Martine Ceberio
Organizer

Constraint Programming and Decision Making Workshop

CoProD'11

The University of Texas at El Paso
March 17th, 2011
Booklet of Abstracts

CoProD'11: 4th International Workshop on Constraint Programming and Decision Making
March 17, 2011 at the University of Texas at El Paso
Union Building
Acacia room (first floor)

8:30 am	Registration and breakfast
8:45 am	Opening of CoProD'11 and welcome by Dr. Ann Gates, UTEP's Associate Vice President for Research
8:50 am – 9:40 am	Constraint-based Assembling of Protein Fragments *by Enrico Pontelli, NMSU, Las Cruces, NM, and Alessandro Dal Palu, University of Parma, Italy*
9:40 am – 10:30 am	Extended BDI-based Model for Human Decision-Making and Social Behavior: Various Applications *by Young Jun Son, University of Arizona, Tucson, AZ*
10:30 am – 10:50 am	Coffee break
10:45 am – 10:55 am	Address to CoProD'11 participants by Dr. Junius Gonzales, UTEP's Provost
10:55 am – 11:45 am	Mixed Variable Surrogate Models for Engineering Analysis *by Patty Hough, Sandia National Laboratories, Livermore, CA*
11:45 am – 12:10 pm	Adding Constraints: A (Seemingly Counterintuitive but) Useful Heuristic in Solving Difficult Problems *by Olga Kosheleva, Martine Ceberio, and Vladik Kreinovich, CS, UTEP*
12:10 pm – 12:30 pm	Finite Difference Equations with the Interval Parameters *by Andrzej Pownuk, Math, UTEP*
12:30 pm – 2:10 pm	Lunch (Andesite room)
2:10 pm – 3:00 pm	Parallel Traversal of Spatio-Temporal Graphs in the Shortest Time under Constraints *by Xiaobai Sun, Duke University, NC*
3:00 pm – 3:50 pm	Predicting the impact of non synonymous Single Nucleotide Polymorphisms on protein function *by Francois Modave, Texas Tech University Health Sciences Center, El Paso, TX*
3:50 pm – 4:15 pm	Under Physics-Motivated Constraints, Generally-Non-Algorithmic Computational Problems Become Algorithmically Solvable *by Vladik Kreinovich, CS, UTEP*
4:15 pm – 4:40 pm	Coffee break
4:40 pm – 5:20 pm	Optimal control applied to a discrete influenza model *by Leticia Velazquez, Math, UTEP*
5:20 pm – 5:45 pm	EEG Diagnosis and Medical Decision Making Using Wavelet Transforms Method and Fuzzy Logic *by Mohmmad A. Obeidat, Amjed Al Fahoum, and Ayman Mansour, Wayne State University, Detroit, MI, and Yarmouk University, Irbid, Jordan*
5:45 pm – 6:10 pm	Why Curvature in L-Curve: Combining Soft Constraints *by Uram Anibal Sosa Aguirre, Martine Ceberio, and Vladik Kreinovich, Math and CS, UTEP*
6:10 pm – 6:30 pm	Closing of CoProD'11: Discussions and survey

Sponsored by: NSF Hosted at: UTEP

CoProD'11 – List of talks
The University of Texas at El Paso
March 17, 2011

Invited talks

1. **Title:** Constraint-based Assembling of Protein Fragments
   Speakers: *Enrico Pontelli, Department of Computer Science, New Mexico State University, Las Cruces, NM, and Alessandro Dal Palú, Department of Mathematics, University of Parma, Italy*

2. **Title:** Extended BDI-based Model for Human Decision- Making and Social Behavior: Various Applications
   Speaker: *Young-Jun Son, Systems and Industrial Engineering, The University of Arizona, Tucson, AZ*

3. **Title:** Mixed Variable Surrogate Models for Engineering Analysis
   Speaker: *Patty Hough, Sandia National Laboratories, Livermore, CA*

4. **Title:** Parallel Traversal of Spatio-Temporal Graphs in the Shortest Time under Constraints
   Speaker: *Xiaobai Sun, Department of Computer Science, Duke University, NC*

5. **Title:** Predicting the impact of non synonymous Single Nucleotide Polymorphisms on protein function
   Speaker: *François Modave, Texas Tech University Health Sciences Center, El Paso, TX*

6. **Title:** Optimal control applied to a discrete influenza model
   Speaker: *Leticia Velazquez, Math Department, The University of Texas at El Paso*

Contributed abstracts

1. **Title:** Adding Constraints: A (Seemingly Counterintuitive but) Useful Heuristic in Solving Difficult Problems
   Authors: *Olga Kosheleva, Martine Ceberio, and Vladik Kreinovich, The University of Texas at El Paso.*

2. **Title:** Finite Difference Equations with the Interval Parameters
   Authors: *Andrzej Pownuk, The University of Texas at El Paso*

