NAME ____ DATE ______________ PERIOD _____

1-1 Enrichment

Bargain Hunt

Use the For Sale signs on this page to solve each problem. If information you need is not given, write "cannot be solved."

2 for $5
JIGSAW PUZZLES
Regularly $3.49 each

Save!
Sweatshirts
$9.99 each
Regularly $11.99

Videotapes!!!
5 for $45.95!

This Week Only!
Model Airplane Kits $3.99
Reg. $4.99–$24.99

Going Out of Business
Handbags—3 for $15

1. Kiko works Saturday mornings at the videotape store. She bought ten videotapes on sale and used a $10 employee discount coupon to help pay for the tapes. How much did she spend in all?

2. Toni bought six handbags at the store that is going out of business. How much did she spend for each handbag?

3. Sid earned $40 working after school. How much money will he have left if he buys a sweatshirt and four jigsaw puzzles?

4. Suzette bought six jigsaw puzzles and a model airplane kit. How much change did she receive from a $20 bill?

5. Last week Norrine bought a model airplane kit for $18.67. How much would she have saved if she had waited until this week to buy the kit?

6. How much would you save if you bought three sweatshirts and two jigsaw puzzles?

1-2 Enrichment

The Four-Digits Problem

Use the digits 1, 2, 3, and 4 to write expressions for the numbers 1 through 50. Each digit is used exactly once in each expression. (There might be more than one expression for a given number.)

You can use addition, subtraction, multiplication (not division), exponents, and parentheses in any way you wish. Also, you can use two digits to make one number, as in 34. A few expressions are given to get you started.

$1 = (3 \times 1) - (4 - 2)$	$18 =$	$35 = 2^{(4+1)} + 3$
$2 =$	$19 = 3(2 + 4) + 1$	$36 =$
$3 =$	$20 =$	$37 =$
$4 =$	$21 =$	$38 =$
$5 =$	$22 =$	$39 =$
$6 =$	$23 = 31 - (4 \times 2)$	$40 =$
$7 =$	$24 =$	$41 =$
$8 =$	$25 =$	$42 =$
$9 =$	$26 =$	$43 = 42 + 1^3$
$10 =$	$27 =$	$44 =$
$11 =$	$28 =$	$45 =$
$12 =$	$29 = 2^{(4+1)} - 3$	$46 =$
$13 =$	$30 =$	$47 =$
$14 =$	$31 =$	$48 =$
$15 = 2(3 + 4) + 1$	$32 =$	$49 =$
$16 =$	$33 =$	$50 =$
$17 =$	$34 =$	

NAME ______________________________ DATE ____________ PERIOD _____

1-3 Enrichment

Nested Expressions

Sometimes more than one set of parentheses are used to group the quantities in an expression. These expressions are said to have "nested" parentheses. The expression below has "nested" parentheses.

$$(4 + (3 \cdot (2 + 3)) + 8) \div 9$$

Expressions with several sets of grouping symbols are clearer if braces such as { } or brackets such as [ ] are used. Here is the same example written with brackets and braces.

$$\{4 + [3 \cdot (2 + 3)] + 8\} \div 9$$

To evaluate expressions of this type, work from the inside out.

$$\begin{aligned} \{4 + [3 \cdot (2 + 3)] + 8\} \div 9 &= \{4 + [3 \cdot 5] + 8\} \div 9 \\ &= [4 + 15 + 8] \div 9 \\ &= 27 \div 9 \\ &= 3 \end{aligned}$$

Evaluate each expression.

1. $3 + [(24 \div 8) \cdot 7] - 20$

2. $[(16 - 7 + 5) \div 2] - 7$

3. $[2 \cdot (23 - 6) + 14] \div 6$

4. $50 - [3 \cdot (15 - 5)] + 25$

5. $12 + \{28 - [2 \cdot (11 - 7)] + 3\}$

6. $\{75 + 3 \cdot [(17 - 9) \div 2]\} \cdot 2$

7. $20 + \{3 \cdot [6 + (56 \div 8)]\}$

8. $\{4 + [5 \cdot (12 - 5)] + 15\} \cdot 10$

9. $\{15 \cdot [(38 - 26) \div 4]\} - 15$

10. $\{[34 + (6 \cdot 5)] \div 8\} + 40$

NAME ____ DATE ______________ PERIOD _____

1-4 Enrichment

Albert Einstein's Famous Theory

When you have solved the puzzle below, the letters in the heavy black boxes will spell the name of an important scientific theory proposed by Albert Einstein. His theory relates mass and energy.

Use these clues to complete the puzzle below.

1. A language of symbols.

2. To evaluate an expression, you ________ the variable with a number.

3. To find a specific numerical value for an algebraic expression.

4. An expression must contain at ________ one operation as well as variables or numbers.

5. The order of ________ helps you to know which operation to do first when evaluating an expression.

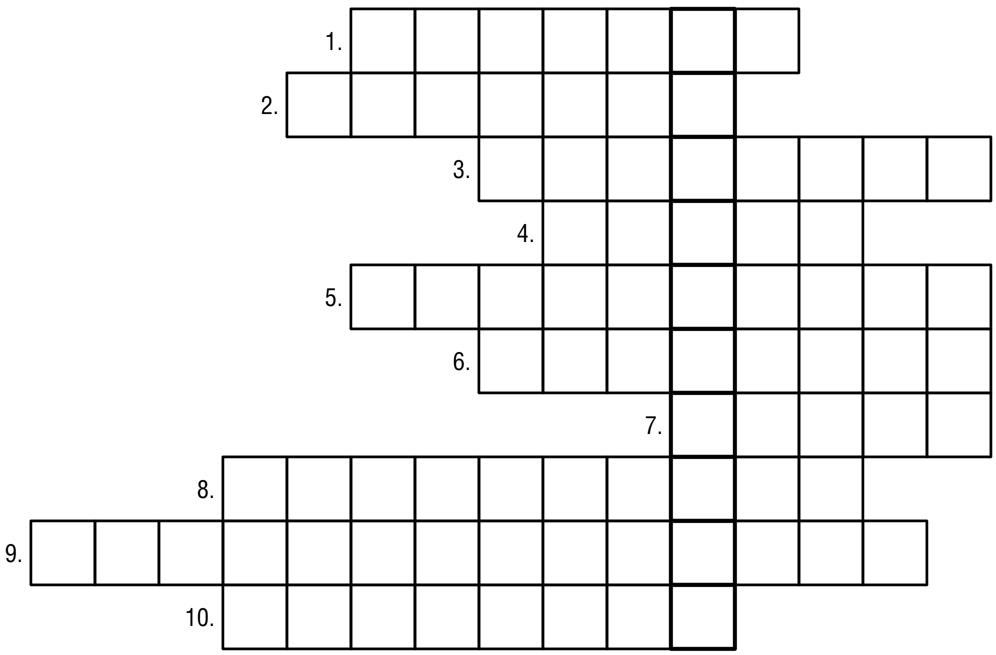
6. A symbol that stands for an unknown quantity.

7. The worth of something.

8. You call $2a + 3b$ an algebraic ________.

9. The process of finding the product of two numbers is called ________.

10. In an algebraic expression, you should ________ or divide before you add or subtract.



1-5 Enrichment

Equations as Models

When you write an equation that represents the information in a problem, the equation serves as a model for the problem. One equation can be a model for several different problems.

Each of Exercises 1–8 can be modeled by one of these equations.

$n + 2 = 10$ $\quad n - 2 = 10$ $\quad 2n = 10$ $\quad \frac{n}{2} = 10$

Choose the correct equation. Then solve the problem.

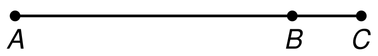
1. Chum earned \$10 for working two hours. How much did he earn per hour?

2. Ana needs \$2 more to buy a \$10 scarf. How much money does she already have?

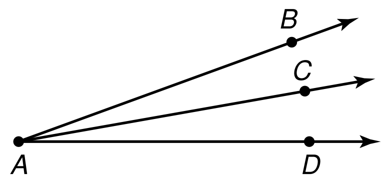
3. Kathy and her brother won a contest and shared the prize equally. Each received \$10. What was the amount of the prize?

4. Jameel loaned two tapes to a friend. He has ten tapes left. How many tapes did Jameel originally have?

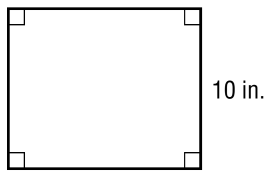
5. In the figure below, the length of $\overline{AC}$ is 10 cm. The length of $\overline{BC}$ is 2 cm. What is the length of $\overline{AB}$?



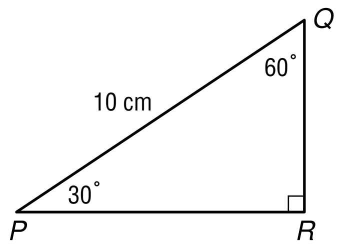
6. Ray $\overline{AC}$ bisects $\angle BAD$. The measure of $\angle BAC$ is 10°. What is the measure of $\angle BAD$?



7. The width of the rectangle below is 2 inches less than the length. What is the length?



8. In the triangle below, the length of $\overline{PQ}$ is twice the length of $\overline{QR}$. What is the length of $\overline{QR}$?



9. **CHALLENGE** On a separate sheet of paper, write a problem that can be modeled by the equation $3a + 5 = 29$.

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1-6 Enrichment

The Yoruba

The Yoruba are an African people who live in southwestern Nigeria and parts of Benin and Togo. No one is sure exactly how long the Yoruba have inhabited this region, but by the year A.D. 1300 they had built many cities surrounded by farms. Through the ages, the Yoruba have made many contributions to the development of art, music, commerce, and mathematics.

At the right, you see the basic numbers of the **Yoruba numeration system**. All other numbers are formed by combining these basic numbers. For example, this is how the Yoruba form the number 36.

$$\begin{aligned}\text{ogun} \times \text{eeji} - \text{eerin} &= 20 \times 2 - 4\\ &= 40 - 4\\ &= 36\end{aligned}$$

So, the Yoruba phrase for 36 is *eerin din logoji*, which means "four less than two twenties."

Yoruba Names for Numbers	
ookan	1
eeji	2
eeta	3
eerin	4
aarun	5
eefa	6
eeje	7
eejo	8
eesan	9
eewaa	10
ogun	20
ogbon	30
igba	200
irinwo	400

Each expression shows how the Yoruba form a certain number. What is the number?

1. a. eeta + ogun
   b. ookan + ogun
   c. eeji + ogbon

2. a. ogun − eeji
   b. ogun − eerin
   c. ogbon − aarun

3. a. ogun × eeji + ookan
   b. ogun × eeta − eeta
   c. ogun × aarun − eerin

4. a. ogun × eeta − eewaa − aarun
   b. ogun × eerin − eewaa + eeji
   c. ogun × aarun − eewaa − ookan

5. Refer to your answers to Exercise 4 above. How do you think the Yoruba form the number 50? 70? 90?

6. **CHALLENGE** How do you think the Yoruba form the number 2,000?

1-7 Enrichment

Nested Magic Squares

A magic square is a square arrangement of numbers in which the sum of the numbers in every row, column, and diagonal is the same number. The numbers 1 through 49 can be arranged to make three nested magic squares. First, the large 7-by-7 outer square is magic. Remove its border and you get a 5-by-5 magic square. Finally, remove the border again to get a 3-by-3 magic square.

In the figure below insert the rest of the numbers 1 through 49 to make three nested magic squares.

46	1	2	3	42	41	40
		24	29	22		

NAME ____ DATE ______________ PERIOD _____

1-8 Enrichment

Using a Measurement Conversion Chart

You may sometimes want to convert customary measurements to metric measurements. For example, suppose you are reading about horses and want to know how long 5 furlongs are.

Start by finding a conversion table such as the one shown here. (Dictionaries often include such tables.)

1 mil	= 0.001 inch	= 0.0254 millimeter
1 inch	= 1,000 mil	= 2.54 centimeters
12 inches	= 1 foot	= 0.3048 meter
3 feet	= 1 yard	= 0.9144 meter
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet	= 1 rod	= 5.029 meters
40 rods	= 1 furlong	= 201.168 meters
8 furlongs, 5,280 feet, 1,760 yards	= 1 (statute) mile	= 1.6093 kilometers
3 miles	= 1 (land) league	= 4.828 kilometers

To change from a large unit to a small unit, multiply. To change from a small unit to a large one, divide.

EXAMPLE 1 **Change 5 furlongs to meters.**

$5 \times 201.168 = 1{,}005.84$
So, 5 furlongs is about 1,000 meters, or 1 kilometer.

Change each measurement to a metric measurement. Round each answer to the nearest tenth.

1. 10 yards

2. 100 leagues

3. 10 inches

4. 100 rods

5. 1,000 mils

6. 10 feet

7. 50 miles

8. 50 furlongs

9. 50 inches

10. 200 feet

11. 200 miles

12. 200 yards

1-9 Enrichment

The Speed of Light

Light travels at approximately 186,000 miles per second. You can use the formula below to find how long it takes light to travel from one place to another.

$$\frac{\text{distance}}{\text{speed of light}} = \text{time}$$

For example, the sun is about 9.3×10^7 miles from Earth. If a gigantic explosion were to occur on the sun, how long would it take to see it from Earth?

$$\frac{93{,}000{,}000}{186{,}000} = 500 \leftarrow \text{ Write } 9.3 \times 10^7 \text{ as } 93{,}000{,}000.$$

It would take about 500 seconds to see the explosion.

Now you need to change seconds to minutes, since minutes is a more sensible unit for time in this case. To change seconds to minutes, divide.

$$\frac{500}{60} \rightarrow \begin{array}{r} 8 \\ 60\overline{)500} \\ \underline{480} \\ 20 \end{array}$$

It would take about 8 minutes to see the explosion from Earth.

Compute each amount of time it takes for light to travel to Earth from each place. Then change seconds to a sensible unit.

	Location	Closest Distance to Earth	Time (in seconds)	Time (in a sensible unit)
1.	moon	2.2×10^5 mi		
2.	Halley's Comet	3.11×10^6 mi		
3.	Mars	3.46×10^7 mi		
4.	Venus	2.57×10^7 mi		
5.	Jupiter	3.67×10^8 mi		
6.	Pluto	2.67×10^9 mi		
7.	nearest star	2.48×10^{13} mi		

NAME ______________________________________ DATE ______________ PERIOD _____

2-1 Enrichment

Breaking the Code

Many secret messages are written in code. One way to construct a code is to use a *substitution alphabet*. For example, the letter A might be coded into Y, the letter B into R, and so on until every letter is coded.

To break a code of this type, it is helpful to know that the letters of the alphabet occur with different frequencies. For example, the letter E occurs an average of 13 times out of every 100 letters. In any message, however, the frequencies will vary.

Letters	Frequency (per hundred)
E	13
T	9
A, O	8
N	7
I, R	6.5
S, H	6

Use the clues below to break this coded message.

"FOZ BUJRSJBKD CJMMJSGDFE," UZQKUAZC VOZUDNSA ONDQZV, "DKE JR FOZ MKSF NM FOZUZ HZJRT FNN QGSO ZYJCZRSZ. POKF PKV YJFKD PKV NYZUDKJC KRC OJCCZR HE POKF PKV JUUZDZYKRF."

1. On another sheet of paper, make a frequency distribution chart for the letters in the message. Which seven letters in the message appear most frequently?

2. The most frequently used letter in the alphabet is E, so write an E underneath each place this letter occurs in the message.

3. The word *the* is very common and appears twice in the message. Use this fact to determine which letters stand for T and H.

4. The word *was* occurs three times in the last sentence. What letters represent W, A, and S in the message?

5. The message is a quote from a famous detective whose last name begins with H. Complete the detective's name and you will have three more letters.

6. What is the message?

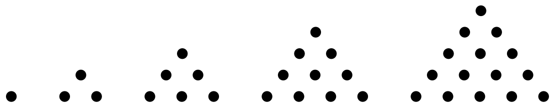
Lesson 2–1

NAME ____ DATE ______________ PERIOD _____

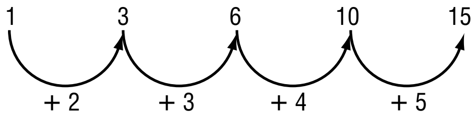
2-2 Enrichment

Number Patterns

The dot diagram below illustrates a number pattern.



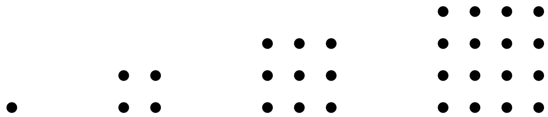
You can discover what number in the pattern comes next by drawing the next figure in the dot pattern. You can also use thinking with numbers. Try to see how two consecutive numbers in the pattern are related.



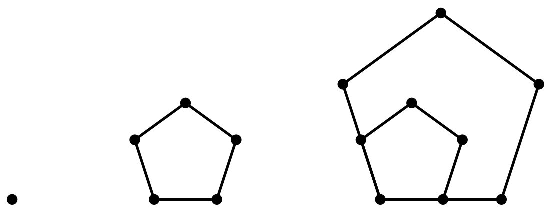
It looks like the next number in the pattern is obtained by adding 6 to 15. The next number in the pattern is 21. You can check this by drawing the next figure in the dot pattern.

Write the next two numbers in the number pattern for each dot diagram.

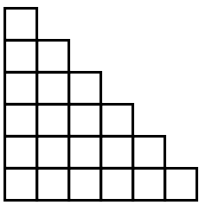
1.



2.



3. A staircase is being built from cubes. How many cubes will it take to make a staircase 25 cubes high?



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2-3 Enrichment

Enhanced Line Plots

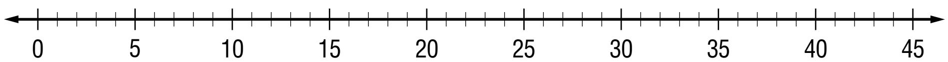
You have learned to create line plots to analyze given data. Sometimes altering a line plot can show even more information about a data set.

SPORTS For Exercises 1–4, use the following data about the Super Bowl.

The National Football League began choosing its champion in the Super Bowl in 1967. The list below shows the margin of victory and the winning league for the first 36 Super Bowl games. In the list, A indicates that the winning team is from the American Football Conference (AFC), N indicates that the winning team is from the National Football Conference.

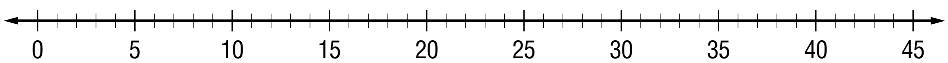
Year	Margin	Year	Margin	Year	Margin	Year	Margin
1	25-N	10	7-N	19	17-N	28	23-N
2	10-A	11	9-A	20	3-A	29	7-A
3	10-N	12	18-A	21	3-A	30	17-A
4	1-N	13	22-N	22	4-A	31	4-N
5	15-A	14	35-N	23	19-N	32	14-N
6	19-N	15	27-A	24	10-N	33	17-A
7	4-A	16	16-A	25	21-N	34	5-N
8	29-A	17	17-N	26	12-A	35	45-N
9	13-N	18	36-N	27	32-N	36	7-A

1. Make a line plot of the data.



2. What do you observe about the winning margins?

3. Make a new line plot for the winning margins by replacing each × with A for an AFC win or N for an NFC win. What do you observe about the winning margins when looking at this enhanced line plot?



2-4 Enrichment

Quartiles

The median is a number that describes the "center" of a set of data. Here are two sets with the same median, 50, indicated by ◯.

25	30	35	40	45	50	55	60	65	70	75
0	10	20	40	50	50	60	70	80	90	100

But, sometimes a single number may not be enough. The numbers shown in the triangles can also be used to describe the data. They are called *quartiles*. The lower quartile is the median of the lower half of the data. It is indicated by ▽. The upper quartile is the median of the upper half. It is indicated by △.

Circle the median in each set of data. Draw triangles around the quartiles.

1.

29	52	44	37	27	46	43	60	31	54	36

2.

1.7	0.4	1.4	2.3	0.3	2.7	2.0	0.9	2.7	2.6	1.2

3.

1,150	1,600	1,450	1,750	1,500	1,300	1,200

4.

5	2	9	7	9	3	7	8	7	2	5	6	9	5	1

Use the following set of test scores to solve the problems.

71 57 29 37 53 41 25 37 53 27
62 55 75 48 66 53 66 48 75 66

5. Which scores are "in the lower quartile"?

6. How high would you have to score to be "in the upper quartile"?

2-5 Enrichment

Back-to-Back Stem-and-Leaf Plots

You can use a **back-to-back stem-and-leaf plot** to compare two sets of data. In this type of plot, the leaves for one set of data are on one side of the stems, and the leaves for the other set of data are on the other side of the stems. Two keys to the data are needed.

MARKETING **For Exercises 1 and 2, use the following data about advertising to preteens and teens.**

Advertisers decide when to advertise their products on television based on when the people who are likely to buy will be watching. The table shows the percents of boys and girls ages 6 to 14 who watch television at different times of day. (Values are rounded to the nearest percent.)

Time	Boys	Girls
Monday–Friday, 6 A.M.–9 A.M	11	9
Monday–Friday, 3 P.M.–5 P.M.	21	22
Monday–Friday, 5 P.M.–8 P.M.	30	29
Monday–Saturday, 8 P.M.–10 P.M. and Sunday, 7 P.M.–10 P.M.	29	27
Saturday, 6 A.M.–8 A.M.	7	4
Saturday, 8 A.M.–1 P.M.	26	23
Saturday, 1 P.M.–5 P.M.	12	8
Saturday, 5 P.M.–8 P.M.	18	12
Sunday, 6 A.M.–8 A.M	3	3
Sunday, 8 A.M.–1 P.M.	10	9
Sunday, 1 P.M.–5 P.M.	12	7
Sunday, 5 P.M.–7 P.M.	15	9

1. Make a back-to-back stem-and-leaf plot of the data by recording the data about boys on the left side of the stems and recording the data about girls on the right side of the stems. Who watches television more often, boys or girls?

2. If you were scheduling advertising for a product aimed at pre-teen girls, when would you advertise? Explain your reasoning.

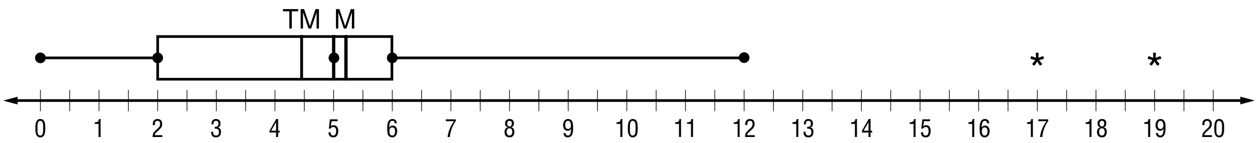
NAME ____ DATE ______________ PERIOD _____

2-6 Enrichment

The Trimmed Mean

Sometimes a mean can be distorted by outliers. To avoid this, exclude any outliers and compute a new mean. This new measure is called the *trimmed mean*.

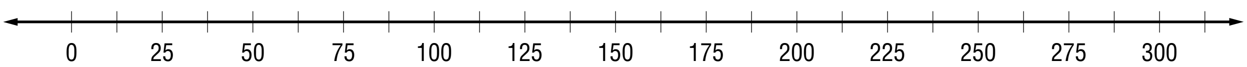
Data Points	0	2	3	4	5	6	9	10	11	12	17	19
Frequency	6	3	6	1	5	5	1	1	1	2	1	1



Construct a box-and-whisker plot for each frequency table. Mark the mean M and the trimmed mean TM.

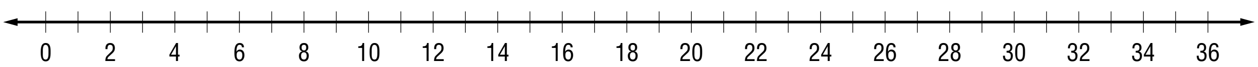
1. Areas of 48 States, Rounded to Nearest 25 Thousand Square Miles

Area (thousands)	0	25	50	75	100	125	150	275
Number of States	8	4	17	10	4	2	2	1



2. Projected Populations in 2010, Rounded to Nearest 2 Million

Population (millions)	0	2	4	6	8	10	12	16	18	22	36
Number of States	6	13	10	7	3	3	2	1	1	1	1



Use reference materials to predict the outliers.

3. Exercise 1

4. Exercise 2

2-7 Enrichment

African Americans in History

A magazine published this list of fifty African Americans who made significant contributions to American history and culture.

