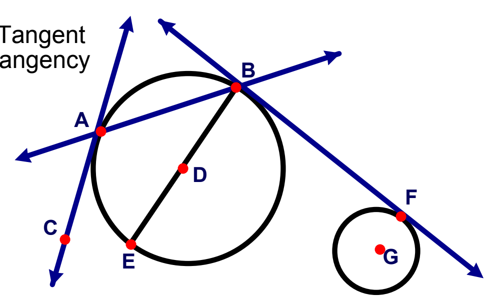
- A Radius
- B Diameter
- C Chord
- D Secant
- Tangent
- Common Tangent
- Point of Tangency
- Center



Slide 179 / 255

82 Which term best describes Line AC?

- A Radius
- B Diameter
- C Chord
- D Secant
- Tangent
- Common Tangent
- Point of Tangency
- Center



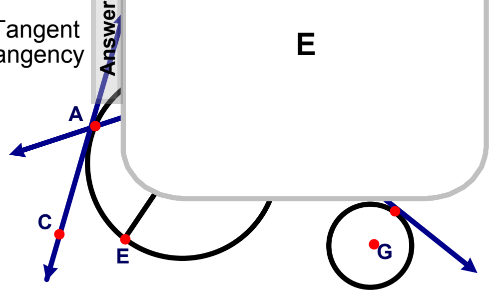
Slide 179 (Answer) / 255

82 Which term best describes Line AC?

- A Radius
- B Diameter
- C Chord
- D Secant
- Tangent
- Common Tangent
- Point of Tangency
- Center

Answer

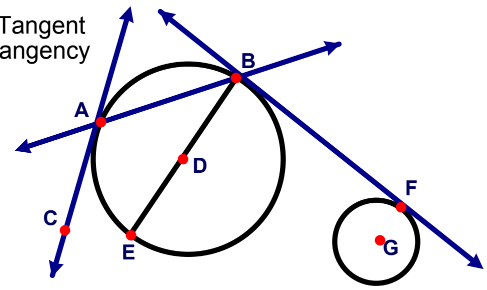
E



Slide 180 / 255

83 Which term best describes Point F?

- A Radius
- B Diameter
- C Chord
- D Secant
- Tangent
- Common Tangent
- Point of Tangency
- Center



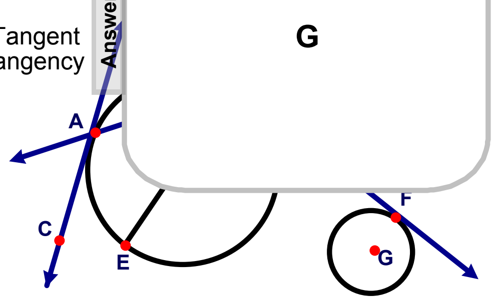
Slide 180 (Answer) / 255

83 Which term best describes Point F?

- A Radius
- B Diameter
- C Chord
- D Secant
- Tangent
- Common Tangent
- Point of Tangency
- Center

Answer

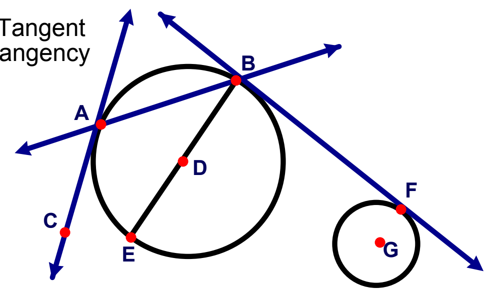
G



Slide 181 / 255

84 Which term best describes Segment BA?

- A Radius
- B Diameter
- C Chord
- D Secant
- Tangent
- Common Tangent
- Point of Tangency
- Center



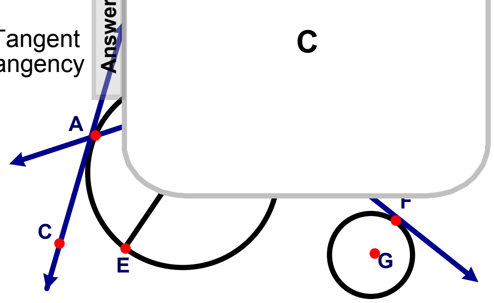
Slide 181 (Answer) / 255

84 Which term best describes Segment BA?

- A Radius
- B Diameter
- C Chord
- D Secant
- Tangent
- Common Tangent
- Point of Tangency
- Center

Answer

C

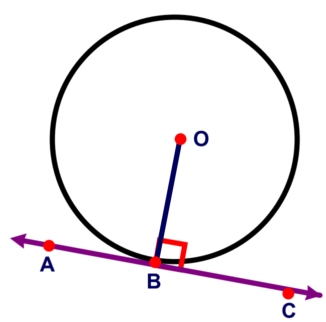


Slide 182 / 255

Tangents and Radii are Perpendicular

A tangent and radius that intersect on a circle are perpendicular.

This is an essential result for mathematics and physics. It appears to be true, but let's prove it.



Slide 182 (Answer) / 255

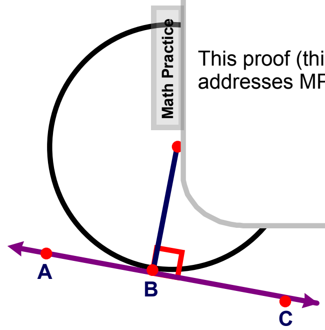
Tangents and Radii are Perpendicular

A tangent and radius that intersect on a circle are perpendicular.

This is an essentia appears to be true, but let's pr

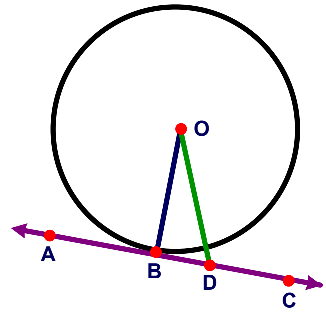
Math Practice

This proof (this slide and the next 3) addresses MP3 & MP6



Slide 183 / 255

Proof that Tangents and Radii are Perpendicular



Let's use an indirect proof.

We'll make an assumption and see if it leads to a contradiction.

Let's assume that another point on $\overleftrightarrow{AC}$ is where a line from Point O is perpendicular to the $\overleftrightarrow{AC}$.

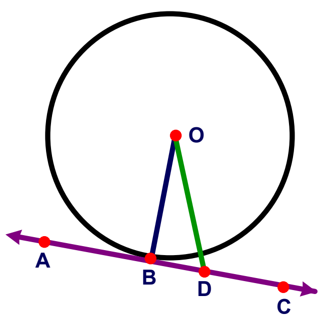
Let's name that point D, so the line perpendicular to $\overleftrightarrow{AC}$ is $\overline{OD}$.

Then ΔODB must be a right triangle with $\overline{OD}$ and $\overline{BD}$ being the legs.

Then $\overline{OB}$ must be the hypotenuse.

Slide 184 / 255

Proof that Tangents and Radii are Perpendicular



If $\overline{OD}$ is a leg of the right triangle and $\overline{OB}$ is the hypotenuse, then $\overline{OB}$ must be longer than $\overline{OD}$.

But, $\overline{OD}$ extends from the center of the circle to beyond the circle.

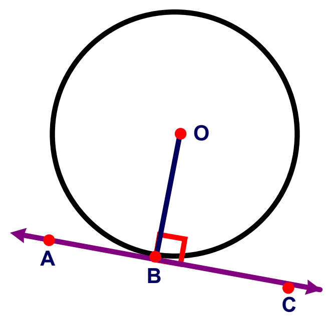
And $\overline{OB}$ only extends from the center of the circle to the circumference of the circle.

So, $\overline{OB}$ must be shorter than $\overline{OD}$.

This is a contradiction, which proves that our original assumption was incorrect.

Slide 185 / 255

Proof that Tangents and Radii are Perpendicular



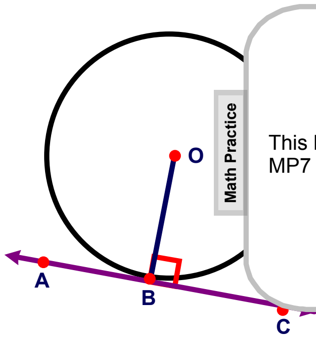
Our assumption was that $\overline{OB}$ was not perpendicular to $\overleftrightarrow{AC}$.

If that is false, then it must be that $\overline{OB}$ is perpendicular to $\overleftrightarrow{AC}$, which is what set out to prove.

Lab - Tangent Lines

Slide 185 (Answer) / 255

Proof that Tangents and Radii are Perpendicular



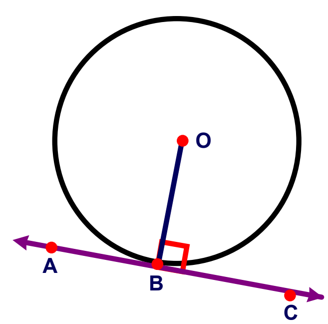
Math Practice

This lab addresses MP3, MP5, MP7 & MP8

Lab - Tangent Lines

Slide 186 / 255

Using an Intersecting Tangents & Radius to Solve Problems



Whenever you are given, or can draw, a circle and a tangent, you can construct a radius to the point of tangency.

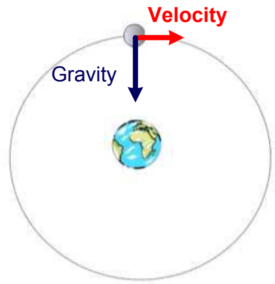
The tangent and radius will form a right angle.

This is often very helpful and what is needed to solve a problem.

Sometimes, that takes the form of creating a right triangle, with all the information that is provided.

Slide 187 / 255

Using an Intersecting Tangent & Radius to Solve Problems



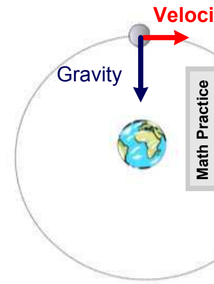
This property of a tangent and intersecting radius shows up often in physics.

For instance, it explains why the force of gravity pulling the moon into its circular orbit is perpendicular to the direction of its orbital velocity.

The force acts along the radius and the motion is tangent to the circular orbit.

Slide 187 (Answer) / 255

Using an Intersecting Tangent & Radius to Solve Problems



Math Practice

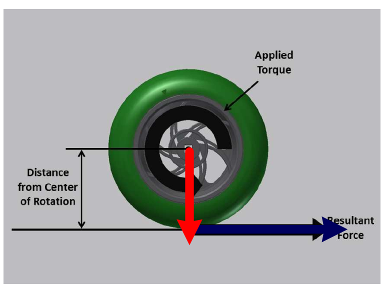
This slide and the next 2 address MP4

nd
p
y the
noon
on of
dius
nt to the
circular orbit.

Slide 188 / 255

Using an Intersecting Tangent & Radius to Solve Problems

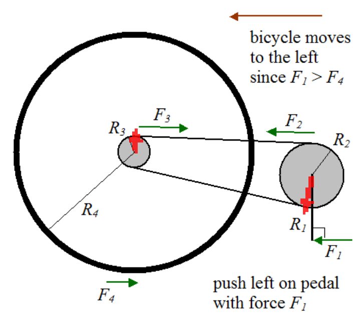
Or, the maximum torque that can be applied on a wheel by a road when braking, or by a tire on a road while accelerating.



Slide 189 / 255

Using an Intersecting Tangent & Radius to Solve Problems

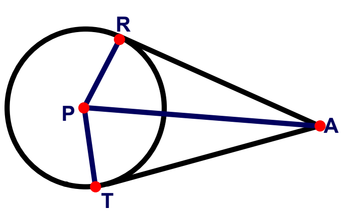
Or, the transfer of force due to a bicycle chain.



Slide 190 / 255

Theorem

Tangent segments from a common external point are congruent.



∠R and ∠T are right angles.

$\overline{PA}$ is congruent to itself by the Reflexive Property.

$\overline{PR}$ and $\overline{PT}$ are congruent because they are radii.

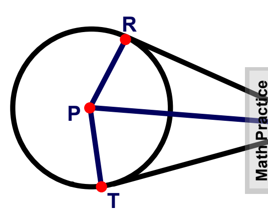
ΔPRA and ΔPTA are congruent due to the Hypotenuse-Leg Theorem.

Therefore, $\overline{AR}$ and $\overline{AT}$ are congruent since they are corresponding parts of congruent triangles.

Slide 190 (Answer) / 255

Theorem

Tangent segments from a common external point are congruent.



Math Practice

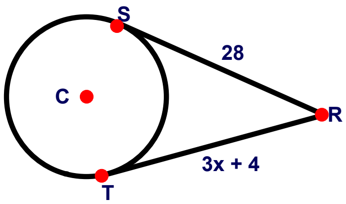
This lab addresses MP3 & MP6

Therefore, AR and AT are congruent since they are corresponding parts of congruent triangles.

Slide 191 / 255

Example

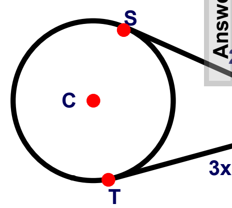
Given: $\overline{RS}$ is tangent to circle C at S and $\overline{RT}$ is tangent to circle C at T. Find the value of x.



Slide 191 (Answer) / 255

Example

Given: $\overline{RS}$ is tangent to circle C ... at T. Find the value of x.



Answer

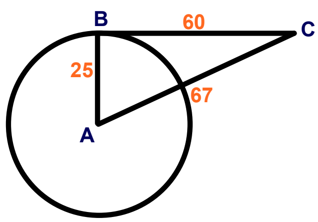
$3x + 4 = 28$
$3x = 24$
$x = 8$

This example addresses MP2

Slide 192 / 255

85 $\overline{AB}$ is a radius of circle A. Is $\overline{BC}$ tangent to circle A?

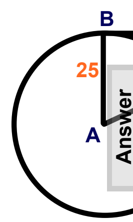
- Yes
- No



Slide 192 (Answer) / 255

85 $\overline{AB}$ is a radius of circle A. Is $\overline{BC}$ tangent to circle A?

- Yes
- No



Answer

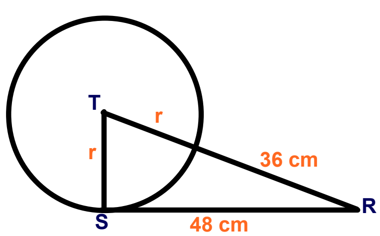
$67^2 = 25^2 + 60^2$
$4489 = 625 + 3600$
$4489 \neq 4225$

No

Slide 193 / 255

86 S is a point of tangency. Find the radius r of circle T.

- A 31.7
- B 60
- C 14
- D 3.5



Slide 193 (Answer) / 255

86 S is a point of tangency. Find the radius r of circle T.

- A 31.7
- B 60
- C 14
- D 3.5

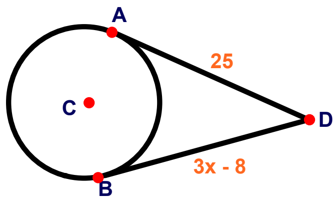
Answer

$(r + 36)^2 = r^2 + 48^2$
$r^2 + 72r + 1296 = r^2 + 2304$
$72r = 1008$
$r = 14$

C

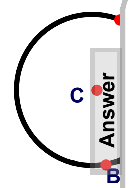
Slide 194 / 255

87 In circle C, $\overline{DA}$ is tangent at A and $\overline{DB}$ is tangent at B. Find the value of x.



Slide 194 (Answer) / 255

87 In circle C, $\overline{DA}$ is tangent at A and $\overline{DB}$ is tangent at B. Find the value of x.

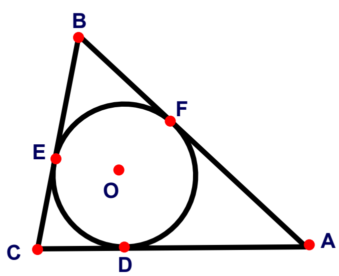


Answer

$3x - 8 = 25$
$3x = 33$
$x = 11$

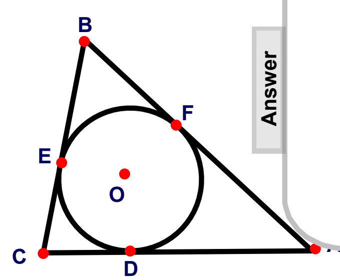
Slide 195 / 255

88 $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, and $\overleftrightarrow{CA}$ are tangents to circle O. $AD = 5$, $AC = 8$, and $BE = 4$. Find the perimeter of triangle ABC.



Slide 195 (Answer) / 255

88 $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, and $\overleftrightarrow{CA}$ are tangents to circle O. $AD = 5$, $AC = 8$, and $BE = 4$. Find the perimeter of triangle ABC.



Answer

$EC = 3$

$BF = 4$

$AF = 5$

$P = 3 + 3 + 4 + 4 + 5 + 5$

$P = 24$

Slide 196 / 255

Angles Intercepted by Tangents and Secants

Tangents and secants can form other angle relationships in circle. Recall the measure of an inscribed angle is 1/2 its intercepted arc.

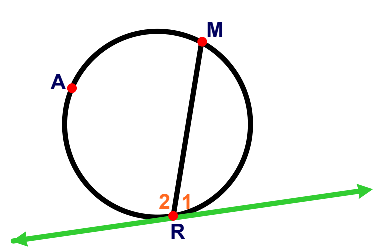
This can be extended to any angle that has its vertex on the circle.

This includes angles formed by two secants, a secant and a tangent, a tangent and a chord, and two tangents.

Slide 197 / 255

Theorem: A Tangent and a Chord

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc.



This is just a special case of the rule that the measure of an inscribed angle is equal to half the measure of its intercepted arc.

In this case,

m∠1 = $\frac{1}{2}$ mRM

m∠2 = $\frac{1}{2}$ mMAR

Slide 198 / 255

Theorem

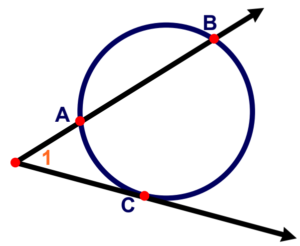
If a tangent and a secant, two tangents, or two secants intersect outside a circle, then the measure of the angle formed is half the difference of their intercepted arcs.

This results in three rules for calculating angles, but they are very similar to one another, so not hard to remember.

Especially, if you see one right after the other.

Slide 199 / 255

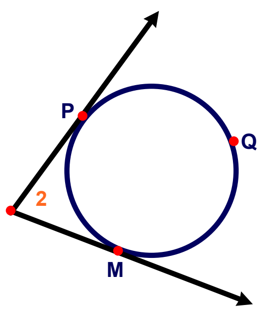
The Intercepted Arc of a Tangent and a Secant



$m\angle 1 = \frac{1}{2}(mBC - mAC)$

Slide 200 / 255

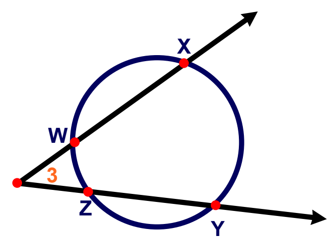
The Intercepted Arc of Two Tangents



$m\angle 2 = \frac{1}{2}(mPQM - mPM)$

Slide 201 / 255

The Intercepted Arc of Two Secants

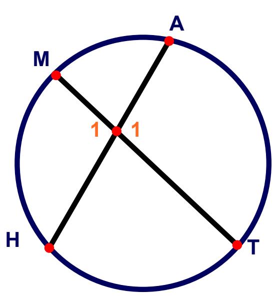


$m\angle 3 = \frac{1}{2}(mXY - mWZ)$

Slide 202 / 255

The Intercepted Arc of Two Chords

If two chords intersect inside a circle, then the measure of each angle is the average of the two intercepted arcs.



$m\angle 1 = \frac{1}{2}(mAT + mMH)$

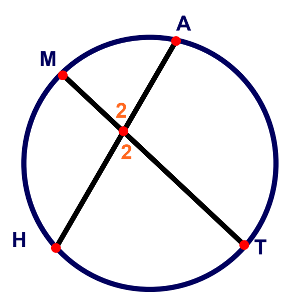
This is equally true for each pair of vertical angles.

The other pair of angles is on the next slide.

Slide 203 / 255

The Intercepted Arc of Two Chords

If two chords intersect inside a circle, then the measure of each angle is the average of the two intercepted arcs.

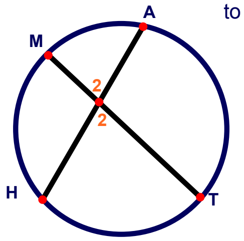


$m\angle 2 = \frac{1}{2}(mMA + mHT)$

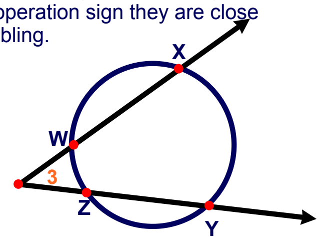
Slide 204 / 255

Chords vs. Secants and/or Tangents

A way to remember whether to add or subtract the two arcs that the segments intersect is to visualize the segments rotating on their intersection point to see which operation sign they are close to resembling.



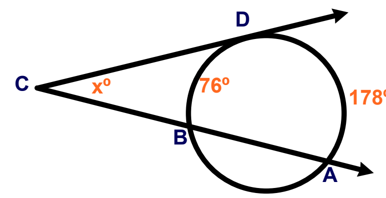
In the diagram above, if you manipulate the 2 segments at the intersection point, they make an addition sign, so add the arcs together before taking 1/2.



In the diagram above, if you manipulate the 2 rays at their intersection point, they overlap each other, making a subtraction sign, so subtract the arcs before taking 1/2.

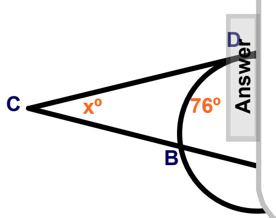
Slide 205 / 255

89 Find the value of x.



Slide 205 (Answer) / 255

89 Find the value of x.



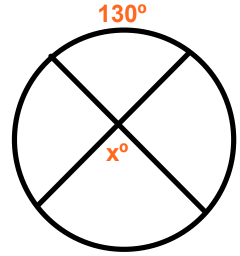
$$m\angle BCD = \frac{1}{2}(m\widehat{AD} - m\widehat{BD})$$

$$x = \frac{1}{2}(178^0 - 76^0)$$

$$x = 51^0$$

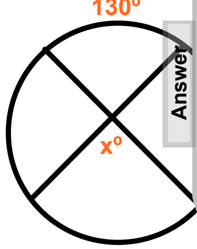
Slide 206 / 255

90 Find the value of x.



Slide 206 (Answer) / 255

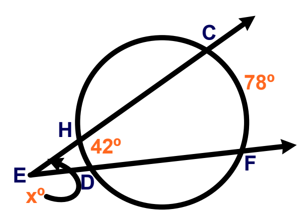
90 Find the value of x.



x = 1/2 (130⁰ + 156⁰)
x = 143⁰

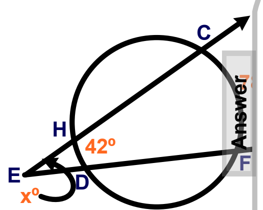
Slide 207 / 255

91 Find the value of x.



Slide 207 (Answer) / 255

91 Find the value of x.



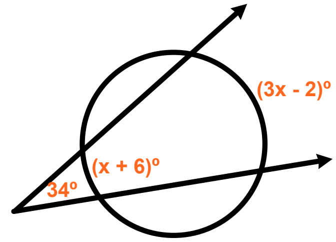
$$x = \frac{1}{2}(78 - 42)$$

$$x = \frac{1}{2}(36)$$

$$x = 18^\circ$$

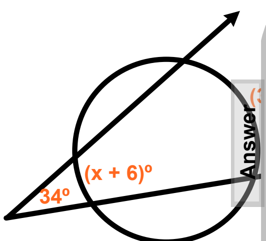
Slide 208 / 255

92 Find the value of x.



Slide 208 (Answer) / 255

92 Find the value of x.



$$34 = \frac{1}{2}(3x - 2 - (x + 6))$$

$$34 = \frac{1}{2}(2x - 8)$$

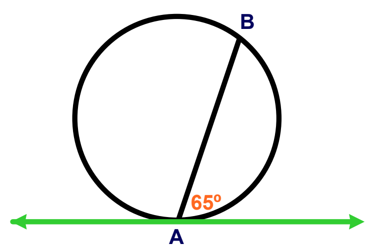
$$68 = 2x - 8$$

$$76 = 2x$$

$$38 = x$$

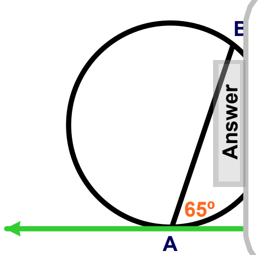
Slide 209 / 255

93 Find mAB.



Slide 209 (Answer) / 255

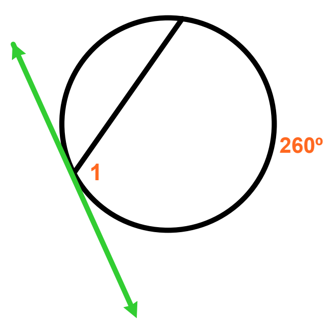
93 Find mAB.



$m\overset{\frown}{AB} = 130^\circ$

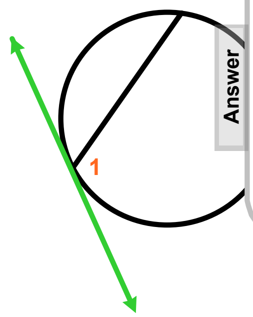
Slide 210 / 255

94 Find $m\angle 1$.



Slide 210 (Answer) / 255

94 Find m∠1.

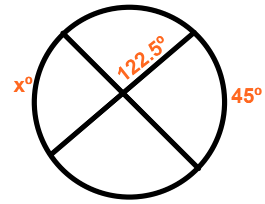


Answer

$$m\angle 1 = 130^\circ$$

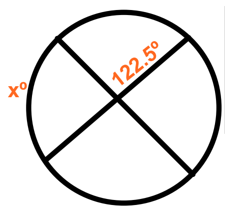
Slide 211 / 255

95 Find the value of x.



Slide 211 (Answer) / 255

95 Find the value of x.



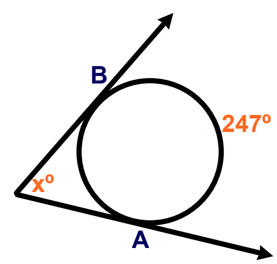
Answer

$$57.5^\circ = \frac{1}{2}(45 + x)$$
$$115 = 45 + x$$
$$70^\circ = x$$

Slide 212 / 255

Example

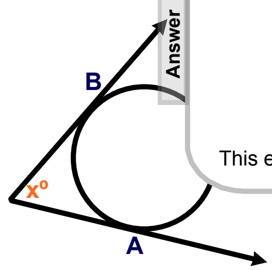
To find the angle, you need the measure of both intercepted arcs. First, find the measure of the minor arc mAB. Then we can calculate the value of the variable x°.



Slide 212 (Answer) / 255

Example

To find the angle, you need the measure of both intercepted arcs. First, find the measure of t... can calculate the value of the varia...



Answer

First find the minor arc.

$$360^0 - 247^0 = 113^0$$

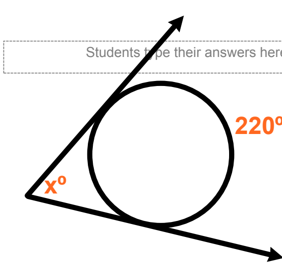
$$x = \frac{1}{2}(247 - 113)$$
$$x = \frac{1}{2}(134)$$
$$x = 67^\circ$$

This example addresses MP2

Slide 213 / 255

96 Find the value of x.

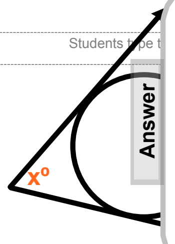
Students type their answers here



Slide 213 (Answer) / 255

96 Find the value of x

Students type t



Answer

First find the minor arc.

$$360^0 - 220^0 = 140^0$$

$$x = \frac{1}{2}(220 - 140)$$

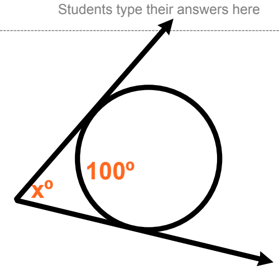
$$x = \frac{1}{2}(80)$$

$$x = 40^\circ$$

Slide 214 / 255

97 Find the value of x.

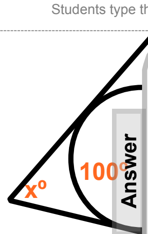
Students type their answers here



Slide 214 (Answer) / 255

97 Find the value of x.

Students type their answers here



Answer

First find the major arc.

$$360^0 - 100^0 = 260^0$$

$$x = \frac{1}{2}(260 - 100)$$

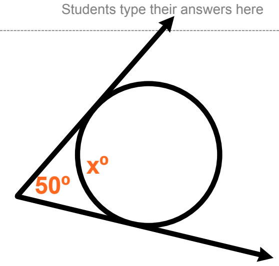
$$x = \frac{1}{2}(160)$$

$$x = 80^\circ$$

Slide 215 / 255

98 Find the value of x

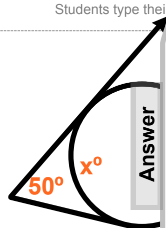
Students type their answers here



Slide 215 (Answer) / 255

98 Find the value of x

Students type their a



Answer

Find the major arc.

$$(360 - x)^\circ$$

$$50 = \frac{1}{2}(360 - x - x)$$

$$50 = \frac{1}{2}(360 - 2x)$$

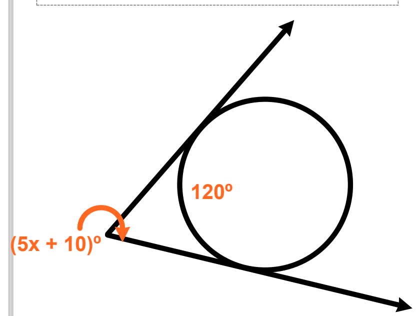
$$50 = 180^\circ - x$$

$$x = 130^\circ$$

Slide 216 / 255

99 Find the value of x.

Students type their answers here



Slide 216 (Answer) / 255

99 Find the value of x.

Students ty

Answer

Find the major arc.

$$360-120=240$$

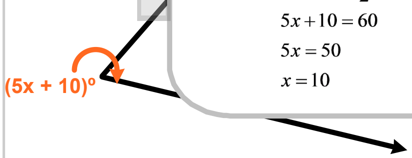
$$5x+10=\frac{1}{2}(240-120)$$

$$5x+10=\frac{1}{2}(120)$$

$$5x+10=60$$

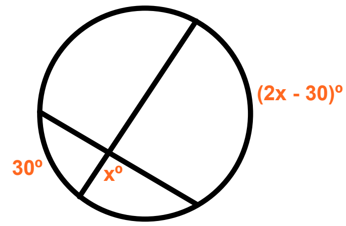
$$5x=50$$

$$x=10$$



Slide 217 / 255

100 Find the value of x.



Slide 217 (Answer) / 255

Slide 218 / 255

Released PARCC Exam Question

The following question from the released PARCC EOY exam uses what we just learned and combines it with what we learned earlier to create a challenging question.

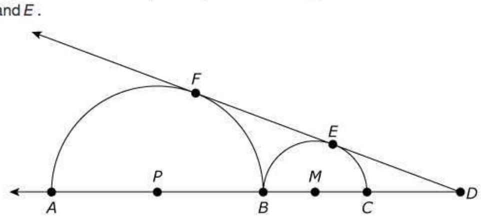
Please try it on your own.

Then we'll go through the process we used to solve it.

Slide 219 / 255

Question 5/25 **Topic: Tangents**

The figure shows two semicircles with centers P and M. The semicircles are tangent to each other at point B, and $\overrightarrow{DE}$ is tangent to both semicircles at F and E.



If $PB = BC = 6$, what is ED?

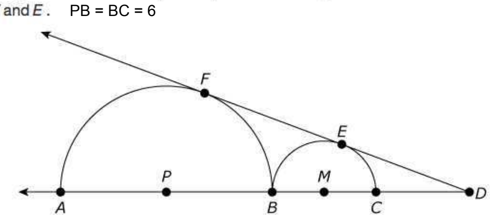
- A. 6
- B. $\sqrt{48}$
- C. 8

Slide 220 / 255

101 What have we learned that will help solve this problem?

- A A tangent and intersecting radius are perpendicular
- B All radii of a circle are congruent
- C Similar triangles have proportional corresponding parts
- D All of the above

The figure shows two semicircles with centers P and M. The semicircles are tangent to each other at point B, and $\overrightarrow{DE}$ is tangent to both semicircles at F and E. PB = BC = 6



Slide 220 (Answer) / 255

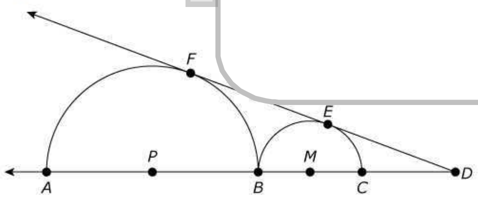
101 What have we learned that will help solve this problem?

- A A tangent and intersecting radius are perpendicular
- B All radii of a circle a
- C Similar triangles ha s
- D All of the above

Answer

D All of the above

The figure shows two semicircles w
tangent to each other at point *B*, a
F and *E*. PB = BC = 6

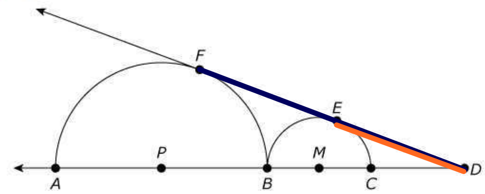


Slide 221 / 255

Question 5/25 **Topic: Tangents**

First, draw a radius from the center of each circle to the point where the given tangent meets the circle, points F and P.

The figure shows two semicircles with centers *P* and *M*. The semicircles are tangent to each other at point *B*, and $\overrightarrow{DE}$ is tangent to both semicircles at *F* and *E*. PB = BC = 6

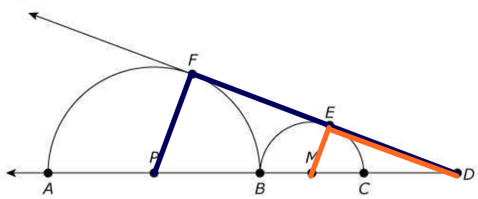


Slide 222 / 255

Question 5/25 **Topic: Tangents**

Now draw ΔFDP and ΔEDM.

The figure shows two semicircles with centers *P* and *M*. The semicircles are tangent to each other at point *B*, and $\overrightarrow{DE}$ is tangent to both semicircles at *F* and *E*. PB = BC = 6

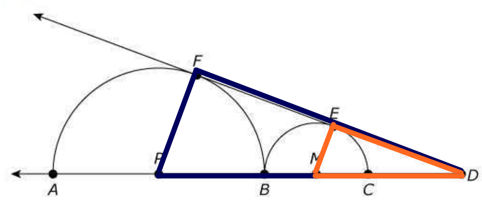


Slide 223 / 255

102 What can you see from this drawing?

- A ΔFDP and ΔEDM are right triangles
- B ΔFDP and ΔEDM are similar
- C The corresponding sides of FDP and EDM are proportional
- D All of the above

The figure shows two semicircles with centers *P* and *M*. The semicircles are tangent to each other at point *B*, and $\overrightarrow{DE}$ is tangent to both semicircles at *F* and *E*. PB = BC = 6



Slide 223 (Answer) / 255

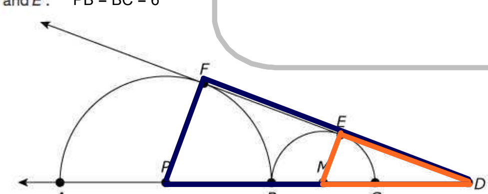
102 What can you see from this drawing?

- A ΔFDP and ΔEDM a
- B ΔFDP and ΔEDM
- C The correspondinc proportional
- D All of the above

Answer

D

The figure shows two semicircle
tangent to each other at point *B*,
F and *E*. PB = BC = 6

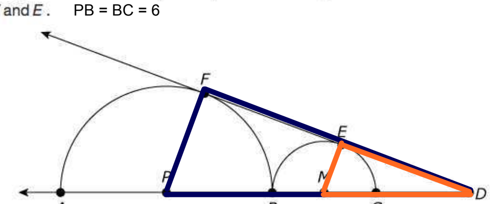


Slide 224 / 255

103 What are the lengths of FP and EM?

- A FP = 6 and EM = 6
- B FP = 3 and EM = 3
- C FP = 6 and EM = 3
- D They can't be found from this

The figure shows two semicircles with centers *P* and *M*. The semicircles are tangent to each other at point *B*, and $\overrightarrow{DE}$ is tangent to both semicircles at *F* and *E*. PB = BC = 6



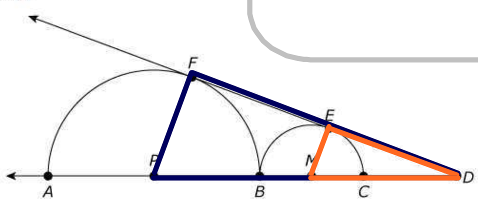
103 What are the lengths of FP and EM?

- A FP = 6 and EM = 6
- B FP = 3 and EM = 3
- C FP = 6 and EM = 3
- D They can't be fou

Answer

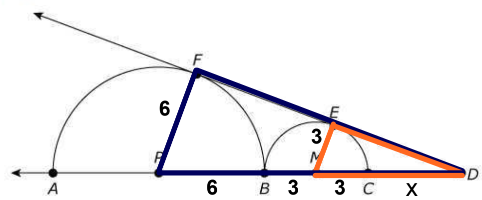
C

The figure shows two semicircles with c
tangent to each other at point B, and $\overrightarrow{I}$
F and E. PB = BC = 6



104 What is the ratio of proportionality, k, between ΔFDP and ΔEDM?

The figure shows two semicircles with centers P and M. The semicircles are tangent to each other at point B, and $\overrightarrow{DE}$ is tangent to both semicircles at F and E. PB = BC = 6

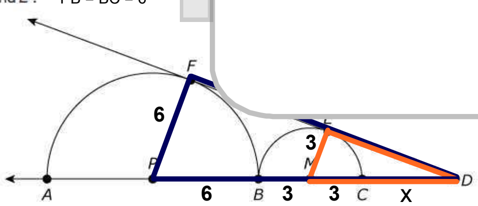


104 What is the ratio of proportionality, k, between ΔFDP and ΔEDM?

Answer

2

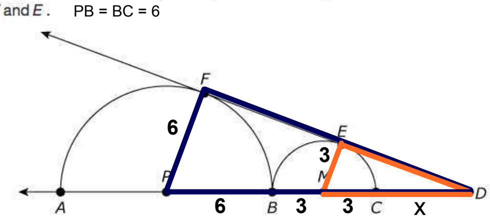
The figure shows two semicircle
tangent to each other at point B
F and E. PB = BC = 6



105 Write an expression for the length of $\overline{PD}$.

- A PD = 12
- B PD = 12 + x
- C PD = 6 + x
- D It can't be found from this

The figure shows two semicircles with centers P and M. The semicircles are tangent to each other at point B, and $\overrightarrow{DE}$ is tangent to both semicircles at F and E. PB = BC = 6



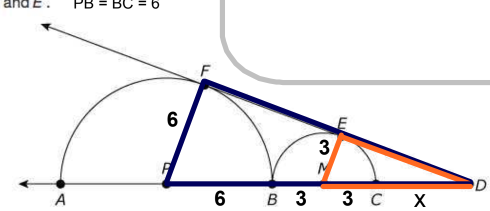
105 Write an expression for the length of $\overline{PD}$.

- A PD = 12
- B PD = 12 + x
- C PD = 6 + x
- D It can't be found f

Answer

B

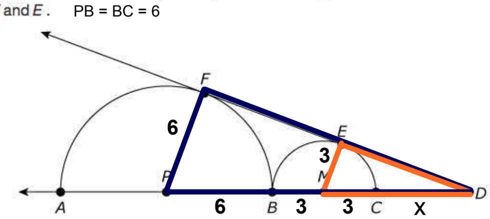
The figure shows two semicirc
tangent to each other at point B,
F and E. PB = BC = 6



106 Write an expression for the length of $\overline{MD}$.

- A MD = 12
- B MD = 3 + x
- C PD = 6 + x
- D It can't be found from this

The figure shows two semicircles with centers P and M. The semicircles are tangent to each other at point B, and $\overrightarrow{DE}$ is tangent to both semicircles at F and E. PB = BC = 6



Slide 227 (Answer) / 255

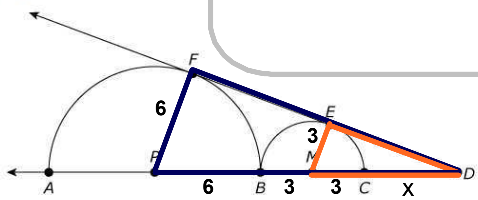
106 Write an expression for the length of $\overline{MD}$.

- A MD = 12
- B MD = 3 + x
- C PD = 6 + x
- D It can't be found fr

Answer

B

The figure shows two semicirc
tangent to each other at point B ,
F and E . PB = BC = 6

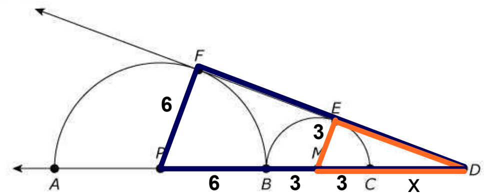


Slide 228 / 255

107 Write a proportion relating MD and PD.

- A $\frac{6}{3} = \frac{12 + x}{3 + x}$
- B $\frac{3}{6} = \frac{12 + x}{3 + x}$
- C $\frac{6}{3} = \frac{12 + x}{3}$
- D It can't be found from this

The figure shows two semicircles with centers P and M . The semicircles are tangent to each other at point B , and $\overrightarrow{DE}$ is tangent to both semicircles at F and E . PB = BC = 6



Slide 228 (Answer) / 255

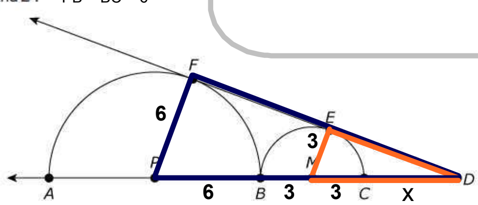
107 Write a proportion relati

- A $\frac{6}{3} = \frac{12 + x}{3 + x}$
- B $\frac{3}{6} = \frac{12 + x}{3 + x}$
- C $\frac{6}{3} = \frac{12 + x}{3}$
- D It can't be found fr

Answer

A

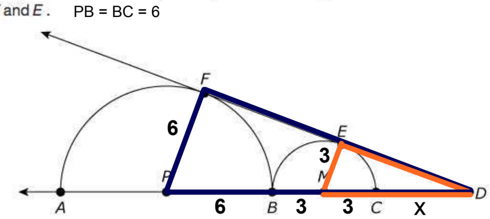
The figure shows two semicircles
tangent to each other at point B ,
F and E . PB = BC = 6



Slide 229 / 255

108 What is the length of $\overline{MD}$?

The figure shows two semicircles with centers P and M . The semicircles are tangent to each other at point B , and $\overrightarrow{DE}$ is tangent to both semicircles at F and E . PB = BC = 6



Slide 229 (Answer) / 255

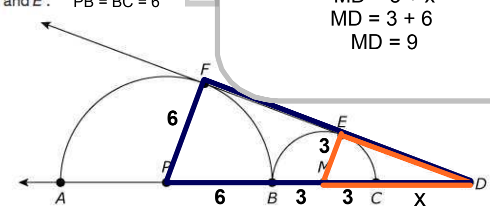
108 What is the length of $\overline{MD}$?

Answer

$$\frac{6}{3} = \frac{12 + x}{3 + x}$$

6(3 + x) = 3(12 + x)
18 + 6x = 36 + 3x
3x = 18
x = 6
MD = 3 + x
MD = 3 + 6
MD = 9

The figure shows two semicirc
tangent to each other at point B ,
F and E . PB = BC = 6

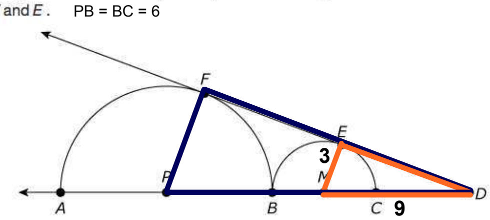


Slide 230 / 255

109 Write an equation to find ED.

- A $ED^2 = 3^2 + 9^2$
- B $9^2 = ED^2 - 3^2$
- C $9^2 = ED^2 + 3^2$
- D It can't be found from this

The figure shows two semicircles with centers P and M . The semicircles are tangent to each other at point B , and $\overrightarrow{DE}$ is tangent to both semicircles at F and E . PB = BC = 6

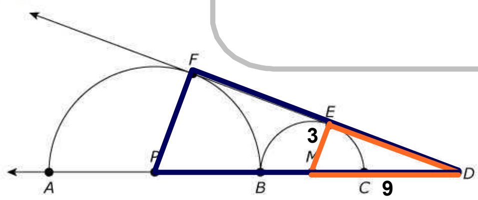


Slide 230 (Answer) / 255

109 Write an equation to find ED.

- A $ED^2 = 3^2 + 9^2$
- B $9^2 = ED^2 - 3^2$
- C $9^2 = ED^2 + 3^2$
- D It can't be found

The figure shows two semicircles ... tangent to each other at point B, ... F and E. PB = BC = 6



Answer

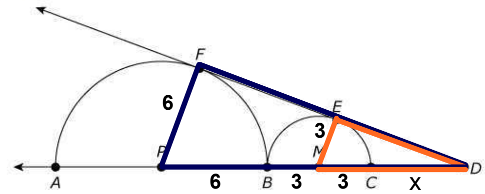
C

Slide 231 / 255

110 What is the length of $\overline{ED}$?

- A 6
- B $\sqrt{48}$
- C 8
- D $\sqrt{72}$

The figure shows two semicircles with centers P and M. The semicircles are tangent to each other at point B, and $\overrightarrow{DE}$ is tangent to both semicircles at F and E. PB = BC = 6

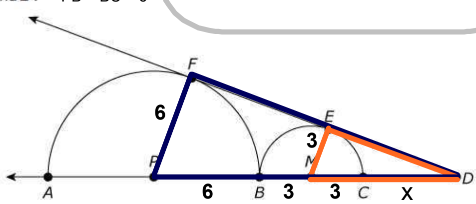


Slide 231 (Answer) / 255

110 What is the length of $\overline{ED}$?

- A 6
- B $\sqrt{48}$
- C 8
- D $\sqrt{72}$

The figure shows two semi... tangent to each other at p... F and E. PB = BC = 6



Answer

D

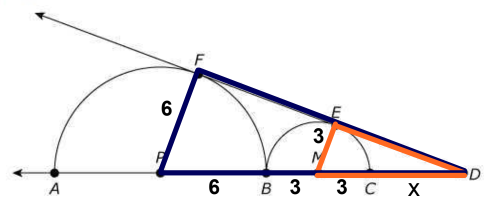
$3^2 + y^2 = 9^2$
$9 + y^2 = 81$
$y^2 = 72$
$ED = \sqrt{72}$

Slide 232 / 255

111 What would be a good way to check your answer.

- A Solve for FD and see if it is equal to ED.
- B Solve for PD and see if it is equal to ED.
- C Solve for FD and see if it is double ED.
- D Solve for FD and see if it is half ED.

The figure shows two semicircles with centers P and M. The semicircles are tangent to each other at point B, and $\overrightarrow{DE}$ is tangent to both semicircles at F and E. PB = BC = 6

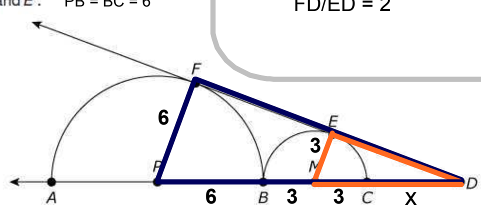


Slide 232 (Answer) / 255

111 What would be a good way to check your answer.

- A Solve for FD and s...
- B Solve for PD and s...
- C Solve for FD and s...
- D Solve for FD and s...

The figure shows two semicirc... tangent to each other at point B, ... F and E. PB = BC = 6



Answer

C

$6^2 + z^2 = 18^2$
$36 + z^2 = 324$
$z^2 = 288$
$FD = \sqrt{288}$

$FD/ED = \sqrt{288}/\sqrt{72}$
$FD/ED = \sqrt{4}$
$FD/ED = 2$

Slide 233 / 255

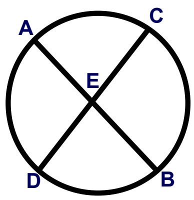
Segments & Circles

Return to the table of contents

Slide 234 / 255

Theorem

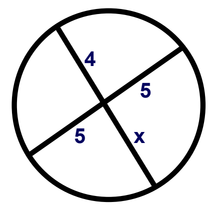
If two chords intersect inside a circle, then the products of the measures of the segments of the chords are equal.



$EA \bullet EB = EC \bullet ED$

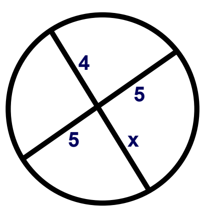
Slide 235 / 255

112 Find the value of x.



Slide 235 (Answer) / 255

112 Find the value of x.

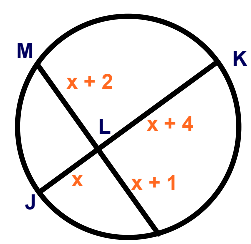


Answer

$$4 \bullet x = 5 \bullet 5$$
$$4x = 25$$
$$x = 6\frac{1}{4}$$

Slide 236 / 255

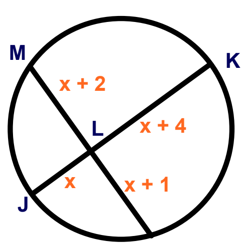
113 Find the length of $\overline{ML}$.



Answer

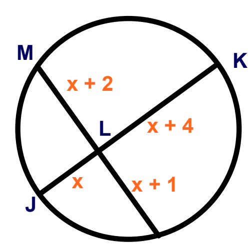
Slide 237 / 255

114 Find the length of $\overline{JK}$.



Slide 237 (Answer) / 255

114 Find the length of $\overline{JK}$.



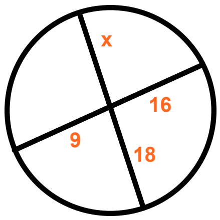
Answer

$$(x+2)(x+1) = x(x+4)$$
$$x^2 + 3x + 2 = x^2 + 4x$$
$$3x + 2 = 4x$$
$$2 = x$$

JK = 2 + (2 + 4) = 8

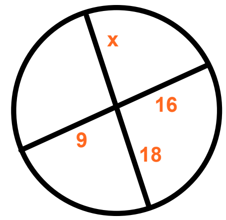
Slide 238 / 255

115 Find the value of x.



Slide 238 (Answer) / 255

115 Find the value of x.



Answer

$9 \times 16 = 18 \times x$

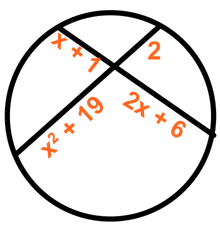
$144 = 18x$

$8 = x$

Slide 239 / 255

116 Find the value of x.

- A -2
- B 4
- C 5
- D 6



Slide 239 (Answer) / 255

116 Find the value of x.

- A -2
- B 4
- C 5
- D 6

Answer

B

$(2x + 6)(x + 1) = 2(x^2 + 19)$

$2x^2 + 8x + 6 = 2x^2 + 38$

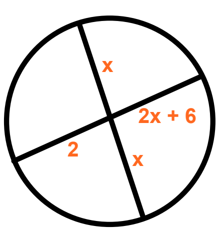
$8x + 6 = 38$

$8x = 32$

$x = 4$

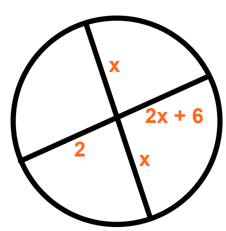
Slide 240 / 255

117 Find the value of x.



Slide 240 (Answer) / 255

117 Find the value of x.

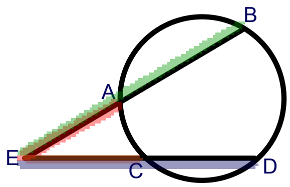


Answer

x = 6

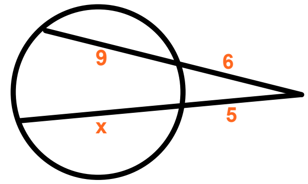
Secant Segment Theorem

If two secant segments are drawn to a circle from an external point, then the product of the measures of one secant segment and its external secant segment equals the product of the measures of the other secant segment and its external secant segment.

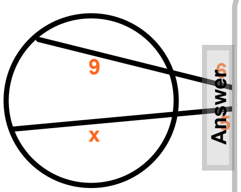


$$EA \bullet EB = EC \bullet ED$$

118 Find the value of x.

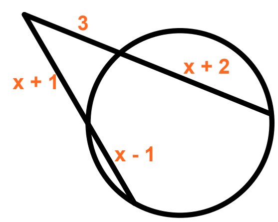


118 Find the value of x.

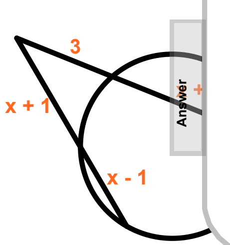


6(9+6) = 5(x+5)
6(15) = 5x + 25
90 = 5x + 25
5x = 65
x = 13

119 Find the value of x.



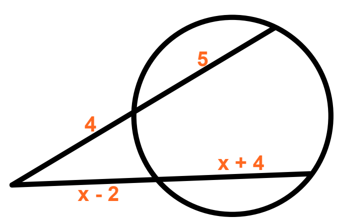
119 Find the value of x.



$$(x+1)(2x) = 3(x+5)$$
$$2x^2 + 2x = 3x + 15$$
$$2x^2 - x - 15 = 0$$
$$(x-3)(2x+5)$$
$$x = 3$$

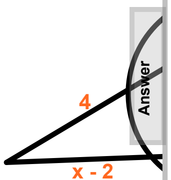
Note: x = -2.5 (other algebraic solution) is not possible in Geometry because you can't have negative segment lengths.

120 Find the value of x.



Slide 244 (Answer) / 255

120 Find the value of x.



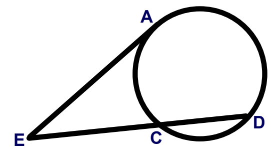
$$4(9) = (x-2)(2x+2)$$
$$36 = 2x^2 - 4x + 2x - 4$$
$$36 = 2x^2 - 2x - 4$$
$$0 = 2x^2 - 2x - 40$$
$$0 = 2(x-5)(x+4)$$
$$x = 5$$

Note: x = -4 (other algebraic solution) is not possible in Geometry because you can't have negative segment lengths.

Slide 245 / 255

Tangent-Secant Theorem

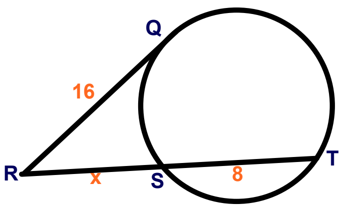
If a tangent segment and a secant segment are drawn to a circle from an external point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment.



$$EA^2 = EC \bullet ED$$

Slide 246 / 255

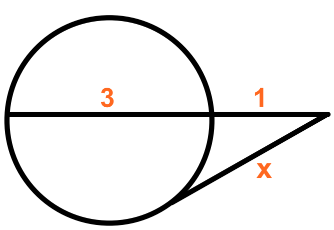
121 Find the length of $\overline{RS}$.



Slide 246 (Answer) / 255

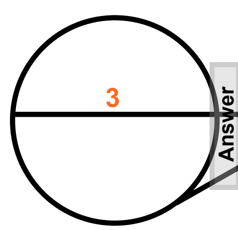
Slide 247 / 255

122 Find the value of x.



Slide 247 (Answer) / 255

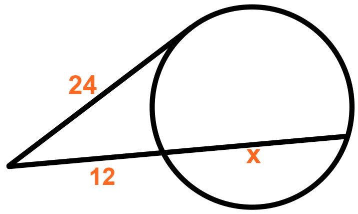
122 Find the value of x.



$$1(4) = x^2$$
$$4 = x^2$$
$$\pm 2 = x$$
$$x = 2$$

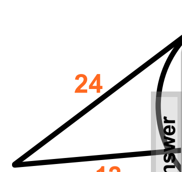
Slide 248 / 255

123 Find the value of x.



Slide 248 (Answer) / 255

123 Find the value of x.



Answer

$12(12+x)=24^2$

$144+12x=576$

$12x=432$

$x=36$

Slide 249 / 255

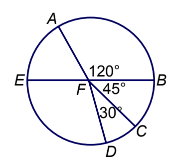
Questions from Released PARCC Examination

Return to the table of contents

Slide 250 / 255

Question 3/7 **Topic: Angles, Arcs and Arc Lengths**

—



Drag and drop each arc length to its subtended central angle.

$\frac{\pi}{2}$ π 2π $\frac{3\pi}{4}$

Subtended Central Angle	Arc Length
∠AFB	
∠BFC	
∠CFD	
∠AFE	

Slide 250 (Answer) / 255

Question 3/7 **Topic: Angles, Arcs and Arc Lengths**

—

Drag and drop eac
to its subtended c

$\frac{\pi}{2}$ π 2π

Answer

$arc\ length(s)=\frac{n°}{360°}\times 2\pi r$

$radius = 3\ units$
$n = Subtended\ Central\ Angle\ given$

$arc\ length\angle AFB=\frac{120°}{360°}\times(2)(3)\pi = 2\pi$

$arc\ length\angle BFC=\frac{45°}{360°}\times(2)(3)\pi = \frac{3\pi}{4}$

$arc\ length\angle CFD=\frac{30°}{360°}\times(2)(3)\pi = \frac{\pi}{2}$

$\angle AFE = 180° - 120° = 60°$

$arc\ length\angle AFE=\frac{60°}{360°}\times(2)(3)\pi = \pi$

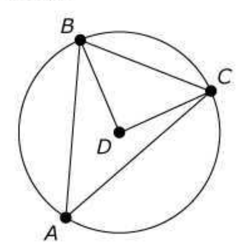
Subtended Central Angle	Arc Length
∠AFB	2π
∠BFC	$\frac{3\pi}{4}$
∠CFD	$\frac{\pi}{2}$
∠AFE	π

∠AFE

Slide 251 / 255

Question 1/25 **Topic: Inscribed Angles**

The figure shows $\triangle ABC$ inscribed in circle *D*.



If $m\angle CBD = 44°$, find $m\angle BAC$.

Enter your answer in the box.

degrees

Slide 251 (Answer) / 255

Question 1/25 **Topic: Inscribed Angles**

The figure shows $\triangle ABC$ inscribed in circle D.

Answer

$m\angle BAC$ = 46 degrees

If $m\angle CBD = 44^\circ$, find $m\angle$

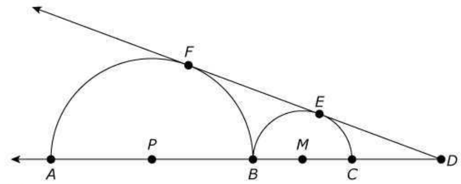
Enter your answer in the box.

______ degrees

Slide 252 / 255

Question 5/25 **Topic: Tangents**

The figure shows two semicircles with centers P and M. The semicircles are tangent to each other at point B, and $\overrightarrow{DE}$ is tangent to both semicircles at F and E.



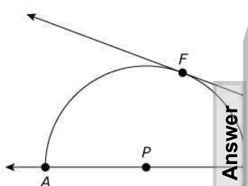
If $PB = BC = 6$, what is ED?

- A. 6
- B. $\sqrt{48}$
- C. 8
- D. $\sqrt{72}$

Slide 252 (Answer) / 255

Question 5/25 **Topic: Tangents**

The figure shows two semicircles with centers P and M. The semicircles are tangent to each other at point B, and $\overrightarrow{DE}$ is tangent to both semicircles at F and E.



If $PB = BC = 6$, what is ED?

- A. 6
- B. $\sqrt{48}$
- C. 8
- D. $\sqrt{72}$

Answer

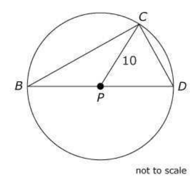
If $PB = BC = 6$, what is ED?

- A. 6
- B. $\sqrt{48}$
- C. 8
- ● D. $\sqrt{72}$

Slide 253 / 255

Question 17/25 **Topic: Inscribed Angles**

The figure shows a circle with center P, a diameter $\overline{BD}$, and inscribed $\triangle BCD$. $PC = 10$. Let $m\angle CBD = (x)^\circ$ and $m\angle BCD = (x+54)^\circ$.



Part A

Find the value of x. Enter your answer in the box.

$x =$ ______

Part B

Select from the drop-down menus to correctly complete the sentence.

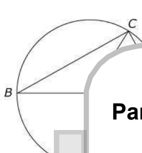
The length of $\overline{CD}$ is ______ because ______.

a. 10	d. ΔCPD is equilateral
b. less than 10	e. m∠CPD < 60°
c. greater than 10	f. m∠CPD > 60°

Slide 253 (Answer) / 255

Question 17/25 **Topic: Inscribed Angles**

The figure shows a circle with center P, a diameter $\overline{BD}$, and inscribed $\triangle BCD$. $PC = 10$. Let $m\angle CBD = (x)^\circ$ and $m\angle BCD = (x+54)^\circ$.



Part A

Find the value of x. Enter your answer in the box.

$x =$ ______

Part B

Select from the drop-down menus to correctly complete the sentence.

The length of $\overline{CD}$ is ______ because

a. 10	d. ΔCPD is equilateral
b. less than 10	e. m∠CPD < 60°
c. greater than 10	f. m∠CPD > 60°

Answer

Part A: $x =$ **36**

Part B:

The length of $\overline{CD}$ is **c. greater than 10** because **f. m∠CPD > 60°**.

Slide 254 / 255

Question 22/25 **Topic: Inscribed Angles**

Point B is the center of a circle, and $\overline{AC}$ is a diameter of the circle. Point D is a point on the circle different from A and C.

Part A

Drag and drop the following choices into the boxes to indicate which statements are always true, sometimes true or never true.

Always True | Sometimes True | Never True

Statements	
$AD > CD$	
$m\angle CBD = \frac{1}{2}(m\angle CAD)$	
$m\angle CBD = 90^\circ$	
$m\angle ABD = 2(m\angle CBD)$	

Question 22/25

Topic: Inscribed Angles

Point B is the center of a circle, and $\overline{AC}$ is a diameter of the circle. Point D is a point on the circle different from A and C .

Part A

Drag and drop the following choices i
sometimes true or never true.

Always True | S

Statemen	
$AD > C$	
$m\angle CBD = \frac{1}{2}(m\angle$	
$m\angle CBD = 90$	
$m\angle ABD = 2(m\angle CBD)$	

Answer

Part A:

Statements	
$AD > CD$	Sometimes True
$m\angle CBD = \frac{1}{2}(m\angle CAD)$	Never True
$m\angle CBD = 90°$	Sometimes True
$m\angle ABD = 2(m\angle CBD)$	Sometimes True

Question 22/25

Topic: Inscribed Angles

Part B

If $m\angle BDA = 20^\circ$, what is $m\angle CBD$?

- A. 20°
- B. 40°
- C. 70°
- D. 140°

Question 22/25

Topic: Inscribed Angles

Part B

If $m\angle BDA = 20$

- A. 20°
- B. 40°
- C. 70°
- D. 140°

Answer

Part B:

- A. 20°
- B. 40° (selected)
- C. 70°
- D. 140°