



**NEW JERSEY CENTER  
FOR TEACHING & LEARNING**

## Progressive Mathematics Initiative®

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**NEW JERSEY CENTER  
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## 8th Grade

### The Number System and Mathematical Operations - Part 1

2015-11-20

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## Table of Contents

- Addition, Natural Numbers & Whole Numbers**
- Addition, Subtraction and Integers**
- Multiplication and Division of Integers**
- Operations with Rational Numbers**
- Converting Repeating Decimals to Fractions**
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- Glossary & Standards**

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## Table of Contents

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Addition, Subtr  
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Teacher Notes

**Vocabulary Words are bolded in the presentation. The text box the word is in is then linked to the page at the end of the presentation with the word defined on it.**

Click on a topic to go to that section.

Slide 3 (Answer) / 206

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## Addition, Natural Numbers & Whole Numbers

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Slide 4 / 206

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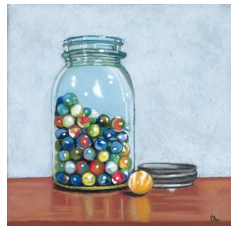
## Natural Numbers

The first numbers developed were the **Natural Numbers**, also called the Counting Numbers.

1, 2, 3, 4, 5, ...

The three dots, (...), means that these numbers continue forever: there is no largest counting number..

Think of counting objects as you put them in a container those are the counting numbers.



Slide 5 / 206

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### Natural Numbers

Natural numbers were used before there was history.

All people use them.

This "counting stick" was made more than 35,000 years ago and was found in Lebombo, Swaziland.

The cuts in this bone record the number "29."



<http://www.taneter.org/math.html>

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### Natural Numbers and Addition

They were, and are, used to count objects

- > goats,
- > bales,
- > bottles,
- > etc.

Drop a stone in a jar, or cut a line in a stick, every time a goat walks past.

That jar or stick is a record of the number.

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### Numbers versus Numerals

Numbers exist even without a numeral, such as the number indicated by the cuts on the Lebombo Bone.

A numeral is the name we give a number in our culture.

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## Numbers versus Numerals



If asked how many tires my car has, I could hand someone the above marbles.

That number is represented by:

- 4 in our Base 10 numeral system
- IV in the Roman numeral system
- 100 in the Base 2 numeral system

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## Whole Numbers

Adding zero to the Counting Numbers gives us the **Whole Numbers**.

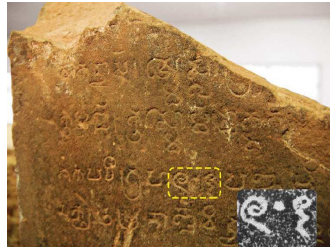
0, 1, 2, 3, 4, ...

Counting numbers were developed more than 35,000 years ago.

It took 34,000 more years to invent zero.

This the oldest known use of zero (the dot), about 1500 years ago.

It was found in Cambodia and the dot is for the zero in the year 605.



<http://www.smithsonianmag.com/history/origin-number-zero-180953392/?no-ist>

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## Why zero took so long to Invent

Horses versus houses.

	<b>3</b>	
	<b>2</b>	
	<b>1</b>	
	<b>0</b>	

zero horses = zero houses

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## Why zero took so long

Would I tell someone I have a herd of zero goats?  
Or a garage with zero cars?  
Or that my zero cars have zero tires?  
Zero just isn't a natural number, but it is a whole number.

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## Addition, Subtraction and Integers

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## Addition and Subtraction

The simplest mathematical operation is addition.  
The inverse of addition is subtraction.  
Two operations are inverses if one "undoes" the other.  
**Inverse operations** is a very important concept, and applies to all mathematics.

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## Addition

The simplest mathem  
The inverse of add  
Two operations are  
**Inverse operations** i  
all mathematics.

Math Practice

**MP.5: Use appropriate tools strategically.**

**When reviewing addition, subtraction, multiplication, and division, number lines will be used to aid the students.**

**If they ever get stuck, ask: How could you use a number line to show your thinking?**

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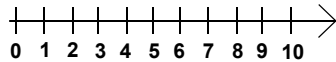
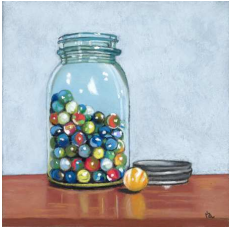
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## Addition and Subtraction

Each time a marble is dropped in a jar we are doing addition.

Each time a marble is removed from a jar, we are doing subtraction.

A number line allows us to think of addition in a new way.



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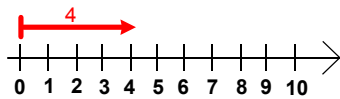
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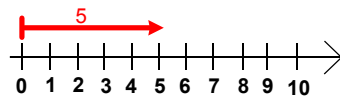
## Adding Whole Numbers

Let's find the sum of 4 and 5 on a number line.

The number +4 is four steps to the right. Starting at 0, takes you to 4.



The number +5 is five steps to the right. Starting at 0, takes you to 5.



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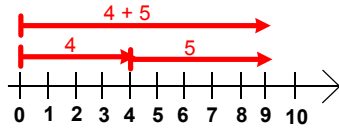
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## Adding Whole Numbers

To find the sum "4 + 5" start at zero and take four steps to the right for the first number.

Then, starting where you ended after those first steps, take five more steps to the right, to represent adding five.

If we were walking, we could look down and see we are standing at 9.



Therefore,  $4 + 5 = 9$ .

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## The Commutative Property of Addition

A mathematical operation is commutative if the order doesn't matter.

In this case, addition would be commutative if  $4 + 5 = 5 + 4$

Let's test that.

We found that  $4 + 5 = 9$

How about  $5 + 4$ ?

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## The Commutative Property of Addition

A mathematical operation is commutative if the order doesn't matter.

In this case, a

Let's test that

We found that

How about

**This slide and the next 3 slides are a proof of the commutative property through the use of number lines.**

**MP.2: Reasoning quantitatively and abstractly.**

**MP.3: Construct viable arguments and critique the reasoning of others.**

**Ask on the 3rd slide: Can you create your own pair of numbers and prove that  $a + b = b + a$ ?**

Math Practice

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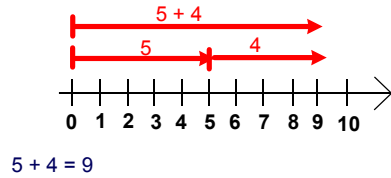
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### The Commutative Property of Addition

First, take five steps to the right.

Then, starting where you ended, take four more steps to the right.

Once more, we could look down and see we are standing at 9.




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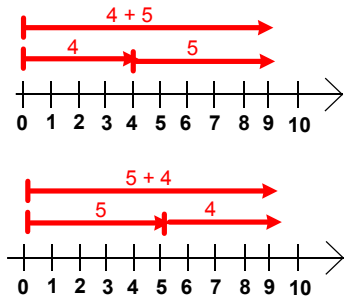
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### The Commutative Property of Addition

So,  $4 + 5 = 5 + 4$

Addition is commutative in this case.




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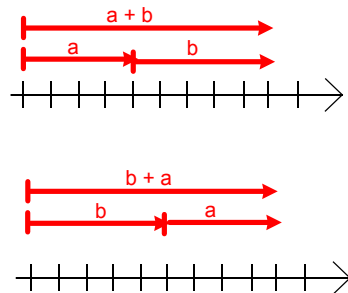
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### The Commutative Property of Addition

But there's nothing special about these numbers.

This is true for any numbers:

$$a + b = b + a$$




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## Inverse Operations

Operations are "inverses" when one of them undoes what the other does.

What would undo adding 5?

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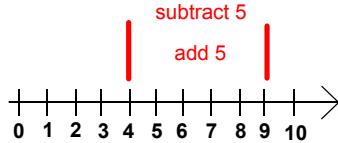
## Addition and Subtraction are Inverses

Addition and subtraction are inverses.

Adding a number and then subtracting that same number leaves you where you started.

Starting at 4, add 5 and then subtract 5.

You end up where you started.




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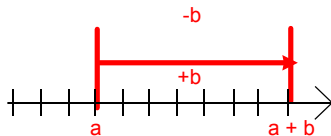
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## Addition and Subtraction are Inverses

This is true for any two numbers.

Start with  $a$ , then add  $b$ , then subtract  $b$ .

You end up with the number you began with:  $a$ .




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## Inverse Operations

We started with the addition question: what number results when we add 5 and 4? The answer is 9.

$$5 + 4 = 9$$

That leads to two new related subtraction questions.

Starting with 9, what number do we get when we subtract 5?

$$9 - 5 = 4$$

Starting with 9, what number do we get when we subtract 4.

$$9 - 4 = 5$$

Subtraction was invented to undo addition, but it now can be used to ask new questions.

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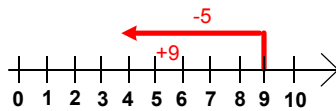
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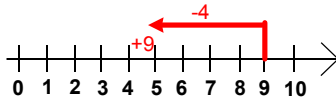
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## Subtracting Whole Numbers

Here's what  $9-5$  looks like on the number line



And, here's  $9-4$




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## We Need More Numbers

Subtraction was invented to undo addition.

But this new operation allows us to ask new questions.

And, the number system to that point couldn't provide answers.

For example:

What is the result of subtracting 7 from 4?

$$4 - 7 = ?$$

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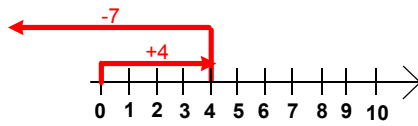
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## Subtracting Whole Numbers

$$4 - 7 = ?$$

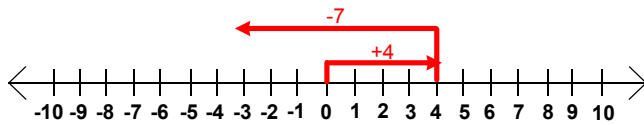


There was no answer to questions like this in the whole number systems.

Which was all there was until about 500 years ago.

## Negative Numbers

$$4 - 7 = ?$$



This led to the invention of negative numbers.

They were called negative from the Latin "negare" which means "to deny," since people denied that such numbers could exist.

They weren't used much until the Renaissance, and were only fully accepted in the 1800's.

If you had trouble with negative numbers, so did most everyone else.

## Integers

Adding the negative numbers to the whole numbers yields the **Integers**.

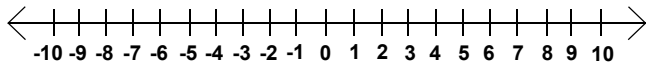
...-3, -2, -1, 0, 1, 2, 3, ...

In this case, "..." at the left and right, means that the sequence continues in both directions forever.

There is no largest integer...nor is there a smallest integer.

## Integers

The below number line shows only the integers.



## Adding and Subtracting Integers

Now that we have a new operation (subtraction) and new numbers (integers), we need new rules.

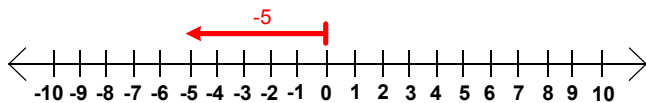
The new rules are based on the old ones.

We have to show how to do addition and subtraction with our newly invented negative integers.

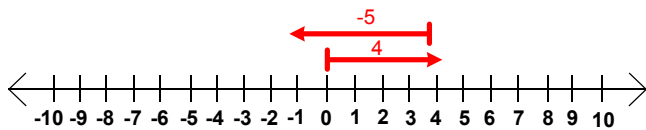
## Adding Integers

First, let's add a positive and negative integer:  $4 + (-5) = ?$

Before we start, we must understand that  $-5$  means to move 5 spaces to the left of 0, as shown below.



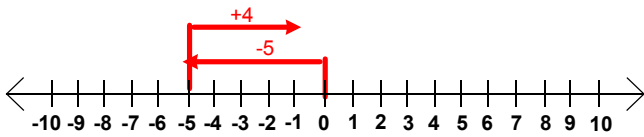
Now, add the  $-5$  to the end of the 4.



Therefore,  $4 + (-5) = -1$ .

### Adding Integers

Addition is commutative, so we get the same answer for  $(-5) + 4 = ?$



Therefore,  $(-5) + 4 = -1$ .

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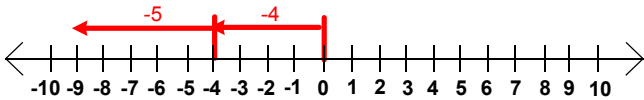
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### Adding Integers

Now let's add two negative integers:  $(-4) + (-5) = ?$

Like the last example, we start off going to the left 4 spaces, but then we continue to the left 5 spaces.



Therefore,  $(-4) + (-5) = -9$ .

Don't memorize "rules" to add positive & negative numbers.  
Instead, sketch the number line to perform each addition problem.  
That way you know that you are correct every time.

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1 Which number line would be used to show  $9 + (-1)$ ?

- A
- B
- C
- D

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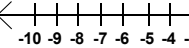
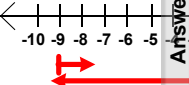
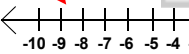
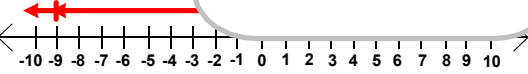
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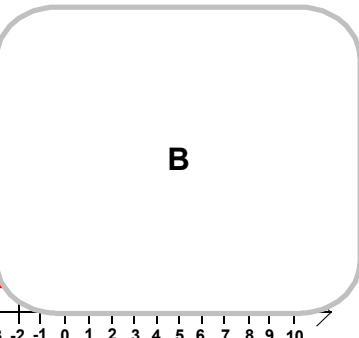
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1 Which number line would be used to show  $9 + (-1)$ ?

- A 
- B 
- C 
- D 




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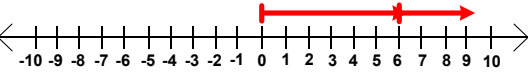
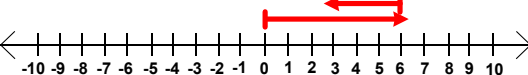
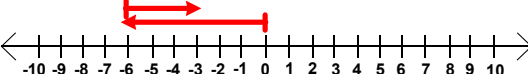
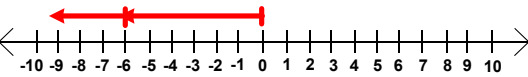
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2 Which number line would be used to show  $(-6) + (-3)$ ?

- A 
- B 
- C 
- D 

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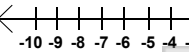
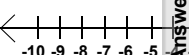

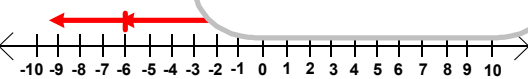
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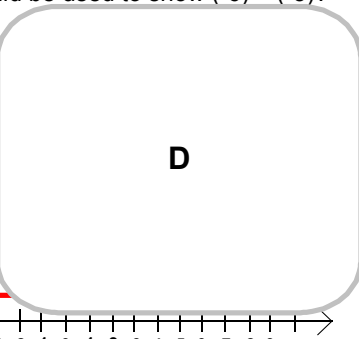
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2 Which number line would be used to show  $(-6) + (-3)$ ?

- A 
- B 
- C 
- D 




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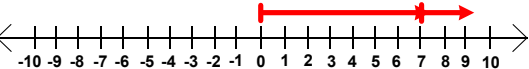
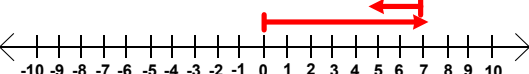

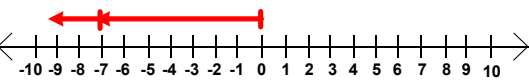
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3 Which number line would be used to show  $(-7) + 2$ ?

- A 
- B 
- C 
- D 

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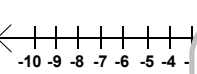
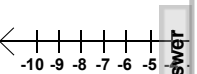
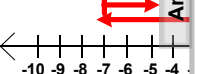
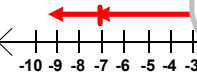
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3 Which number line would be used to show  $(-7) + 2$ ?

- A 
  - B 
  - C 
  - D 
- C

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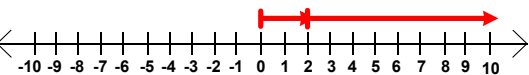
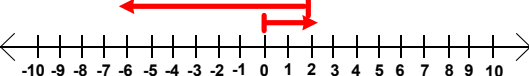
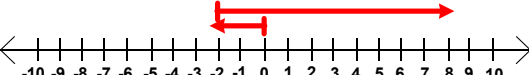
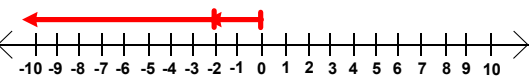
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4 Which number line would be used to show  $2 + 8$ ?

- A 
- B 
- C 
- D 

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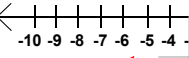
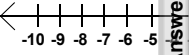
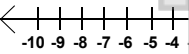
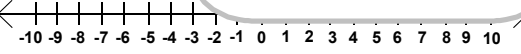
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4 Which number line would be used to show  $2 + 8$ ?

- A 
- B 
- C 
- D 

**Answer**  
**A**

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5 )  $11 + (-4) =$

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5 )  $11 + (-4) =$

**Answer**  
**7**

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6 )  $-4 + (-4) =$

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6 )  $-4 + (-4) =$

Answer

**-8**

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7 )  $17 + (-20) =$

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7 )  $17 + (-20) =$

Answer

**-3**

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8 )  $-15 + (-30) =$

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8 )  $-15 + (-30) =$

Answer

**-45**

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9 )  $-5 + 10 =$

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9 )  $-5 + 10 =$

Answer

5

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10 )  $11 + (-4) =$

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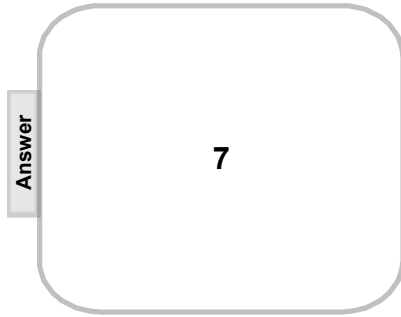
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10 )  $11 + (-4) =$




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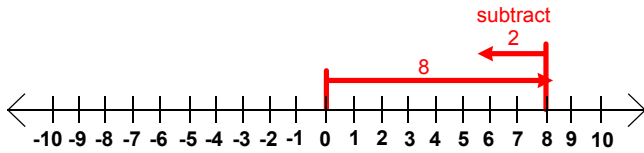
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### Subtracting Integers

$$8 - 2 = 6$$

Since 8 is positive, we need to travel to the right 8 steps.

Next, instead of moving to the right 2 spaces, as in addition, subtraction moves in the opposite direction, which means we move 2 spaces to the left.



Our answer is 6.

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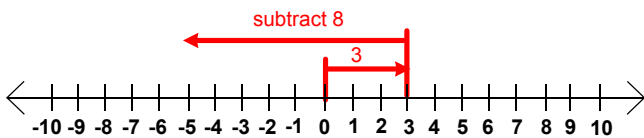
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### Subtracting Integers

How would we show  $3 - 8$ ?

As in the last example, we would start off going right 3 and then left 8.



Therefore,  $3 - 8 = -5$

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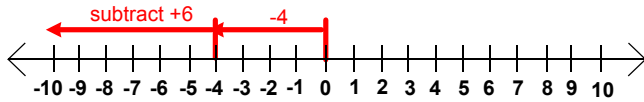
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## Adding & Subtracting Integers

How would we show  $(-4) - 6$ ?

Remember that  $(-4)$  is shown by going 4 spaces to the left.

Subtracting 6, take us 6 more spaces to the left.



Therefore,  $(-4) - 6 = -10$

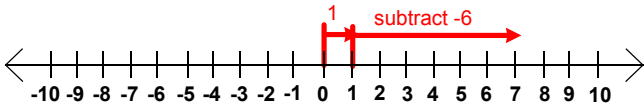
## Adding & Subtracting Integers

The most potentially confusing case is subtracting a negative integer.

How would we show  $1 - (-6)$ ?

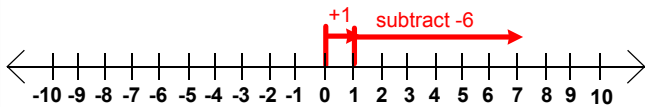
First, we would go 1 space to the right, because of the 1.

Adding  $-6$  spaces would take us to the left, but we must do the opposite since we are subtracting, moving 6 steps to the right.



Therefore,  $1 - (-6) = 7$

## Adding & Subtracting Integers



Therefore,  $1 - (-6) = 7$

When we subtracted  $-6$ , it was the same as adding  $+6$ .

This is because  $-6$  is six spaces to the left, and so when you go in the opposite direction, you go 6 spaces to the right.

The same as adding 6.

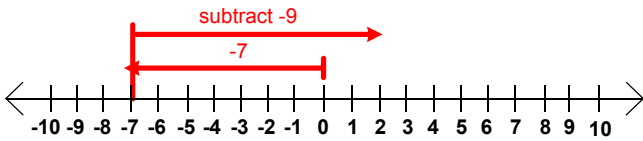
Subtracting a negative number is the same as adding a positive one: "two minuses make a plus".

### Adding & Subtracting Integers

How would we show  $-7 - (-9)$ ?

First, go 7 spaces to the left, because we're adding  $-7$ .

If we were adding  $-9$  we would go 9 spaces to the left, but since we are subtracting  $-9$ , we must go in the opposite direction, 9 to the right.



Therefore,  $-7 - (-9) = 2$

Let's look at this another way.

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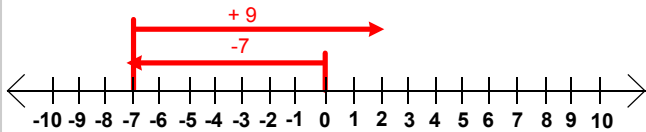
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### Adding & Subtracting Integers

We could rewrite  $-7 - (-9)$  to be  $-7 + 9$ , since subtracting a negative number is the same as adding a positive one.

That yields the same answer:  $+2$




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11 Which number line would be used to show  $2 - 8$ ?

- A
- B
- C
- D

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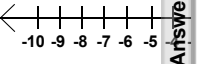
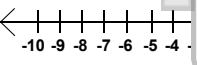
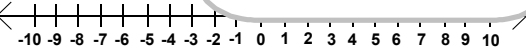
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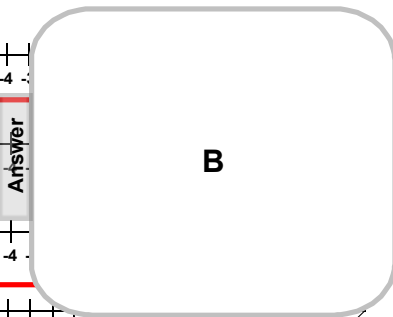
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11 Which number line would be used to show  $2 - 8$ ?

- A 
- B 
- C 
- D 




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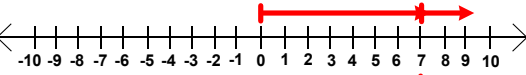

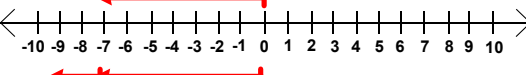
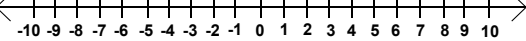
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12 Which number line would be used to show  $(-7) - 2$ ?

- A 
- B 
- C 
- D 

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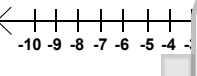
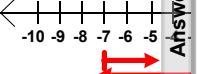
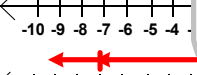
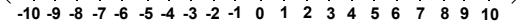
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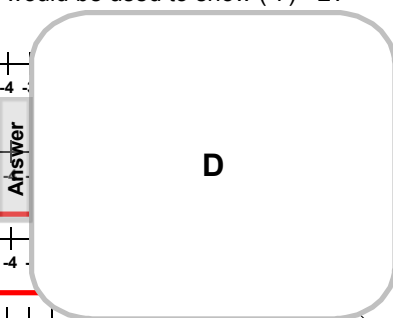
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12 Which number line would be used to show  $(-7) - 2$ ?

- A 
- B 
- C 
- D 




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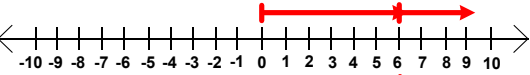
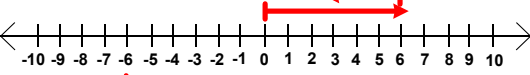


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13 Which number line would be used to show  $(-6) - (-3)$ ?

- A 
- B 
- C 
- D 

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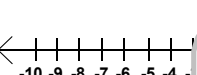
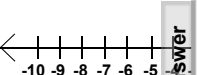
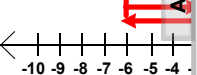
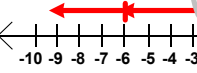
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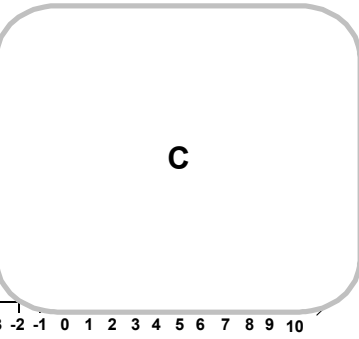
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13 Which number line would be used to show  $(-6) - (-3)$ ?

- A 
- B 
- C 
- D 




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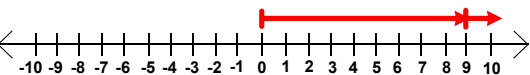
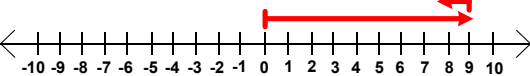
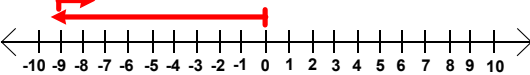
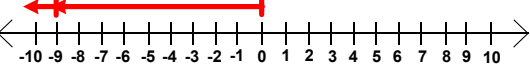
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14 Which number line would be used to show  $9 - (-1)$ ?

- A 
- B 
- C 
- D 

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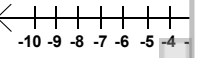
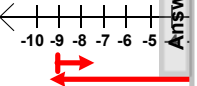
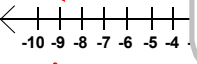
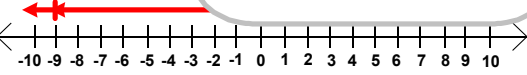
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14 Which number line would be used to show  $9 - (-1)$ ?

- A 
- B 
- C 
- D 

**A**

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15 )  $-2 - 6 =$

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15 )  $-2 - 6 =$

**Answer**  
**-8**

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16 )  $4 - 10 =$

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16 )  $4 - 10 =$

Answer

**-6**

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17 )  $3 - (-8) =$

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17 )  $3 - (-8) =$

**Answer**

**11**

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18 )  $-5 - (-3) =$

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18 )  $-5 - (-3) =$

**Answer**

**-2**

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19 )  $-7 - 8 =$

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19 )  $-7 - 8 =$

Answer

**-15**

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20 )  $-7 - 2 =$

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20 )  $-7 - 2 =$

Answer

**-9**

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21 Convert the subtraction problem into an addition problem.

$-8 - (-4)$

- A  $8 + 4$
- B  $8 + (-4)$
- C  $-8 + 4$
- D  $-8 + (-4)$

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21 Convert the subtraction problem into an addition problem.

$-8 - (-4)$

- A  $8 + 4$
- B  $8 + (-4)$
- C  $-8 + 4$
- D  $-8 + (-4)$

Answer

**C**

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22 Convert the subtraction problem into an addition problem.

$$-1 - 9$$

- A  $-1 + 9$
- B  $-1 + (-9)$
- C  $1 + 9$
- D  $1 + (-9)$

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22 Convert the subtraction problem into an addition problem.

$$-1 - 9$$

- A  $-1 + 9$
- B  $-1 + (-9)$
- C  $1 + 9$
- D  $1 + (-9)$

Answer

**B**

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23 Convert the subtraction problem into an addition problem.

$$12 - (-5)$$

- A  $12 + 5$
- B  $12 + (-5)$
- C  $-12 + 5$
- D  $-12 + (-5)$

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23 Convert the subtraction problem into an addition problem.

$$12 - (-5)$$

- A  $12 + 5$
- B  $12 + (-5)$
- C  $-12 + 5$
- D  $-12 + (-5)$

Answer

**A**

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### Is Subtraction Commutative?

Subtraction would be commutative if  $a - b = b - a$

Let's test that with some numbers

$$4 - 5 = -1$$

$$5 - 4 = 1$$

Since  $-1 \neq 1$ , then  $4 - 5 \neq 5 - 4$

In general,  $a - b \neq b - a$

Subtraction is not commutative.

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## Multiplication and Division of Integers

[Return to Table of Contents](#)

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## Multiplication

Multiplication can be indicated by putting a dot between two numbers, or by putting the numbers into parentheses.

(We won't generally use "x" to indicate multiplication since that letter is used a lot in algebra for variables.)

So multiplying 3 times 2 will be written as:

$$3 \cdot 2$$

or

$$(3)(2)$$

## Multiplication

Multiplication can be indicated by putting a dot between two numbers, or by putting the numbers into parentheses.

(We won't generally use "x" to indicate multiplication since that letter is used a lot in algebra for variables.)

So multiplying 3

Math Practice

**MP.7: Look for and make use of structure.**

**Ask: How is multiplication related to addition?**

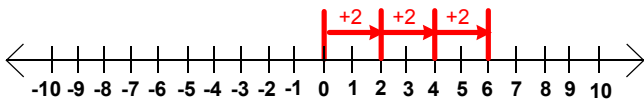
**Note: answer to this question is shown on the next slide.**

## Multiplication

Multiplication is repeated addition.

So, to find the product of  $3 \cdot 2$  we would add the number 2 to itself three times:

$$3 \cdot 2 = 2 + 2 + 2$$





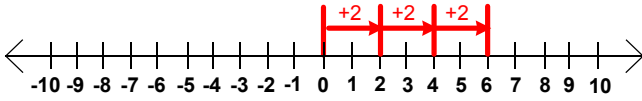
### Multiplication is Commutative

Since addition is commutative...

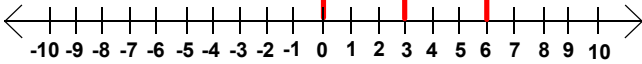
And multiplication is just repeated addition,

multiplication is commutative:

$$3 \cdot 2 = 2 + 2 + 2 = 6$$



$$2 \cdot 3 = 3 + 3 = 6$$




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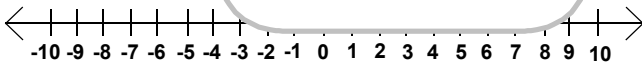
### Multiplication is Commutative

Since addition is commutative...

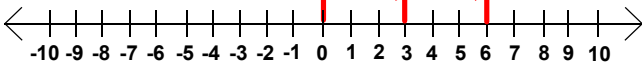
And multiplication is just repeated addition,

multiplication is commutative:

$$3 \cdot 2 = 2 + 2 + 2 = 6$$



$$2 \cdot 3 = 3 + 3 = 6$$



Teacher's Note




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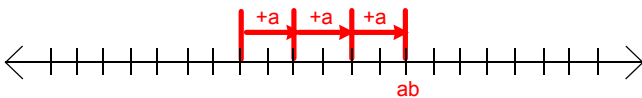
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### Multiplication is Commutative

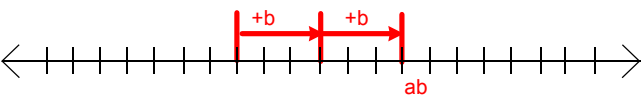
$$a \cdot b = b \cdot a$$

Adding a number "a" to itself "b" times yields the same result as adding a number "b" to itself "a" times.

Adding "a" to itself "b" times yields "ab".



Adding "b" to itself "a" times also yields "ab".




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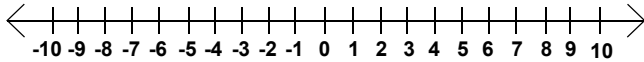
## Multiplying Negative Integers

It works the same way when you multiply a negative number by a positive number.

So,  $3 \cdot (-2)$  just indicates to add  $-2$  to itself three times:

$$3 \cdot (-2) = (-2) + (-2) + (-2) = -6$$

$-2$     $-2$     $-2$



Would  $(-3) \cdot 2$  give us the same answer of  $-6$ ? Explain your answer.

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## Multiplying Negative Integers

It works the same way when you multiply a negative number by a positive number.

So,  $3 \cdot (-2)$  just indicates

Yes, in multiplication, the order in which you multiply numbers does not matter. Therefore,

$$\begin{aligned} 3 \cdot (-2) &= (-3) \cdot 2 \\ &= (-3) + (-3) \\ &= -6 \end{aligned}$$

Answer



Would  $(-3) \cdot 2$  give us the same answer of  $-6$ ? Explain your answer.

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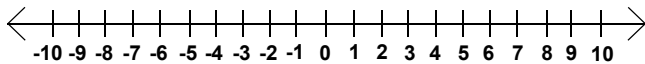
## Multiplying Negative Integers

Since multiplication is commutative,

$$(-3) \cdot 2 = 2 \cdot (-3) = (-3) + (-3) = -6$$

So, this just becomes adding  $-3$  to itself 2 times.

$-3$     $-3$



Multiplying a positive and negative integer results in a negative integer:

$$(a)(-b) = (-a)(b) = -ab$$

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### Multiplying Positive & Negative Integers

When multiplying two numbers, if both are positive, the answer is positive.

If one is negative and the other is positive, the answer is negative.

How about if both numbers being multiplied are negative?

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### Multiplying Positive & Negative Integers

How do we interpret  $(-3)(-2)$ ?

Since,  $(3)(-2)$  means to add  $-2$  to itself 3 times

We could interpret  $(-3)(-2)$  to mean to subtract  $(-2)$  from itself three times.

We already learned that addition and subtraction are inverses, so subtracting  $-2$  is the same as adding 2.

So,  $(-3)(-2)$  indicates to add 2 to itself 3 times.

So,  $(-3)(-2) = (3)(2) = 6$

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### Multiplying & Dividing Integers

While showing how to multiply integers, we have come across some "shortcuts" for determining the sign of our product.

What generalizations can you make about the sign of your product?

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## Multiplying & Dividing Integers

While showing how to multiply integers across some "shortcuts" for the product.

What general rule can you use to determine the sign of the product?

Answer & Math Practice

- + times + = +
- + times - = -
- times + = -
- times - = +

The question on this slide addresses MP8: Look for and express regularity in repeated reasoning.

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24 What will be the sign of the product  $(-6) \cdot 8$ ?

- Positive
- Negative

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24 What will be the sign of the product  $(-6) \cdot 8$ ?

- Positive
- Negative

Answer

**B**

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25 What will be the sign of the product  $(-4)(-9)$ ?

- Positive
- Negative

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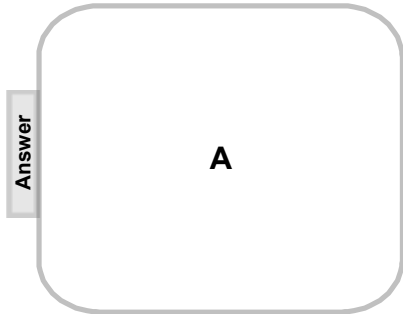
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25 What will be the sign of the product  $(-4)(-9)$ ?

- Positive
- Negative



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26 Find the product:  $9 \cdot (-11)$

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26 Find the product:  $9 \cdot (-11)$

Answer

**-99**

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27 Find the product:  $(-12)(-11)$

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27 Find the product:  $(-12)(-11)$

Answer

**132**

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28 Find the product:  $(-7) \cdot 9$

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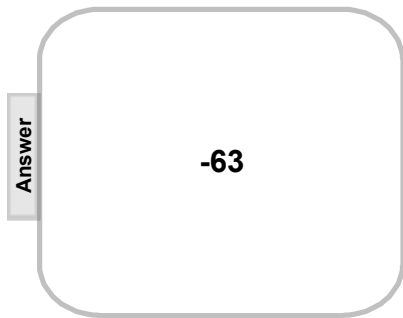
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28 Find the product:  $(-7) \cdot 9$



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29 Find the product:  $(-3) \cdot (-9)$

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29 Find the product:  $(-3) \cdot (-9)$

Answer

27

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### Inverse Operations

$$7(4) = 28$$

This equation provides the answer "28" to the multiplication question "what is the product of 7 and 4".

What are the two inverse questions that can be asked and answered based on the above multiplication fact?

Which mathematical operation is the inverse of multiplication?

DISCUSS!

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### In

Math Practice

**MP.1: Make sense of problems & persevere in solving them.**

**Ask: What information are you given?**

**What are the questions asking?**

**How can the equation given help you determine the inverse operation of multiplication?**

This equation provides the answer "28" to the multiplication question "what is the product of 7 and 4".

What are the two inverse questions that can be asked and answered based on the above multiplication fact?

Which mathematical operation is the inverse of multiplication?

DISCUSS!

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## Inverse Operations

$$7(4) = 28$$

There are two division questions are the inverse of this are:

$$28 \div 4 = 7$$

This provides the answer "7" to the question "what is 28 divided by 4".

$$28 \div 7 = 4$$

This provides the answer "4" to the question "what is 28 divided by 7".

## Inverse Operations

Division asks:

If I divide something into pieces of equal size, what will be the size of each piece?

For instance,  $15 \div 3$  asks if I divide 15 into 3 pieces, what will be the size of each piece?

The answer is 5, since  $5 + 5 + 5 = 15$

Three equal pieces of 5 will add to equal 15.

## Multiplying & Dividing Integers

Since multiplication and division are so closely related, we can get the rules for division using the rules of multiplication.

For example,  $\frac{30}{6} = 5$  because 5 is the number you multiply by 6 to get 30.

In turn,  $30 = 6 \cdot 5$ , so  $\frac{30}{6} = 5$

### Multiplying & Dividing Integers

because -5 is the number you multiply by 6 to get 30.

are so closely related, we can get the of multiplication.

the number you multiply by 6 to

In turn,  $-30 = 6 \cdot (-5)$ , so  $\frac{-30}{6} = (-5)$

the questions on this slide address P6 & MP7.

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### Multiplying & Dividing Integers

What is  $\frac{30}{-6}$ ? Why?

What is  $\frac{-30}{-6}$ ? Why?

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### Multiplying & Dividing Integers

What is  $\frac{30}{-6}$ ? Why? -5 because -5 is the number you multiply by -6 to get 30.

In turn,  $30 = (-6) \cdot (-5)$ , so  $\frac{30}{-6} = (-5)$

What is  $\frac{-30}{-6}$ ? Why? 5 because 5 is the number you multiply by -6 to get -30.

In turn,  $-30 = (-6) \cdot (5)$ , so  $\frac{-30}{-6} = (5)$

Answer

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## Multiplying & Dividing Integers

What generalizations can you make about the sign of the quotient?

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## Multiplying & Dividing Integers

What generalization about the quotient?

Answer

+ over + = +  
+ over - = -  
- over + = -  
- over - = +

The question on this slide addresses MP8

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30 What is the sign of the quotient of  $-69 \div (-3)$ ?

- Positive
- Negative

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30 What is the sign of the quotient of  $-69 \div (-3)$ ?

- Positive
- Negative

Answer

**A**

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31 What is the sign of the quotient of  $52 \div (-4)$ ?

- Positive
- Negative

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31 What is the sign of the quotient of  $52 \div (-4)$ ?

- Positive
- Negative

Answer

**B**

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32 Find the quotient of  $-65 \div (-5)$ .

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32 Find the quotient of  $-65 \div (-5)$ .

Answer **13**

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33 Find the quotient of  $-126 \div (-3)$ .

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33 Find the quotient of  $-126 \div (-3)$ .

Answer

**42**

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34 Find the quotient of  $104 \div (-4)$ .

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34 Find the quotient of  $104 \div (-4)$ .

Answer

**-26**

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35 Find the quotient of  $-88 \div (11)$ .

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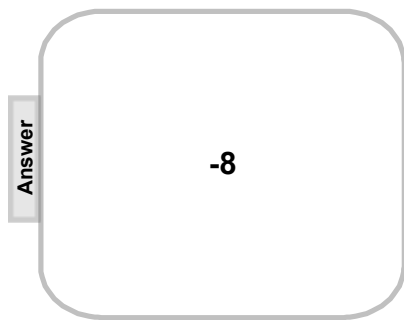
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35 Find the quotient of  $-88 \div (11)$ .



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## Operations with Rational Numbers

[Return to Table of Contents](#)

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## New Numbers: Fractions

Just as subtraction led to a new set of numbers: negative integers.

Division leads to a new set of numbers: **fractions**.

This results when you ask questions like:

$$1+2 = ?$$

$$1+3 = ?$$

$$2+3 = ?$$

$$1+1,000,000 = ?$$

## New Numbers: Fractions

$1+2 = ?$  asks the question:

If I divide 1 into 2 equal pieces, what will be the size of each?

The answer to this question cannot be found in the integers.

New numbers were needed.

## Fractions

The space between any two integers can be divided by any integer you choose...as large a number as you can imagine.

There are as many fractions between any pair of integers as there are integers.

Fractions can be written as the ratio of two numbers:

$$\frac{2}{3}, \frac{1}{4}, \frac{1}{8}, \frac{1}{3}, \frac{4}{5}, \frac{7}{5}, \frac{80}{4} \text{ etc.}$$

Or in decimal form by dividing the numerator by the denominator:

$$-0.\overline{666}, -0.25, -0.125, 0.\overline{333}, 0.8, 1.4, 20, \text{ etc.}$$

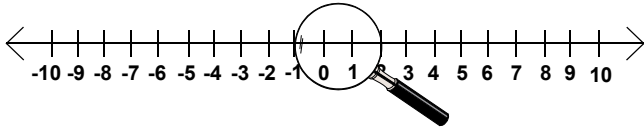
The bar over "666" and "333" means that pattern repeats forever.



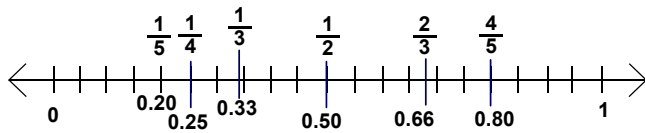
## Fractions

There are an infinite number of fractions between each any integers.

Looking closely between 0 and 1, we can locate a few of them.



It's easier to find their location when they are in decimal form since it's clear which integers they're between...and closest to.




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## Rational Numbers

Rational Numbers are numbers that can be expressed as a ratio of two integers.

This includes all the fractions, as well as all the integers.

What are a few ways you could write 5 as a ratio of two integers?

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## Rational Numbers

Rational Numbers are numbers that can be expressed as a ratio of two integers.

This includes all the fractions, as well as all the integers.

What are a few ways you could write 5 as a ratio of two integers?

**Math Practice**

MP6 - Attend to precision.

Ask:

- What mathematical terms apply in this situation?
- How are you showing the meaning of the terms with your representation?

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### Rational Numbers

Fractions can be written in "fraction" form or decimal form.

When written in decimal form, rational numbers are either:

Terminating, such as

$$-\frac{1}{2} = -0.500000000000 = -0.5$$

Repeating, such as

$$\frac{1}{7} = 0.142857142857142857... = 0.\overline{142857}$$

Or,

$$\frac{1}{3} = 0.3333333333333333... = 0.\overline{33}$$

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### Fractions and the Negative Sign

When we have a negative fraction, the negative sign can be in different places.

The following all are negative one-half.

$$-\frac{1}{2} \quad \frac{-1}{2} \quad -\left(\frac{1}{2}\right) \quad \frac{1}{-2} \quad \left(-\frac{1}{2}\right)$$

*Why are they all negative?*

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### Fractions and the Negative Sign

When we have a negative fraction, the negative sign can be in different places.

$$-\frac{1}{2} \quad \frac{-1}{2} \quad -\left(\frac{1}{2}\right) \quad \frac{1}{-2} \quad \left(-\frac{1}{2}\right)$$

**Answer**

**Because the fraction bar signifies division. When dividing a positive by a negative, or a negative by a positive, your answer is negative. A number is also negative if you perform the operation of division and then make the fraction negative.**

**Note: This topic is new to 7th Grade.**

*Why are they all negative?*

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## Fractions and the Negative Sign

These two fractions equal positive one-half.

$$\frac{-1}{-2} \quad -\left(-\frac{1}{2}\right)$$

*Why are they both positive?*

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## Fractions and the Negative Sign

These

Answer

**Because the fraction bar signifies division. When dividing a negative by a negative, your answer is positive.**

**A negative can also be considered the opposite sign of the original number, so the opposite of negative one half is positive one half.**

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## Dividing by Zero

One number that is not defined by our numbers and mathematical operations is dividing any number by zero.

The result of that division is "undefined."

This will be critical later when we are working with equations or simplifying fractions.

Dividing by zero is undefined since there is no way to say how many times zero can go into any number.

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## Review of Operations with Rational Numbers

The following formative assessment questions are review from 7th grade. If further instruction is need, see the presentation at:

<http://njctl.org/courses/math/7th-grade/numbers-and-operations-7th-grade/>

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36 Convert the subtraction problem into an addition problem.

$$-3.7 - (-10.1)$$

- A  $-3.7 + 10.1$
- B  $3.7 + (-10.1)$
- C  $-3.7 + (-10.1)$
- D  $3.7 + 10.1$

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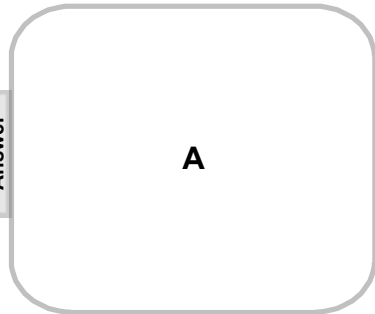
36 Convert the subtraction problem into an addition problem.

$$-3.7 - (-10.1)$$

- A  $-3.7 + 10.1$
- B  $3.7 + (-10.1)$
- C  $-3.7 + (-10.1)$
- D  $3.7 + 10.1$

Answer

**A**



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37 Convert the subtraction problem into an addition problem.

$$-9 - 3\frac{1}{2}$$

- A  $-9 + 3\frac{1}{2}$
- B  $9 + (-3\frac{1}{2})$
- C  $-9 + (-3\frac{1}{2})$
- D  $9 + 3\frac{1}{2}$

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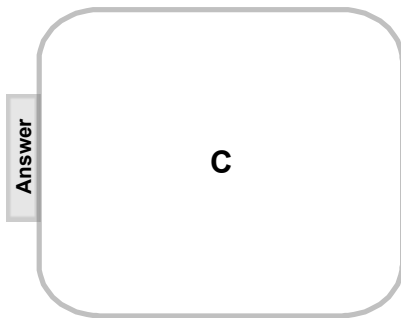
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37 Convert the subtraction problem into an addition problem.

$$-9 - 3\frac{1}{2}$$

- A  $-9 + 3\frac{1}{2}$
- B  $9 + (-3\frac{1}{2})$
- C  $-9 + (-3\frac{1}{2})$
- D  $9 + 3\frac{1}{2}$



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38 Convert the subtraction problem into an addition problem.

$$6.5 - (-3.2)$$

- A  $-6.5 + (-3.2)$
- B  $-6.5 + 3.2$
- C  $6.5 + (-3.2)$
- D  $6.5 + 3.2$

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38 Convert the subtraction problem into an addition problem.

$$6.5 - (-3.2)$$

- A  $-6.5 + (-3.2)$
- B  $-6.5 + 3.2$
- C  $6.5 + (-3.2)$
- D  $6.5 + 3.2$

Answer

**D**

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39 )  $-10.5 + 6.2 =$

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39 )  $-10.5 + 6.2 =$

Answer

**-4.3**

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40 )  $-7.3 - (-4) =$

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40 )  $-7.3 - (-4) =$

**Answer**

**11.3**

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41 )  $4\frac{1}{2} - 7\frac{2}{3}$

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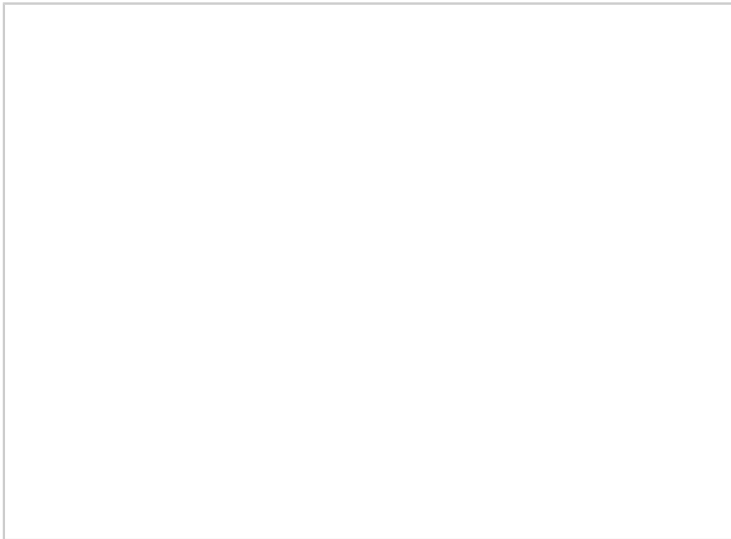
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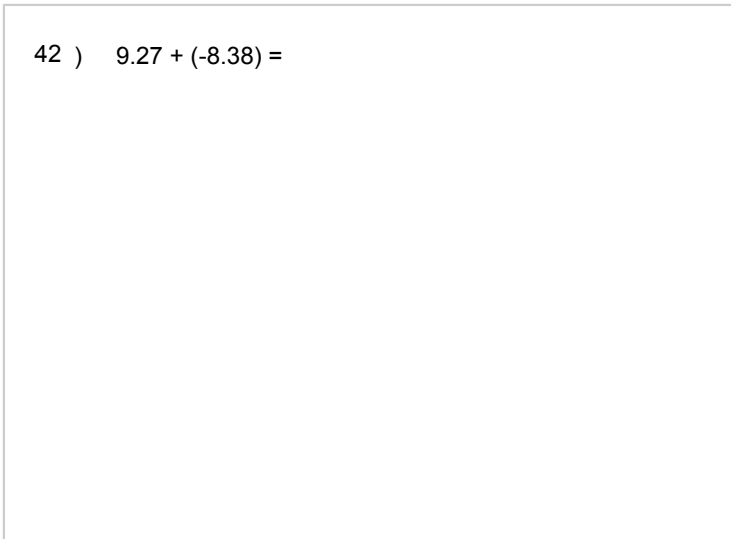
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42 )  $9.27 + (-8.38) =$



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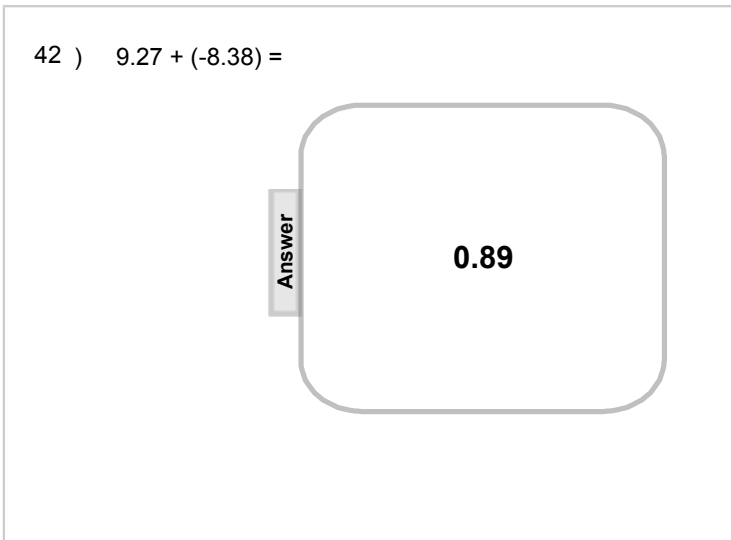
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42 )  $9.27 + (-8.38) =$

**Answer**  
**0.89**



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43 )  $-4.2 + (-5.9) =$

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43 )  $-4.2 + (-5.9) =$

Answer

**-10.1**

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44 )  $-2 - (-3.95) =$

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44 )  $-2 - (-3.95) =$

Answer

**1.95**

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45 )  $5 - 6 + (-7.5) =$

Hint: Remember addition and subtraction is solved left to right in the order of operations!

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45 )  $5 - 6 + (-7.5) =$

Answer

**-8.5**

Hint: Remember addition and subtraction is solved left to right in the order of operations!

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46 )  $19 + (-12) - 11 =$

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46 )  $19 + (-12) - 11 =$

Answer

**-4**

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47 )  $-2.3 + 4.1 + (-12.7) =$

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47 )  $-2.3 + 4.1 + (-12.7) =$

Answer

**-10.9**

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48 )  $-8.3 - (-3.7) + 5.2 =$

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48 )  $-8.3 - (-3.7) + 5.2 =$

Answer

**0.6**

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49 )  $16\frac{1}{2} - (-9) - 21\frac{2}{5}$

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49 )  $16\frac{1}{2} - (-9) - 21\frac{2}{5}$

Answer

$$4\frac{1}{10}$$

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50 )  $5(-4.82) =$

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50 )  $5(-4.82) =$

Answer

**-24.10**

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51 )  $(3.2)(-6.4) =$

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51 )  $(3.2)(-6.4) =$

Answer

**20.48**

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52 )  $(-5.12)(-9) =$

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52 )  $(-5.12)(-9) =$

Answer

**46.08**

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53 )  $4\frac{1}{3}\left(-2\frac{2}{5}\right) =$

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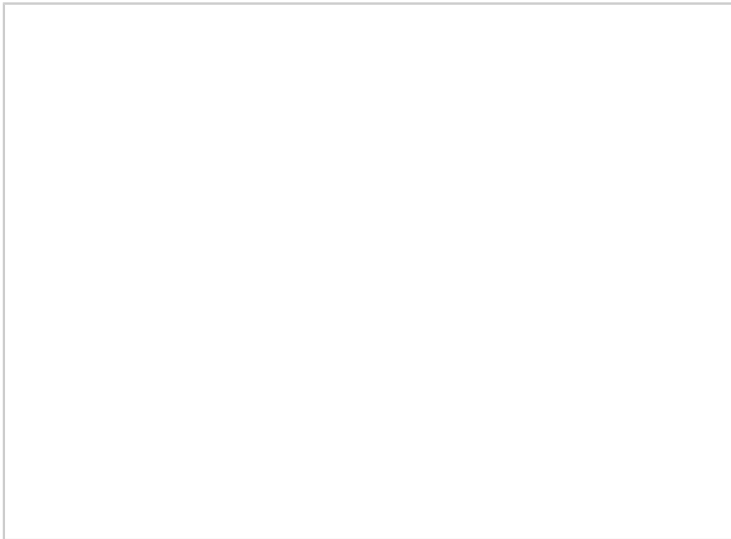
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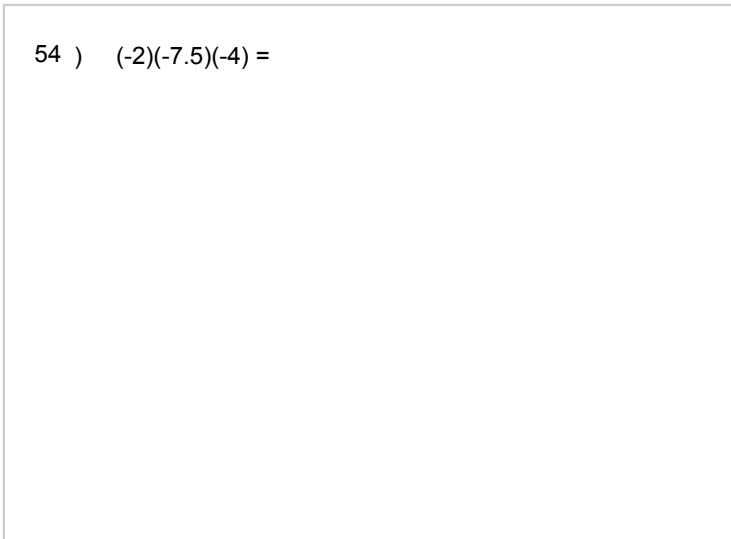
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54 )  $(-2)(-7.5)(-4) =$



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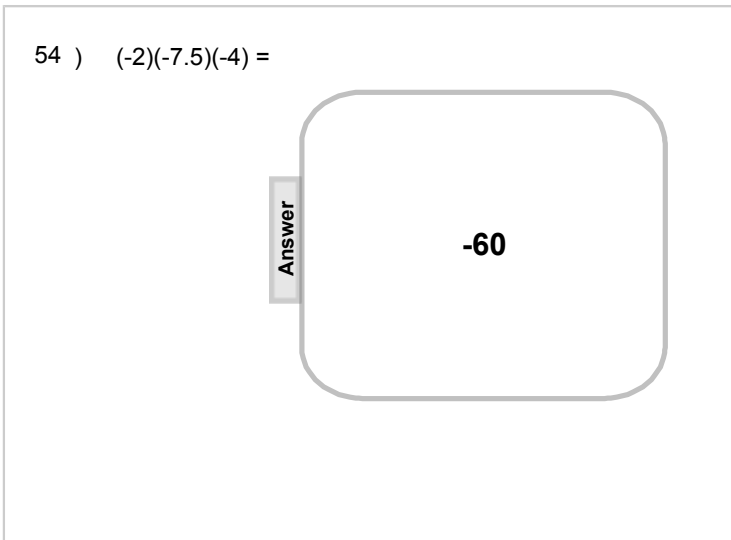
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54 )  $(-2)(-7.5)(-4) =$

Answer

**-60**

A rounded rectangular box with a thin black border. On the left side, there is a small grey tab with the word "Answer" written vertically in black. In the center of the box, the number "-60" is written in a bold, black font.

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55 )  $4\frac{1}{2}(-2)(5\frac{1}{4})$

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56 )  $(-2.5)(-4.1)(3) =$

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56 )  $(-2.5)(-4.1)(3) =$

Answer

**30.75**

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57 Jane has entered a baking contest. Jane uses 3.1 ounces of flour to make one cinnamon roll. How many ounces of flour does Jane need to make 7 cinnamon rolls?

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57 Jane has entered a baking contest. Jane uses 3.1 ounces of flour to make one cinnamon roll. How many ounces of flour does Jane need to make 7 cinnamon rolls?

Answer  
& Math Practice

**21.7 ounces**

**MP.4: Model with mathematics.**

**Ask: What connections do you see?**

**Write a number sentence to describe this situation.**

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58 Timmy is shipping 4 boxes of shirts. Each box weighs 6.3 pounds. If it cost 5.20 per pound to ship. How much does Timmy have to spend to ship them?

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58 Timmy is shipping 4 boxes of shirts. Each box weighs 6.3 pounds. If it cost 5.20 per pound to ship. How much does Timmy have to spend to ship them?

**\$131.04**

Answer  
& Math Practice

**MP.4: Model with mathematics.**

**Ask: What connections do you see?**

**Write a number sentence to describe this situation.**

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59 Find the value of:

$$\frac{-25.5}{5}$$

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59 Find the value of:

$$\frac{-25.5}{5}$$

Answer

**-5.1**

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60 Find the value of:

$$-7 \div \frac{1}{3}$$

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60 Find the value of:

$$-7 \div \frac{1}{3}$$

Answer

**-21**

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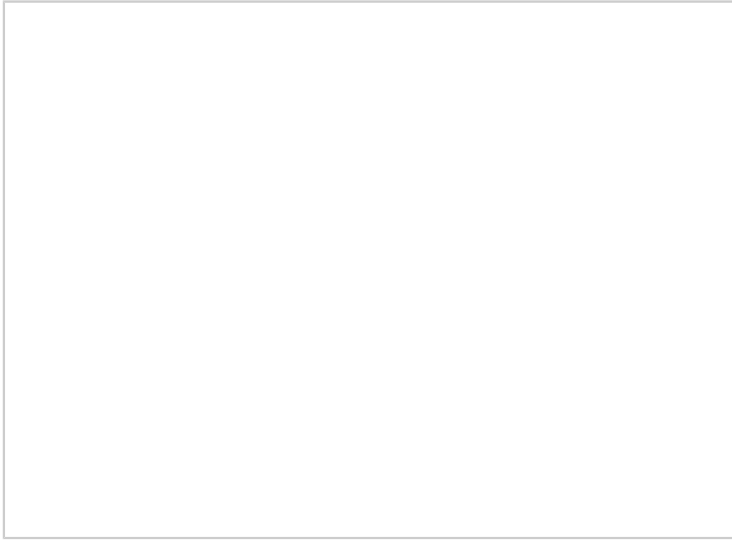
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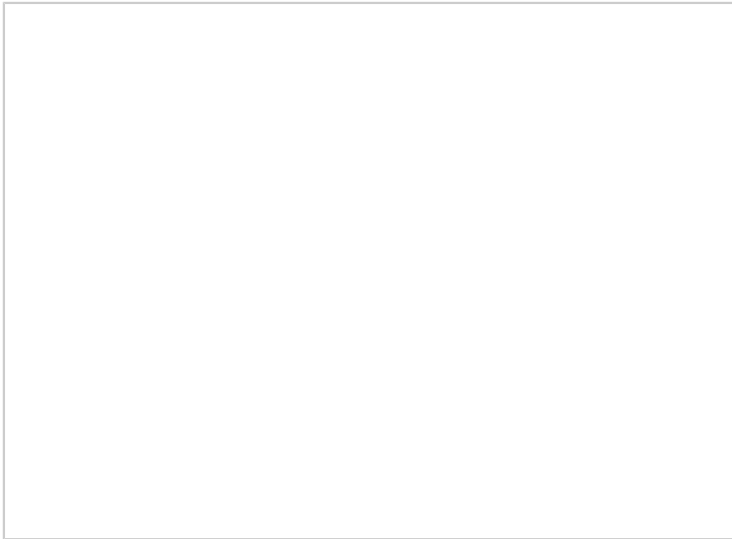
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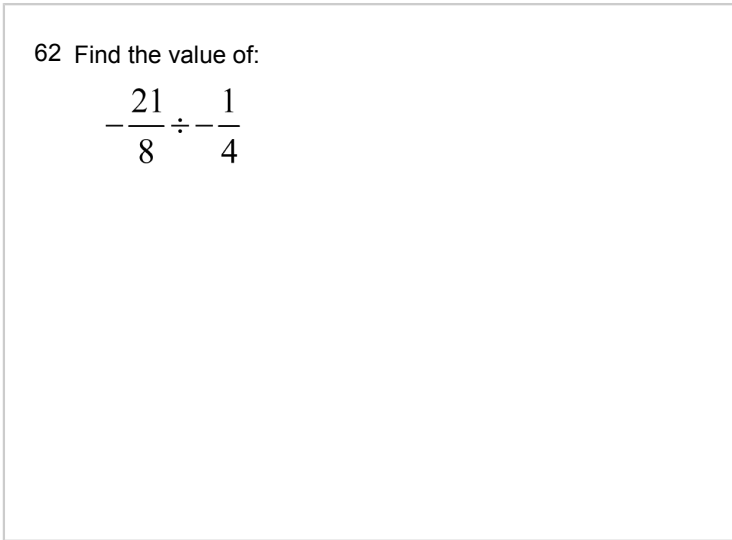
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62 Find the value of:

$$-\frac{21}{8} \div -\frac{1}{4}$$



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62 Find the value of:

$$-\frac{21}{8} \div -\frac{1}{4}$$

Answer

$$10\frac{1}{2}$$

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63 Find the value of:

$$19.375 \div (-6.25)$$

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63 Find the value of:

$$19.375 \div (-6.25)$$

Answer

$$-3.1$$

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64 Find the value of:

$$\frac{45}{-3} \div (-5)$$

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64 Find the value of:

$$\frac{45}{-3} \div (-5)$$

**Answer**

**3**

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65 Kobe put 8 toy cars in a row. The line of cars was 16.4 meters long. How long was each car?

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65 Kobe put 8 toy cars in a row. The line of cars was 16.4 meters long. How long was each car?

Answer

**2.05 meters**

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66 Olivia squeezed  $\frac{3}{4}$  of a gallon of orange juice. She split the orange juice equally into 6 cups. How many gallons was in each cup?

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66 Olivia squeezed  $\frac{3}{4}$  of a gallon of orange juice. She split the orange juice equally into 6 cups. How many gallons was in each cup?

Answer

**$\frac{1}{8}$  of a gallon**

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## Converting Repeating Decimals to Fractions

Return to  
Table of  
Contents

## How can you convert Rational Numbers into Decimals?

Use long division!

Divide the numerator by the denominator.

If the decimal terminates or repeats, then you have a rational number.

If the decimal continues forever, then you have an irrational number.

## Converting Fractions to Repeating Decimals

Example 1:  $\frac{7}{33}$

Long Division Review

$$\begin{array}{r}
 .2121\dots \\
 33 \overline{)7.0000} \\
 \underline{-66} \phantom{00} \\
 40 \phantom{00} \\
 \underline{-33} \phantom{00} \\
 70 \phantom{00} \\
 \underline{-66} \phantom{00} \\
 40 \phantom{00} \\
 \underline{-33} \phantom{00} \\
 7
 \end{array}$$

67 Determine the decimal equivalent of  $\frac{8}{11}$

- A  $0.\bar{8}$
- B  $0.7\bar{2}$
- C  $0.\bar{72}$
- D .1375

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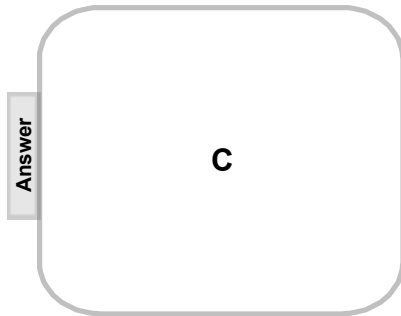
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67 Determine the decimal equivalent of  $\frac{8}{11}$

- A  $0.\bar{8}$
- B  $0.7\bar{2}$
- C  $0.\bar{72}$
- D .1375



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68 Determine the decimal equivalent of  $\frac{23}{33}$

- A  $0.14\bar{3}$
- B  $0.\bar{143}$
- C  $0.6\bar{9}$
- D  $0.\bar{69}$

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68 Determine the decimal equivalent of  $\frac{23}{33}$

- 0.14 $\bar{3}$
- 0.14 $\bar{3}$
- 0.6 $\bar{9}$
- 0.6 $\bar{9}$

Answer

**D**

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69 Determine the decimal equivalent of  $\frac{5}{111}$

- A 0.04 $\bar{5}$
- B 0.04 $\bar{5}$
- C 0.222
- D 0. $\bar{2}$

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69 Determine the decimal equivalent of  $\frac{5}{111}$

- A 0.04 $\bar{5}$
- B 0.04 $\bar{5}$
- C 0.222
- D 0. $\bar{2}$

Answer

**A**

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70 Determine the decimal equivalent of  $\frac{80}{333}$

- A 0.41625
- B  $0.2\overline{40}$
- C  $0.\overline{240}$
- D  $0.\overline{41625}$

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70 Determine the decimal equivalent of  $\frac{80}{333}$

- A 0.41625
- B  $0.2\overline{40}$
- C  $0.\overline{240}$
- D  $0.\overline{41625}$

Answer

**C**

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71 What decimal is the equivalent of  $\frac{6}{11}$  ?

- A  $0.18\overline{3}$
- B  $0.183$
- C  $0.5\overline{4}$
- D  $0.\overline{54}$

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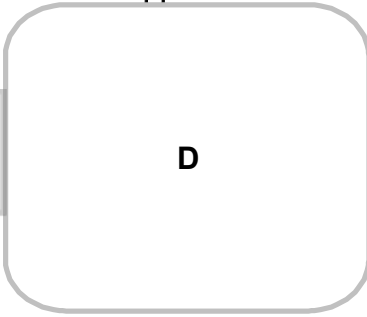
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71 What decimal is the equivalent of  $\frac{6}{11}$  ?

- 0.18 $\bar{3}$
- 0.18 $\bar{3}$
- 0.5 $\bar{4}$
- 0.5 $\bar{4}$

Answer



From PARCC EOY sample test non-calculator #2

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### Repeating Decimals to Fractions

A rational number is a number that can be written as a simple fraction.

The decimal expansion of a rational number either terminates or repeats.

A repeating decimal is written using a "bar line". The bar line appears over the digit(s) that repeat.

Ex:  $0.\bar{3} = 0.333333\dots$ ,  $0.1\bar{2} = 0.121212\dots$ ,  $1.8\bar{3} = 1.833333\dots$

Since a repeating decimal is a rational number, it can be written as a fraction.

But how do we do that?

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### Repeating Decimals to Fractions

In groups, write  $0.1\bar{5}$  as a fraction.

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## Steps to Convert a Decimal to a Fraction

1. Set  $x$  equal to the repeating decimal.
2. Determine how many digits repeat.
3. Multiply both sides of the equation by 10 to that power.
4. Rewrite the equation in partially expanded form.
5. Replace the repeating decimal with  $x$ .
6. Solve the 2-step equation.
7. Simplify.

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## Convert $0.\overline{15}$ to a Fraction

### Steps

- |   |  |
|---|--|
| 1. Set $x$ equal to the repeating decimal.                  | 1. $x = 0.151515\dots$   |
| 2. Determine how many digits repeat.                        | 2. $x = 0.\overline{15}$<br>2 digits repeat  |
| 3. Multiply both sides of the equation by 10 to that power. | 3. $10^2x = 10^2(0.151515\dots)$<br>$100x = 100(0.151515\dots)$<br>$100x = 15.1515\dots$ |
| 4. Rewrite the equation in partially expanded form.         | 4. $100x = 15 + 0.1515\dots$   |
| 5. Replace the repeating decimal with $x$ .                 | 5. $100x = 15 + x$   |
| 6. Solve the 2-step equation.                               | 6. $100x = 15 + x$<br>$\frac{99x}{99} = \frac{15}{99} - \frac{x}{99}$                    |
| 7. Simplify.  | 7. $x = \frac{15}{99} = \frac{5}{33}$  |

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## Convert $0.\overline{4}$ to a Fraction

### Steps

- |   |  |
|---|--|
| 1. Set $x$ equal to the repeating decimal.                  | 1. $x = 0.444\dots$  |
| 2. Determine how many digits repeat.                        | 2. $x = 0.\overline{4}$<br>1 digit repeats                     |
| 3. Multiply both sides of the equation by 10 to that power. | 3. $10x = 10(0.444\dots)$<br>$10x = 4.444\dots$                |
| 4. Rewrite the equation in partially expanded form.         | 4. $10x = 4 + 0.444\dots$                                      |
| 5. Replace the repeating decimal with $x$ .                 | 5. $10x = 4 + x$   |
| 6. Solve the 2-step equation.                               | 6. $10x = 4 + x$<br>$\frac{9x}{9} = \frac{4}{9} - \frac{x}{9}$ |
| 7. Simplify.  | 7. $x = \frac{4}{9}$   |

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**Fraction Form**

Write 0.5050... as a fraction.

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**Fraction Form**

Write 0.5050... as a fraction.

Answer  $\frac{50}{99}$

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**Fraction Form**

Write 0.2727... as a fraction.

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### Fraction Form

Write 0.2727... as a fraction.

Answer

$$\frac{27}{99} = \frac{3}{11}$$

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72 What is the fraction form of  $0.\bar{8}$  ?

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72 What is the fraction form of  $0.\bar{8}$  ?

Answer

$$\frac{8}{9}$$

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73 What is the fraction form of  $0.\overline{26}$  ?

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73 What is the fraction form of  $0.\overline{26}$  ?

Answer

$$\frac{26}{99}$$

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74 What is the fraction form of  $0.\overline{3}$  ?

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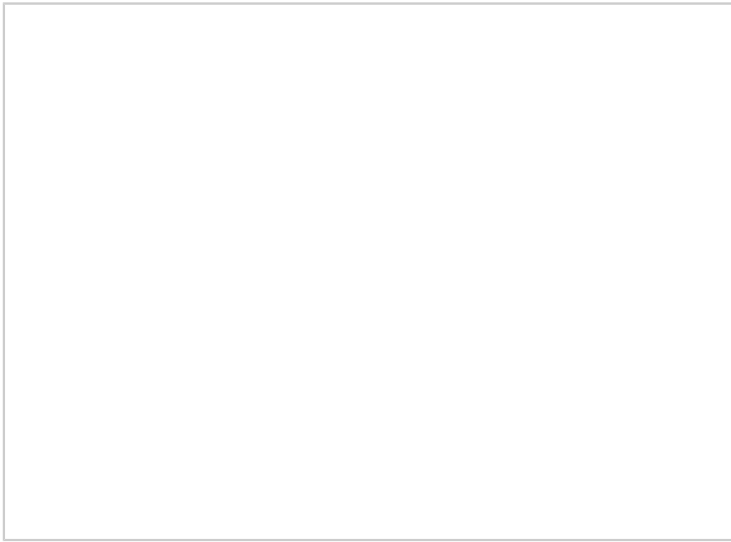
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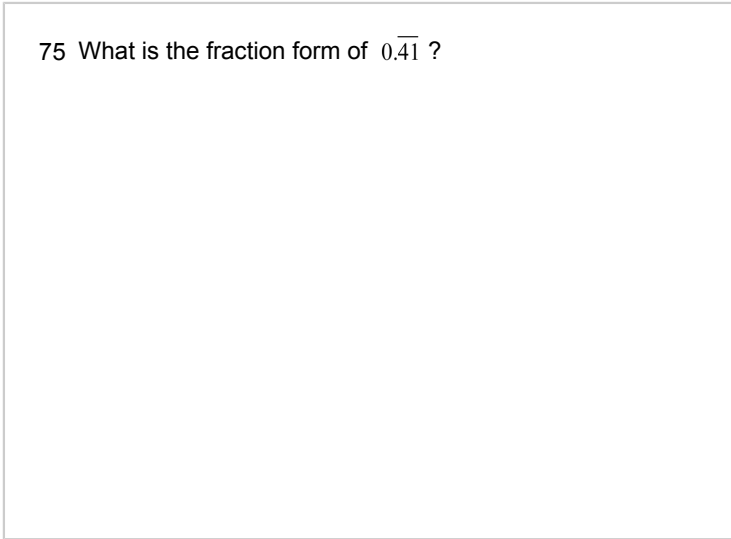
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75 What is the fraction form of  $0.\overline{41}$  ?



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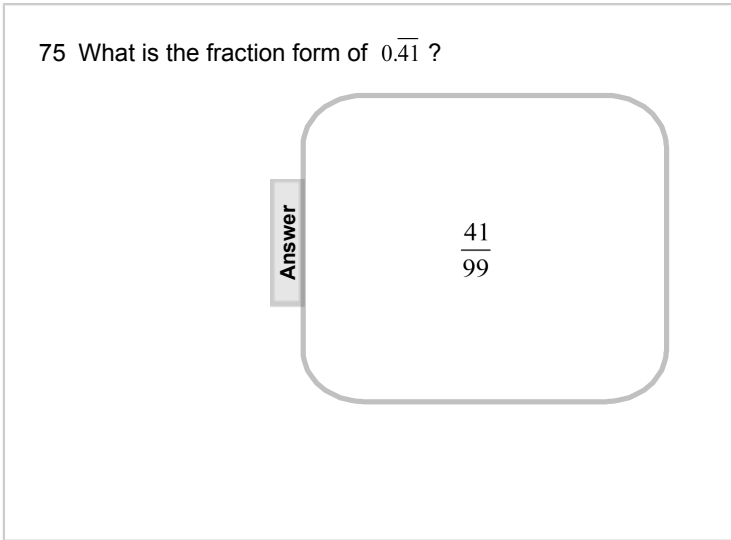
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75 What is the fraction form of  $0.\overline{41}$  ?

Answer

$$\frac{41}{99}$$


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76 What is the fraction form of  $0.\overline{534}$  ?

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76 What is the fraction form of  $0.\overline{534}$  ?

Answer

$$\frac{534}{999}$$

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### Patterns

Look over your answers for the last several questions. Do you notice any patterns?

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### Patterns

Look over  
questi

Answer

The digits that repeat will become the numerator. The number of digits that repeat will become the number of 9's in the denominator.

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### Convert $0.\overline{9}$ to a Fraction

Interesting Example:

1.  $x = 0.999\dots$
2.  $x = 0.\overline{999}$   
    1 digit repeats
3.  $10x = 10(0.999\dots)$   
    $10x = 9.999\dots$
4.  $10x = 9 + 0.999\dots$
5.  $10x = 9 + x$
6.  $10x = 9 + x$   
    $\frac{-x}{9} = \frac{-x}{9}$   
    $\frac{9x}{9} = \frac{9}{9}$
7.  $x = 1$

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## Exponents, Squares, Square Roots & Perfect Squares

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### Powers of Integers

Just as multiplication is repeated addition, exponents are repeated multiplication.

For example,  $3^5$  reads as "3 to the fifth power" =  $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

In this case "3" is the base and "5" is the exponent.

The base, 3, is multiplied by itself 5 times.

### Powers of Integers

When evaluating exponents of negative numbers, keep in mind the meaning of the exponent and the rules of multiplication.

For example,  $(-3)^2 = (-3)(-3) = 9$ ,

is the same as  $(3)^2 = (3)(3) = 9$ .

However,  $(-3)^2 = (-3)(-3) = 9$

is NOT the same as  $-3^2 = -(3)(3) = -9$ ,

Similarly,  $(3)^3 = (3)(3)(3) = 27$

is NOT the same as  $(-3)^3 = (-3)(-3)(-3) = -27$ ,

### Powers of Integers

When evaluating the meaning

For example

is the same

However

is NOT the same

Similarly,

is NOT the same as

$$(-3)^3 = (-3)(-3)(-3) = -27,$$

Math Practice

**MP.6: Attend to precision.**

mind

**Emphasize the difference between  $-4^2$  and  $(-4)^2$  w/ even powers. In  $-4^2$ , only the 4 is getting squared and the negative stays in front, whereas in  $(-4)^2$ , -4 is being multiplied by itself, making the answer positive.**

**Ask: What does the exponent (which ever number it is) mean?**

**How do you know the sign of your answer?**

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77 What is  $(-7)^2$ ?

49

-49

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77 What is  $(-7)^2$ ?

49

-49

Answer

**A**

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78 What is  $-8^2$ ?

- 64
- 64

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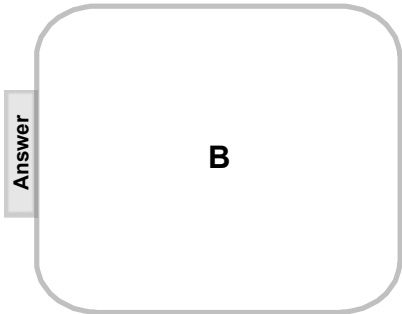
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78 What is  $-8^2$ ?

- 64
- 64



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79 What is  $-2^4$ ?

- 16
- 16

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79 What is  $-2^4$ ?

- 16
- 16

Answer

**B**

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80 What is  $(-2)^6$ ?

- 64
- 64

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80 What is  $(-2)^6$ ?

- 64
- 64

Answer

**A**

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81 Evaluate:  $4^3$

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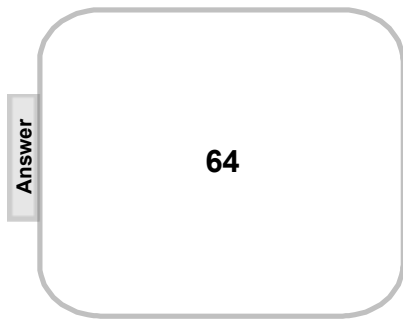
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81 Evaluate:  $4^3$



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82 Evaluate:  $(-2)^7$

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82 Evaluate:  $(-2)^7$

Answer

**-128**

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83 Evaluate:  $(-3)^4$

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83 Evaluate:  $(-3)^4$

Answer

**81**

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### Special Term: Squares

A number raised to the second power can be said to be "squared."  
That's because the area of a square of length x is  $x^2$ : "x squared."

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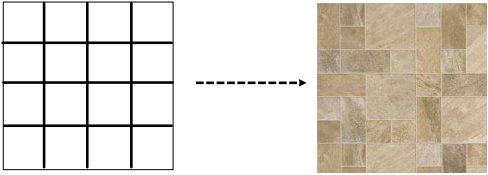
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### Area of a Square

The **area** of a figure is the number of square units needed to cover the figure.  
The area of the square below is 16 square units because 16 square units are needed to COVER the figure...



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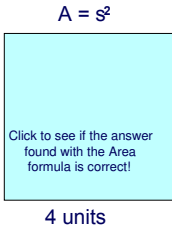
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### Area of a Square

The area (A) of a square can be found by squaring its side length, as shown below:



The area (A) of a square is labeled as square units, or units<sup>2</sup>, because you cover the figure with squares...

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84 What is the area of a square with sides of 5 inches?

- A 16 in<sup>2</sup>
- B 20 in<sup>2</sup>
- C 25 in<sup>2</sup>
- D 30 in<sup>2</sup>

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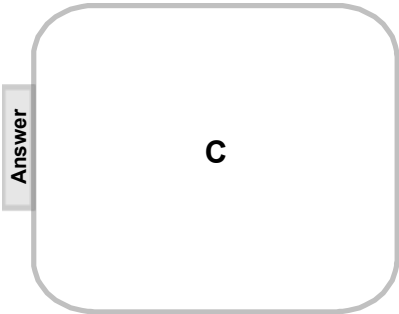
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84 What is the area of a square with sides of 5 inches?

- A 16 in<sup>2</sup>
- B 20 in<sup>2</sup>
- C 25 in<sup>2</sup>
- D 30 in<sup>2</sup>



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85 What is the area of a square with sides of 6 inches?

- A 16 in<sup>2</sup>
- B 20 in<sup>2</sup>
- C 24 in<sup>2</sup>
- D 36 in<sup>2</sup>

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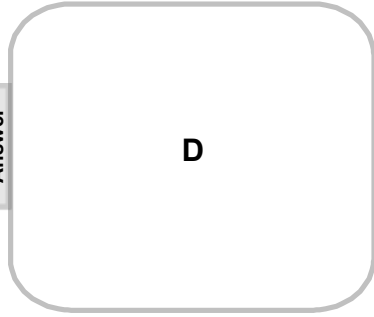
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85 What is the area of a square with sides of 6 inches?

- A 16 in<sup>2</sup>
- B 20 in<sup>2</sup>
- C 24 in<sup>2</sup>
- D 36 in<sup>2</sup>

Answer

**D**



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86 If a square has an area of 9 ft<sup>2</sup>, what is the length of a side?

- A 2 ft
- B 2.25 ft
- C 3 ft
- D 4.5 ft

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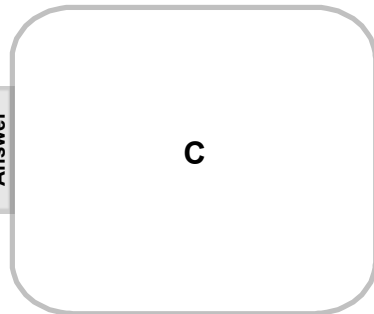
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86 If a square has an area of 9 ft<sup>2</sup>, what is the length of a side?

- A 2 ft
- B 2.25 ft
- C 3 ft
- D 4.5 ft

Answer

**C**



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87 What is the area of a square with a side length of 16 in?

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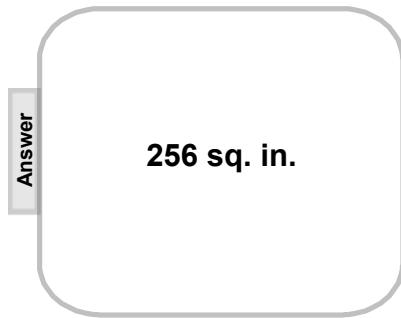
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87 What is the area of a square with a side length of 16 in?



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88 What is the side length of a square with an area of 196 square feet?

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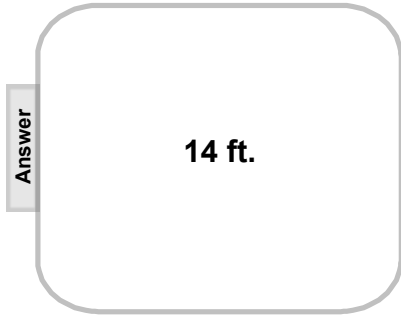
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88 What is the side length of a square with an area of 196 square feet?




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### Square

When you **square** a number you multiply it by itself.

$5^2 = \underline{\quad} = \underline{\quad}$  so the square of 5 is  $\underline{\quad}$

You can indicate squaring a number with an **exponent** of 2, by asking for the square of a number, or by asking for a number squared.

What is the square of seven? click  $\underline{\quad}$

What is nine squared? click  $\underline{\quad}$

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### Squares

Make a list of the numbers 1-15 and then square each of them.

Your paper should be set up as follows:

<u>Number</u>	<u>Square</u>
1	1
2	4
3	

(and so on)

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## The Root as an Inverse Operation

Performing an operation and then the inverse of that operation returns us to where we started.

We already saw that if we add 5 to a number and then subtract 5, we get back to the original number.

Or, if we multiply a number by 7 and then divide by 7, we get back to the original number.

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## The Root as an Inverse Operation

Inverses of exponents are a little more complicated for two reasons.

First, there are two possible inverse operations.

The equation  $16 = 4^2$  provides the answer 16 to the question: what is 4 raised to the power of 2?

One inverse operation is shown by:  $4 = \sqrt{16}$

This provides the answer 4 to the question: What number raised to the power of 2 yields 16?

This shows that the square root of 16 is 4.

It's true since  $(4)(4) = 16$

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## Logs as an Inverse Operation

The other inverse operation will not be addressed until Algebra 2.

Just for completeness, that inverse operation is  $2 = \log_4 16$ .

It provides the answer 2 to the question:

To what power must 4 be raised to get 16.

You'll learn more about that in Algebra II, but you should realize it's the other possible inverse operation.

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## The Root as an Inverse Operation

Number	Square
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225

The numbers in the right column are squares of the numbers in the left column.

If you want to "undo" squaring a number, you must take the **square root** of the number. The symbol for square root is called the radical sign and it looks like this:



So, the numbers in the left column are the square roots of the numbers in the right column. For example:

$$\sqrt{81} = 9$$

## Perfect Square

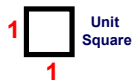
Number	Square
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225

When the square root of a number is a whole number, the number is called a **perfect square**.

Since all of the numbers in the right hand column have whole numbers for their square roots, this is a list of the first 15 perfect squares.

## A Closer Look at Perfect Squares

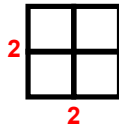
A number is a perfect square if you can take that quantity of 1x1 unit squares and form them into a square.



For Example:

4 is a perfect square, because you can take 4 unit squares and form them into a 2x2 square.

(Notice that the square root of 4 is the length of one of its sides, since that side times itself equals 4.)



*Is 5 a perfect square? Explain how you know.*

### Find the Square Root

You may refer to your chart if you need to.

$\sqrt{4} = \square$

$\sqrt{49} = \square$

$\sqrt{64} = \square$

$\sqrt{1} = \square$

$\sqrt{0} = \square$

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89 What is  $\sqrt{1}$  ?

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89 What is  $\sqrt{1}$  ?

**Answer**

**1**

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90 What is  $\sqrt{81}$  ?

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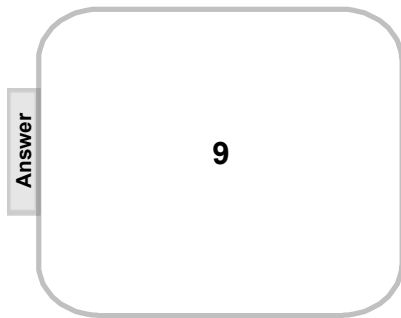
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90 What is  $\sqrt{81}$  ?



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91 What is the square of 15?

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91 What is the square of 15?

Answer

225

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92 What is  $\sqrt{256}$  ?

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92 What is  $\sqrt{256}$  ?

Answer

16

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93 What is  $13^2$ ?

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93 What is  $13^2$ ?

Answer

**169**

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94 What is  $\sqrt{196}$  ?

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94 What is  $\sqrt{196}$  ?

Answer

**14**

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95 What is the square of 18?

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95 What is the square of 18?

Answer

**324**

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96 What is 11 squared?

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96 What is 11 squared?



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97 What is 20 squared?

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97 What is 20 squared?

Answer

400

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## Glossary & Standards

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Teacher Notes

**Vocabulary Words are bolded in the presentation. The text box the word is in is then linked to the page at the end of the presentation with the word defined on it.**

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# Area

The number of square units needed to cover a figure.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Area = 16 square units

4 units
Area = 4 units x 4 units Area = 16 square units

4 units
Area = (4 units) <sup>2</sup> Area = 16 units <sup>2</sup>

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# Exponent

A quick way to write repeated multiplication.

also known as Power	What is $7^2$ ? $\frac{7}{1} \times \frac{7}{2} = 49$ $7^2 = 49$	ten to the power of 3 $10^3 =$ $\frac{10}{1} \times \frac{10}{2} \times \frac{10}{3} =$ 1,000
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# Fractions

Numbers created through division written as the ratio of two numbers.

$\frac{2}{3}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{3}$ $\frac{4}{5}$ $\frac{7}{5}$ $\frac{80}{4}$	Dividing by zero is not allowed.
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## Integers

Positive numbers, negative numbers  
and zero

..., -2, -1, 0, 1, 2, ...	$\mathbb{Z}$ symbol for integers
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## Inverse Operation

The operation that reverses the  
effect of another operation.

Addition $+$ Subtraction $-$ Multiplication $\times$ Division $\div$	$-5 + x = 5$ $+5$ $+5$ <hr/> $x = 10$	$11 = 3y + 2$ $-2$ $-2$ <hr/> $9 = 3y$ $\div 3$ $\div 3$ <hr/> $3 = y$
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## Natural Numbers

Counting numbers

1, 2, 3, 4, ...	$\mathbb{N}$ symbol for natural numbers
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## Perfect Square

A number made by squaring a whole number.

49 ✓	169 ✓	24 ✗
$\sqrt{49} = 7$	$\sqrt{169} = 13$	$\sqrt{24} = 4.898\dots$
$(7)(7) = 49$	$(13)(13) = 169$	4.89897... ≠ whole number

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## Square

To multiply a number by itself.

What is 4 squared?	What is the square of 6?	What is 10 squared?
$4^2 =$	$6^2 =$	$10^2 =$
$4 \times 4 = (4)(4) =$	$6 \times 6 = (6)(6) =$	$10 \times 10 = (10)(10) =$
16	36	100

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## Square Root


A value that, when multiplied by itself, gives the number. The result of undoing the squaring of a number.

Symbol: $\sqrt{\quad}$ "radical sign"	$\sqrt{64} = 8$ $(8)(8) = 64$ $8 \times 8 = 64$	$\sqrt{121} = 11$ $(11)(11) = 121$ $11 \times 11 = 121$
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# Whole Numbers

Counting numbers including 0

0, 1, 2, 3, ...	 symbol for whole numbers
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## Standards for Mathematical Practice

- MP1 Making sense of problems & persevere in solving them.
- MP2 Reason abstractly & quantitatively.
- MP3 Construct viable arguments and critique the reasoning of others.
  
- MP4 Model with mathematics.
- MP5 Use appropriate tools strategically.
- MP6 Attend to precision.
- MP7 Look for & make use of structure.
- MP8 Look for & express regularity in repeated reasoning.

Click on each standard to bring you to an example of how to meet this standard within the unit.



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