50 Influential Figures in African-American History	
Robert S. Abbott (1870–1940)	James Weldon Johnson (1871–1938)
Richard Allen (1760–1831)	Ernest E. Just (1883–1941)
Louis Armstrong (1900–1971)	Joe Louis (1914–1981)
Ella Baker (1903–1986)	Martin Luther King, Jr. (1929–1968)
James Baldwin (1924–1987)	Malcolm X (1925–1965)
Benjamin Banneker (1736–1806)	Benjamin E. Mays (1894–1984)
Ida B. Wells-Barnett (1862–1931)	Jesse Owens (1913–1980)
Mary McLeod Bethune (1875–1955)	Adam Clayton Powell, Jr. (1908–1972)
Ralph J. Bunche (1904–1971)	A. Philip Randolph (1889–1979)
George Washington Carver (1861?–1943)	Paul Robeson (1898–1976)
Martin R. Delany (1812–1885)	Jackie Robinson (1919–1972)
Frederick Douglass (1817–1895)	Mary Church Terrell (1863–1954)
Charles R. Drew (1904–1950)	Howard Thurman (1900–1981)
W. E. B. Du Bois (1868–1963)	William Monroe Trotter (1872–1934)
Paul Laurence Dunbar (1872–1906)	Sojourner Truth (1797?–1883)
Edward Kennedy Ellington (1899–1974)	Harriet Tubman (1821?–1913)
Marcus Garvey (1887–1940)	Henry McNeal Turner (1834–1915)
Prince Hall (1735?–1807)	Nat Turner (1800–1831)
Fannie Lou Hamer (1917–1977)	David Walker (1785–1830)
W. C. Handy (1873–1958)	Madame C. J. Walker (1867–1919)
Frances E. W. Harper (1825–1911)	Booker T. Washington (1856–1915)
Charles H. Houston (1895–1950)	Phillis Wheatley (1753?–1784)
Langston Hughes (1902–1967)	Daniel Hale Williams (1856–1931)
Zora Neale Hurston (1901?–1960)	Carter G. Woodson (1875–1950)
Jack Johnson (1878–1946)	Richard Wright (1908–1960)

1. On a separate sheet of paper, construct a histogram that displays the years of birth for these fifty people. Organize the data in twenty-year intervals, such as 1841–1860 and 1861–1880. If there is a question mark next to a person's year of birth, use that year in the histogram.

2. Refer to the histogram you constructed in Exercise 1. In which interval were the most people born? In which interval were the fewest born? What historical events can you associate with the times these people were living?

2-8 Enrichment

Choosing a Representative Sample

Statisticians often use **samples** to represent larger groups. For example, television ratings are based on the opinions of a few people who are surveyed about a program. The people surveyed are just part of the whole group of people who watched the program. When using samples, people taking surveys must make sure that their samples are representative of the larger group in order to ensure that their conclusions are not misleading.

ADVERTISING A company that makes athletic shoes is considering hiring a professional basketball player to appear in their commercials. Before hiring him, they are doing research to see if he is popular with teens. Would they get good survey results from taking a survey about the basketball player from each of these surveys?

1. 200 teens at a basketball game of the basketball player's team

2. 25 teens at a shopping mall

3. 500 students at a number of different middle and high schools

Decide whether each location is a good place to find a representative sample for the selected survey. Justify your answer.

4. number of hours of television watched in a month at a shopping mall

5. favorite kind of entertainment at a movie theater

6. whether families own pets in an apartment complex

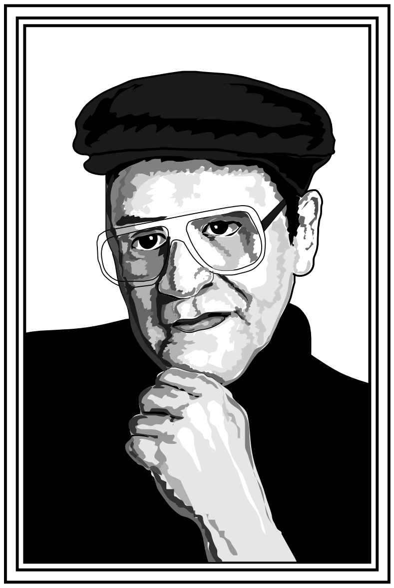
7. taste test of a soft drink at a grocery store

8. favorite teacher in a school cafeteria

9. teenagers' favorite magazine at five different high schools

3-1 Enrichment

Jaime Escalante



Jaime Escalante (1930–) was born in La Paz, Bolivia, and came to the United States in 1963. For ten years, he worked at odd jobs to support himself and his family while pursuing his dream—becoming certified to teach high school mathematics in California. As a mathematics teacher, he has become well known for his ability to inspire students to succeed in mathematics at levels they never thought possible. In 1988, the story of Mr. Escalante and a group of his students was the subject of the popular motion picture *Stand and Deliver*.

Mr. Escalante teaches concepts students must master if they are to succeed in high school and college mathematics. One of these is the concept of absolute value. For instance, a student should be able to solve an equation like $|y| = 6$ quickly using mental math. Here's how.

You know that $|6| = 6$ and $|-6| = 6$.

So, the equation $|y| = 6$ has *two* solutions: 6 and -6.

Solve each equation. (*Hint*: One equation has no solution.)

1. $|a| = 8$

2. $|r| = 0$

3. $|j| = -3$

4. $|t| + 1 = 15$

5. $10 - |m| = 3$

6. $|c| - 4 = 16$

7. $5|z| = 60$

8. $12 \div |g| = 4$

9. $48 = 8|x|$

10. $2|d| + 3 = 5$

11. $4|p| - 9 = 59$

12. $7|z| + 12 = 12$

13. Suppose that the value of x can be selected from the set $\{-2, -1, 0, 1, 2\}$. Find all of the solutions of the equation $|x| = x$.

14. One of these statements is false. Which one is it? Explain.

a. The absolute value of every integer is positive.

b. There is at least one integer whose absolute value is zero.

c. The absolute value of an integer is never negative.

Lesson 3–1

3-2 Enrichment

Quantitative Comparisons

An unusual type of problem is found on some standardized multiple-choice tests. This problem type is called the *quantitative comparison*.

In each quantitative comparison question, you are given two quantities, one in Column A and one in Column B. You are to compare the two quantities and shade one of four circles on an answer sheet.

Shade circle A	if the quantity in Column A is greater;
Shade circle B	if the quantity in Column B is greater;
Shade circle C	if the two quantities are equal;
Shade circle D	if the relationship cannot be determined from the information given.

Shade the correct oval to the left of each problem number.

		Column A	Column B
Ⓐ Ⓑ Ⓒ Ⓓ	**1.**	0.006 + 2	0.002 + 6
Ⓐ Ⓑ Ⓒ Ⓓ	**2.**	ten billion dollars	1,000 million dollars
Ⓐ Ⓑ Ⓒ Ⓓ	**3.**	20 inches	the perimeter of a square with an area of 25 square inches
Ⓐ Ⓑ Ⓒ Ⓓ	**4.**	half of one third	one fifth
Ⓐ Ⓑ Ⓒ Ⓓ	**5.**	the greatest possible product of two odd positive numbers less than 20	the greatest possible product of two even positive numbers less than 20
Ⓐ Ⓑ Ⓒ Ⓓ	**6.**	0.000000001	$-x$ is x if greater than 0
Ⓐ Ⓑ Ⓒ Ⓓ	**7.**	$\|x\|$	$\|x + 1\|$
Ⓐ Ⓑ Ⓒ Ⓓ	**8.**	$\|y\|$	$\|-y\|$
Ⓐ Ⓑ Ⓒ Ⓓ	**9.**	$2\|x\|$ if $x \neq 0$	$\|x\|$ if $x \neq 0$
Ⓐ Ⓑ Ⓒ Ⓓ	**10.**	$-x$ if x is less than 0	$\|x\|$ if x is less than 0

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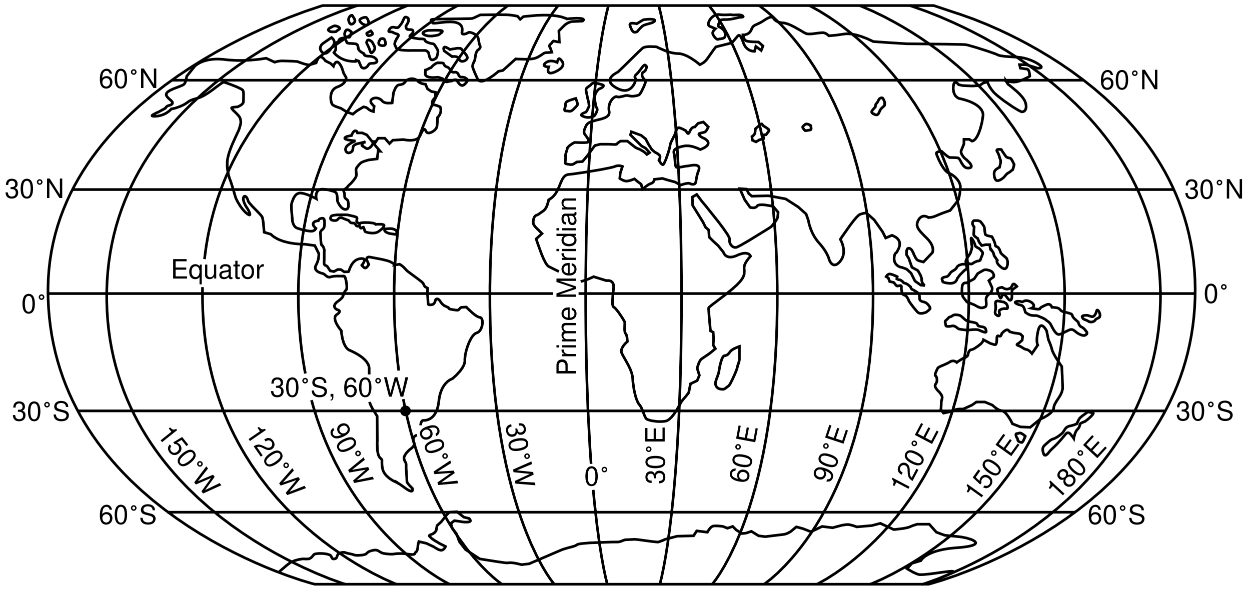
3-3 Enrichment

Latitude and Longitude

This world map shows some of the latitude and longitude lines. Latitude is measured in degrees north and south of the equator. Longitude is measured in degrees east and west of the prime meridian, a line passing through Greenwich, England. (Greenwich is a suburb of London.)

The latitude is usually given first. For example, the location of 30°S, 60°W is lower South America.



Name a place near each location. Use an atlas or other reference source to check your answers.

1. 30°N, 30°W

2. 30°S, 30°E

3. 60°N, 120°W

4. 15°N, 150°W

5. 30°S, 140°E

6. 25°N, 100°W

7. 40°N, 120°W

8. 45°N, 90°W

9. 40°N, 5°W

10. 60°N, 45°W

11. 35°N, 140°E

12. 0°, 60°E

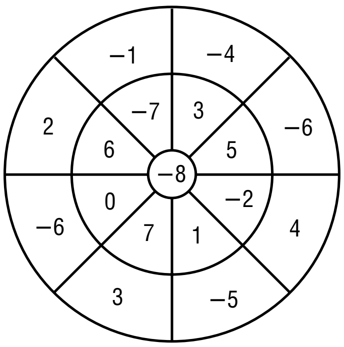
Lesson 3–3

3-4 Enrichment

Dartboard Puzzles

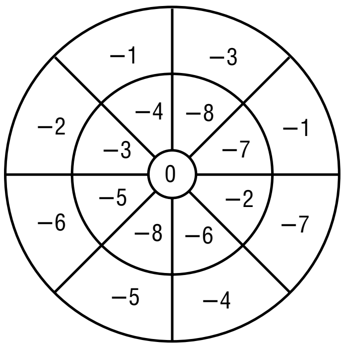
Three darts are thrown. Each dart must land on a different space in order to count. Find the highest and the lowest possible scores.

1.



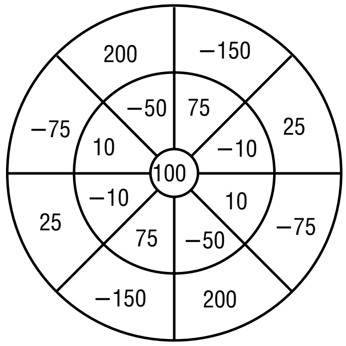
highest score:
lowest score:

2.



highest score:
lowest score:

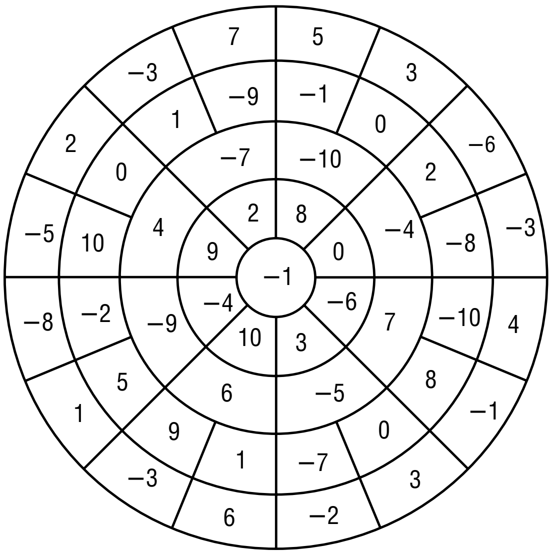
3.



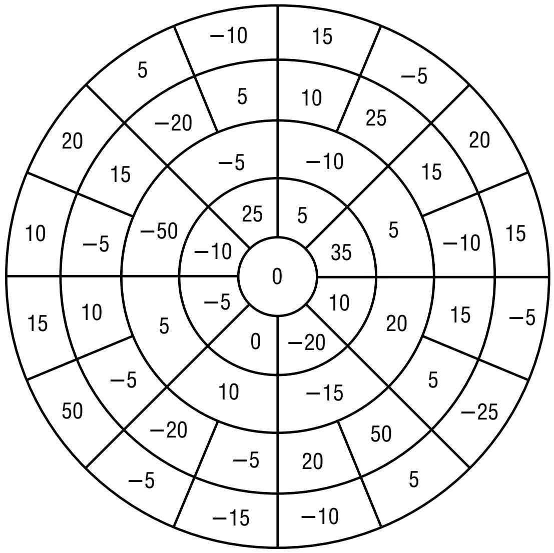
highest score:
lowest score:

In these problems, five darts are thrown. Each dart must land on a different space in order to count. Solve each puzzle.

4. Find three ways to make the score −5.



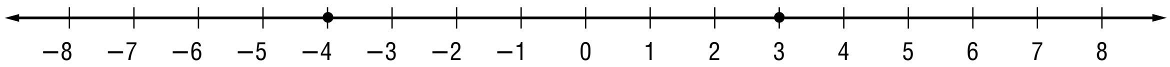
5. Find three ways to make the score 0.



3-5 Enrichment

Distance on the Number Line

To find the distance between two points on a number line, subtract their coordinates. Then, take the absolute value of the difference.



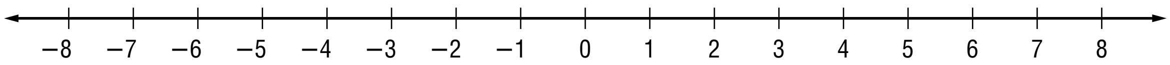
$$-4 - 3 = -7$$
$$|-7| = 7$$

You can also find the distance by finding the absolute value of the difference of the coordinates.

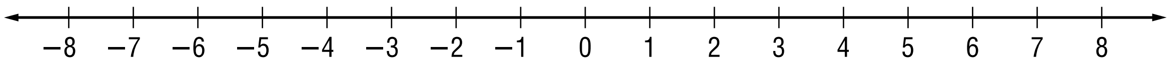
$$|-4 - 3| = 7$$

Graph each pair of points. Then write an expression using absolute value to find the distance between the points.

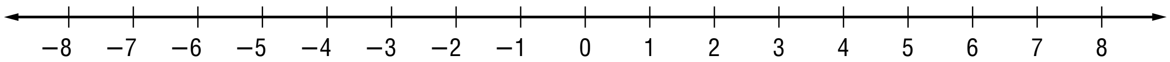
1. A at -5 and B at 2



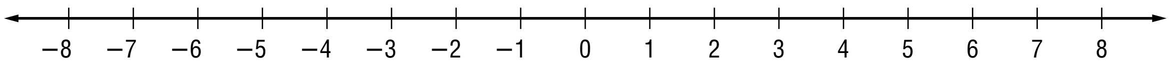
2. C at -7 and D at -1



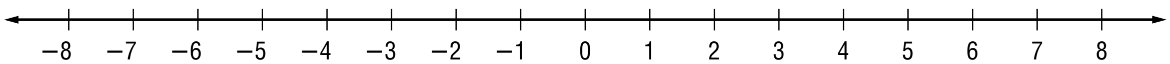
3. E at -5 and F at 5



4. W at 0 and X at 6



5. Y at -4 and Z at 0

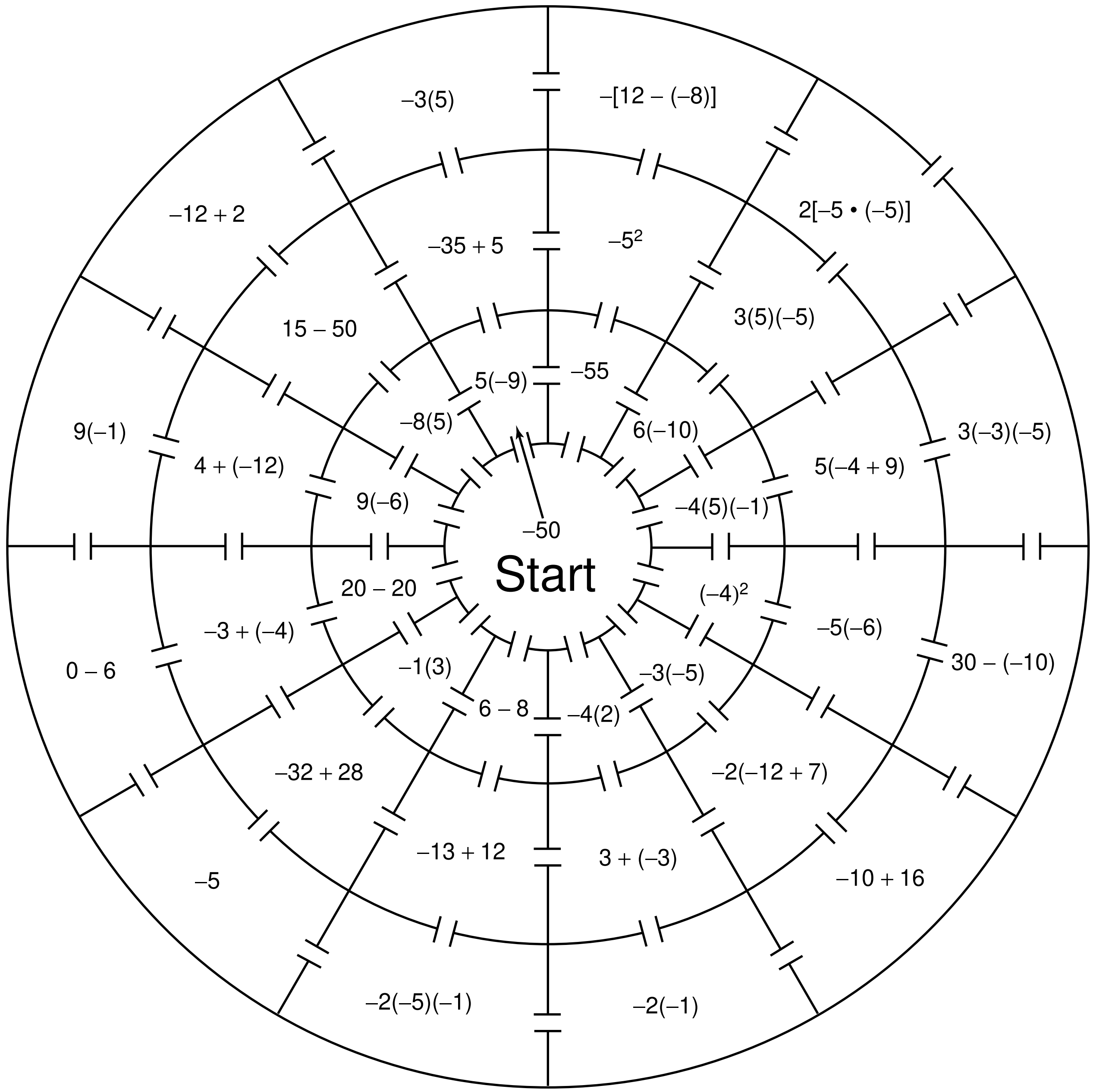


NAME ______________________________________ DATE ______________ PERIOD _____

3-6 Enrichment

Integer Maze

Find your way through the maze by moving to the expression in an adjacent section with the next highest value.



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3-7 Enrichment

Lesson 3–7

Division by Zero?

Some interesting things happen when you try to divide by zero. For example, look at these two equations.

$$\frac{5}{0} = x \qquad \frac{0}{0} = y$$

If you can write the equations above, you can also write the two equations below.

$$0 \cdot x = 5 \qquad 0 \cdot y = 0$$

However, there is no number that will make the left equation true. This equation has no solution. For the right equation, *every* number will make it true. The solutions for this equation are "all numbers."

Because division by zero leads to impossible situations, it is not a "legal" step in solving a problem. People say that division by zero is undefined, or not possible, or simply not allowed.

Describe the solution set for each equation.

1. $4x = 0$

2. $x \cdot 0 = 0$

3. $x \cdot 0 = x$

4. $\frac{0}{x} = 0$

5. $\frac{0}{x} = x$

6. $\frac{0}{x} = 5$

What values for x must be excluded to prevent division by 0?

7. $\frac{1}{x^2}$

8. $\frac{1}{x - 1}$

9. $\frac{1}{x + 1}$

10. $\frac{0}{2x}$

11. $\frac{1}{2x - 2}$

12. $\frac{1}{3x + 6}$

Explain what is wrong with this "proof."

13. Step 1 $0 \cdot 1 = 0$ and $0 \cdot (-1) = 0$

Step 2 Therefore, $\frac{0}{0} = 1$ and $\frac{0}{0} = -1$.

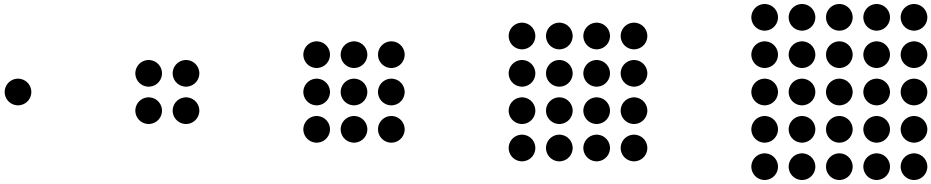
Step 3 Therefore, $1 = -1$.

NAME ______________________________ DATE ____________ PERIOD _____

Enrichment

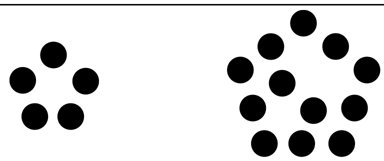
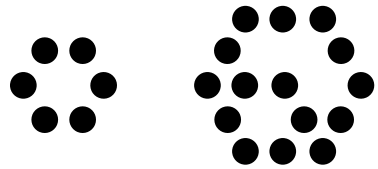
Expressions for Figurate Numbers

Figurate numbers are numbers that can be shown with dots arranged in specific geometric patterns. Below are the first five square numbers.



The expression n^2 will give you the number of dots in the nth square number. The variable n takes on the values 1, 2, 3, 4, and so on. So, to find the 10th square number, you would use 10 for n.

1. Match each set of dot patterns with its name and expression. Write exercise numbers in the boxes to show the matchings.

	Dot Patterns for Second and Third Numbers	Name of Figurate Number	Expression
a.		pentagonal ☐	$n(2n - 1)$ ☐
b.		hexagonal ☐	$\frac{n(n + 1)}{2}$ ☐
c.		triangular ☐	$\frac{n(3n - 1)}{2}$ ☐

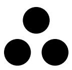
Use the algebraic expressions on this page to compute each number. Then make a drawing of the number on a separate sheet of paper.

2. 6th square

3. 4th triangular

4. 4th pentagonal

5. 4th hexagonal



6. 5th triangular

7. 5th pentagonal

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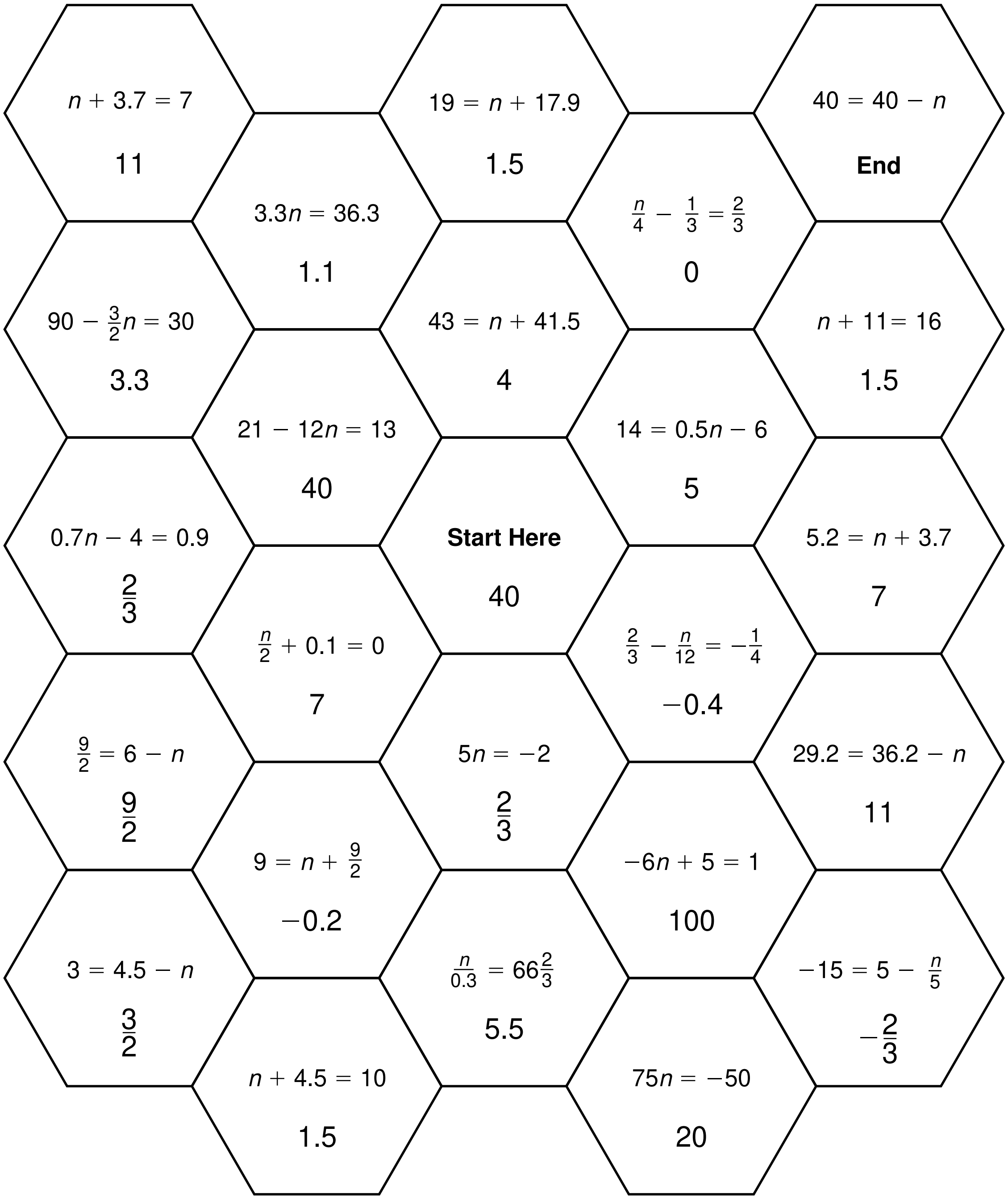
4-2 Enrichment

Equation Hexa-maze

This figure is called a *hexa-maze* because each cell has the shape of a hexagon, or six-sided figure.

To solve the maze, start with the number in the center. This number is the solution to the equation in one of the adjacent cells. Move to that cell. The number in the new cell will then be the solution to the equation in the next cell. At each move, you may only move to an adjacent cell. Each cell is used only once.



NAME ____ DATE ______________ PERIOD _____

4-3 Enrichment

Equations of the form $y = ax$ and $y = x \div a$ can be used to show how one quantity varies with another. Here are two examples.

Driving at a speed of 50 miles per hour, the distance you travel d varies directly with the time you are one the road t. The longer you drive, the farther you get. $d = 50t$

It is also the case that the time t varies directly with the distance d. The farther you drive, the more time it takes. $t = \frac{d}{50}$

Complete the equation for each situation. Then describe the relationship in words.

1. If you go on a diet and lose 2 pounds a month, after a certain number of months m, you will have lost p pounds.

2. You and your family are deciding between two different places for your summer vacation. You plan to travel by car and estimate you will average 55 miles per hour. The distance traveled d will result in a travel time of t hours.

3. You find that you are spending more than you had planned on renting video movies. It costs $2.00 to rent each movie. You can use the total amount spent a to find the number of movies you have rented m.

4. You spend $30 a month to take the bus to school. After a certain number of months m, you will have spent a total of dollars d on transportation to school.

5. You are saving money for some new athletic equipment and have 12 weeks before the season starts. The amount you need to save each week s will depend on the cost c of the equipment you want to buy.

Lesson 4–3

NAME ________________________ DATE ____________ PERIOD _____

4-4 Enrichment

Some equations contain two or more expressions that are called *like terms*. For example, in the equation $3a + 2a + 4 = 14$, the expressions $3a$ and $2a$ are like terms. When you see like terms, you can combine them into one expression.

$3a + 2a = 5a$

When you solve an equation containing like terms, combine them first before continuing to solve the equation. To solve $3a + 2a + 4 = 14$, proceed as follows.

$$3a + 2a + 4 = 14$$

Combine like terms.

$$5a + 4 = 14$$
$$5a + 4 - 4 = 14 - 4$$
$$5a = 10$$
$$\frac{5a}{5} = \frac{10}{5}$$
$$a = 2$$

Solve each equation. Then locate the solution on the number line below. Place the letter corresponding to the answer on the line at the right of the exercise.

1. $3x + 4x + 3 = -39$ ______

2. $-3x - 2 + 5x = 12$ ______

3. $-5 - 4x + 7x = 1$ ______

4. $-\frac{1}{2}x + 6x - 2 = 20$ ______

5. $-2.4x + 1.2 + 1.2x = 4.8$ ______

6. $\frac{1}{3}(6 - x) = -1$ ______

7. $1 = -\frac{1}{4}x + 5 + \frac{3}{4}x$ ______

8. $7x + (-2x) + x = 42$ ______

9. $\frac{2}{5}(5x + 5x) = -20$ ______

−9	−8	−7	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7	8	9
W	I	H	F	L	J	O	P	N	V	G	C	D	T	M	K	A	B	R

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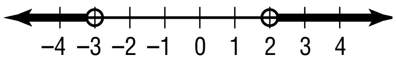
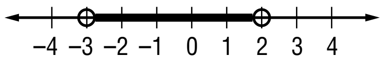
4-5 Enrichment

Lesson 4–5

Statements that consist of two or more inequalities are called **compound inequalities**. When you graph a compound inequality, you need to pay special attention to the words that connect the inequalities.



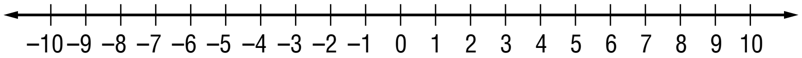
The graph includes all numbers that are *either* less than -3 *or* greater than 2.



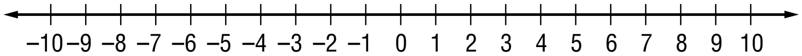
The graph includes all numbers that are *both* greater than -3 *and* less than 2.

Graph each compound inequality.

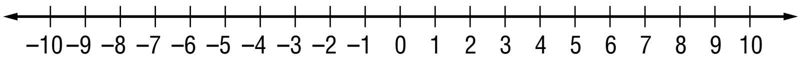
1. $h > -5$ and $h < 4$



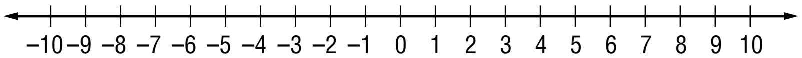
2. $q < -7$ or $q > 6$



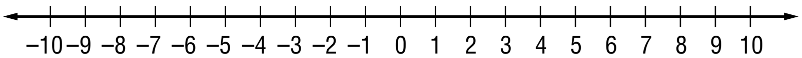
3. $x \geq 0$ and $x \leq 8$



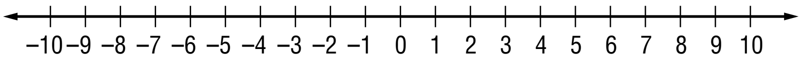
4. $k \geq 4$ or $k \leq -2$



5. $r \leq -3$ or $r > 0$



6. $a < 8$ and $a \geq -4$



7. **CHALLENGE** Describe the graph of each inequality.

a. $m < -4$ and $m > 4$

b. $m > -4$ or $m < 4$

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4-6 Enrichment

To solve equations containing two variables, find ordered pair solutions for the equation by selecting values for x and completing a table. Although any value can be selected for x, values usually selected include -2, -1, 0, 1, and 2.

For example, to solve the equation $y = 2x$ given below in Exercise 1, first select values for x, then complete a table.

Ordered pair solutions for the equation $y = 2x$ include $(-2, -4)$, $(-1, -2)$, $(0, 0)$, $(1, 2)$, and $(2, 4)$.

Match each equation with the point whose coordinates are a solution of the equation. Then, at the bottom of the page, write the letter of the point on the line directly above the number of the equation *each time it appears*. (The first one has been done as an example.) If you have matched the equations and solutions correctly, the letters below will reveal a message.

1. $y = 2x$	$A(-3, 8)$	$N(-1, 0)$
2. $y = x - 3$	$B(0, 2)$	$O(3, 0)$
3. $y = -x + 1$	$C(-2, 1)$	$P(1, 5)$
4. $y = 3x - 2$	$D(0, -5)$	$Q(0, 6)$
5. $y = -2x - 4$	$E(-1, -5)$	$R(1, 6)$
6. $y = x + (-2)$	$F(1, 3)$	$S(2, 1)$
7. $y = -4x - 1$	$G(0, -4)$	$T(-2, 3)$
8. $y = \frac{1}{2}x$	$H(-1, 3)$	$U(1, 2)$
9. $y = x + 3$	$I(2, 0)$	$V(-3, 5)$
10. $y = 7x + 7$	$J(0, 4)$	$W(0, -7)$
11. $y = -2x - 6$	$K(-3, 1)$	$X(-3, -3)$
12. $y = -x + 5$	$L(-4, 2)$	$Y(1, 8)$
13. $y = -5x + 8$	$M(-2, 2)$	$Z(0, -8)$
14. $y = -x$		

___	___	___	___	___	___	___	___	___	___	___		___	___		___	___	___
14	12	3	7	4	14	12	3	6	9	8		6	8		3	7	4

___	___	___	___	U	___	___	___		___	___		___	___	___	___	___	___	___
11	12	10	5	1	12	5	4		2	13		8	9	6	4	10	9	4

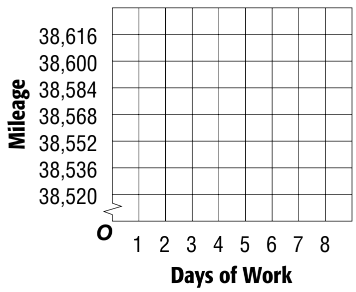
4-7 Enrichment

Rate of Change

The graph of any given relationship is a straight line when the rate of change is constant for equal time intervals. The rate of change is sometimes called the slope of the line. If the rate of change is negative, the y-values decrease as the x-values increase. If the rate of change is positive, the y-values increase as the x-values increase. You can find the rate of change by reading a graph, by looking at the data in a table, or by finding the slope.

For Exercises 1 and 2, use the following sets of data to find and interpret the rate of change.

1. Ingrid's family purchased a used car that Ingrid was allowed to drive only to work. The car originally had 38,520 miles on it, and no one else in the family used it during the school year.



a. Ingrid worked at a movie theatre 8 miles from home. Plot on graph paper the point that represents the number of miles on the odometer after the first day Ingrid used the car.

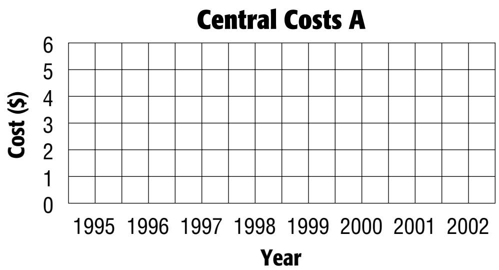
b. Then plot the point that represents the mileage after Ingrid has used the car for 4 days. What is the rate of change in miles per day?

c. Continue to plot points that represent the mileage after any given number of days. Describe the pattern you see.

d. Calculate the rate of change in miles per day, first using the number of miles after 5 days of work and then again after 10 days of work. What can you conclude?

2. Find the slope, or rate of change in price, for each pair of points plotted on the following two graphs. What real-world situations could the points represent? Describe what each rate of change tells you.

a.



Lesson 4–7

NAME ______________________________________ DATE ______________ PERIOD _____

5-1 Enrichment

Perfect Numbers

A positive integer is *perfect* if it equals the sum of its factors that are less than the integer itself.

If the sum of the factors (excluding the integer itself) is greater than the integer, the integer is called *abundant*.

If the sum of the factors (excluding the integer itself) is less than the integer, the integer is called *deficient*.

The factors of 28 (excluding 28 itself) are 1, 2, 4, 7, and 14.
Since $1 + 2 + 4 + 7 + 14 = 28$, 28 is a perfect number.

Complete the table to classify each number as perfect, abundant, or deficient.

	Number	Divisors (Excluding the Number Itself)	Sum	Classification
1.	14			
2.	6			
3.	12			
4.	20			
5.	10			

Show that each number is perfect.

6. 496

7. 8,128

8. **CHALLENGE** 33,550,336

Lesson 5–1

5-2 Enrichment

Sundaram's Sieve

This arrangement of numbers is called Sundaram's Sieve. Like the Sieve of Eratosthenes, Sundaram's arrangement can be used to find prime numbers.

4	7	10	13	16	19	22	25	28	31
7	12	17	22	27	32	37	42	47	52
10	17	24	31	38	45	52	59	66	73
13	22	31	40	49	58	67	76	85	94
16	27	38	49	60	71	82	93	104	115

Here's how to use Sundaram's Sieve to find prime numbers. If a number, n, is not in the Sieve, then $2n + 1$ is a prime number. If a number, n, is in the Sieve, then $2n + 1$ is not a prime number.

32 is in the sieve. $2 \times 32 + 1 = 65$ 65 is not prime.

35 is not in the sieve. $2 \times 35 + 1 = 71$ 71 is prime.

1. Does the sieve give all primes up to 99? all the composites?

2. Sundaram's Sieve is constructed from arithmetic sequences. Describe the pattern used to make the first row.

3. How is the first column constructed?

4. How are the second through fifth rows constructed?

5. How would you add a sixth row to the sieve?

6. Use Sundaram's Sieve to find 5 four-digit prime numbers. You will need to add more numbers to the sieve to do this.

5-3 Enrichment

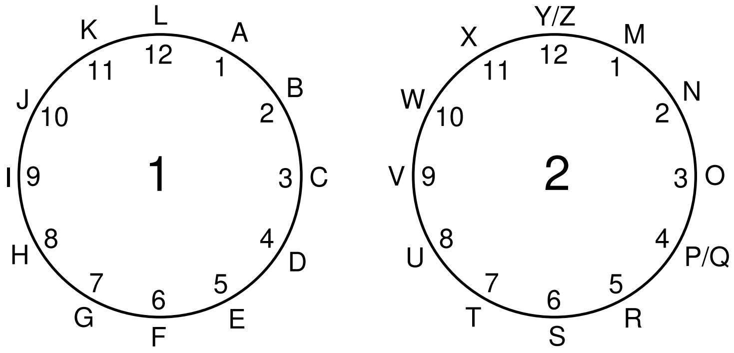
A Two-Clock Code

Two clock faces can be used to create coded secret messages.

To encode a message, write each letter of the message as a fraction. Use the hour next to the letter as the denominator and the number in the center of that clock as the numerator.

For example, the letter G will be encoded as the fraction $\frac{1}{7}$. The letter R becomes $\frac{2}{5}$.

Notice the Y and Z are both written with the same fraction. The same is true for P and Q.



1. Decode this message. The result will be a "secret" from a well-known poem written by Henry Wadsworth Longfellow.

$$\frac{1}{9}\ \frac{1}{6}\ \frac{2}{7}\ \frac{1}{8}\ \frac{1}{5}\ \frac{1}{2}\ \frac{2}{5}\ \frac{1}{9}\ \frac{2}{7}\ \frac{1}{9}\ \frac{2}{6}\ \frac{1}{8}\ \frac{2}{1}\ \frac{1}{1}\ \frac{2}{5}\ \frac{1}{3}\ \frac{1}{8}\ \frac{1}{2}\ \frac{2}{12}\ \frac{1}{12}\ \frac{1}{1}\ \frac{2}{2}\ \frac{1}{4}\ \frac{2}{3}\ \frac{2}{5}\ \frac{2}{6}\ \frac{1}{5}\ \frac{1}{1}$$

$$\frac{1}{6}\ \frac{2}{5}\ \frac{2}{3}\ \frac{2}{1}\ \frac{2}{7}\ \frac{1}{8}\ \frac{1}{5}\ \frac{2}{7}\ \frac{2}{3}\ \frac{2}{10}\ \frac{2}{2}\ \frac{2}{7}\ \frac{2}{3}\ \frac{2}{2}\ \frac{1}{9}\ \frac{1}{7}\ \frac{1}{8}\ \frac{2}{7}\ \frac{1}{8}\ \frac{1}{1}\ \frac{2}{2}\ \frac{1}{7}\ \frac{1}{1}$$

$$\frac{1}{12}\ \frac{1}{1}\ \frac{2}{2}\ \frac{2}{7}\ \frac{1}{5}\ \frac{2}{5}\ \frac{2}{2}\ \frac{1}{1}\ \frac{1}{12}\ \frac{2}{3}\ \frac{1}{6}\ \frac{2}{7}\ \frac{1}{9}\ \frac{2}{2}\ \frac{2}{7}\ \frac{1}{8}\ \frac{1}{5}\ \frac{1}{2}\ \frac{1}{5}\ \frac{1}{12}\ \frac{1}{6}\ \frac{2}{5}\ \frac{2}{12}\ \frac{1}{1}\ \frac{2}{5}\ \frac{1}{3}\ \frac{1}{8}$$

$$\frac{2}{3}\ \frac{1}{6}\ \frac{2}{7}\ \frac{1}{8}\ \frac{1}{5}\ \frac{2}{2}\ \frac{2}{3}\ \frac{2}{5}\ \frac{2}{7}\ \frac{1}{8}\ \frac{1}{3}\ \frac{1}{8}\ \frac{2}{8}\ \frac{2}{5}\ \frac{1}{3}\ \frac{1}{8}\ \frac{2}{7}\ \frac{2}{3}\ \frac{2}{10}\ \frac{1}{5}\ \frac{2}{5}\ \frac{1}{1}\ \frac{2}{6}\ \frac{1}{1}\ \frac{2}{6}\ \frac{1}{9}\ \frac{1}{7}\ \frac{2}{2}\ \frac{1}{1}\ \frac{1}{12}$$

$$\frac{1}{12}\ \frac{1}{9}\ \frac{1}{7}\ \frac{1}{8}\ \frac{2}{7}\ \frac{2}{3}\ \frac{2}{2}\ \frac{1}{5}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{2}\ \frac{2}{12}\ \frac{1}{12}\ \frac{1}{1}\ \frac{2}{2}\ \frac{1}{4}\ \frac{1}{1}\ \frac{2}{2}\ \frac{1}{4}\ \frac{2}{7}\ \frac{2}{10}\ \frac{2}{3}\ \frac{1}{9}\ \frac{1}{6}\ \frac{1}{2}\ \frac{2}{12}\ \frac{2}{6}\ \frac{1}{5}\ \frac{1}{1}$$

2. Use the two-clock code to create a secret message of your own.

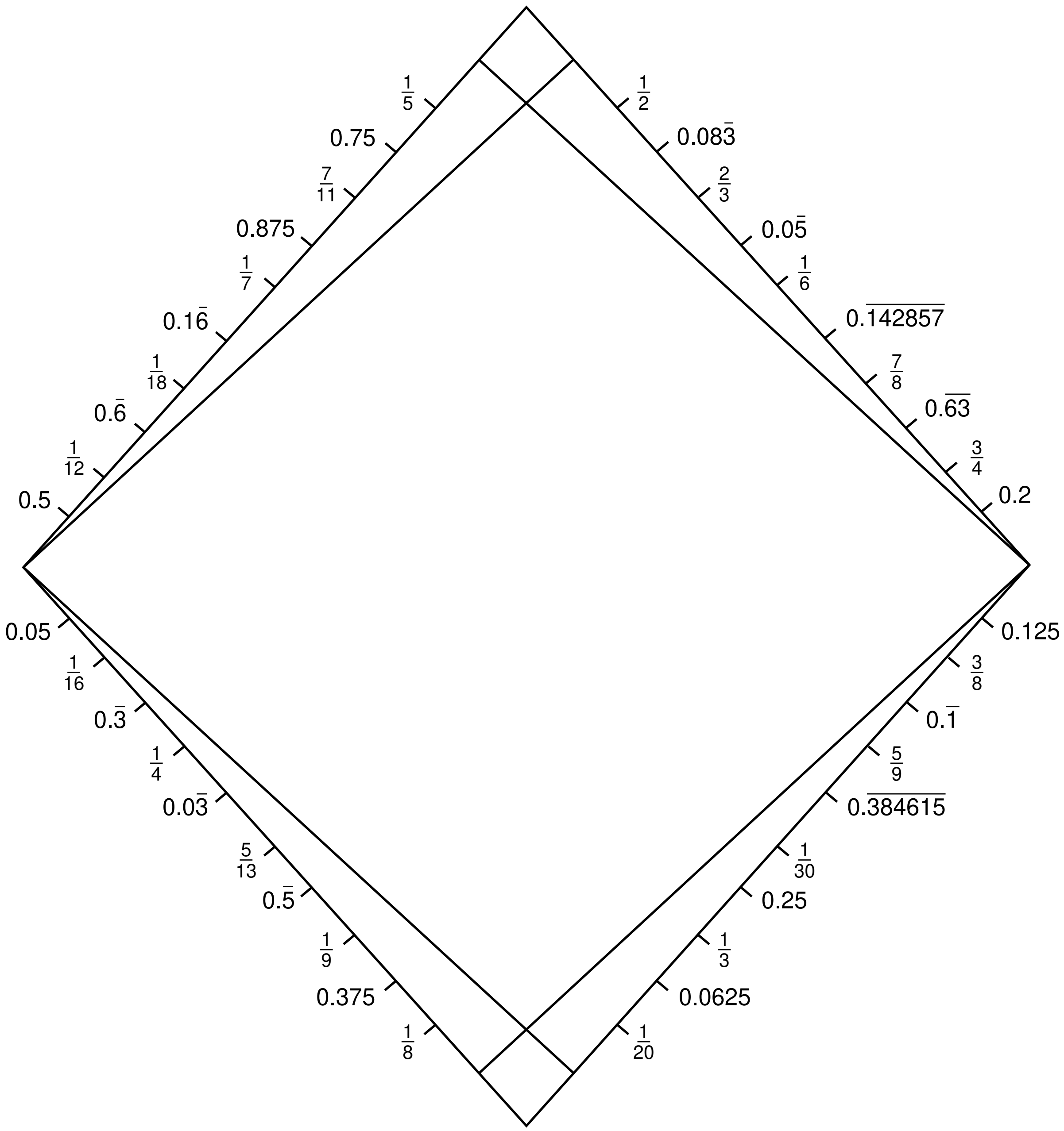
Lesson 5–3

NAME ____ DATE ______________ PERIOD _____

5-4 Enrichment

Making a Line Design

Connect each pair of equivalent numbers with a straight line segment. Although you will draw only straight lines, the finished design will appear curved!



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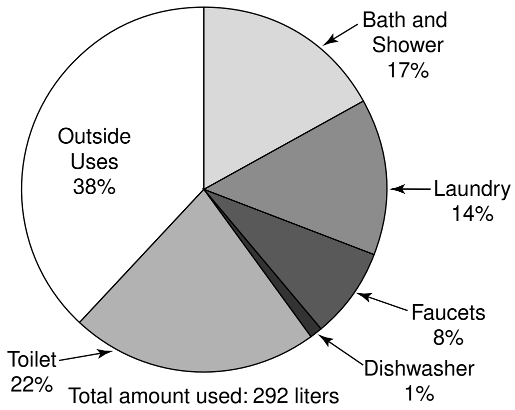
5-5 Enrichment

Margarita Colmenares

Margarita Colmenares is an environmental engineer. She is a native of Los Angeles and a 1981 graduate of Stanford University. In 1989, she became the first woman president of the Society of Hispanic Professional Engineers. Ms. Colmenares was recently appointed to direct an office at the U.S. Department of Education. She has a special interest in education and has traveled extensively to talk to student groups about careers in engineering.

Environmental engineers like Colmenares use mathematics to predict the effect that our actions will have on our environment. They may also recommend ways to protect the environment. On this page, you will consider some data and recommendations concerning water usage.

Daily Water Usage in the United States (Per Person)



Refer to the graph above.

1. Which one category accounts for more than $\frac{1}{3}$ of the water usage?

2. Estimate the fraction of a person's daily water usage that is for bath and shower.

Use the graph above. Estimate the amount of water used in each category.

3. outside uses

4. bath and shower

5. toilet

6. laundry

7. dishwasher

8. faucets

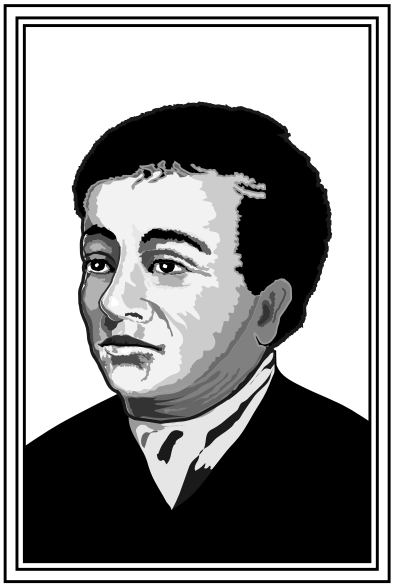
In each situation, what percent of the water used can be saved by following the recommendation?

9. Using a water-saving shower head can save 65 liters of water out of the 130 liters normally used in a five-minute shower.

10. Turning off the water while brushing your teeth can reduce the water used from 20 liters to 2 liters.

5-6 Enrichment

African-American Scientists and Inventors



When you buy a pair of shoes, you usually have a wide variety of styles, sizes, and prices to choose from. It is the work of an African-American inventor, Jan Matzeliger (1852–1889), that makes this possible. In 1882, Matzeliger patented a *lasting machine* that could shape the upper portion of a shoe and attach it to the sole in a fraction of the time it took to do the job by hand. Using this machine, shoe manufacturers were able to increase production and reduce prices dramatically.

African Americans have made many significant contributions to mathematics, science, and invention. By solving the percent problems and matching the problem and the correct solution, you will learn more about just a few of them.

Solutions

A.	20	Benjamin Banneker
B.	21	Marjorie Lee Browne
C.	18	Lewis Latimer
D.	17.5	Jane Cooke Wright

1. 35% of 50 is what number?
This physician researched and tested chemotherapy as a method of treating cancer. In 1952, she became head of the Cancer Research Foundation at Harlem Hospital.

2. What percent of 75 is 15?
This mathematician was part of the team of surveyors who created the street plan for Washington, D.C. in the late eighteenth century.

3. 4.5% of 400 is what number?
In 1876, this engineer drew up the plans that accompanied Alexander Graham Bell's application for a patent on the telephone.

4. 120% of what number is 25.2?
In 1949, she became one of the first two African-American women to earn a doctorate in mathematics. She was head of the mathematics department at North Carolina Central University from 1951 to 1970.

5-7 Enrichment

A Cross-Number Puzzle

Use the clues at the bottom of the page to complete the puzzle. You are to write one digit in each box.

A 2	9	3		B			C		D
5				E					
8		F	G		H		I		
J							K		
		L				M			
N					O			P	
		Q					R		

Across

C largest number less than 200 that is divisible by 29

E square of first prime greater than 20

F least common multiple of 3 and 11

H next term in sequence 61, 122, 244, 488

J greatest common factor of 141 and 329

K the eighth power of 2

L least common multiple of 2, 7, and 13

M numerator of fraction equal to 0.8125

N least common multiple of 86 and 5

O smallest prime greater than 60

P largest two-digit prime

Q next term in sequence 4, 15, 26, 37

R largest two-digit composite less than 40

Down

B smallest number divisible by 3 and 5

D the sixth power of 4

G least common multiple of 2 and 179

H the number of two-digit positive integers

I smallest number over 600 divisible by 89

L smallest three-digit number divisible by 13.

M the smallest two-digit prime number

N largest prime factor of 82

O perfect square between 60 and 70

P largest two-digit number divisible by 3

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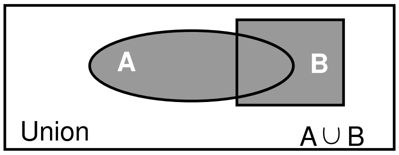
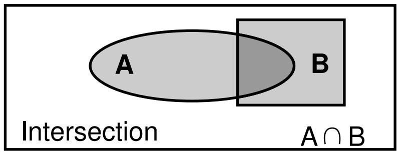
5-8 Enrichment

Intersection and Union of Sets

The darker shaded areas in the Venn diagrams show the *union* and *intersection* of sets A and B.





For example, if $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$, then their union and intersection are written as:

Union: $A \cup B = \{1, 2, 3, 4, 5, 6\}$ Intersection: $A \cap B = \{3, 4\}$

Draw a Venn diagram for sets A and B. Then write the numbers included in $A \cup B$ and $A \cap B$. In Exercises 2 and 4, record the numbers as decimals.

1. $A = \{$integers between 0 and 7$\}$
$B = \{$factors of 12$\}$

2. $A = \{$one-place decimals between 0 and 0.5$\}$
$B = \{$fractions with 1, 2, 3, or 4 as numerator and 5 as a denominator$\}$

3. $A = \{$perfect squares between 0 and 30$\}$
$B = \{$odd whole numbers less than 10$\}$

4. $A = \left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}$
$B = \{0.\overline{1}, 0.\overline{2}, 0.\overline{3}, 0.\overline{4}, 0.\overline{5}, 0.\overline{6}, 0.\overline{7}, 0.\overline{8}, 0.\overline{9}\}$

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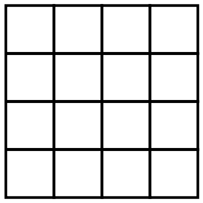
6-1 Enrichment

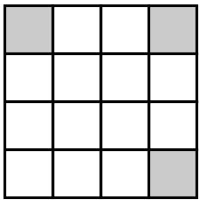
Fractional Areas



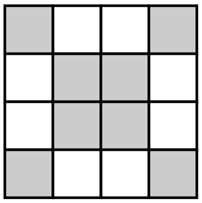
The figure at the right shows one square inch. Each small square equals $\frac{1}{16}$ of a square inch.

Write a fraction or mixed number for the shaded area of each drawing.

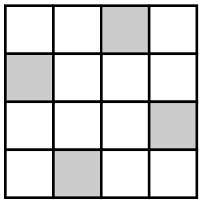
1.



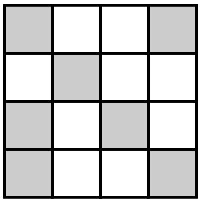
2.



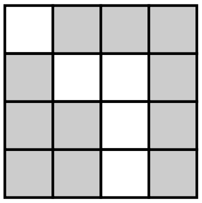
3.



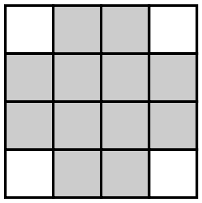
4.



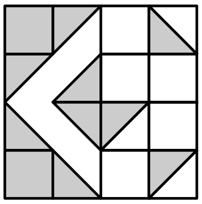
5.



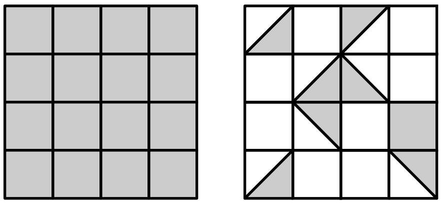
6.



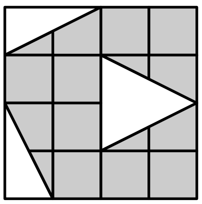
7.



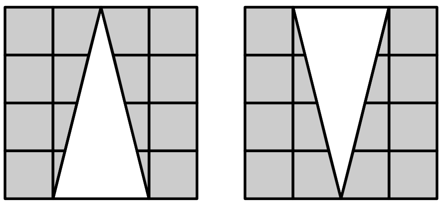
8.



9.



10.



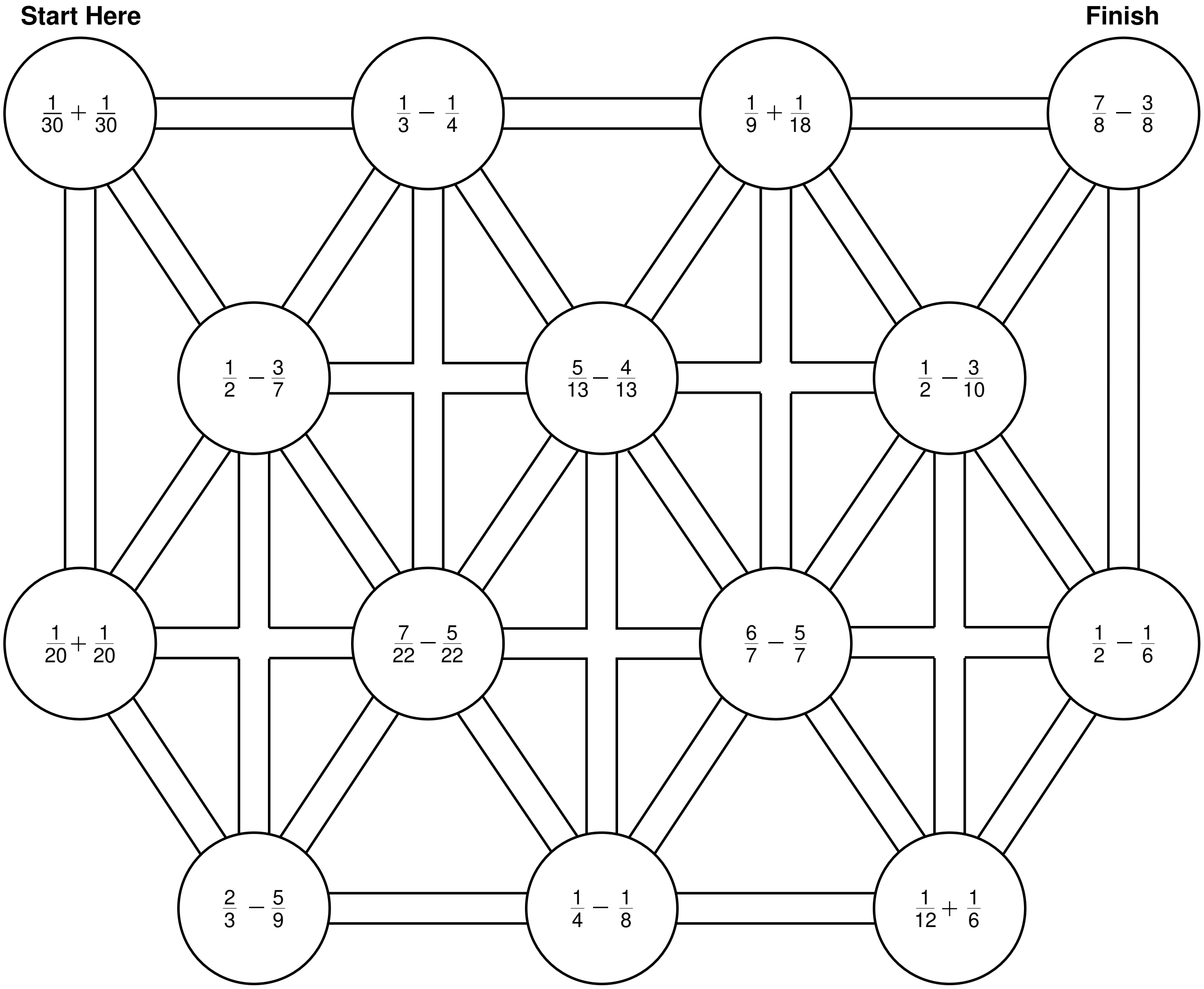
Lesson 6–1

NAME ______________________________________ DATE ____________ PERIOD _____

6-2 Enrichment

Fractions Maze

To solve this maze, start at the upper left corner. Then, draw a line to the next circle with the smallest sum or difference. The answers written in order will form a pattern.



Describe the pattern in the fractions along the line you drew from start to finish.

6-3 Enrichment

Arithmetic Sequences of Fractions

Each term in an *arithmetic* sequence is created by adding or subtracting the same number to the term before. The number added or subtracted is called the *common difference*.

The sequence below is an increasing arithmetic sequence with a common difference of $\frac{1}{4}$.

$$\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}, 1\frac{1}{8}$$

Below is a decreasing arithmetic sequence with a common difference of $1\frac{1}{5}$.

$$7\frac{3}{5}, 6\frac{2}{5}, 5\frac{1}{5}, 4, 2\frac{4}{5}$$

Write the common difference for each arithmetic sequence.

1. $\frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \frac{7}{8}, 1, 1\frac{1}{8}$

2. $1\frac{1}{3}, 3\frac{5}{6}, 6\frac{1}{3}, 8\frac{5}{6}$

3. $4\frac{1}{2}, 4\frac{2}{5}, 4\frac{3}{10}, 4\frac{1}{5}$

4. $11, 9\frac{2}{3}, 8\frac{1}{3}, 7, 5\frac{2}{3}$

Write the next term in each arithmetic sequence.

5. $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{11}{12}, 1$

6. $\frac{13}{20}, \frac{11}{20}, \frac{9}{20}, \frac{7}{20}$

7. $5\frac{1}{5}, 5\frac{7}{10}, 6\frac{1}{5}, 6\frac{7}{10}$

8. $4\frac{11}{12}, 3\frac{3}{4}, 2\frac{7}{12}, 1\frac{5}{12}$

Write the first five terms in each sequence.

9. This increasing sequence starts with $\frac{1}{6}$ and has a common difference of $1\frac{1}{5}$.

10. This decreasing sequence starts with $6\frac{1}{3}$ and has a common difference of $\frac{3}{4}$.

Lesson 6–3

NAME ______________________ DATE ____________ PERIOD _____

6-4 Enrichment

Changing Measures of Length

Fractions and mixed numbers are frequently used with customary measures.

The problems on this page will give you a chance to practice using multiplication of fractions as you change measures of lengths to different equivalent forms.

12 inches (in.)	=	1 foot (ft)
3 feet	=	1 yard (yd)
$5\frac{1}{2}$ yards	=	1 rod (rd)
320 rods	=	1 mile (mi)

Use a fraction or a mixed number to complete each statement. Refer to the table above as needed.

1. 12 ft 6 in. = $\boxed{}$ ft

2. 1 rod = $\boxed{}$ ft

3. $\frac{5}{8}$ yd = $\boxed{}$ in.

4. 10 ft = $\boxed{}$ yd

5. 7 yd 2 ft = $\boxed{}$ yd

6. 1,540 yd = $\boxed{}$ mi

7. 1,000 rd = $\boxed{}$ mi

8. 27 in. = $\boxed{}$ yd

Use a whole number to complete each statement. Refer to the table above as needed.

9. $10\frac{1}{2}$ ft = 10 ft $\boxed{}$ in.

10. $12\frac{1}{2}$ yd = $\boxed{}$ in.

11. 1 mi = $\boxed{}$ ft

12. 1 mi = $\boxed{}$ yd

13. $\frac{1}{10}$ mi = $\boxed{}$ yd

14. $\frac{3}{4}$ ft = $\boxed{}$ in.

15. 10 rd = $\boxed{}$ ft

16. $\frac{3}{8}$ mi = $\boxed{}$ ft

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6-5 Enrichment

Lesson 6–5

Trail Blazers

Each puzzle on this page is called a **trail blazer**. To solve it, you must find a trail that begins at any one of the small squares and ends at the goal square, following these rules.

1. The sum of all the fractions on the trail must equal the number in the goal square.
2. The trail can only go horizontally or vertically.
3. The trail cannot retrace or cross itself.

$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{3}{16}$	$\frac{1}{8}$
$\frac{1}{8}$	$\frac{1}{16}$	$\frac{5}{8}$
	1 ← goal square	

When you are solving a trail blazer, try to eliminate possibilities. For instance, in the puzzle at the right, you know that you cannot include $\frac{3}{4}$ using $\frac{3}{4} + \frac{1}{4} = 1$ because you can't reach the goal box. $\frac{3}{4} + \frac{1}{2} = 1\frac{1}{4}$ will not work either as the goal for the entire trail is only 1.

1.

$\frac{1}{12}$	$\frac{2}{3}$	$\frac{5}{6}$
$\frac{1}{2}$	$\frac{5}{12}$	$\frac{1}{6}$
$\frac{7}{12}$	$\frac{1}{3}$	$\frac{1}{12}$
	1	

2.

$\frac{4}{5}$	$\frac{1}{5}$	$\frac{3}{5}$
$\frac{7}{10}$	$\frac{1}{2}$	$\frac{2}{5}$
$\frac{9}{10}$	$\frac{3}{10}$	$\frac{1}{10}$
	$2\frac{1}{2}$	

3.

$\frac{7}{8}$	$\frac{2}{3}$	$\frac{3}{8}$
$\frac{5}{12}$	$\frac{3}{4}$	$\frac{5}{8}$
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{3}$
	$3\frac{1}{2}$	

4.

$\frac{3}{8}$	$\frac{3}{4}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{2}$
$\frac{1}{8}$	$\frac{7}{8}$	$\frac{1}{4}$	$\frac{7}{8}$	$\frac{1}{4}$
$\frac{1}{2}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{1}{8}$	$\frac{1}{2}$
$\frac{7}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$
$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$
		6		

5.

$\frac{1}{20}$	$\frac{17}{20}$	$\frac{1}{4}$	$\frac{2}{5}$	$\frac{1}{10}$
$\frac{4}{5}$	$\frac{1}{2}$	$\frac{11}{20}$	$\frac{3}{20}$	$\frac{1}{5}$
$\frac{7}{10}$	$\frac{1}{20}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{19}{20}$
$\frac{1}{5}$	$\frac{13}{20}$	$\frac{3}{10}$	$\frac{3}{5}$	$\frac{1}{2}$
$\frac{9}{20}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{7}{20}$	$\frac{1}{4}$
		4		

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6-6 Enrichment

Continued Fractions

The expression at the right is an example of a *continued fraction*. Although continued fractions may look complicated, they are just a combination of addition and division. Here is one way to simplify a continued fraction.

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{9}}}$$

$$\begin{aligned}
1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{9}}} &= 1 + \left[1 \div \left(1 + \left[1 \div \left(1 + \frac{1}{9}\right)\right]\right)\right] \\
&= 1 + \left[1 \div \left(1 + \left[1 \div \frac{10}{9}\right]\right)\right] \\
&= 1 + \left[1 \div \left(1 + \frac{9}{10}\right)\right] \\
&= 1 + \left[1 \div \frac{19}{10}\right] \\
&= 1 + \frac{10}{19} \\
&= \frac{29}{19}
\end{aligned}$$

Write each continued fraction as an improper fraction.

1. $1 + \cfrac{1}{3 + \frac{1}{3}}$

2. $2 + \cfrac{1}{2 + \frac{1}{2}}$

3. $1 + \cfrac{2}{3 + \frac{2}{3}}$

4. $1 + \cfrac{3}{3 + \frac{1}{4}}$

5. $5 + \cfrac{1}{1 + \frac{1}{5}}$

6. $2 + \cfrac{2}{2 + \frac{2}{5}}$

7. $1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{2}}}$

8. $1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{3}}}$

9. $1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{5}}}$

10. $2 + \cfrac{1}{2 + \cfrac{1}{2 + \frac{1}{2}}}$

11. $3 + \cfrac{1}{3 + \cfrac{2}{1 + \frac{1}{3}}}$

12. $6 + \cfrac{1}{1 + \cfrac{1}{3 + \frac{1}{3}}}$

6-7 Enrichment

Changing Measurements with Factors of 1

Multiplying an expression by the number 1 does not change its value. This property of multiplication can be used to change measurements.

Let's say you wanted to change 4.5 hours to seconds. Start by multiplying 4.5 by the number 1 written in the form $\frac{60 \text{ minutes}}{1 \text{ hour}}$. This first step changes 4.5 hours to minutes.

$$4.5 \text{ hours} \times \frac{60 \text{ minutes}}{1 \text{ hour}}$$

Now, multiply by the number 1 again. This time use the fact that $1 = \frac{60 \text{ seconds}}{1 \text{ minute}}$.

$$4.5 \text{ } \cancel{\text{hours}} \times \frac{60 \text{ } \cancel{\text{minutes}}}{1 \text{ } \cancel{\text{hour}}} \times \frac{60 \text{ seconds}}{1 \text{ } \cancel{\text{minute}}} = 16{,}200 \text{ seconds}$$

Complete by writing the last factor and the answer. You may need to use a table of measurements to find the factors.

1. Change 5 pints to fluid ounces.

$5 \text{ pints} \times \frac{2 \text{ cups}}{1 \text{ pint}} \times$

2. Change 0.8 miles to inches.

$0.8 \text{ mile} \times \frac{5{,}280 \text{ feet}}{1 \text{ mile}} \times$

3. Change 4 square yards to square inches.

$4 \text{ yd}^2 \times \frac{9 \text{ ft}^2}{1 \text{ yd}^2} \times$

4. Change 12 bushels to pints.

$12 \text{ bushels} \times \frac{4 \text{ pecks}}{1 \text{ bushel}} \times \frac{8 \text{ quarts}}{1 \text{ peck}} \times$

5. Change one-half of an acre to square inches.

$\frac{1}{2} \text{ acre} \times \frac{4{,}840 \text{ yd}^2}{1 \text{ acre}} \times \frac{9 \text{ ft}^2}{1 \text{ yd}^2} \times$

6-8 Enrichment

Squares and Rectangles

The perimeter and area of a square are found by using the formulas $P = 4s$ and $A = s^2$. The perimeter and area of a rectangle are found by using formulas $P = 2\ell + 2w$ and $A = \ell w$.

Use these formulas to help answer the following questions.

1. A piece of rope 72 inches long must be cut into two pieces. Each piece of rope will be used to form a square. Where should the rope be cut if the perimeter of one square must be $\frac{1}{3}$ of the perimeter of the other square?

2. A piece of rope 60 inches long must be cut into two pieces. Each piece of rope will be used to form a rectangle. Where should the rope be cut if the perimeter of one rectangle must be twice as great as the perimeter of the other rectangle?

3. A piece of rope 100 inches long must be cut into two pieces. Each piece of rope will be used to form a square. Where should the rope be cut if the sides of one square must be 4 inches longer than the sides of the other square?

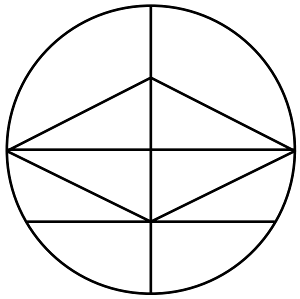
4. A piece of rope 80 centimeters long must be cut into two pieces. Each piece of rope will be used to form a rectangle. Where would the rope be cut if the area of one rectangle must be $\frac{1}{4}$ of the area of the other rectangle?

5. A piece of rope 144 inches long must be cut into two pieces. Each piece of rope will be used to form a square. Where should the rope be cut if the area of one square must be 9 times greater than the area of the other square?

NAME ______________________________ DATE ____________ PERIOD _____

6-9 Enrichment

A Circle Puzzle



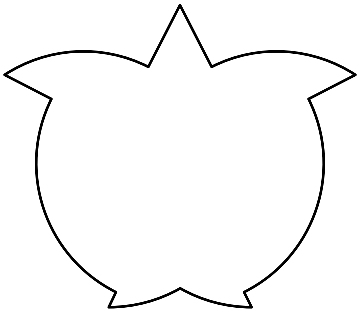
The circle at the right has been divided into ten pieces. Notice that the vertical diameter is marked off into four congruent segments.

Trace the circle and cut it into ten parts to make a set of puzzle pieces.

1. Separate the pieces and put them back together to form the circle. Try this first without looking at the solution.

The puzzle pieces can be used to make many shapes. Use all ten pieces to make each shape shown. Record your solutions.

2.



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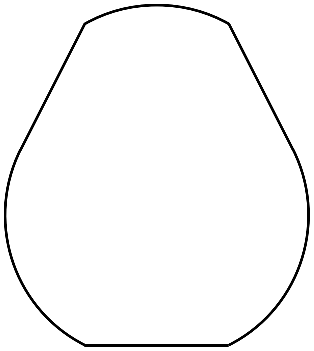

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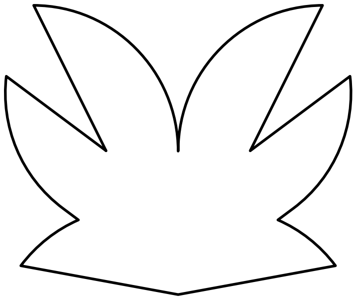

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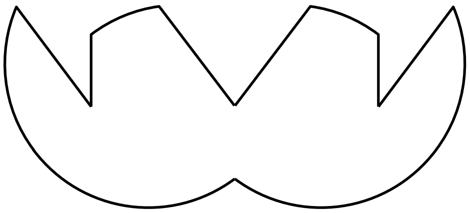

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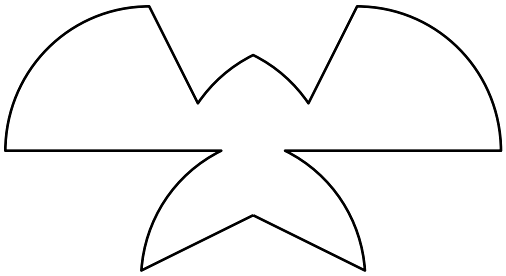

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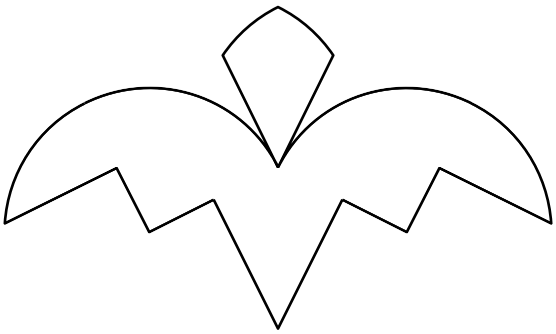

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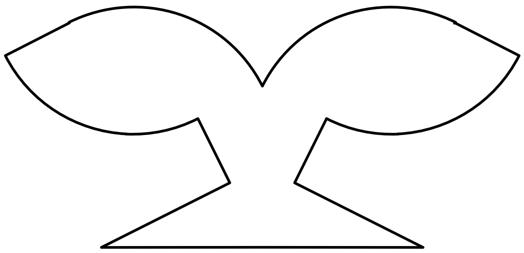


NAME ______________________________ DATE ____________ PERIOD _____

7-1 Enrichment

Chien-Shiung Wu

American physicist Chien-Shiung Wu (1912–1997) was born in Shanghai, China. In 1936, she came to the United States to further her studies in science. She received her doctorate in physics in 1940 from the University of California, and became known as one of the world's leading physicists. In 1975, she was awarded the National Medal of Science.

Wu is most famous for an experiment that she conducted in 1957. The outcome of the experiment was considered the most significant discovery in physics in more than seventy years. The exercise below will help you learn some facts about it.

Choose the value that makes each equation correct. The word or phrase following the solution will complete the statement correctly.

1. $\frac{10}{6} = \frac{m}{15}$ At the time of the experiment, Wu was a professor at ___?___.

m = 25: Columbia University
m = 9: Stanford University

2. $\frac{8}{2} = \frac{t}{2.5}$ The site of the experiment was the ___?___ in Washington, D.C.

t = 10: National Bureau of Standards
t = 6.4: Smithsonian Institution

3. $\frac{5}{y} = \frac{12}{3}$ The experiment involved a substance called ___?___.

y = 20: carbon 14
y = 1.25: cobalt 60

4. $\frac{5}{4} = \frac{4}{n}$ In the experiment, the substance was cooled to ___?___.

n = 5: –100°C
n = 3.2: –273°C

5. $\frac{6}{c} = \frac{0.3}{9}$ The experiment proved that a physical reaction could ___?___.

c = 180: have a "right side" and a "left side"
c = 0.2: occur at a temperature called "absolute zero"

6. $\frac{24}{5.4} = \frac{6}{j}$ Before Wu's experiment, scientists had believed that all physical reactions ___?___.

j = 21.6: occurred at temperatures greater than −250°C
j = 1.35: were perfectly symmetric

7-2 Enrichment

An Educated Consumer

Choosing a checking account is something that most people do at some point in their lives. Because checking accounts vary from institution to institution, and from one type of account to another, you will need to consider the options associated with each account before choosing one of them.

Suppose a bank offers two kinds of checking accounts.

Account A: a $0.20 charge for writing each check and no service charge

Account B: a $0.10 charge for writing each check and a monthly service charge of $1.50

1. Which account would cost less if a person were to write 10 checks in a month?

2. Which account would cost less if a person were to write 20 checks in a month?

3. Using the guess-and-check strategy, find the number of checks that would have to be written for the cost of Account A to equal the cost of Account B. What is that cost?

4. Which account would cost less if a person were to write 250 checks in a year? By how much?

5. Diana Durbin wrote 300 checks in one year. Her total charge for the use of the account that year was $72.00. The bank charges $0.15 for writing one check and charges a fixed amount each month for the use of the account. What is that monthly service charge?

NAME ____ DATE ______________ PERIOD _____

7-3 Enrichment

What Am I?

Solve each proportion. Then, starting at the box marked with the heavy outline, draw an arrow to the adjacent box containing the variable with the least value. (You may move horizontally or vertically. You may use each box at most once.)

	$\frac{3.5}{\frac{1}{4}} = \frac{o}{2}$	$\frac{n}{37} = \frac{54}{55\frac{1}{2}}$	$\frac{5}{e} = \frac{2\frac{1}{2}}{21}$	$\frac{16.5}{36} = \frac{11}{a}$
$\frac{z}{2} = \frac{4}{1}$	$\frac{0.7}{h} = \frac{2.1}{108}$	$\frac{0.3}{0.4} = \frac{18}{o}$	$\frac{3}{5.82} = \frac{n}{48.5}$	$\frac{p}{55} = \frac{\frac{2}{5}}{1}$
$\frac{a}{5} = \frac{9}{15}$	$\frac{2}{7} = \frac{p}{14}$	$\frac{0.5}{i} = \frac{24}{384}$	$\frac{z}{32} = \frac{7}{8}$	$\frac{43.2}{18} = \frac{u}{5}$
$\frac{4}{o} = \frac{20}{30}$	$\frac{3.4}{6.8} = \frac{2.5}{r}$	$\frac{\frac{2}{9}}{\frac{1}{3}} = \frac{t}{18}$	$\frac{\frac{1}{3}}{z} = \frac{\frac{2}{3}}{60}$	$\frac{72}{\frac{1}{2}} = \frac{t}{\frac{1}{4}}$
$\frac{p}{\frac{1}{2}} = \frac{16}{2}$	$\frac{o}{24} = \frac{10.5}{36}$	$\frac{3.5}{5.5} = \frac{7}{r}$	$\frac{2\frac{1}{2}}{9} = \frac{5}{l}$	**Stop here.** $\frac{e}{\frac{1}{5}} = \frac{12.5}{\frac{1}{2}}$
$\frac{600}{150} = \frac{o}{3.5}$	$\frac{1\frac{2}{5}}{1.4} = \frac{r}{20}$	$\frac{0.2}{o} = \frac{0.5}{35}$	$\frac{3}{8} = \frac{6}{d}$	$\frac{\frac{1}{4}}{i} = \frac{3}{96}$

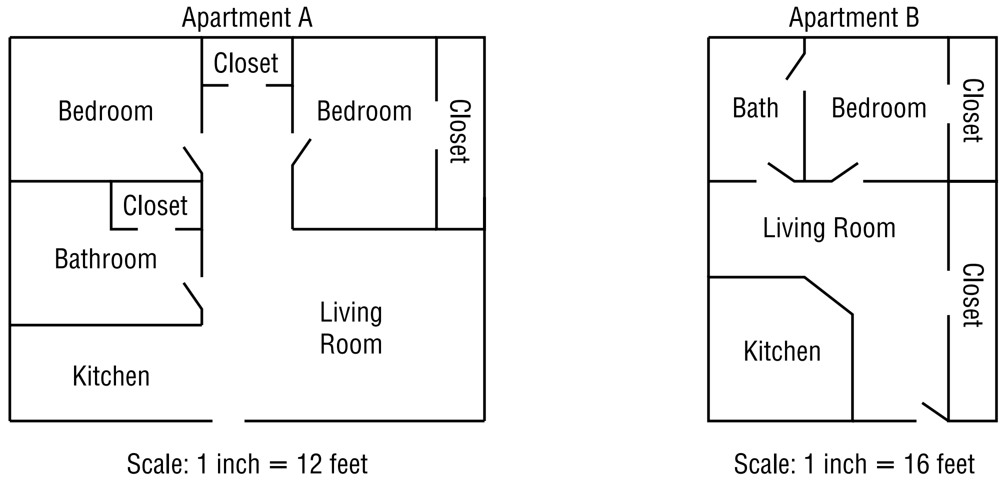
Now fill in the table below with the letters in the order in which you found them. Now you can say what I am.

NAME ______________________________ DATE ______________ PERIOD _____

7-4 Enrichment

Scale Drawings

Use the scale drawings of two different apartments to answer the questions.



1. Which apartment has the greater area?

2. What is the difference in square feet between Apartment A and Apartment B?

3. How much more closet space is offered by Apartment B than Apartment A?

4. How much more bathroom space is offered by Apartment B than Apartment A?

5. A one-year lease for Apartment A costs $450 per month. A one-year lease for Apartment B costs $525 per month. Which apartment offers the greatest value in terms of the cost per square foot?

NAME ____ DATE ______________ PERIOD _____

7-5 Enrichment

Shaded Regions

The fractions or percents listed below each represent one of the shaded regions.

Match each fraction or percent with the shaded region it represents.

1. $\frac{1}{2}$

2. $\frac{25}{64}$

3. $\frac{11}{16}$

4. 25%

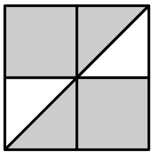
5. $\frac{3}{4}$

6. $62\frac{1}{2}\%$

7. $\frac{29}{64}$

8. 37.5%

9. $\frac{7}{16}$

a.


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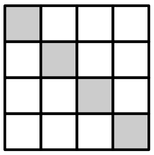

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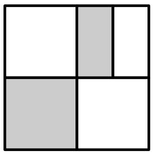
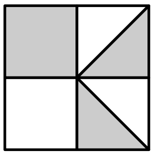
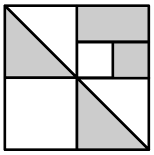
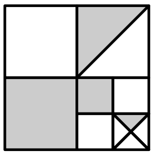
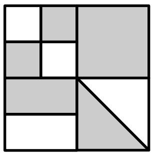
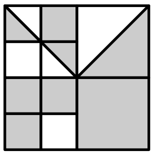


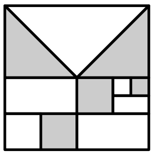
d.


e.


f.


g.


h.


i.


7-6 Enrichment

Juan de la Cierva

Helicopters became widely used in the early 1950s. However, did you know that a similar aircraft was developed in Spain nearly thirty years earlier? The inventor was Juan de la Cierva (1895–1936), and for many years his aircraft were used in rescue work. The modern helicopter is faster and more versatile, but it retains many features of Cierva's design.

Fill in the blanks below to find what Cierva called his aircraft. On the line next to the decimal, fraction, or mixed number, write the letter matching the answer. If you have found the percents correctly, the letters read downward will spell out the name of the aircraft.

1. $\frac{3}{2}$	________	**A** 150%
2. 0.006	________	**G** 0.029%
3. 3.2	________	**I** 0.006%
4. 2.9	________	**O** 350%
5. 0.00029	________	**O** 290%
6. 0.00006	________	**R** 0.5%
7. $\frac{1}{200}$	________	**T** 320%
8. $3\frac{1}{2}$	________	**U** 0.6%

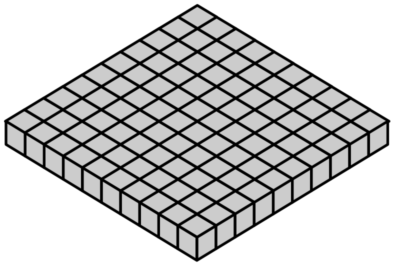
NAME ______________________________________ DATE ______________ PERIOD _____

7-7 Enrichment

Model Behavior

When a block is painted and then separated into small cubes, some of the faces of the cubes will have paint on them and some will not.

For each set of blocks determine the percent of cubes that are painted on the given number of faces.



1. 0 faces

2. 1 face

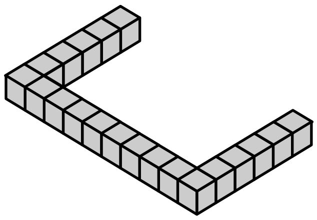
3. 2 faces

4. 3 faces

5. 4 faces

6. 5 faces

7. 6 faces



8. 0 faces

9. 1 face

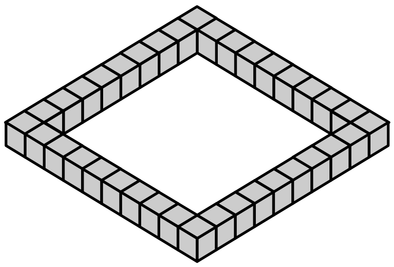
10. 2 faces

11. 3 faces

12. 4 faces

13. 5 faces

14. 6 faces



15. 0 faces

16. 1 face

17. 2 faces

18. 3 faces

19. 4 faces

20. 5 faces

21. 6 faces

Lesson 7–7

NAME ______________________ DATE ____________ PERIOD _____

7-8 Enrichment

Working Backward

Working backward can be a helpful problem-solving tool, especially in problems where the answer is given and information you would expect to have is omitted.

A large corporation reports that ________ % of its employees exercise on a regular basis. If 2,120 employees exercise regularly, how many employees does the corporation have? Answer: 2,650 employees

Use the percent proportion to solve for the missing percent.

$$\frac{2{,}120}{2{,}650} = \frac{x}{100}$$
$$2{,}120 \times 100 = 2{,}650x$$
$$x = 80$$

80% of the employees exercise on a regular basis.

Write the missing information for each exercise.

1. A progressive community states that 96% of its households recycle materials at least once monthly. If ________ households recycle at least once monthly, how many households are in the community? Answer: 15,480 households

2. The purchase price of a cassette tape deck is $139.00. The sales tax rate is ________ %. Find the cost of the cassette tape deck. Answer: $148.73

3. In a seventh grade class, 60% of the students participate in extra-curricular activities. The class has ________ students. How many students participate in extra-curricular activities? Answer: 15 students

4. Claims by a manufacturer state that 3 out of 4 people prefer their product when compared to a similar product of another manufacturer. If ________ people were surveyed, how many did not prefer the product? Answer: 35

5. Seventy percent of the students entering a certain high school complete their studies and graduate. If ________ students did not complete their studies and graduate, how many students earned a diploma? Answer: 455

6. A middle school survey discovered that 15% of the student body watched two hours or less of television each week and 45% watched ten or more hours each week. If ________ students were surveyed, how many students watched between 2 and 10 hours of television each week? Answer: 48.

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8-1 Enrichment

Made in the Shade

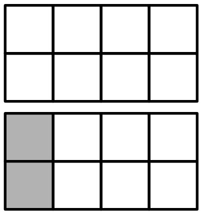
To shade 25% of the figure below, ask yourself how many of the eight squares need to be shaded. Then use the percent proportion to find the answer.

$$\frac{x}{8} = \frac{25}{100}$$

$$100x = 8 \times 25$$

$$\frac{100x}{100} = \frac{200}{100}$$

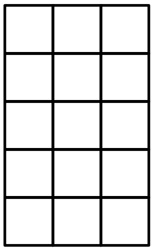
$$x = 2$$



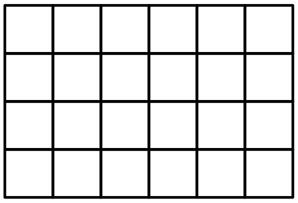
If you shade two squares, you have shaded 25% of the figure.

Shade the indicated percent of each diagram.

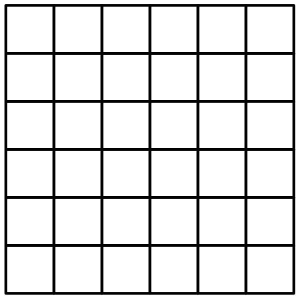
1. Shade 40%.



2. Shade 37.5%.

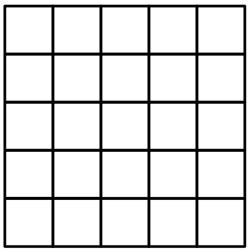


3. Shade $16\frac{2}{3}$%.

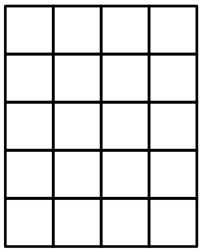


Shade the indicated percent of each diagram. You will need to divide the squares in each diagram into smaller squares.

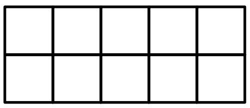
4. Shade 30%.



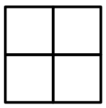
5. Shade 62.5%.



6. Shade 27.5%.



7. Shade 28.125%.



8-2 Enrichment

Just the Facts

Use the percent proportion and your calculator to help discover some interesting facts about the United States. Round your answers to the nearest tenth of a percent.

1. The United States produced 84,412 million eggs in 2000. The state of Ohio produced 8,163 million eggs—more than any other state. What percent of the eggs produced in the United States were produced outside Ohio?

2. The resident population of the United States in 2000 was 281,421,906 people. Of these people, 33,871,648 lived in California. What percent of the population was not living in California?

3. In the 2000 presidential election, 50,459,211 people voted for George W. Bush, and 51,003,894 people voted for Al Gore. Of the people who voted for these two candidates, what percent voted for George W. Bush?

4. During the period 1990–1998, Las Vegas, Nevada, was the fastest-growing metropolitan area in the United States. Its population grew from 852,646 to 1,321,546 people. By what percent did the population increase during this period?

5. The amount Americans had in personal savings in 1990 was 208.7 billion dollars. In 2000, it was 67.7 billion dollars. By what percent did personal savings decrease from 1990 to 2000?

6. In 1990, the average annual personal income in the United States was \$19,614. In 2000, it rose to \$30,069. By what percent did the average annual personal income increase from 1990 to 2000?

7. During the period 1998–2008, computer engineering is expected to be the fastest-growing occupation in the United States. In 1998, 299 thousand computer engineers were employed. There are projected to be 622 thousand employees in 2008. By what percent will the number of computer engineers increase from 1998 to 2008?

8-3 Enrichment

Table of Random Digits

A table of random digits can be used to simulate probability experiments. This table of random digits contains 50 digits.

8	1	5	8	6	7	9	9	8	0
9	9	3	7	3	3	1	8	7	4
7	3	0	9	9	2	4	6	2	4
4	0	5	2	9	9	6	3	8	2
8	4	2	1	6	3	7	0	3	1

8	1	5	8	6	7	9	9	8	0
9	9	3	7	3	3	1	8	7	4
7	3	0	9	9	2	4	6	2	4
4	0	5	2	9	9	6	3	8	2
8	4	2	1	6	3	7	0	3	1

For example, how often might someone expect a coin to land heads up two consecutive times or more in 50 tosses? Our table can be used to make this prediction. Since the table contains the digits 0 through 9, let's say a toss of heads represents the digits 0, 2, 4, 6, and 8, or $\frac{1}{2}$ of the possible digits that appear in the table.

Using the table to imitate the 50 tosses, we must look for the digits 0, 2, 4, 6, and 8 that occur two or more times consecutively. These have been circled in the table at the right above, and we would expect to toss two or more consecutive heads 4 times in 50 trials.

Use the table of random digits to answer the following questions.

1. How many times might a coin toss of 3 or more consecutive tails occur in 50 trials? (*Hint:* Let 1, 3, 5, 7, and 9 represent a toss of tails.)

2. How many times might a coin toss of 4 or more consecutive heads occur in 50 trials? (*Hint:* Let 0, 2, 4, 6, and 8 represent a toss of heads.)

3. Letting the digits 1, 3, 5, 7, and 9 represent a coin toss of tails, what is the maximum number of consecutive tails that could be expected in 50 tosses?

4. Letting the digits 0, 2, 4, 6, and 8 represent a coin toss of heads, what is the maximum number of consecutive heads that could be expected in 50 tosses?

Lesson 8–3

8-4 Enrichment

A Taxing Exercise

People who earn income are required by law to pay taxes. The amount of tax a person owes is computed by first subtracting the amount of all *exemptions* and *deductions* from the amount of income, then using a tax table like this.

Schedule X–Use if your filing status is **Single**

If the amount on Form 1040, line 37, is: *Over—*	*But not over—*	Enter on Form 1040, line 38	*of the amount over—*
$0	$20,350	------------------15%	$0
20,350	49,300	$3,052.50 + 28%	20,350
49,300	--------	11,158.50 + 31%	49,300

Compute each person's income. Subtract $5,550 for each person's exemption and deduction. Then use the tax rate schedule to compute the amount of federal tax owed.

1. A cashier works 40 hours each week, earns $7.50 per hour, and works 50 weeks each year.

2. A newspaper carrier works each day, delivers 154 papers daily, and earns $0.12 delivering each paper.

3. A baby-sitter earns $3.50 per hour per child. During a year, the baby-sitter works with two children every Saturday for 8 hours and with three children every other Sunday for 6 hours.

4. While home from college for the summer, a painter earns $17.00 per hour, working 45 hours each week for 15 weeks.

5. Working before and after school in the school bookstore, an employee works 2.5 hours each day for 170 days and earns $4.60 per hour.

6. After graduating from college, a computer programmer accepts a position earning $2,450 monthly.

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8-5 Enrichment

Missing Fact Match

The problems on this page are missing a key fact and cannot be solved.

Find the missing fact in Column 2 that completes each problem in Column 1. After each missing fact has been matched to its problem in Column 1, find each answer.

Problem	Missing Fact	Answer
1. The school band held a fund-raiser by selling band buttons. Each button sold for $1.50, which included a 20% profit. How much profit did the fund-raiser earn?	_____ The team had 12 players.	**1.** __________
2. The athletic department received a bill of $153.36, including tax, for extra uniforms. Find the cost of the uniforms before tax.	_____ Regular price is $22.50.	**2.** __________
3. If everyone is present, there are 25 students in a mathematics class. How many students were in class on Monday?	_____ Paper products are 25% off.	**3.** __________
4. The volleyball team stopped after the game to eat. The bill was $57.60, not including a 15% tip. If the bill was split equally among the players, what was each player's share?	_____ They reached 150% of their goal.	**4.** __________
5. The school bookstore does not tax supplies and is having a spring sale. Find the cost of two spiral notebooks that regularly sell for $1.40 each.	_____ Sales tax on clothing is 8%.	**5.** __________
6. A department store advertises 40% off jeans in a back-to-school sale. If sales tax is 5%, what is the cost of two pairs of jeans?	_____ Total sales were 1,200 buttons.	**6.** __________
7. Last season, the goal of the basketball team was to win 12 games. How many games did they win last season?	_____ Monday's attendance was 92%.	**7.** __________

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8-6 Enrichment

Taking an Interest

When interest is paid on both the amount of the deposit and any interest already earned, interest is said to be **compounded**. You can use the formula below to find out how much money is in an account for which interest is compounded.

$$A = P(1 + r)^n$$

In the formula, P represents the principal, or amount deposited, r represents the rate applied each time interest is paid, n represents the number of times interest is given, and A represents the amount in the account.

EXAMPLE **A customer deposited $1,500 in an account that earns 8% per year. If interest is compounded and earned semiannually, how much is in the account after 1 year?**

Use the formula $A = P(1 + r)^n$.

Since interest is earned semiannually, $r = 8 \div 2$ or 4% and $n = 2$.

$$A = 1{,}500(1 + 0.04)^2 \quad \text{Use a calculator.}$$
$$= 1{,}622.40$$

After 1 year, there is $1,622.40 in the account.

Use the compound interest formula and a calculator to find the value of each of these investments. Round each answer to the nearest cent.

1. $2,500 invested for 1 year at 6% interest compounded semiannually

2. $3,600 invested for 2 years at 7% interest compounded semiannually

3. $1,000 invested for 5 years at 8% interest compounded annually

4. $2,000 invested for 6 years at 12% interest compounded quarterly

5. $4,800 invested for 10 years at 9% interest compounded annually

6. $10,000 invested for 15 years at 7.5% interest compounded semiannually

9-1 Enrichment

Coin-Tossing Experiments

If a coin is tossed 3 times, there are 8 possible outcomes. They are listed in the table below.

Number of Heads	0	1	2	3
Outcomes	TTT	HTT	HHT	HHH
		THT	THH	
		TTH	HTH	

Once all the outcomes are known, the probability of any event can be found. For example, the probability of getting 2 heads is $\frac{3}{8}$. Notice that this is the same as getting 1 tail.

1. A coin is tossed 4 times. Complete this chart to show the possible outcomes.

Number of Heads	0	1	2	3	4
Outcomes	TTTT				

2. What is the probability of getting all tails?

3. Now complete this table. Make charts like the one in Exercise 1 to help find the answers. Look for patterns in the numbers.

Number of Coin Tosses	2	3	4	5	6	7	8
Total Outcomes							
Probability of Getting All Tails							

4. What happens to the number of outcomes? the probability of all tails?

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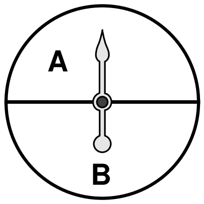
9-2 Enrichment

Probabilities and Regions

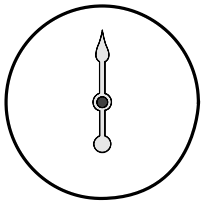
The spinner at the right can be used to indicate that the probability of landing in either of two regions is $\frac{1}{2}$.

$$P(A) = \frac{1}{2} \quad P(B) = \frac{1}{2}$$

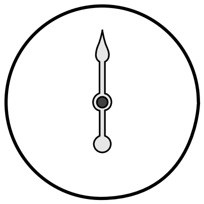


Read the description of each spinner. Using a protractor and ruler, divide each spinner into regions that show the indicated probability.

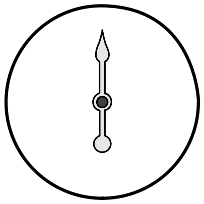
1. Two regions A and B: the probability of landing in region A is $\frac{3}{4}$. What is the probability of landing in region B?



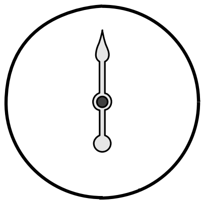
2. Three regions A, B, and C: the probability of landing in region A is $\frac{1}{2}$ and the probability of landing in region B is $\frac{1}{4}$. What is the probability of landing in region C?



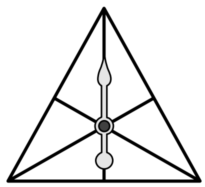
3. Three regions A, B, and C: the probability of landing in region A is $\frac{3}{8}$ and the probability of landing in region B is $\frac{1}{8}$. What is the probability of landing in region C?



4. Four regions A, B, C, and D: the probability of landing in region A is $\frac{1}{16}$, the probability of landing in region B is $\frac{1}{8}$, and the landing probability of in region C is $\frac{1}{4}$. What is the probability of landing in region D?



5. The spinner at the right is an equilateral triangle, divided into regions by line segments that divide the sides in half. Is the spinner divided into regions of equal probability?



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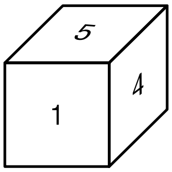
9-3 Enrichment

Curious Cubes

If a six-faced number cube is rolled any number of times, the theoretical probability of the number cube landing on any given face is $\frac{1}{6}$.

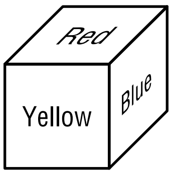
Each number cube below has six faces and has been rolled 100 times. The outcomes have been tallied and recorded in a frequency table. Based on the data in each frequency table, what can you say are probably on the unseen faces of each cube?

1.



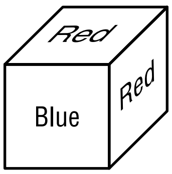
Outcome	Tally
1	15
2	14
3	18
4	16
5	19
6	18

2.



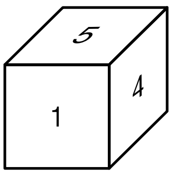
Outcome	Tally
blue	17
red	30
yellow	53

3.



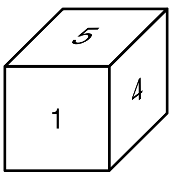
Outcome	Tally
red	30
blue	16
blank	54

4.



Outcome	Tally
1	34
4	32
5	34

5.



Outcome	Tally
1	14
5	13
4	18
2	16
blank	39

9-4 Enrichment

Permutation Puzzles

When you change the order of a set of objects in a permutation by switching the places of two items next to one another, you *transpose* two items in the permutation. In the permutation at the left below, switch B and C to get the permutation at the right below.

$$A\ B\ C\ D \longrightarrow A\ C\ B\ D$$

For each arrangement at the left show how to switch two letters at a time to get the arrangement at the right. Show your switches in drawings.

1.

A	B
C	D

D	B
A	C

2.

A	B	C
D	E	F

B	F	C
A	D	E

3.

A	B	C
D	E	F
G	H	I

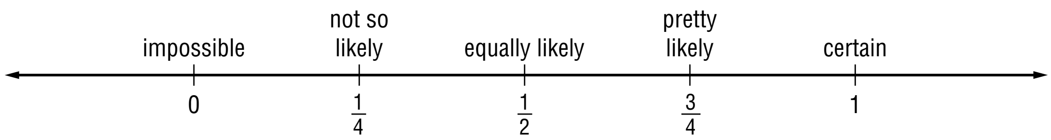
B	C	I
A	E	F
D	G	H

NAME ______________________________ DATE ____________ PERIOD _____

9-5 Enrichment

From Impossible to Certain Events

A probability is often expressed as a fraction. As you know, an event that is impossible is given a probability of 0 and an event that is certain is given a probability of 1. Events that are neither impossible nor certain are given a probability somewhere between 0 and 1. The probability line below shows relative probabilities.



Determine the probability of an event by considering its place on the diagram above.

1. Medical research will find a cure for all diseases.

2. There will be a personal computer in each home by the year 2010.

3. One day, people will live in space or under the sea.

4. Wildlife will disappear as Earth's human population increases.

5. There will be a fifty-first state in the United States.

6. The sun will rise tomorrow morning.

7. Most electricity will be generated by nuclear power by the year 2010.

8. The fuel efficiency of automobiles will increase as the supply of gasoline decreases.

9. Astronauts will land on Mars.

10. The percent of high school students who graduate and enter college will increase.

11. Global warming problems will be solved.

12. All people in the United States will exercise regularly within the near future.

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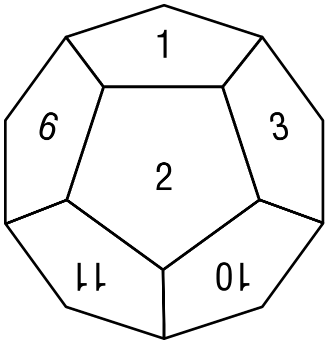
9-6 Enrichment

Rolling a Dodecahedron



A **dodecahedron** is a solid. It has twelve faces, and each face is a pentagon.

At the right, you see a dodecahedron whose faces are marked with the integers from 1 through 12. You can roll this dodecahedron just as you roll a number cube. With the dodecahedron, however, there are *twelve* equally likely outcomes.

Refer to the dodecahedron shown at the right. Find the probability of each event.

1. *P*(5)

2. *P*(odd)

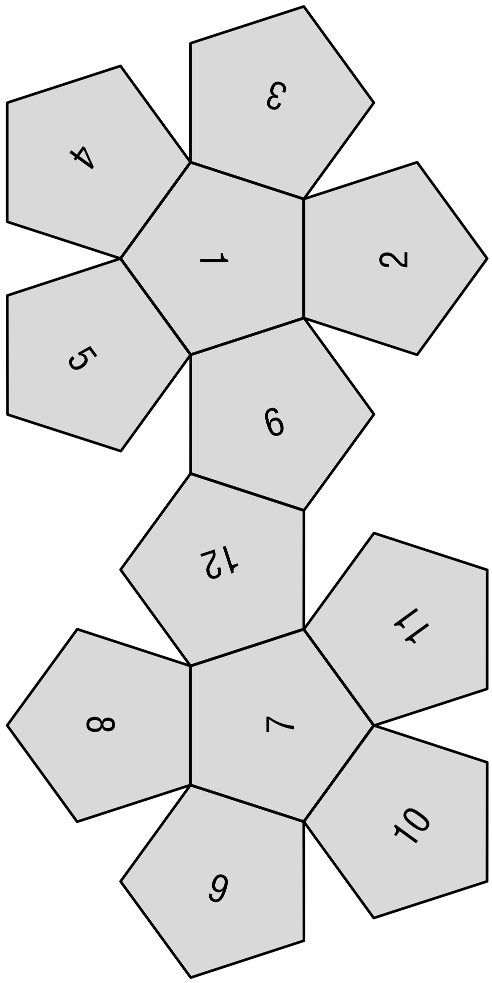
3. *P*(prime)

4. *P*(divisible by 5)

5. *P*(less than 4)

6. *P*(fraction)

You can make your own dodecahedron by cutting out the pattern at the right. Fold along each of the solid lines. Then use tape to join the faces together so that your dodecahedron looks like the one shown above.



7. Roll your dodecahedron 100 times. Record your results on a separate sheet of paper, using a table like this.

Outcome	Tally	Frequency
1		
2		

8. Use your results from Exercise 7. Find the experimental probability for each of the events described in Exercises 1–6.

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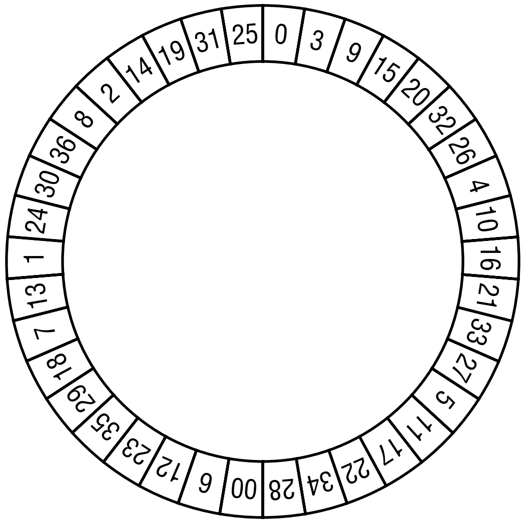
9-7 Enrichment

Independent Events



The game of roulette is played by dropping a ball into a spinning, bowl-shaped wheel. When the wheel stops spinning, the ball will come to rest in any of 38 locations.

On a roulette wheel, the eighteen even numbers from 2 through 36 are colored red and the eighteen odd numbers from 1 through 35 are colored black. The numbers 0 and 00 are colored green.

To find the probability of two independent events, the results of two spins, find the probability of each event first.

$$P(\text{red}) = \frac{18}{38} \text{ or } \frac{9}{19}$$

$$P(\text{black}) = \frac{18}{38} \text{ or } \frac{9}{19}$$

Then multiply.

$$P(\text{red, then black}) = \frac{9}{19} \times \frac{9}{19} \text{ or } \frac{81}{361}$$

Find each probability.

1. black, then black

2. prime number, then a composite number

3. a number containing at least one 0, then a number containing at least one 2

4. red, then black

5. the numbers representing your age, month of birth, and then day of birth

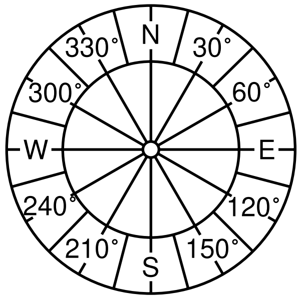
NAME ______________________________________ DATE ______________ PERIOD _____

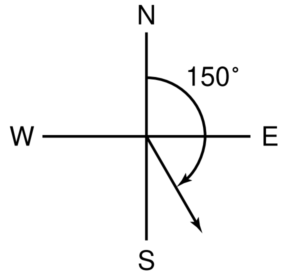
Enrichment

Compass Directions

There are 360° in a complete rotation. The directions north, east, south, and west are shown on the compass at the right.

To find the direction a boat or airplane is heading, measure clockwise from north around the compass. The example shows a heading of 150°.

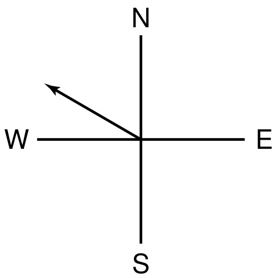




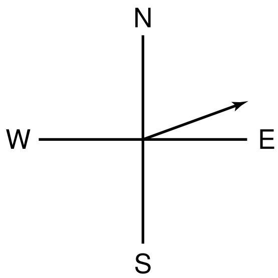
150° = 90° + 60°

Use a protractor. Write the compass heading in degrees for each diagram.

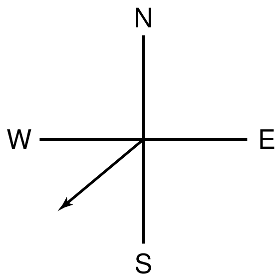
1.



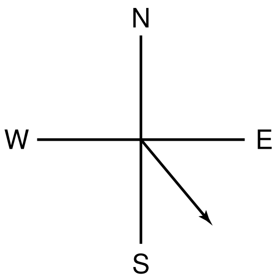
2.



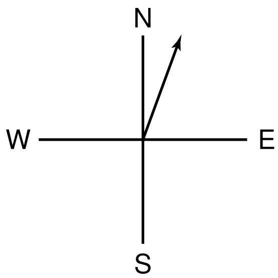
3.



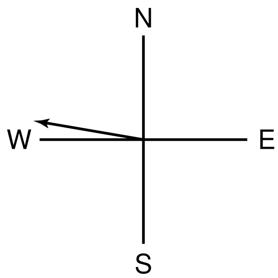
4.



5.



6.



The drawing at the right is called a compass rose. Use the compass rose to translate each direction into degrees.

7. East

8. North

9. Northeast (NE)

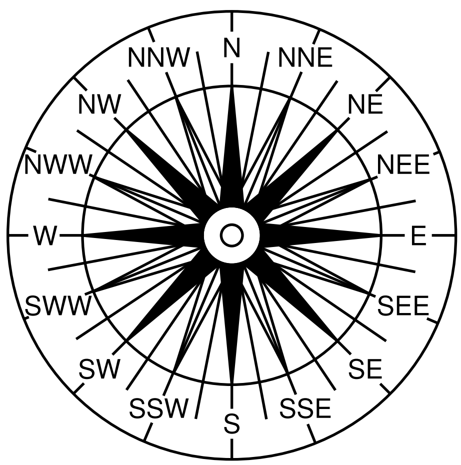
10. Southwest (SW)

11. South

12. Southeast (SE)

13. Northwest (NW)

14. North northeast (NNE)



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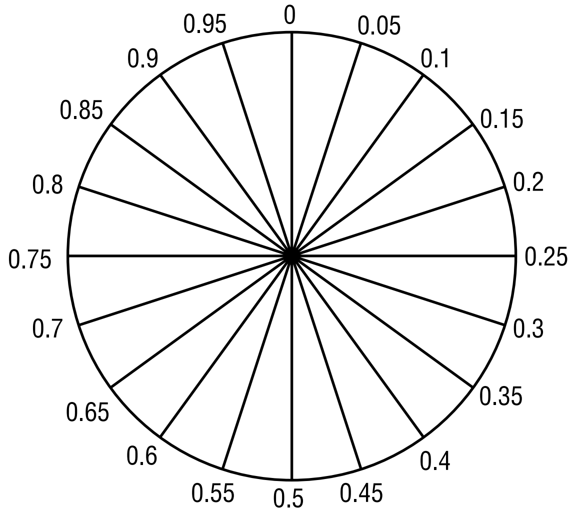
10-2 Enrichment

Relative Frequency and Circle Graphs

The **relative frequency** tells how the frequency of one item compares to the total of all the frequencies. Relative frequencies are written as fractions, decimals, or percents.

For example, in Exercise 1 below, the total of all the frequencies is 50. So, the relative frequency of the grade A is 8 ÷ 50, or 0.16.

The circle at the right is divided into 20 equal parts. You can trace this circle and then use relative frequencies to make circle graphs.

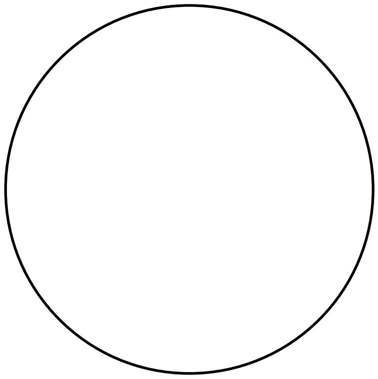


Complete each chart to show the relative frequencies. Then sketch a circle graph for the data. Use decimals rounded to the nearest hundredth.

1. History Grades for 50 Students

Grade	Frequency	Relative Frequency
A	8	0.16
B	16	
C	18	
D	6	
F	2	

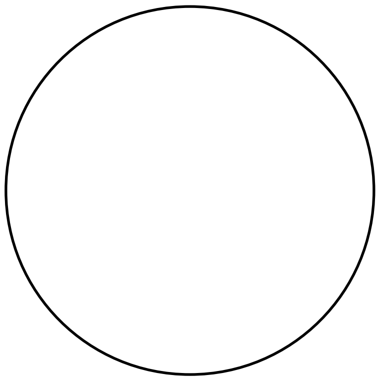
History Grades for 50 Students



2. Steve's Budget

Item	Amount Spent	Relative Spending
Telephone	$26	
Movies	$46	
Books	$24	
Car	$38	
Other	$66	

Steve's Budget



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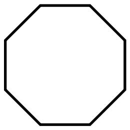
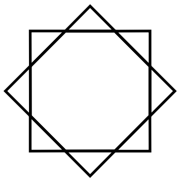
10-3 Enrichment

Star Polygons

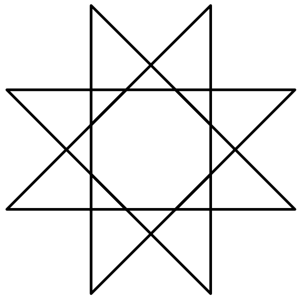
Any polygon can be turned into a *star polygon* by extending its sides. A star polygon is also called a *stellated* polygon.



Octagon



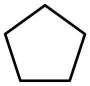
Extend the sides to make the first star.



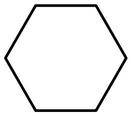
Extend the sides again to make a second star.

Make a star by extending the sides of each polygon.

1. pentagon

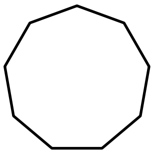


2. hexagon

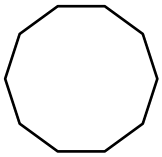


Trace each polygon on a separate sheet of paper. Then, make three different stars by extending the sides three times.

3. nonagon

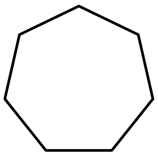


4. decagon

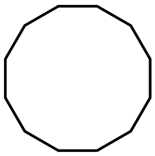


Show all the different stars that can be made from each polygon.

5. heptagon



6. dodecagon



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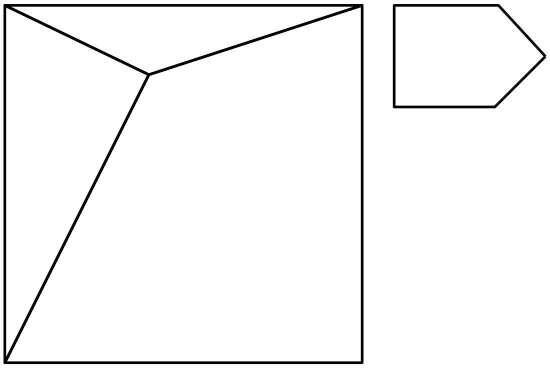
10-4 Enrichment

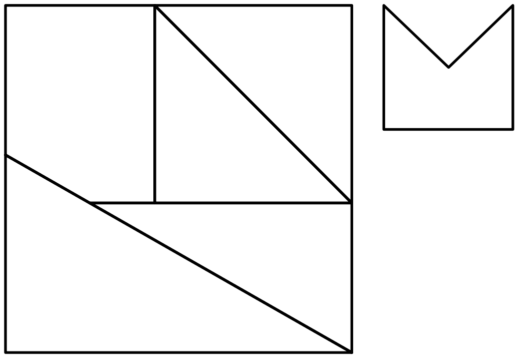
Dissecting Squares

In a *dissection puzzle*, the pieces of one shape are rearranged to make a different shape. Draw a square and then make a set of pieces to solve each dissection puzzle. Record your answers.

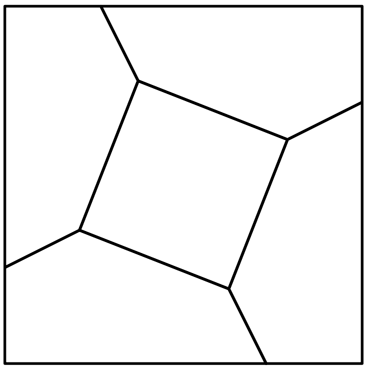
1. Rearrange the pieces to make a figure shaped like the one at the right.



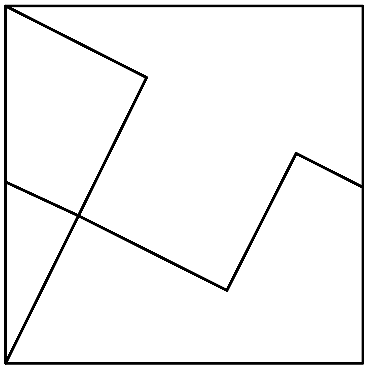
2. Rearrange the pieces to make a figure shaped like the one at the right.



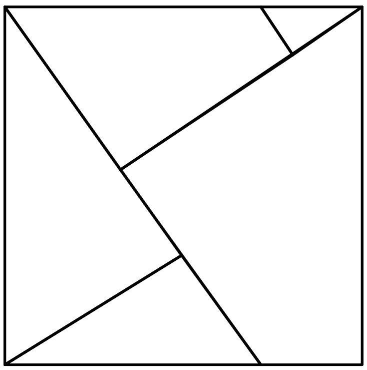
3. Rearrange the pieces to make an octagon with sides of equal length.



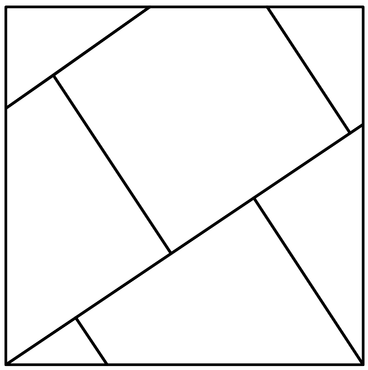
4. Rearrange the pieces to make a figure shaped like a plus sign.



5. Rearrange the pieces to make two new squares.



6. Rearrange the pieces to make three squares of equal size.

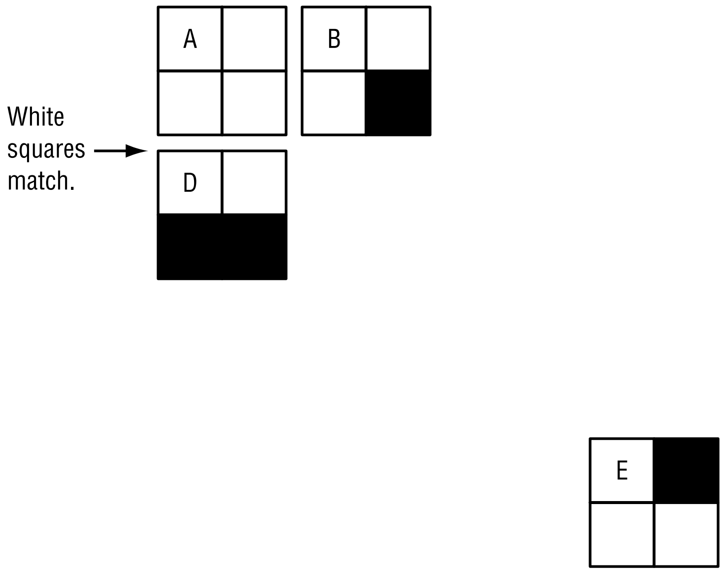


10-5 Enrichment

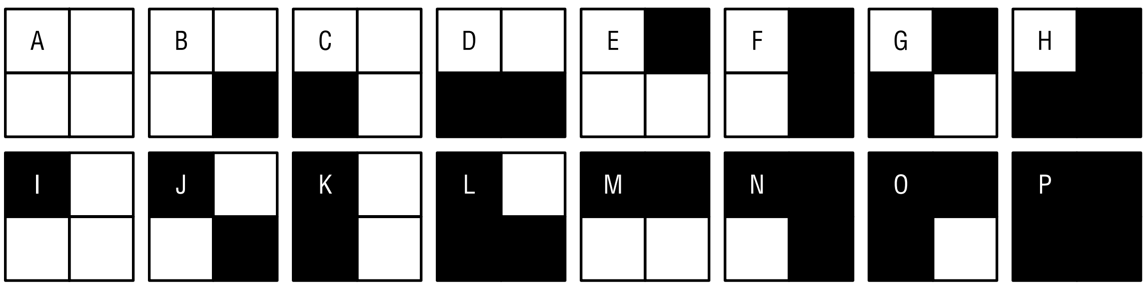
The Colormatch Square

To work this puzzle, cut out the 16 tiles at the bottom of this page. The goal of the puzzle is to create a square so that the sides of any pair of adjacent tiles match. You are not allowed to rotate any of the tiles.

1. Complete the solution to the colormatch square puzzle below.



2. Find at least one other solution in which the A tile is in the upper left corner.



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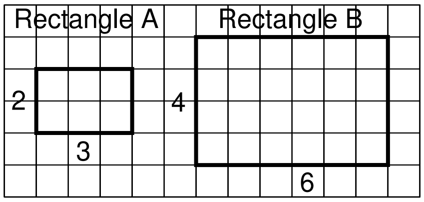
10-6 Enrichment

Similar Figures and Areas

The areas of two similar figures are related in a special way. Suppose that rectangle A is 2 units by 3 units and rectangle B is 4 units by 6 units.



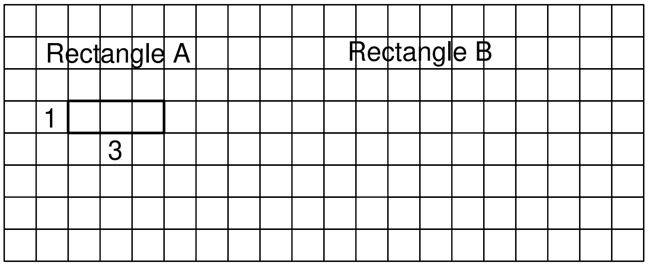
The area of rectangle A is $2 \times 3 = 6$ units^2.

The area of rectangle B is $4 \times 6 = 24$ units^2.

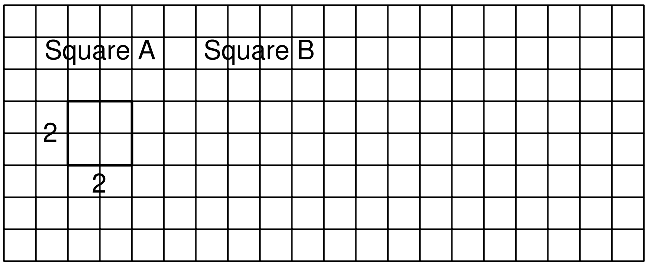
The lengths of the sides of rectangle B are twice those of rectangle A and the area of rectangle B is four times that of rectangle A.

Sketch figure B similar to figure A and satisfying the given condition.

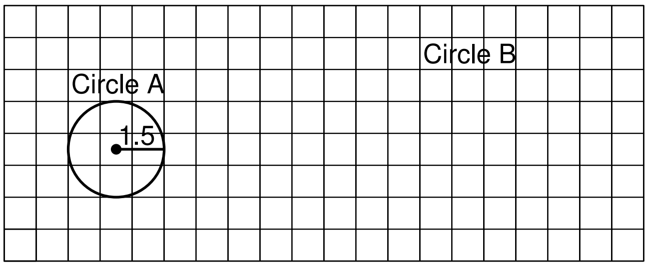
1. Rectangle B has sixteen times the area of rectangle A.



2. Square B has an area that is 4 times that of square A.



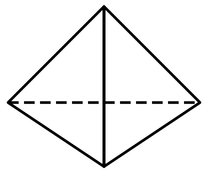
3. Circle B has an area four times that of circle A.



10-7 Enrichment

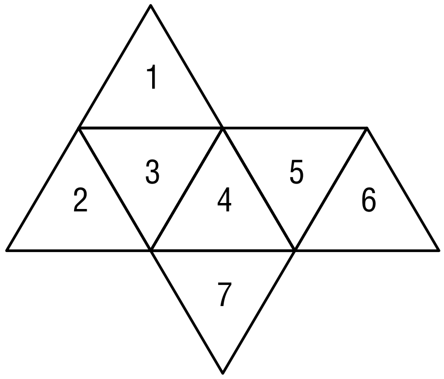
Tessellated Patterns for Solid Shapes

Tessellations made from equilateral triangles can be used to build three-dimensional shapes. In Exercise 1, you should get a shape like the one shown at the right. It is called a pyramid.

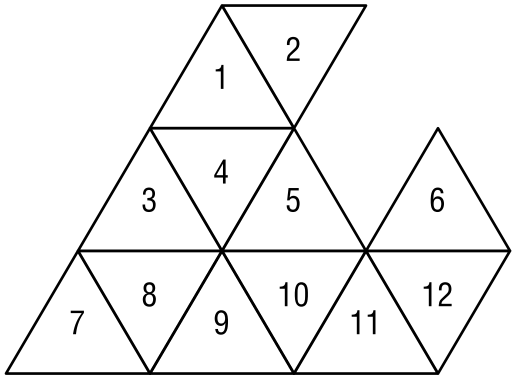


Copy each pattern. Crease the pattern along the lines. Then follow directions for folding the pattern. Use tape to secure the folded parts. When you have finished each model, describe it in words.

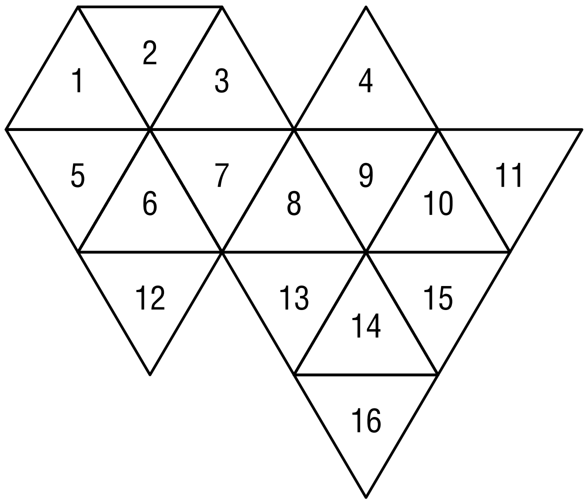
1. Fold 5 over 1.
Repeat, in this order:
fold 6 over 7,
fold 2 over 6.



2. Cut between 4 and 5. Then fold 5 over 3.
Repeat in this order:
fold 6 over 5,
fold 7 over 12, and
fold 2 over 9.



3. Cut between 1 and 2 and between 14 and 15. Then fold 15 over 14.
Repeat, in this order:
fold 1 over 2,
fold 4 over 3,
fold 11 over 1,
fold 16 over 5, and
fold 12 over 13.



Lesson 10–7

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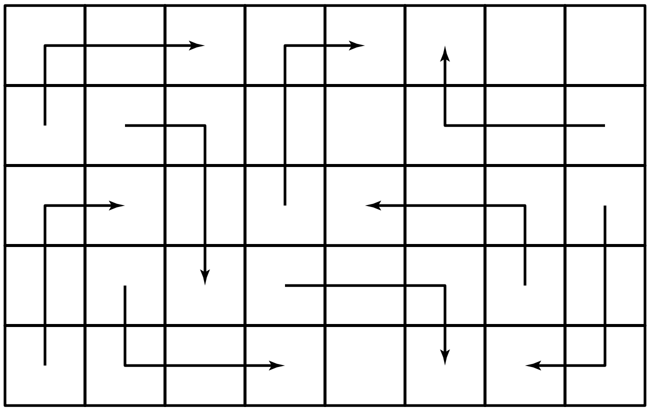
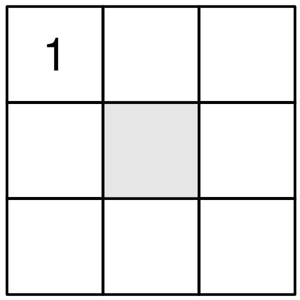
10-8 Enrichment

Chess Moves

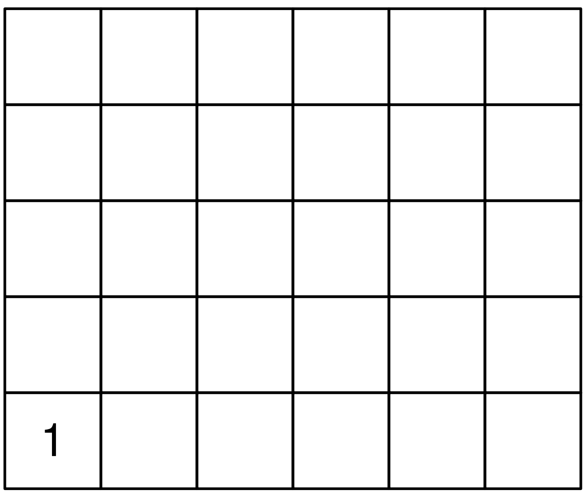
In the game of chess, a knight can move several different ways. It can move two spaces vertically or horizontally, then one space at a 90° angle. It can also move one space vertically or horizontally, then two spaces at a 90° angle. Several examples of a knight's moves are indicated on the grid at the right.



1. Use the diagram at the right. Place a knight or other piece in the square marked 1. Move the knight so that it lands on each of the remaining white squares only once. Mark each square in which the knight lands with 2, then 3, and so on.



2. Use the diagram below. Place a knight or other piece in the square marked 1. Move the knight so that it lands on each of the remaining squares only once. Mark each square in which the knight lands with 2, then 3, and so on.



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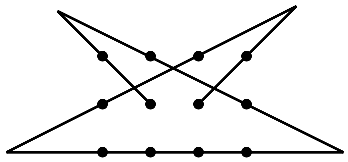
10-9 Enrichment

The Twelve Dot Puzzle

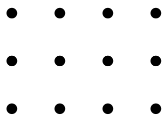


In this puzzle, a broken line made up of 5 segments must pass through each of 12 dots. The line cannot go through a dot more than once, although it may intersect itself. The line must start at one dot and end at a different dot.

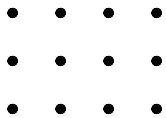
One solution to this puzzle is shown at the right. Two solutions to the puzzle are not “different” if one is just a reflection or rotation of the other.

Find 18 other solutions.

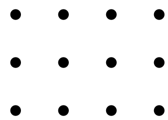
1.



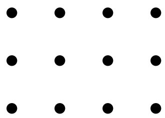
2.



3.



4.



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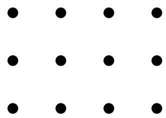

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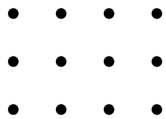
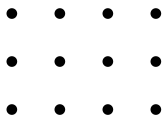


7.



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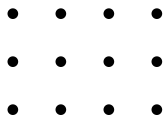
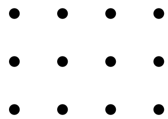
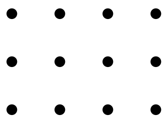


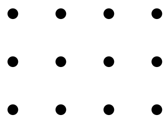
9.



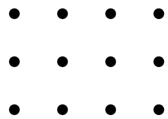
10.



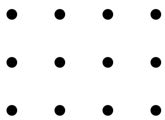
11.



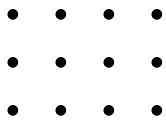
12.



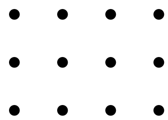
13.



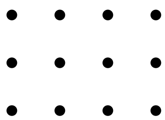
14.



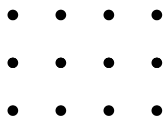
15.



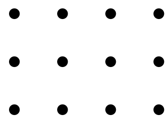
16.



17.



18.



Lesson 10–9

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11-1 Enrichment

The Geometric Mean

The square root of the product of two numbers is called their **geometric mean**. The geometric mean of 12 and 48 is $\sqrt{12 \cdot 48} = \sqrt{576}$ or 24.

Find the geometric mean for each pair of numbers.

1. 2 and 8

2. 4 and 9

3. 9 and 16

4. 16 and 4

5. 16 and 36

6. 12 and 3

7. 18 and 8

8. 2 and 18

9. 27 and 12

Lesson 11–1

Recall the definition of a **geometric sequence**. Each term is found by multiplying the previous term by the same number. A missing term in a geometric sequence equals the geometric mean of the two terms on either side.

Find the missing term in each geometric sequence.

10. 4, 12, $\boxed{?}$, 108, 324

11. 10, $\boxed{?}$, 62.5, 156.25, 390.625

12. 1, 0.4, $\boxed{?}$, 0.064, 0.0256

13. 700, 70, 7, 0.7, $\boxed{?}$, 0.007

14. 6, $\boxed{?}$, 24

15. 18, $\boxed{?}$, 32

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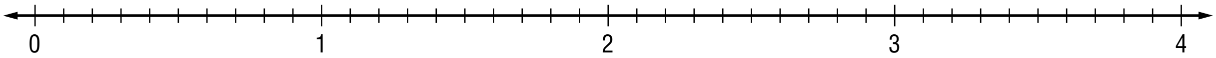
11-2 Enrichment

World Series Records

Each problem gives the name of a famous baseball player. To find who set each record, graph the points on the number line.

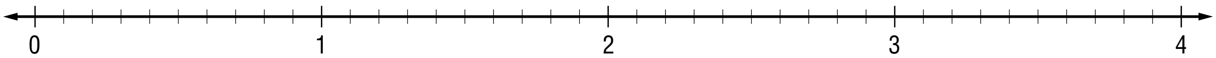
1. pitched 23 strikeouts in one World Series

U at $\sqrt{3}$, X at 3.3, K at 0.75, O at $\frac{3}{2}$, F at $\sqrt{6}$, A at $2\frac{7}{8}$



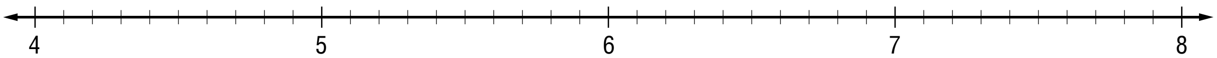
2. 71 base hits in his appearances in World Series

B at $\sqrt{5}$, R at $\sqrt{12}$, A at 3.75, G at $\frac{16}{13}$, E at $\frac{5}{2}$, Y at 0.375, R at $\frac{13}{4}$, I at 1.6, and O at $0.\overline{7}$



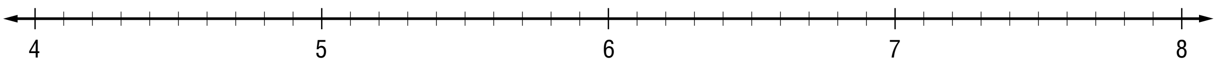
3. 10 runs in a single World Series

N at $\sqrt{60}$, K at $\sqrt{30}$, A at 4.3, S at 6.2, C at $\frac{46}{9}$, O at $\sqrt{45}$, and J at $\sqrt{17}$



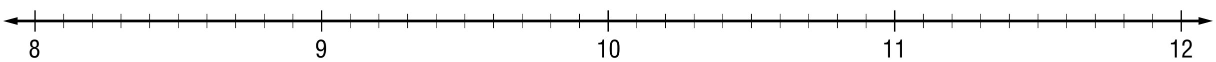
4. batting average of 0.625 in a single World Series

E at $\sqrt{32}$, U at $6\frac{5}{6}$, A at $\frac{14}{3}$, T at $\sqrt{55}$, B at 5.3, R at $\sqrt{40}$, H at 7.75, B at $\frac{21}{5}$



5. 42 World Series runs in his career

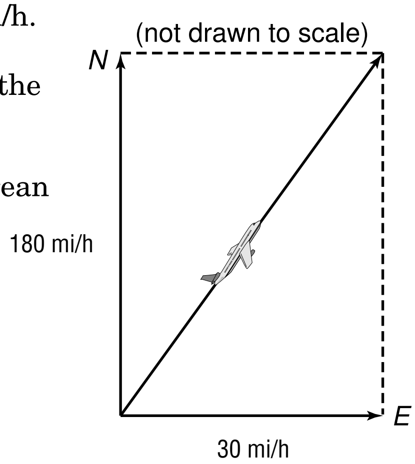
E at $\sqrt{140}$, Y at 9.6, I at 8.6, E at $\sqrt{90}$, A at $\frac{21}{2}$, M at $\sqrt{70}$, C at $8\frac{7}{8}$, M at $\sqrt{100}$, N at 10.7, K at $9\frac{1}{11}$, T at $\sqrt{120}$, L at 11.4



11-3 Enrichment

Pythagoras in the Air

In the diagram at the right, an airplane heads north at 180 mi/h. But, the wind is blowing towards the east at 30 mi/h. So, the airplane is really traveling east of north. The middle arrow in the diagram shows the actual direction of the airplane.



The actual speed of the plane can be found using the Pythagorean Theorem.

$$\begin{aligned}\sqrt{30^2 + 180^2} &= \sqrt{900 + 32{,}400}\\ &= \sqrt{33{,}300}\\ &\approx 182.5\end{aligned}$$

The plane's actual speed is about 182.5 mi/h.

Find the actual speed of each airplane. Round answers to the nearest tenth. (You might wish to draw a diagram to help you solve the problem.)

1. An airplane travels at 240 mi/h east. A wind is blowing at 20 mi/h toward the south.

2. An airplane travels at 620 mi/h west. A wind is blowing at 35 mi/h toward the south.

3. An airplane travels at 450 mi/h south. A wind is blowing at 40 mi/h toward the east.

4. An airplane travels at 1,200 mi/h east. A wind is blowing at 30 mi/h toward the north.

Lesson 11–3

NAME ______________________________ DATE ____________ PERIOD _____

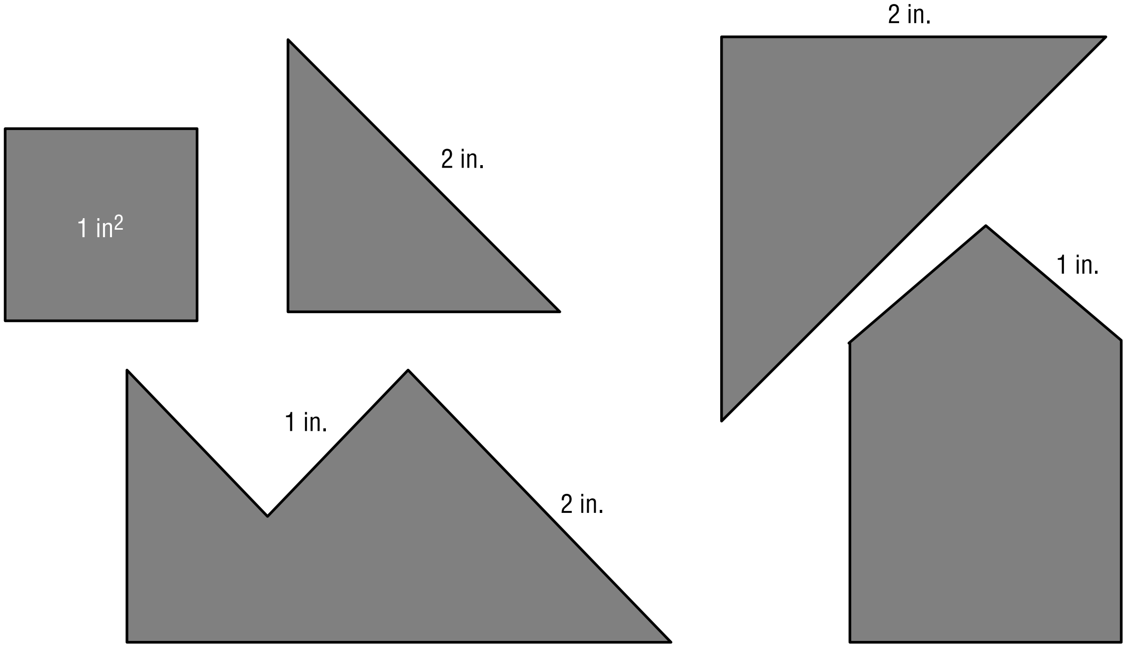
11-4 Enrichment

Two Area Puzzles

Cut out the five puzzle pieces at the bottom of this page. Then use them to solve these two puzzles.

1. Use all five puzzle pieces to make a square with an area of 9 square inches. Record your solution below.

2. Use the four largest pieces to make a square with an area of 8 square inches. Record your solution below.



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11-5 Enrichment

Heron's Formula

A formula named after Heron of Alexandria, Egypt, can be used to find the area of a triangle given the lengths of its sides.

Heron's formula states that the area A of a triangle whose sides measure a, b, and c is given by

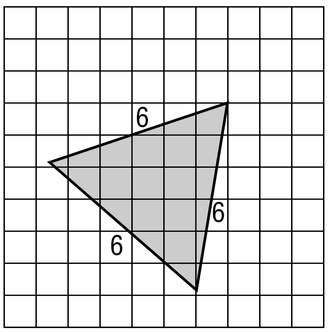
$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where s is the semiperimeter:

$$s = \frac{a+b+c}{2}.$$

Estimate the area of each triangle by finding the mean of the inner and outer measures. Then use Heron's Formula to compute a more exact area. Give each answer to the nearest tenth of a square unit.

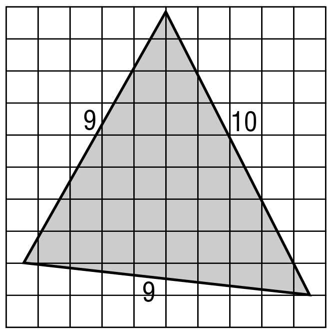
1.



Estimated area:

Computed area:

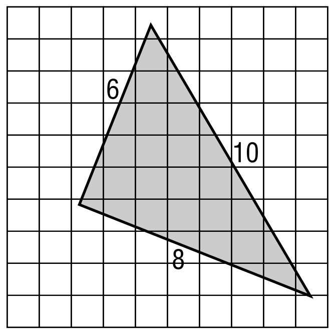
2.



Estimated area:

Computed area:

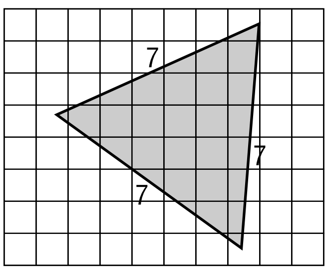
3.



Estimated area:

Computed area:

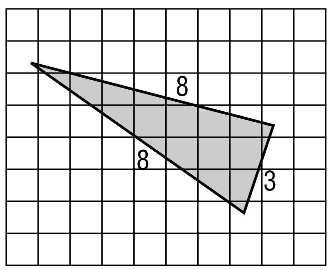
4.



Estimated area:

Computed area:

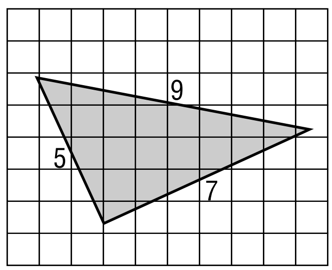
5.



Estimated area:

Computed area:

6.



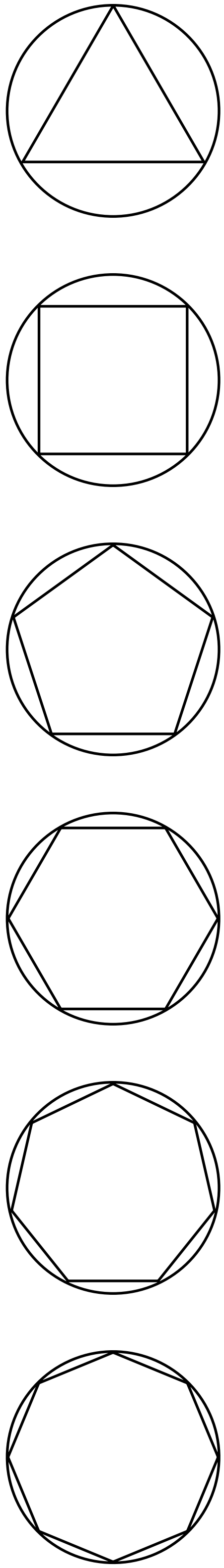
Estimated area:

Computed area:

NAME ______________________________ DATE ____________ PERIOD _____

11-6 Enrichment

Seki Kowa



Japanese mathematician Seki Kowa (c. 1642–1708) is called The Arithmetical Sage because of his many contributions to the development of mathematics in Japan. Before Seki, mathematics in Japan was considered a form of art to be enjoyed by intellectuals in their leisure time. Seki demonstrated the practical uses of mathematics and introduced social reforms that made it possible for anyone, not just intellectuals, to study mathematics.

One of Seki's contributions to mathematics was his calculation of a value of π that was correct to eighteen decimal places.

$$\pi \approx 3.141592653589793238\ldots$$

Seki had noticed the phenomenon that you see at the right: as the number of sides of a regular polygon increases, the polygon looks more and more like a circle. So, Seki calculated the following ratio for polygons of increasingly many sides.

$$\frac{\text{perimeter of regular polygon}}{\text{diameter of circle drawn around the polygon}}$$

As the number of sides of the polygon gets larger, this ratio must get closer to the ratio of the circumference of the circle to the diameter of the circle. This ratio, of course, is π.

You are given information below about a regular polygon and the circle drawn around the polygon. Use a calculator to find Seki's ratio. (Give as many decimal places as there are in your calculator display.) What do you notice about your answers?

1. length of one side = 5
number of sides = 6
diameter of circle = 10

2. length of one side ≈ 4.5922
number of sides = 8
diameter of circle = 12

3. length of one side ≈ 3.7544
number of sides = 20
diameter of circle = 24

4. length of one side ≈ 37.5443
number of sides = 20
diameter of circle = 240

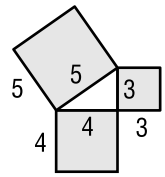
5. length of one side ≈ 1.6754
number of sides = 150
diameter of circle = 80

6. length of one side ≈ 2.6389
number of sides = 500
diameter of circle = 420

11-7 Enrichment

Extending the Pythagorean Theorem

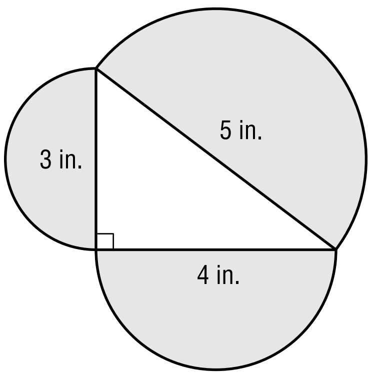
The Pythagorean Theorem says that the sum of the areas of the two smaller squares is equal to the area of the largest square. Show that the Pythagorean Theorem can be extended to include other shapes on the sides of a triangle. To do so, find the areas of the two smaller shapes. Then, check that their sum equals the area of the largest shape.



1. area of smallest shape:

area of middle shape:

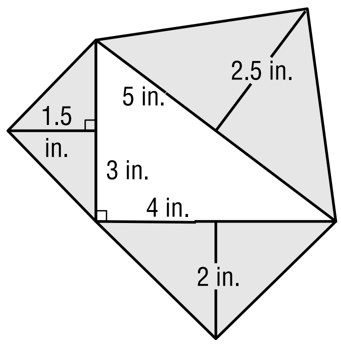
area of largest shape:



2. area of smallest shape:

area of middle shape:

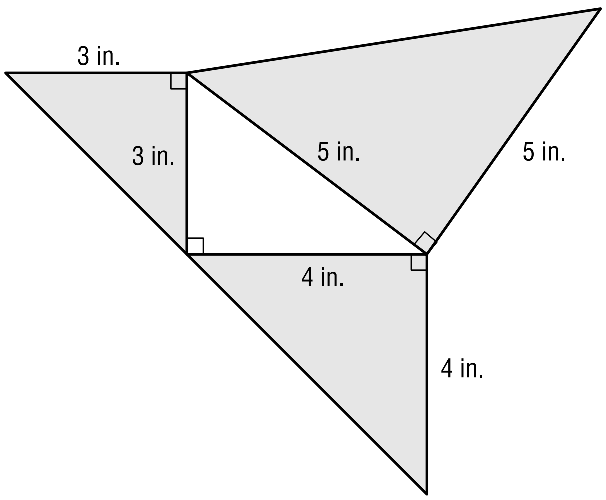
area of largest shape:



3. area of smallest shape:

area of middle shape:

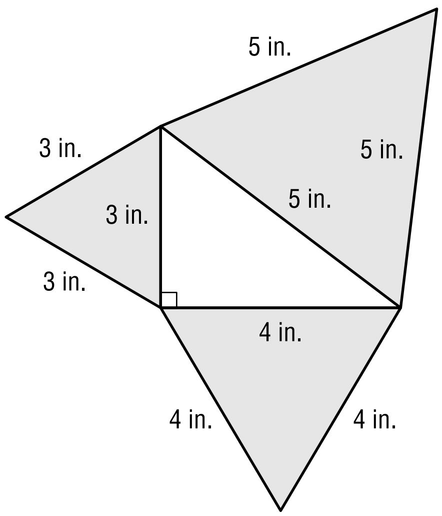
area of largest shape:



4. area of smallest shape:

area of middle shape:

area of largest shape:



Lesson 11–7

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11-8 Enrichment

Area Formulas for Regular Polygons

Recall that the sides of a regular polygon are all the same length. Here are some area formulas for four of the regular polygons. The variable s stands for the length of one side.

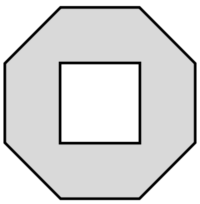
triangle	pentagon	hexagon	octagon
$A = \frac{s^2}{4}\sqrt{3}$	$A = \frac{s^2}{4}\sqrt{25 + 10\sqrt{5}}$	$A = \frac{3s^2}{2}\sqrt{3}$	$A = 2s^2(\sqrt{2} + 1)$

Find the area of each polygon with the side of given length. Use a calculator and round each answer to the nearest tenth.

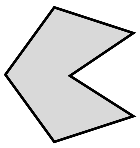
	Length of a Side	Triangle	Pentagon	Hexagon	Octagon
1.	1 cm				
2.	2 cm				
3.	3 cm				
4.	4 cm				

Now use the table above to find the area of each shaded region below. Unless otherwise specified, each segment is 1 centimeter long.

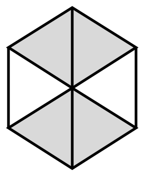
5.



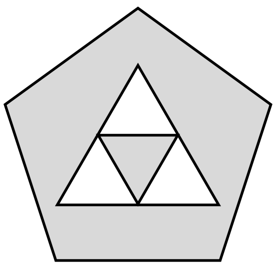
6.



7.

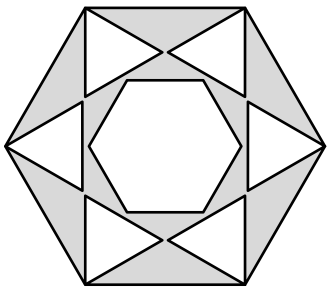


8.



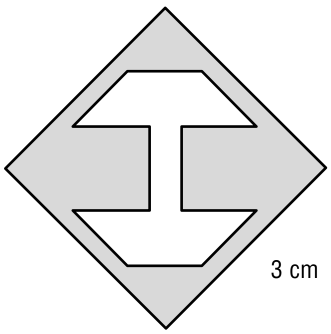
2 cm

9.



2 cm

10.



3 cm

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12-1 Enrichment

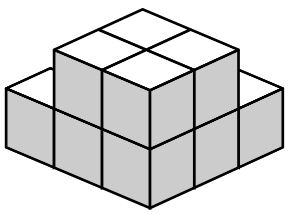
Counting Cubes

The figures on this page have been built by gluing cubes together. Use your visual imagination to count the total number of cubes as well as the number of cubes with glue on 1, 2, 3, 4, or 5, or 6 faces.

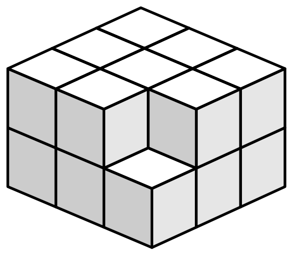
Complete this chart for the figures below.

Figure	Total Number of Cubes	Number of Faces with Glue on Them					
		1 face	2 faces	3 faces	4 faces	5 faces	6 faces
1							
2							
3							
4							
5							
6							

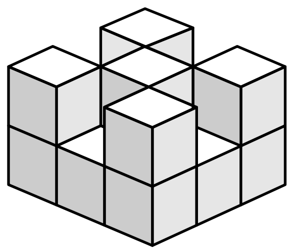
1.



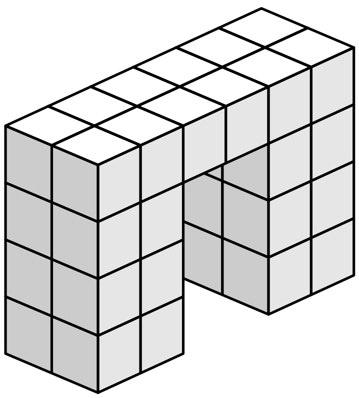
2.



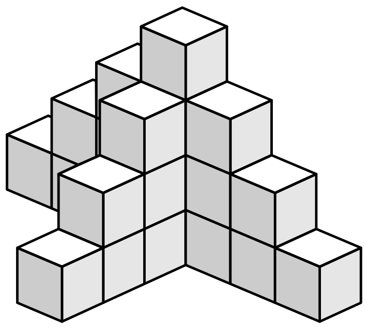
3.



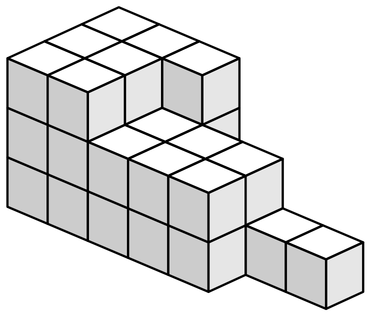
4.



5.



6.

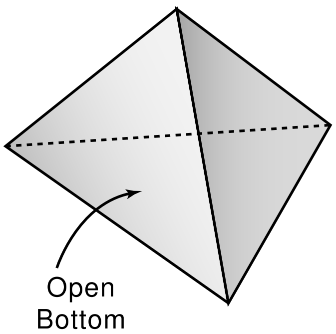


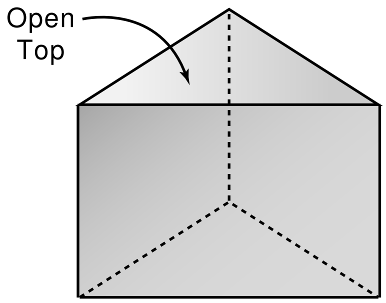
Lesson 12–1

12-2 Enrichment

Volumes of Pyramids

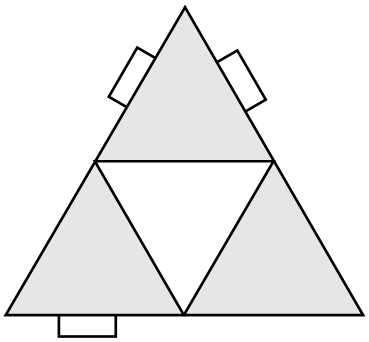
A pyramid and a prism with the same base and height are shown below.

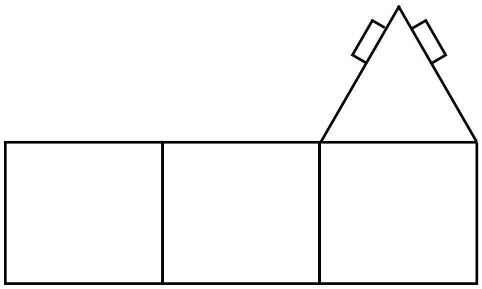




The exercises on this page will help you discover how their volumes are related.

Enlarge and make copies of the two patterns below to make the open pyramid and the open prism shown above. (Each equilateral triangle should measure 8 centimeters on a side.)



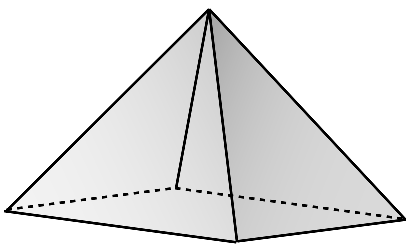


1. Describe the bases of the two solids.

2. How do the heights of the solids compare?

3. Fill the open pyramid with sand or sugar. Pour the contents into the open prism. How many times must you do this to fill the open prism?

4. Describe how you would find the volume of the pyramid shown at the right.



5. Generalize: State a formula for the volume of a pyramid.

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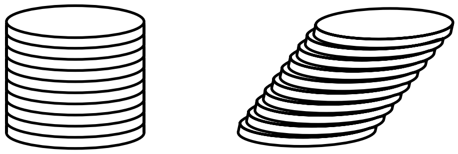
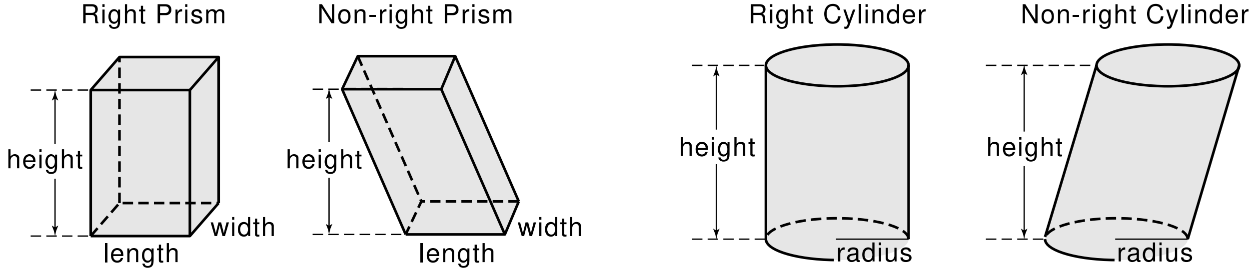
12-3 Enrichment

Volumes of Non-Right Solids

Imagine a stack of ten pennies. By pushing against the stack, you can change its shape as shown at the right. But, the volume of the stack does not change.

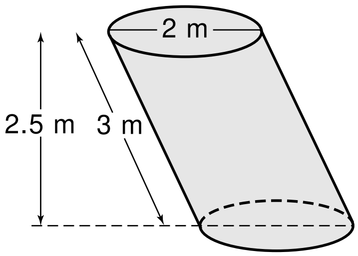


The diagrams below show prisms and cylinders that have the same volume but do not have the same shape.

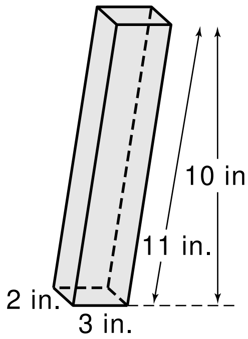


Find the volume of each solid figure. Round to the nearest tenth.

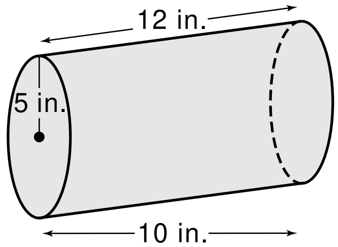
1.



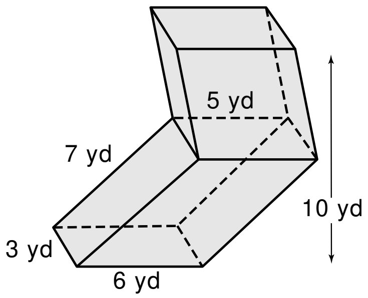
2.



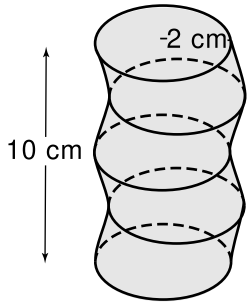
3.



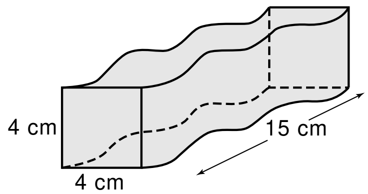
4.



5.



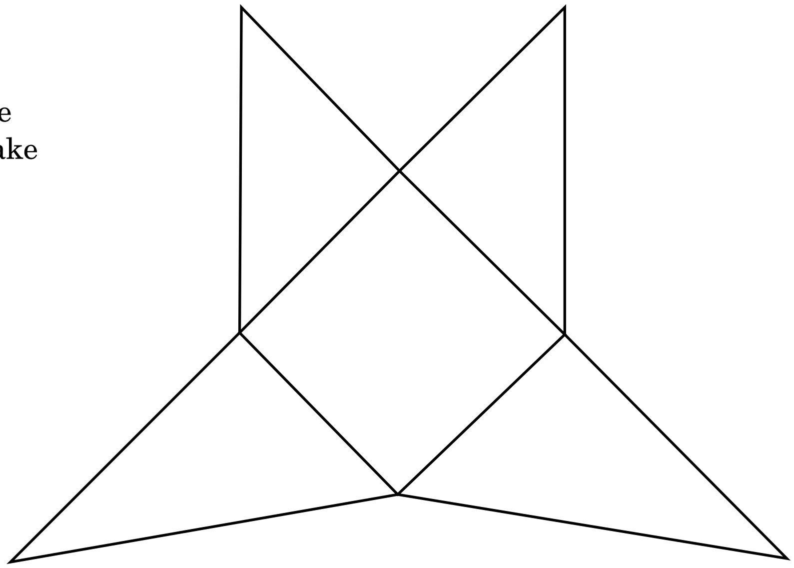
6.



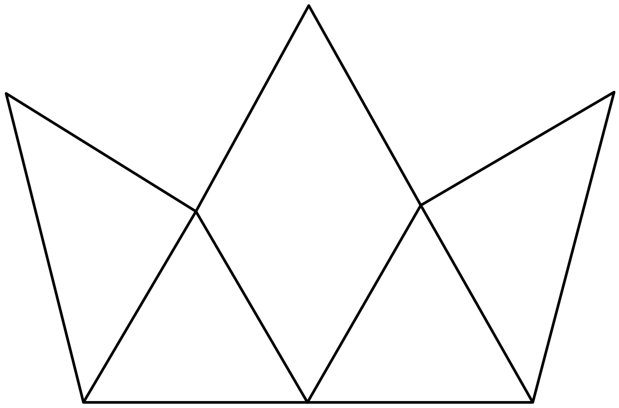
12-4 Enrichment

Pattern Puzzles

1. Make three copies of this pattern. Fold each pattern to make a pyramid. Then, put the three pyramids together to make a cube. Draw a sketch of the completed cube.



2. Make four copies of this pattern. Fold each pattern to make a solid figure. Then, put the four solids together to make a pyramid. Make a sketch of the finished pyramid.



3. Find the surface area of the cube in Exercise 1.

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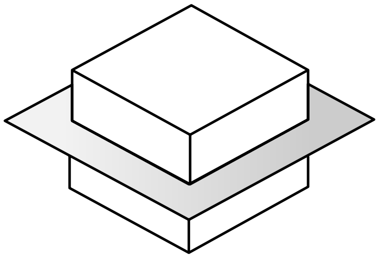
12-5 Enrichment

Cross Sections

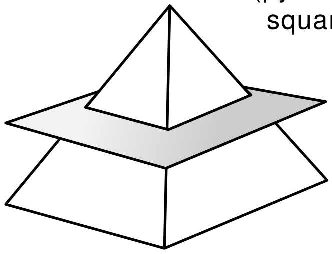
In each diagram on this page, a plane cuts through a solid figure. The intersection of the plane with the solid figure is called a *cross section*.

Sketch the cross section formed in each diagram.

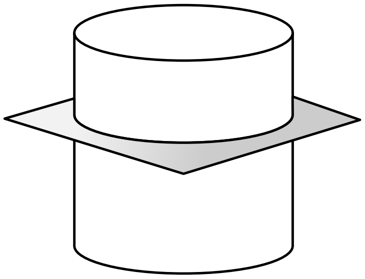
1.



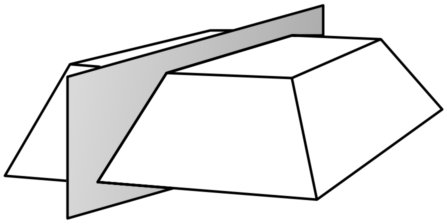
2. (pyramid with a square base)



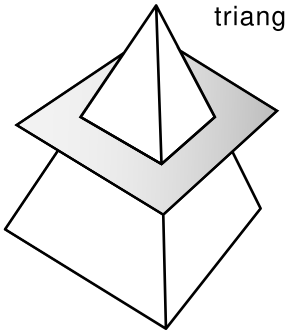
3.



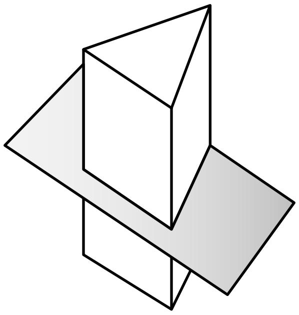
4.



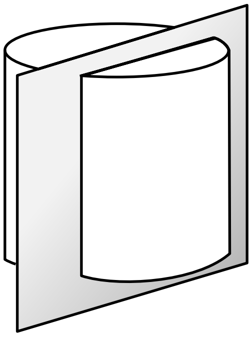
5. (pyramid with a triangular base)



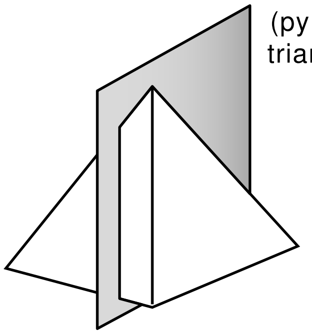
6.



7.



8. (pyramid with a triangular base)



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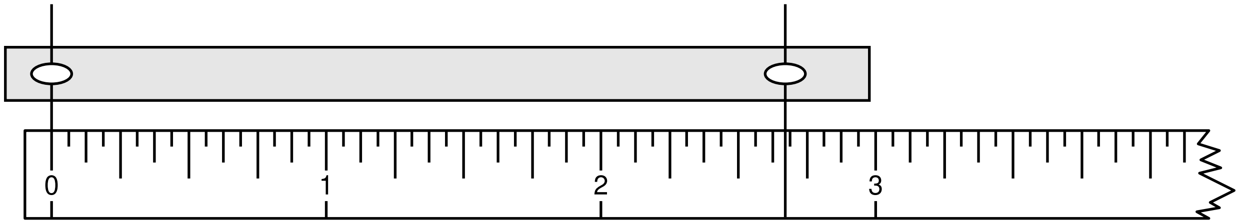
12-6 Enrichment

Absolute Error

The **absolute error** of a measurement is defined to be one-half the smallest unit used in making the measurement. For example, this drawing shows the distance between the centers of the two holes in a piece of metal.



If the distance were measured to the nearest quarter of an inch, the absolute error would be one-eighth of an inch. The symbol ± means "plus or minus." This symbol is often used to report measurements.

$2\frac{3}{4}$ in. ± $\frac{1}{8}$ in.

This way of reporting measurements helps to show how accurate the measurement is. The actual measurement will lie somewhere in this interval.

$$2\frac{3}{4} \text{ in.} - \frac{1}{8} \text{ in.} < m < 2\frac{3}{4} \text{ in.} + \frac{1}{8} \text{ in.}$$

$$2\frac{5}{8} \text{ in.} < m < 2\frac{7}{8} \text{ in.}$$

Write each reported measurement using an interval. Use *m* to represent the actual measurement.

1. 25,000 ± 500 voters

2. 15 ± 0.5 kg

3. 750 ± 25 customers

4. 75 ± 5 mi

5. 14 ± $\frac{1}{2}$ gal

6. $7\frac{1}{4}$ ± $\frac{1}{4}$ in.

Name the unit of measure used to make each measurement.

7. 32 ± $\frac{1}{2}$ ft

8. 23 ± 0.5 m

9. $5\frac{1}{4}$ ± $\frac{1}{8}$ mi

10. 14 ± 0.5 cm

11. $2\frac{3}{8}$ ± $\frac{1}{16}$ in.

12. 8 ± $\frac{1}{2}$ yd