3. **Title:** Under Physics-Motivated Constraints, Generally-Non-Algorithmic Computational Problems Become Algorithmically Solvable
   Authors: *Vladik Kreinovich, The University of Texas at El Paso*

4. **Title:** EEG Diagnosis and Medical Decision Making Using Wavelet Transforms Method and Fuzzy Logic
   Authors: *Mohmmad A. Obeidat[1], Amjed Al Fahoum[2], and Ayman Mansour[1], [1] Wayne State University, Detroit, MI, and [2] Yarmouk University, Irbid, Jordan*

5. **Title:** Why Curvature in L-Curve: Combining Soft Constraints
   Authors: *Uram Anibal Sosa Aguirre, Martine Ceberio, and Vladik Kreinovich, The University of Texas at El Paso*

Constraint-based Assembling of Protein Fragments

Enrico Pontelli[1] and Alessandro Dal Palú[2]

[1]Department of Computer Science, New Mexico State University, Las Cruces, NM
[2]Department of Mathematics, University of Parma, Italy

In this presentation, we overview a recent approach that makes use of constraint programming techniques to tackle the problem of predicting the structure of a protein starting from its primary sequence. The approach relies on the construction of a database of fragments from the observations of known proteins, and the use of Constraint programming to assemble fragments in the 3-d space, minimizing an energy function. We present a preliminary design and some preliminary experimental results.

Extended BDI-based Model for Human Decision-Making and Social Behavior: Various Applications

Young-Jun Son

Systems and Industrial Engineering, The University of Arizona, Tucson, AZ 85721, USA
son@sie.arizona.edu

Abstract. An extended Belief-Desire-Intention (BDI) modeling framework is presented that the author has been developing for human decision-making and social behavior, effectively integrating engineering-, psychology-, and economics-based models. To mimic realistic human behaviors, attributes of the BDI framework are reverse-engineered from human-in-the-loop experiments conducted in the Cave Automatic Virtual Environment (CAVE). We will discuss various applications that the proposed modeling framework has been applied, such as 1) evacuation behaviors under a terrorist bomb attack, 2) pedestrian behaviors in the Chicago Loop area, 3) workforce assignment in a multi-organizational social network for community-based software development, 4) pedestrian behaviors in a shopping mall, 5) evacuation behaviors under fire in a factory, and 6) error detection and resolution by people in a complex manufacturing facility.

Keywords: BDI, human decision behavior, planning, Bayesian belief network.

1 Introduction

The goal of this extended abstract is to describe an extended Belief-Desire-Intention (BDI) modeling framework [1] and [2] (see Fig. 1(a)) that has been developed by the author for human decision-making and social behavior, effectively integrating engineering-, psychology-, and economics-based models. BDI [3] and [4] is a model of the human reasoning process, where a person's mental state is characterized by three major components: beliefs, desires, and intentions. Later, Zhao and Son [5] extended the decision-making module (corresponding to the intention component) of the original BDI model to include three detailed submodules: (1) a deliberator, (2) a real-time planner, and (3) a decision executor in the decision-making (intention) module, where this extension was necessary to accommodate both the decision-making and decision-planning functions in a unified framework. In addition, an emotional module containing a confidence index and time pressure also has been appended to represent these aspects of human psychology. The emotional module affects and is affected by the three other mental modules, that is, beliefs, desires, and decision making. While Zhao and Son [5] provided a conceptual extension of the BDI

model, Lee and Son [1] proposed actual algorithms and techniques that have been employed and further developed to realize submodules for the extended model.

The submodules of the extended BDI modeling framework [1] and [2] are based on a Bayesian Belief Network (BBN), Decision-Field-Theory (DFT), and a Probabilistic Depth-First Search (PDFS) technique, and a key novelty of the framework is its ability to represent both the human decisionmaking and decision-planning functions in a unified framework.

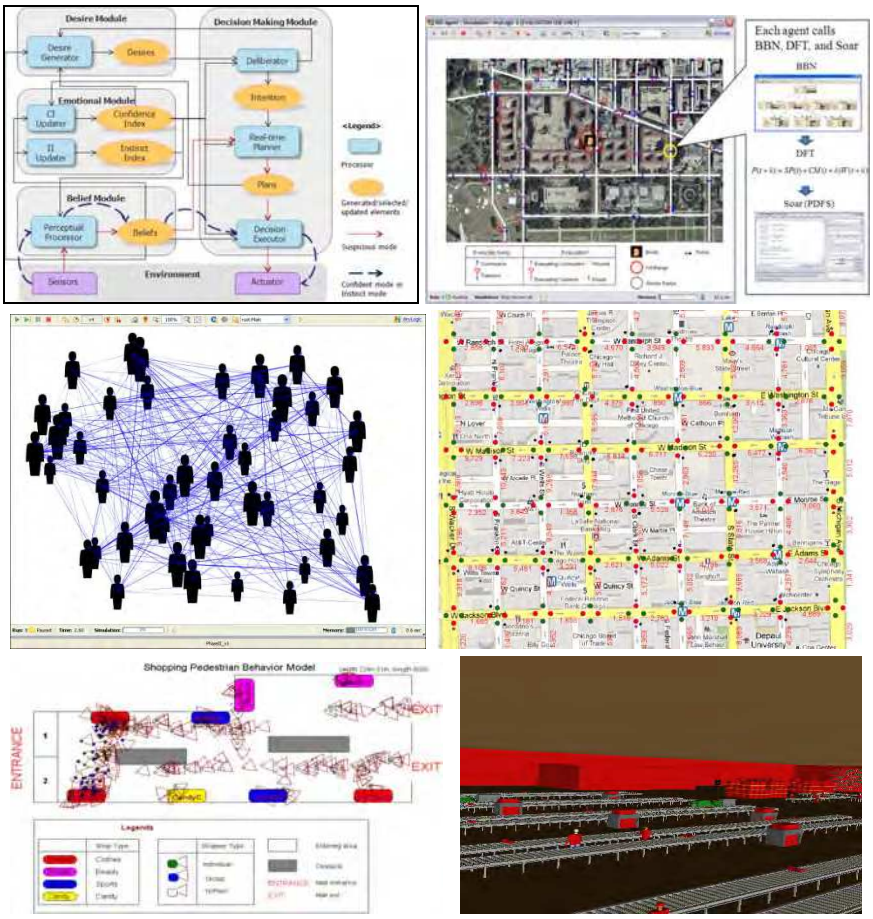

Fig. 1. (a) Components of extended BDI framework [1] and [2]; (b) Snapshot of emergency evacuation simulation [2]; (c) Snapshot of multi-organizational social network simulation [6]; (d) Snapshot of pedestrian behaviors in Chicago Loop area [7]; (e) Snapshot of pedestrian behaviors in a shopping mall [8]; (f) Snapshot of evacuation behaviors under fire in factory [9].

The proposed modeling framework has been sucessfully demonstrated for a human's behaviors under various applications, such as 1) evacuation behaviors under

a terrorist bomb attack (see Fig. 1(b)), 2) workforce assignment in a multi-organizational social network for community-based software development (see Fig. 1(c)), 3) pedestrian behaviors in the Chicago Loop area (see Fig. 1(d)), 4) pedestrian behaviors in a shopping mall (see Fig. 1(e)), 5) evacuation behaviors under fire in a factory (see Fig. 1(f)), and 6) error detection and resolution by people in a complex manufacturing facility.

To mimic realistic human behaviors, attributes of the BDI framework are reverse-engineered from human-in-the-loop experiments conducted in the Cave Automatic Virtual Environment (CAVE). For emergency evacuation scenario as an example [1] and [2], each subject is asked to evaluate the risk and the evacuation time of three available paths (i.e., right, forward, and left) depending on the various environmental observations (i.e., fire, smoke, police, and crowd) at each intersection. Also, each subject is asked to select one of the three available paths. The data collected on the relationship between the environment and the subject's evaluation was used to construct a BBN in the form of a conditional probability distribution. The constructed BBN infers 1) subjective evaluations for each attribute (e.g., risk and time) of each given option and 2) subjective weights of attention corresponding to each attribute, and the DFT calculates preference values of the options based on those matrices of evaluations and weights. Then, the simulated environment and agents conforming to the proposed BDI framework has been implemented. The constructed simulation has been used to test the impact of various factors (e.g., demographics, number of police officers, information sharing via speakers) on evacuation performance (e.g., average evacuation time, percentage of casualties). In the presentation, major modeling issues for each of the above mentioned applications will be addressed in detail.

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Mixed Variable Surrogate Models for Engineering Analysis

Patty Hough

Sandia National Laboratories, Livermore, CA

Modeling and simulation have become common tools in the design and analysis of engineered systems. Depending on the complexity of the system in question and the phenomena of interest, these simulations can take anywhere from a few minutes of wall clock time on a few CPUs to several days of wall clock time on hundreds of CPUs. Furthermore, exploration of the model parameter space (e.g., optimization or uncertainty quantification) is often required to complete the analysis. As a result, the computational requirements can easily exceed the allotted time and resources. One approach to alleviating the computational load is the use of surrogate models. Surrogate models are computationally inexpensive approximations that greatly enhance the tractability of simulation-based engineering analysis. They have been highly successful in solving continuous problems; however, an increasing number of problems include categorical or discrete variables, and surrogate modeling techniques for mixed variable domains are in their infancy and are largely untested. In this talk, I will give an example of how surrogates are used in engineering analysis and briefly describe three approaches for constructing surrogates in mixed variable domains: categorical regression, Treed Gaussian Processes (TGP), and Adaptive Component Selection and Smoothing Operator (ACOSSO) splines. I will describe our efforts to evaluate and compare these approaches, including the principles and metrics we used and the characteristics of the test functions we considered. Finally, I will present our numerical results and discuss our observations regarding the merits of each approach.

Parallel Traversal of Spatio-Temporal Graphs in the Shortest Time under Constraints

Xiaobai Sun

Department of Computer Science
Duke University, NC

We present a new approach and its analysis for parallel traversal of spatio-temporal graphs in the shortest time under constraints in hardware resource and software support. Many large-scale computations in scientific or engineering studies can be cast as that of traversing large graphs that capture the relationships, in space and time, between the computation nodes and often have irregular structures. We introduce and consider as a model problem the parallel computation of the celebrated fast multipole method (FMM). There are persistent efforts in the last two decades on parallel FMM for increasingly large computation problems. Recently, thread programming on existing and emerging computers with global address space provides and promises new and friendly means to exploit the complex spatio-temporal relationships. The new approach has unleashed, on multi-core processors in particular, substantial and previously untapped potential in Parallelizing the FMM, especially when the graph is highly irregular as a result of adaption to the sample sparsity, which had remained until now a major challenge. We present also experimental results.

Predicting the impact of non synonymous Single Nucleotide Polymorphisms on protein function

François Modave

Texas Tech University Health Sciences Center
El Paso, TX

A non-synonymous single nucleotide polymorphism (SNP) is a mutation of the genetic code that produces a different amino acid. It can be either a missense, in which case, a polypeptide with a changed amino acid is produced, or nonsense, where the mutation leads to an early stop codon. Numerous mutations result in no (obvious) phenotype change. However, the 122,000 SNPs in the human SNP database represent more than 50% of the mutations involved in human diseases. For instance, sickle-cell anemia is the result of a simple A to T mutation on the beta-globin gene. In the event of a homozygous mutation, life expectancy is significantly reduced, unlike in the heterozygous case.

Therefore, being able to predict which mutations are innocuous and which mutations are dangerous or deleterious is critical. In this talk, we will present two well-known algorithms and softwares (SIFT and PolyPhen2) that are used to predict the effect of SNPs on protein function, and will discuss their limitations, as well as some of the issues associated with bioinformatics softwares in general. We will then discuss research directions to develop reliable methods to accurately predict how SNPS affect protein function.

Optimal control applied to a discrete influenza model

Invited speaker: Leticia Velazquez
List of all authors: Leticia Velazquez, Paula A. Gonzalez-Parra, Sunmi Lee, Carlos Castillo-Chavez

Math Department
The University of Texas at El Paso

Discrete time Susceptible - Asymptomatic - Infectious - Treated - Recovered (SAITR) model is introduced in the context of influenza transmission. We evaluate the potential effect of control measures such as social distancing and antiviral treatment on the dynamics of a single outbreak.

Optimal control theory is applied to identify the best way of reducing morbidity and mortality at a minimal cost. The problem is solved by using a discrete version of Pontryagin's maximum principle. Numerical results show that dual strategies have stronger impact in the reduction of the final epidemic size.

We present some preliminar results by using interior points methods as another strategy to solve the problem.

Adding Constraints – A (Seemingly Counterintuitive but) Useful Heuristic in Solving Difficult Problems

Olga Kosheleva, Martine Ceberio, and Vladik Kreinovich

University of Texas at El Paso
El Paso, TX 79968, USA
{olgak,mceberio,vladik}@utep.edu

Abstract. Intuitively, the more constraints we impose on a problem, the more difficult it is to solve it. However, in practice, difficult-to-solve problems sometimes get solved when we impose additional constraints and thus, make the problems seemingly more complex. In this methodological paper, we explain this seemingly counter-intuitive phenomenon, and we show that, dues to this explanation, additional constraints can serve as a useful heuristic in solving difficult problems.

Keywords: constraints, algorithmic problems, heuristics

Commonsense intuition: the more constraints, the more difficult the problem. Intuitively, the more constraints we impose on a problem, the more difficult it is to solve it.

For example, if a university has a vacant position of a lecturer in Computer Science Department, and we want to hire a person with a PhD in Computer Science to teach the corresponding classes, then this hiring is a reasonably easy task. However, once we impose constraints: that the person has several years of teaching experience at similar schools and has good evaluations to show for this experience, that this person's research is in the area close to the classes that he or she needs to teach, etc., then hiring becomes a more and more complicated task.

If a person coming to a conference is looking for a hotel to stay, this is usually an easy problem to solve. But once you start adding constraints on how far this hotel is from the conference site, how expensive it is, how noisy it is, etc., the problems becomes difficult to solve.

Similarly, in numerical computations, unconstrained optimization problems are usually reasonably straightforward to solve, but once we add constraints, the problems often become much more difficult.

Sometimes constraints help: a seemingly counterintuitive phenomenon. In practice, difficult-to-solve problems sometimes get solved when we impose additional constraints and thus, make the problems seemingly more complex.

Sometimes this easiness to solve is easy to explain. For example, when a traveler prefers a certain hotel chain, and make this chain's brand name a constraint,

then making reservations in a small town is usually not a difficult problem to solve, because in this town, there is usually only one hotel from this chain.

However, in other cases, the resulting easiness-to-solve is not so easy to explain.

Many such examples come from mathematicians solving practical problems. For example, in application problems, mathematicians often aim for an optimal control or an optimal design. To a practitioner, this desire for the exact optimum may seem like a waste of time. Yes, it is desirable to find an engineering design with the smallest cost under the given constraints – or, vice versa, with the best performance under the given cost constraints – but since we can predict the actual consequences of each design only approximately, wasting time to exactly optimize the approximately optimize the approximately known function does not seem to make sense. If we only know the objective function $f(x)$ with accuracy $\varepsilon > 0$ (e.g., 0.1), then once we are within ε of the maximum, we can as well stop.

In some cases, it is sufficient to simply satisfy some constraint $f(x) \geq f_0$ for some value f_0. However, from the algorithmic viewpoint, often, the best way to solve this problem is to find the maximum of the function $f(x)$ on a given domain – by equating partial derivatives of $f(x)$ to 0. If there is a value x for which $f(x) \geq f_0$, then definitely $\max\limits_{y} f(y) \geq f_0$, so the place x where the function $f(y)$ attains its maximum satisfies the desired constraint. In other words, by imposing an additional constraint – that not only $f(x) \geq f_0$, but also that $f(x) = \max\limits_{y} f(y)$ – we make the problem easier to solve.

In theoretical mathematics, a challenging hypothesis often becomes proven when instead of simply looking for its proof, we look for proofs that can be applied to other cases as well – in other words, when we apply an additional constraint of generalizability; see, e.g., [16] and references therein.

Similarly, interesting results about a physical system become proven in the realm of rigorous mathematics, while, due to the approximate character of the model, arguments on the physical level of rigor would be (and often are) sufficient.

In engineering and science, often, problems get solved when someone starts looking not just for a solution but for a solution that satisfies additional constraints of symmetry, beauty, etc. – or when a physicist looks for a physical theory that fits his philosophical view of the world; a large number of examples how the search for a beautiful solution helped many famous mathematicians and physicists – including Bolzmann and Einstein – are described in [8].

In software design, at first glance, additional constraints imposed by software engineering – like the need to have comments, the need to have simple modules, etc. – seem to make a problem more complicated, but in reality, complex designs often become possible only after all these constraints are imposed.

This phenomenon extends to informal problems as well. For example, in art, many great objects have been designed within strict requirements on shape, form, etc. – under the constraints of a specific reasonable regulated style of music, ballet, poetry, painting, while free-form art while seemingly simpler and less restrictive, does not always lead to more impressive art objects. Some people

find personal happiness when accepting well-regulated life rules – e.g., within a traditional religious community – while they could not find personal happiness in their earlier freer life.

How can we explain this seemingly counter-intuitive phenomenon?

Analysis of the problem. By definition, when we impose an additional constraint, this means that some alternatives which were originally solutions to the problem, stop being such solutions – since we impose extra constraints, constraints that are not always satisfied by all original solutions.

Thus, the effect of adding a constraint is that the number of solution decreases. At the extreme, when we have added the largest possible number of constraints, we get a unique solution.

It turns out that this indeed explains why adding constraints can make the problems easier.

Related known results: the fewer solutions, the easier to solve the problem. Many numerical problems are, in general, algorithmically undecidable: for example, no algorithm can always find a solution to an algorithmically defined system of equation or find a location of the maximum of an algorithmically defined function; see, e.g., [1, 2, 4–6, 17, 18, 22].

The proofs of most algorithmic non-computability results essentially use functions which have several maxima and/or equations which have several solutions. It turned out that this is not an accident: uniqueness actually implies algorithmic computability. Such a result was first proven in [19], where an algorithm was designed that inputs a constructive function of one or several real variables on a bounded set that attains its maximum on this set at exactly one point – and computes this global maximum point. In [20], this result was to constructive functions on general constructive compact spaces.

In [12, 14], this result was applied to design many algorithms: from optimal approximation of functions to designing a convex body from its metric to constructive a shortest path in a curved space to designing a Riemannian space most tightly enclosing unit spheres in a given Finsler space [7]. Several efficient algorithms based on uniqueness have been described in [9–11].

On the other hand, it was proven that a general algorithm is not possible for functions that have exactly two global maxima or systems that have exactly two solutions; see, e.g., [12–15, 17].

Moreover, there are results showing that for every m, problems with exactly m solutions are, in general, more computationally difficult than problems with $m - 1$ solutions; see, e.g., [21].

Resulting recommendation. The above discussion leads to the following seemingly counter-intuitive recommendation: If a problem turns out to be too complex to solve, maybe a good heuristic is to add constraints and make it more complex.

For example, if the problem that we have difficulty solving is an applied mathematical problem, based on an approximate description of reality, maybe a good idea is not to simplify this problem but rather to make it more realistic.

This recommendation may sound counter-intuitive, but applied mathematicians know that often, learning more about the physical or engineering problem helps to solve it.

This can also be applied to education. If students have a hard time solving a class of problems, maybe a good idea is not to make these problems easier, but to make them more complex. Again, at first glance, this recommendation may sound counter-intuitive, but in pedagogy, it is a known fact: if a school is failing, the solution is usually not to make classes easier – this will lead to a further decline in knowledge. Anecdotal evidence shows that a turnaround happens when a new teacher starts giving students more complex more challenging problems – and this boosts their knowledge.

This recommendation is in line with a general American idea – that to be satisfying, the job, among other things, must be a challenge.

Caution. Of course, it is important not to introduce so many constraints that the problem simply stops having solutions at all. Since it is difficult to guess which level of constraints will lead to inconsistency, it may be a good idea to simultaneously several different versions of the original problem, with different number of constraints added – this way, we will hopefully be able to successfully solve one of them.

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Finite Difference Equations with the Interval Parameters

Andrzej Pownuk

Math Department
The University of Texas at El Paso

The Finite Difference Method is one of the most general approach for solution of partial differential equations (PDE). In many situations parameters of the equations are uncertain. There are many definitions of the solution set. In this presentation the united solution set will be applied. The interval solution will be calculated by using adaptive approximation method. Both explicate and implicate finite difference method will be discussed. In order to speed up the calculations special domain specific language (DSL) will be applied. Several numerical examples and applications to structural mechanics will be presented.

Under Physics-Motivated Constraints, Generally-Non-Algorithmic Computational Problems Become Algorithmically Solvable

Vladik Kreinovich

Department of Computer Science
University of Texas at El Paso
El Paso, TX 79968, USA
vladik@utep.edu
http://www.cs.utep.edu/vladik

Abstract. It is well known that many computational problems are, in general, not algorithmically solvable: e.g., it is not possible to algorithmically decide whether two computable real numbers are equal, and it is not possible to compute the roots of a computable function. We propose to constraint such operations to certain "sets of typical elements" or "sets of random elements".

In our previous papers, we proposed (and analyzed) physics-motivated definitions for these notions. In short, a set T is a *set of typical elements* if for every definable sequences of sets A_n with $A_n \supseteq A_{n+1}$ and $\bigcap\limits_n A_n = \emptyset$, there exists an N for which $A_N \cap T = \emptyset$; the definition of a *set of random elements* with respect to a probability measure P is similar, with the condition $\bigcap\limits_n A_n = \emptyset$ replaced by a more general condition $\lim\limits_n P(A_n) = 0$.

In this paper, we show that if we restrict computations to such typical or random elements, then problems which are non-computable in the general case – like comparing real numbers or finding the roots of a computable function – become computable.

Keywords: constraints, computable problems, random elements, typical elements

Physically meaningful computations with real numbers: a brief reminder. In practice, many quantities such as weight, speed, etc., are characterized by real numbers. To get information about the corresponding value x, we perform measurements. Measurements are never absolute accurate. As a result of each measurement, we get a measurement result $\widetilde{x}$; for each measurement, we usually also know the upper bound Δ on the (absolute value of) the measurement error $\Delta x \stackrel{\text{def}}{=} \widetilde{x} - x$: $|x - \widetilde{x}| \le \Delta$.

To fully characterize a value x, we must measure it with a higher and higher accuracy. As a result, when we perform measurements with accuracy 2^{-n} with $n = 0, 1, \ldots$, we get a sequence of rational numbers r_n for which $|x - r_n| \le 2^{-n}$.

From the algorithmic viewpoint, we can view this sequence as an oracle that, given an integer n, returns a rational number r_n. Such sequences represent real numbers in computable analysis; see, e.g., [9, 10].

First negative result. In computable analysis, several negative results are known. For example, it is known that no algorithm is possible that, given two numbers x and y, would check whether these numbers are equal or not.

Computable functions and relative negative results. Similarly, we can define a function $f(x)$ from real numbers to real numbers as a mapping that, given an integer n, a rational number x_m and its accuracy m, produces either a message that this information is insufficient, or a rational number y_n which is 2^{-n}-close to all the values $f(x)$ for $d(x, x_m) \le 2^{-m}$ – and for which, for every x and for each desired accuracy n, there is an m for which a rational number y_n is produced. We can also define a computable function $f(x_1, \ldots, x_k)$ of several real variables (and, even more generally, a function on a computable compact).

Several negative results are known about computable functions as well. For example,

- while there is an algorithm that, given a function $f(x)$ on a computable compact set K (e.g., on a box $[\underline{x}_1, \overline{x}_1] \times \ldots \times [\underline{x}_k, \overline{x}_k]$ in k-dimensional space), produces the values $\max\{f(x) : x \in K\}$,
- no algorithm is possible that would always return a point x at which this maximum is attained (and similarly, with minimum).

From the physicists' viewpoint, these negative results seem rather theoretical. From the purely mathematical viewpoint, if two quantities coincide up to 13 digits, they may still turn to be different: for example, they may be 1 and $1 + 10^{-100}$.

However, in the physics practice, if two quantities coincide up to a very high accuracy, it is a good indication that they are actually equal. This is how physical theories are confirmed: if an experimentally observed value of a quantity turned out to be very close to the value predicted based on a theory, this means that this theory is (triumphantly) true. This is, for example, how General Relativity has been confirmed.

This is how discoveries are often made: for example, when it turned out the speed of the waves described by Maxwell equations of electrodynamics is very close to the observed speed of light c, this led physicists to realize that light is formed of electromagnetic waves.

How physicists argue. A typical physicist argument is that while numbers like $1 + 10^{-100}$ (or $c \cdot (1 + 10^{-100})$) are, in principle, possible, they are abnormal (not typical).

When a physicist argues that second order terms like $a \cdot \Delta x^2$ of the Taylor expansion can be ignored in some approximate computations because Δx is small, the argument is that

– while abnormally high values of a (e.g., $a = 10^{40}$) are mathematically possible,
– typical (= not abnormal) values appearing in physical equations are usually of reasonable size.

How to formalize the physicist's intuition of typical (not abnormal). A formalization of this intuition was proposed and analyzed in [1–7]. Its main idea is as follows. To some physicist, all the values of a coefficient a above 10 are abnormal. To another one, who is more cautious, all the values above 10 000 are abnormal. Yet another physicist may have another threshold above which everything is abnormal. However, for every physicist, there is a value n such that all value above n are abnormal.

This argument can be generalized as a following property of the set T of all typical elements. Suppose that we have a monotonically decreasing sequence of sets $A_1 \supseteq A_2 \supseteq \ldots$ for which $\bigcap\limits_n A_n = \emptyset$ (in the above example, A_n is the set of all numbers $\geq n$). Then, there exists an integer N for which $T \cap A_N = \emptyset$.

We thus say that T is a *set of typical elements* if for every definable decreasing sequence $\{A_n\}$ for which $\bigcap\limits_n A_n = \emptyset$, there exists an N for which $T \cap A_N = \emptyset$.

Comment. Of course, to make this definition precise, we must restrict definability to a *subset* of properties, so that the resulting notion of definability will be defined in ZFC itself (or in whatever language we use); for details, see, e.g., [3].

Relation to randomness. The above notion of typicality is related to the randomness. Indeed, a usual definition of a random sequence (see, e.g., [8]) is based on the idea that a sequence is random if it satisfies all the probability laws – like the law of large numbers, the central limit theorem, etc. A probability law is then described as a definable property that is satisfied with probability 1, i.e., as a complement to a definable set S of probability measure 0 ($P(S) = 0$). Thus, we can say that a sequence is random if it does not belong to any definable set of measure 0. (If we use different languages to formalize the notion "definable", we get different versions of Kolmogorov-Martin-Löf randomness.)

Informally, this definition means that (definable) events with probability 0 cannot happen. In practice, physicists also assume that events with a *very small* probability cannot happen. It is not possible to formalize this idea by simply setting a threshold $p_0 > 0$ below which events are not possible – since then, for N for which $2^{-N} < p_0$, no sequence of N heads or tails would be possible at all. However, we know that for each monotonic sequence of properties A_n with $\lim p(A_n) = 0$ (e.g., $A_n =$ "we can get first n heads"), there exists an N above which a truly random sequence cannot belong to A_N. In [1–7], we thus propose to describe a set R as a set of random elements if it satisfies the following property: for every definable decreasing sequence $\{A_n\}$ for which $\lim P(A_n) = 0$, there exists an N for which $R \cap A_N = \emptyset$.

It turns out that properties of T and R are related:

– every set of random elements is also a set of typical elements, and

– for every set of typical elements T, the difference $T - R_K$, where R_K is the set of the elements random in the usual Komogorov-Martin-Löf sense, is a set of random elements [2].

Physically interesting consequences of these definitions. These definitions have useful consequences [1–7].

For example, when the universal set X is a metric space, both sets T and R are *pre-compact* – with the consequence that all inverse problems become well-defined: for any 1-1 continuous function $f : X \to X$, the restriction of the inverse function to T is also continuous. This means that, in contrast to ill-defined problem, if we perform measurements accurately enough, we can reconstruct the state of the system with any desired accuracy.

Another example is a justification of physical induction: crudely speaking, there exists an N such that if for a typical sequence, a property is satisfied in the first N experiments, then it is satisfied always.

New results: when we restrict ourselves to typical elements, algorithms become possible. In this paper, we analyze the computability consequences of the above definitions. Specifically, we show that most negative results of computability analysis disappear if we restrict ourselves to typical elements.

For example, for every set of typical pairs of real numbers $T \subseteq \mathbb{IR}^2$, there exists an algorithm, that, given real numbers $(x, y) \in T$, decides whether $x = y$ or not. To prove it, consider a decreasing sequence of definable sets

$$A_n = \{(x, y) : 0 < d(x, y) < 2^{-n}\}.$$

By definition of T, there exists an N such that $A_N \cap T = \emptyset$. Thus, if we compute $d(x, y)$ with accuracy $2^{-(N+1)}$ and get a value $< 2^{-N}$, this means that $x = y$ – otherwise $x \neq y$.

Similar (but somewhat more complex) arguments lead to

– an algorithm that, given a typical function $f(x)$ on a computable compact K, computes a value x at which $f(x)$ attains its maximum,
– an algorithm that, given a typical function $f(x)$ on a computable compact K that attains a 0 value somewhere on K, computes a value x at which $f(x) = 0$,
– etc.

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EEG Diagnosis and Medical Decision Making Using Wavelet Transforms Method and Fuzzy Logic

Mohmmad A. Obeidat [1], Amjed Al Fahoum [2], and Ayman Mansour [1]

1 Department of Electrical Engineering ,Wayne State University, Detroit, MI-48202, USA
maobaidat76@yahoo.com , manour@wayne.edu
2 Department of systems engineering and medical bioinformatics ,Yarmouk University, Irbid, Jordan

Abstract.

The recognition of an abnormal activity of the brain functionality is a vital issue. To determine the type of the abnormal activity either a brain image or brain signal are usually considered. Imaging localizes the defect within the brain area and relates this area with some body functionalities. However, some functions may be disturbed without affecting the brain. In this case, imaging may not provide the symptoms of the problem. A cheaper yet efficient approach that can be utilized to detect abnormal activity is the measurement and analysis of the electroencephalogram (EEG) signals. The main goal of this work is to come up with a new method to facilitate the classification of the abnormal and disorder activities within the brain directly using EEG signal processing, which makes it possible to be applied in an on-line monitoring system.

Classification of EEG abnormality may be approached using different analysis methods and classifiers. Among these methods are wavelet decomposition, Fourier analysis, and phase space. Four different approaches are proposed in this work to classify EEG abnormal activity. These approaches depend on transforming the signal into another domain to easily extract significant features that allow proper

classification. The main goal is to yield an efficient classification yet using a simple, fast, and reliable classifier. The first approach is based on selecting features using wavelet coefficients only. The second approach is a combination between wavelet and phase space. The third approach is based on phase space reconstruction only. Finally the fourth approach uses features extracted from Fourier domain using fuzzy logic. All the proposed approaches showed good results and were able to detect the normal EEG with 96 % accuracy. However, abnormal activity detection accuracy varied among the different proposed classifier. The results obtained using these classifiers are comparable to those obtained from recent published works.

The data used in this research is long and it is taken from multichannels of the brain. The first technique ,wavelet transform, showed very high accuracy reached to 96%. The wavelet transform used in this technique has many advantages over other transforms, like Fourier transform, such that it can preserve both frequency and timing information and it can simultaneously extract both low frequency and high frequency signal with different frequency resolutions.

The second technique, reconstruction phase space plot of wavelet based coefficients, showed high frequency 90%. In the third technique, phase space of original signal without transformation, detecting the seizure is fast and simple, but the frequency ranges can't be detected, the accuracy still 90%. The fourth technique , Fourier transform and fuzzy logic, showed accuracy of distinguishing between normal and abnormal of 95%, the classifier depends on the percentages of points that represents the phase of the system and the direction of the points from the center. From the above, this research used two main theory representations for non-linear analysis summarizes in wavelet transform and phase space, these two categories are

highly used in recent researches in studying the nonstationary signals like EEG signal. The phase space is used to represent the state space of the system in time domain, while the wavelet transform helps in finding the relationship between time and frequency at any level. The results in this research are taken using Mat Lab software.

Keywords: Wavelet, