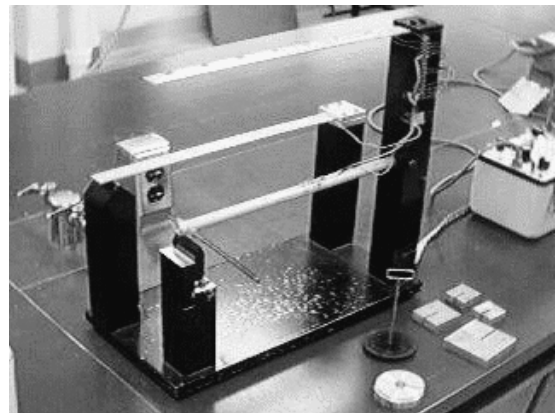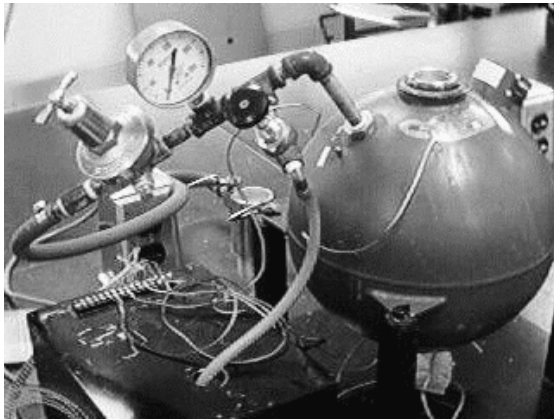# Strain Measurement

Motivation

The main idea of this experiment is to revise the concept of stress, strain, their relation to the applied load in different types of structures, idea of stress and strain transformation, and methods to calculate the strain in any required direction generated by the loads in a direction different from the required one.

Before performing this lab you MUST be familiar with:

1. Basic notion of stress and strain.
2. Plane stress and plane strain. Principal normal and shear stresses. Relation between Principal stresses and stresses at any other orientation.
3. Strain gage and its working principle.
4. Strain rosette and its utility.
5. Plane stress and strain transformation.
6. Mohr's circle for stress and strain.
7. Transformation from stress to strain in 1-D, 2-D and 3-D.
8. Working principle of Wheatstone bridge.

INDIVIDUALLY submit the solution to the following problem in the pre-lab report and before you start the experiment. Assume steel material properties in SI units.
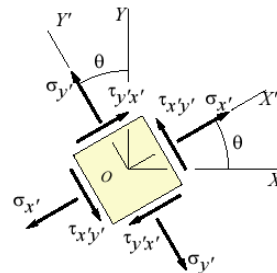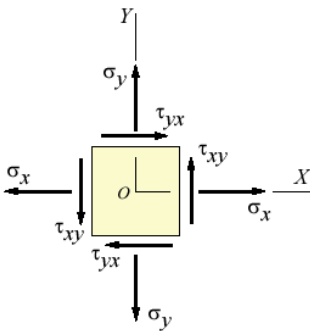
Original stress state is (use MPa for σ_x, σ_y, τ_{xy}, and angle in degrees) according to

$(\sigma_x, \sigma_y, \tau_{xy}, \theta)$ => (mm+10, dd, yy, 30+dd) corresponding to your date of birth mm, dd, yy, (last two digits for year):

1. Transform the state at the angle defined above using stress transformation equations
2. Draw Mohr's circle for stress and use it to find the transformed stresses.
3. Calculate the original longitudinal strains.
4. Calculate longitudinal strains in new transformed orientation

Make sure you understand each setup and find the correct equations to be used for analysis based on theoretical prerequisites

Introduction

A short lived engineer would be one that never considered stress or strain. Stress is present in all structures either static or dynamic, at least on earth anyway. Stress is typically a key element that dictates every design. Strain is the resulting deformation that a structure experiences under a given stress. For most materials considered in mainstream engineering, stress is directly proportional to strain. Within the design process a preliminary survey produces likely candidates for use as materials and the expected stress-strain quantities are calculated. At this point two key issues exist;

(1) How is the stress calculated for complex shapes such as curved cross sections?

(2) How are the strains measured once the structure has been built?

Analytical solutions are available for almost every type of structure imaginable. These equations produce equations for normal or shear stress as a function of loading and geometry. The equations are based entirely upon geometric quantities and don't depend on empirical data for a solution. In this experiment, we challenge those equations. Given various structures, loads are applied and measurements are made that will either coincide or diverge from values produced using theoretically derived equations.

Now, all that is left to find is a means to measure strain. Most people, with the aid of a ruler, can measure 1/32" within ± 1/64". That's fine for extremely high loadings or extremely weak materials, but many materials might fail at say 3500 μstrain, which would be well below 1/20 of the resolution of a ruler. To solve this problem several instruments can be used, one of which is the strain gage in a Wheatstone bridge configuration. This configuration allows measurements, in some cases, as low as +/- 1μstrain. With resolution this good the question arises "Is this small quantity significantly useful in most applications?" Typically, this resolution is not needed and the closest desired resolution may be +/- 5μstrain. It all depends on the application and the goal for the analysis.

In this experiment, the student loads actual structures and examines the strains produced by the loading. Later, the student calculates the strains expected by geometrically derived equations and compares the difference in the two values. At the end, the student can provide evidence if the equations can accurately predict strains present in a structure and also if the strain gage/strain box combination is useful in the measurement of these strains.

Theory

Foil strain gages are essential elements in the measurement of small displacements of materials. Strain gages are constructed from a thin plastic film coated with a copper film layer. The copper film layer is etched away in a certain pattern to give the strain gage lines of conductive material. The direction of these conductive lines is the direction in which the strain is to be measured.

The strain gage is mounted to a material with some type of adhesive. This can be epoxy, cyanoacrilate, or numerous other bonding agents depending on the host material. Strain gauges are mounted with the plastic film backing next to the material that is being measured. Once again, the lines of conductive material run in the direction of the strain that is to be measured.

When the host material is elongated (or compressed) the lines of copper are stretched as well. With Poisson effects in mind it is easy to image the cross sectional area of the lines getting smaller and the overall length of the gage increasing. These two effects give a change in resistance to the strain gage.

The change is resistance (strain) allows the transformation of a physical quantity into an electrical quantity. The change in strain typically may be only 1/10 Ohm at most. This presents a big challenge if one was to simply use an Ohm meter to gauge strain. A Wheatstone bridge, shown in figure 1, can be used in order to make this change in resistance meaningful. The gage is inserted in one "leg" of the bridge. The bridge is assumed to be balanced at a "no strain condition." From there, when strain is applied, the complementary leg of the bridge is adjusted in order to re-balance the bridge. The amount of change in resistance used to re-balance the bridge is directly proportional to the strain that is being applied to the test material.
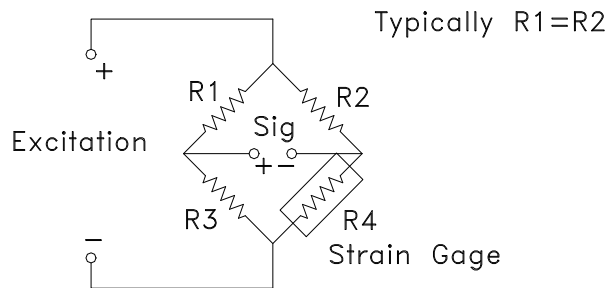

Figure 1 Wheatstone bridge with strain gage in place.

$$R_4 = \frac{R_2}{R_1} R_3 \tag{1}$$

Equation (1) gives the relationship to R_4 as a function of the remaining resistor. In most bridge configurations, R_3 is made close to R_4 and R_1 is made close to R_2. It's easy to imagine that by correct selection of the resistors, R_1 and R_2 that a multiplier effect occurs in the R_2/R_1 ratio. For example, if R_4 is 100 Ohms and changes 1/10 Ohm, then the R_2/R_1 ratio has to change by 1.001 for the bridge to stay in equilibrium. If, say $R_2 = R_1 = 100$ kOhms, then the change in R_2 has to be 100 Ohms. The quantity 100 Ohms is much easier to determine than 1/10 Ohm and also provides more accuracy in doing so.

Manufacturers provide two important pieces of information for the strain gages they sell; the resistance and the gage factor of the gage. The gage factor is a value that relates change in size to change in resistance. No strain gage is usable for measuring strain without these values. These values are used in conjunction with the change in resistance value mentioned above to determine the strain in the tested material. Equation 2 identifies the relation of strain, resistance and gage factor where theoretically R_0 is the initial resistance of the strain gage. However, in actual implementation one must consider the resistance of the wiring connecting the strain gage to the measuring instrument and use the measured initial resistance in place of R_0. ΔR is the change in resistance that occurs in the strain gage since the wiring remains the same.

$$\Delta \varepsilon = \frac{1}{K} \frac{\Delta R}{R_0} \tag{2}$$

An interesting use of the strain gage is in a load cell. Typically load cells are used to measure loads in one direction. In actual set-ups, load cells are calibrated in two ways. The first way is in which the load cell bridge is calibrated by a shunt calibration technique. This is where a known precision resistor is placed across the terminals of the strain gage. A formula determines what the reading should be across the bridge. The value of the output is then interpreted from that reading to give a reading in lb, kg, etc.

The second method of calibration is known as dead weight calibration. This type of calibration is significantly easier and also aids the user in believing the reading produced from the set-up. The load cell (bridge) is balanced from a no-load state. Now increments of weights are added and the potential across the bridge is recorded. The corresponding weight to potential values are recorded and used as a formula for determining unknown loads. This may seem like more physical work, but once again it provides a sense of security in seeing that the loads placed on the cell have a certain conformable value.

The first structure examined in this experiment is the cantilever beam. A beam under bending can be characterized by equation (3).

$$\frac{1}{\rho} = \frac{M}{EI} \tag{3}$$

The radius of curvature is given by equation (4)

$$\frac{1}{\rho} = \frac{d^2y/dx^2}{\left(1+(dy/dx)^2\right)^{3/2}} \tag{4}$$

where y is the deflection in the y direction at any given point x along the beam. For many problems, the deflection is very small. This means that the denominator can be neglected in most cases. Combining equations (3) and (4) yields

$$\frac{M}{EI} = \frac{d^2y}{dx^2} \tag{5}$$

This further reduces to a convenient form of the equation for stress in the cantilever beam.

$$\sigma = \frac{My}{I} \tag{6}$$

An equation of the strain in the beam can be written considering any point in the beam. Equation (3) has been derived through fundamentals of mechanics. The parameters in these two equations involve discrete physical values with no inclusion of "mysterious" correction factors. What's significant about this point is that theory will predict (very closely) what we actually see in the loading of the structure. The case of the cantilever beam is a simple introduction to this argument. The same principles and analyses applied to the cantilever beam can be applied to more complicated structures. This results in a compounding of the confirmation of theoretically derived equations through experimentation. Solutions for the remaining structures can be found in most solid mechanics textbooks and therefore no additional information will be provided in this manual for their identification.

Also, there should be some observation about the usability and reliability of the relatively crude instrumentation involved in the experiment. In most cases, strain values differ at most by 5 μstrain from the actual values. In most of the experiments here, that relates to much less than an ounce of resolution. In the laboratory most load cells typically fall within 0.5 % error.

Apparatus

1) Mounted foil strain gages
2) Data Logger Agilent Data Acquisition system 34970A and computer system
3) Cantilever beam fixed at other end
4) Cantilever beam
5) Tubular beam
6) Pressure vessel (spherical) with air regulator
7) Assorted weights with hanger

Weights (Measure and Record these values)

Holder

Weight #1

Weight #2

Weight #3

Weight #4

Weight #5 (bucket)

Spherical Pressure Vessel

Material	Stainless Steel
Thickness	1/8" +/- 1/64"
Outside diameter	10" +/- 1/8"
E	28 Mpsi
G	10.6 Mpsi

For all Rectangular Beams:

Measure the dimensions and note the uncertainty using digital vernier caliber

Material	Aluminum Cantilever	Aluminum Simply Supported	Steel Cantilever	Steel Simply Supported
Thickness, **t**				
Width, **b**				
E				
G				

CANTILEVER and SIMPLY SUPPORTED BEAMS SETUPS

NOTE: MAKE SURE YOU MEASURE ALL THE DIMENSIONS AS SHOWN IN THE FIGURES BELOW
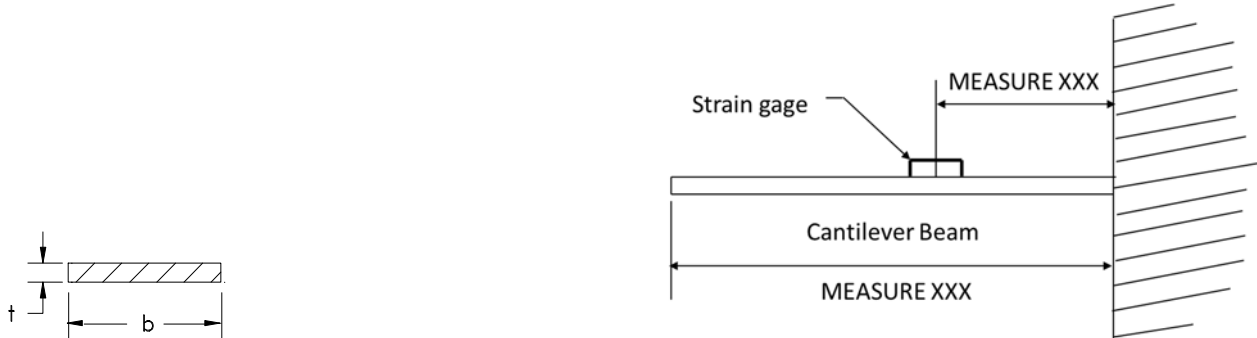
Measure also:

Gage to support Left   =   in

Gage to support Right =   in

Support to support   =   in

TUBULAR BEAM SETUP

NOTE: THERE ARE TWO SETUPS – HOLLOW TUBE AND SOLID TUBE

MAKE SURE YOU GET THE DIMENSIONS FOR THE CORRECT SETUP

Material   6061-T6 Aluminum

R_o

R_l

Arm, L

Objective

1) The confirmation of theoretically derived equations through the use of actual testing.
2) To evaluate the use of the strain gage/strain indicator as a tool in the measurement of strain.

Requirements

1) Make all the calculations indicated in the data table. The calculated strains should correspond and be compared to the measured data.
2) Calculate the maximum percent difference between the theoretical and the experimental data.
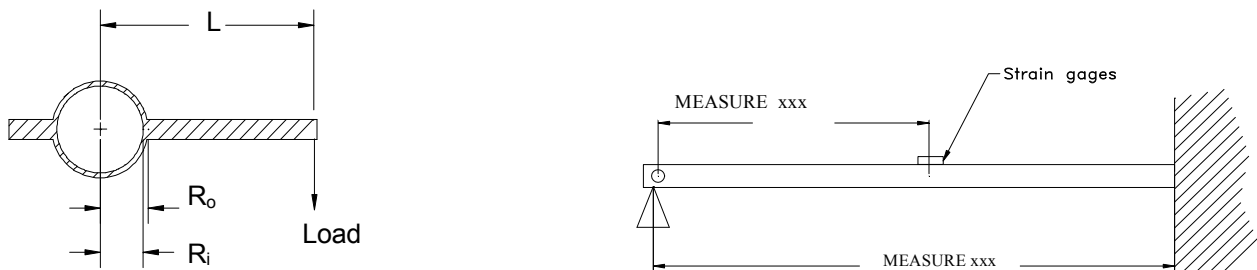3) For the cantilever beam and fixed supported beam, plot the measured and calculated strain as a function of the load's distance from the gage.
4) Compare the results between the theoretical and Digital and Analog systems.
5) Perform uncertainty analysis using one set of reading from each of the table in the data sheet.

Procedure

Turn on the Data Logger. Obtain from the instructor, the amount of load to be used for the beam and the tube. Also, obtain the pressure to be used in pressure vessel. Data such as gage factors, structural properties, operating instructions, and structure dimensions should be on the information sheet.

Make sure you measure correctly any quantities needed especially lengths/distances as well as note the gage factors for the various strain gages.

Analog System

Cantilever Beam

1) Study the experimental setup and read the available operating instructions
2) On the Portable Strain Indicator box, power on the device by moving the power switch to AC
3) Select Channel 7 and adjust the strain indicator to the correct gage factor
4) Zero the indicator prior to loading. With NO load on the cantilever beam, adjust the strain reading to 0000 by using the large black knob and turn the balance knob until the galvanometer is centered. You might have to experiment with the sensitivity knob as well.
5) Apply the load at the required positions. Re-center the galvanometer needle by using the big black knob (not the balance knob). The display should change as this knob is being turned
6) Repeat for remaining points identified on beam.

Digital System

1) Set the Channel switch on the Switch and Balance Unit to the Open position.
2) Turn on computer and login with **Lab User Account**
3) From the desktop, click on **Benchlink**
4) **Open** configuration file and **download** to data acquisition unit.
5) Click on the green button titled **Scan and Log Data**

6) Click on the **Start/Stop** button, when the 34970A unit begins clicking, to begin the data collection process and record data for about 30 seconds (the 34970A unit scans automatically every 8 seconds). Record the initial resistance values for the channels (shown in **DATA** column in Benchlink software) as indicated on the data sheets

7) **Record** the changed resistance values after about 60 seconds of changing each load (shown in DATA column in Benchlink software).

8) Before noting the readings down verify that the data make sense (e.g. is the data following a trend that you expect given a particular set-up)

Cantilever Beam & Simply Supported Beam (2 different setups)

1) Apply the load at the required locations

2) **Record** the changed resistance values after about 60 seconds of changing each load (shown in DATA column in Benchlink software).

3) Repeat for remaining locations.

Tubular Beam

1) Apply a load to the lever arm as instructed by the assistant. Make sure the wedge is securely placed at the end of the tube or the wedge is not present

2) **Record** the changed resistance values after about 60 seconds of changing each load (shown in DATA column in Benchlink software).

Pressure Vessel (Consult with instructor for correct operation of the pressure vessel)

1) Record the initial resistance values (shown in **DATA** column in Benchlink software)

2) Close the air release valve (black knob) and open the inlet regulator valve (T-handle) until the correct amount of pressure is obtained

3) Record the resistance values (shown in **DATA** column in Benchlink software)

4) **Close (CCW) the air regulator valve** and open the outlet valve to empty the tank (this is very important to prevent plastic deformation of the vessel due to sustained high pressure exposure).

Nomenclature

b	=	thickness	R	=	resistance or radius
E	=	modulus of elasticity	t	=	thickness
F	=	force	x	=	distance parallel to beam
G	=	shear modulus	y	=	distance from neutral axis
I	=	moment of inertia	ε	=	strain
K	=	gage factor	ρ	=	radius of curvature (of neutral axis)
L	=	length of arm	σ	=	stress
M	=	moment			
P	=	pressure			

Subscripts

1-4 = resistor number
0 = unstrained
i = inside
o = outside

References

1) Figliola, R. S. and Beasly, D. E., Theory and Design for Mechanical Measurements, John Wiley and Sons, New York, 1991.
2) Popov, Introduction to Solid Mechanics, Prentice Hall, 1968.
3) Shigley, J. E. and Mischke, C. R., Mechanical Engineering Design, 5th ed., McGraw-Hill Book Co., 1989.
4) Gere, J. M. and Timoshenko, S. P., Mechanics of Materials, 2nd ed., Wadsworth, Inc., 1984.

Data Sheets

Cantilever Beam <u>Analog</u> – Material _____ - Channel #7

Load F= _____ lb or _____ Kg   Gage Factor = _____

Distance from Gage to Load (approximately) (record actual distance) (in.)	Measured μstrain	Theoretically Calculated μstrain	% Difference
14			
12			
10			
8			
6			
4			

Maximum percent difference = _____   %, (between calculated and measured strain)

Student Names:

Date:

Instructor's Initials

Cantilever Beam <u>Digital</u> – Material _____- Channel 101

Load F=_____ lb or _____ Kg   Gage Factor =

<u>Strain Gage Initial Resistance</u>

Manufacturer =   Ohms   Measured using DAQ system =   Ohms

NOTE: Record both the resistance of the strain gage and the initial resistance measured without any load

Distance from Gage to Load (in.)	Measured Resistance Loaded (Ohms)	Experimentally Calculated μstrain (based on SG initial resistance)	Experimentally Calculated μstrain (based on measured resistance)	Theoretically Calculated μstrain	% Difference (SG initial resistance)	% Difference (measured resistance)

Maximum percent difference (based on SG resistance) =   %, (between calculated and measured strain)

Maximum percent difference (based on read resistance) =   % (between calculated and measured strain)

Student Names:

Date:

Instructor's Initials

Strain Measurement   3

Cantilever Beam Simply Supported <u>Digital</u> – Material _____ - Channel 101

Load F=_____ lb or _____ Kg   Gage Factor =

<u>Strain Gage Initial Resistance</u>

Manufacturer =   Ohms   Measured using DAQ system =   Ohms

NOTE: Record both the resistance of the strain gage and the initial resistance measured without any load

Distance from Gage to Load (in.)	Measured Resistance Loaded (Ohms)	Experimentally Calculated μstrain (based on SG initial resistance)	Experimentally Calculated μstrain (based on measured resistance)	Theoretically Calculated μstrain	% Difference (SG initial resistance)	% Difference (measured resistance)

Maximum percent difference (based on SG resistance) =   %, (between calculated and measured strain)

Maximum percent difference (based on read resistance) =   % (between calculated and measured strain)

Student Names:

Date:

Instructor's Initials

Torsion Tubular Beam with Wedge <u>Digital</u> – Material _____

Load F=_____ lb or _____ Kg   Gage Factor =

<u>Strain Gage Initial Resistance</u>

Manufacturer =   Ohms   Measured using DAQ system =   Ohms

NOTE: Record both the resistance of the strain gage and the initial resistance measured without any load

Channel	Initial – Non loaded Measured Resistance (Ohms)	Loaded Measured Resistance (Ohms)	Experimentally Calculated μstrain (based on measured resistance)	Theoretically Calculated μstrain	% Difference (measured resistance)
113					
114					
115					

Maximum percent difference (based on read resistance) =   % (between calculated and measured strain)

Student Names:

Date:

Instructor's Initials

Torsion Tubular Beam <u>without</u> Wedge <u>Digital</u> – Material _____

Load F=_____ lb or _____ Kg   Gage Factor =

<u>Strain Gage Initial Resistance</u>

Manufacturer =   Ohms   Measured using DAQ system =   Ohms

NOTE: Record both the resistance of the strain gage and the initial resistance read without any load

Channel	Initial – Non loaded Measured Resistance (Ohms)	Loaded Measured Resistance (Ohms)	Experimentally Calculated μstrain (based on measured resistance)	Theoretically Calculated μstrain	% Difference (measured resistance)
113					
114					
115					

Maximum percent difference (based on read resistance) =   % (between calculated and measured strain)

Student Names:

Date:

Instructor's Initials

Spherical Pressure Vessel <u>Digital</u> – Material _____

Pressure P=_____ psi or _____ KPa (between 70 to 80 psi)

Gage Factor = 2.09 ± 0.5% for all strain gages

Channel	Initial – Non pressurized Measured Resistance (Ohms)	Pressurized Measured Resistance (Ohms)	Experimentally Calculated μstrain	Theoretically Calculated μstrain	% Difference (measured resistance)
107					
108					
111					

Maximum percent difference (based on read resistance) = _____ % (between calculated and measured strain)

Student Names:

Date:

Instructor's Initials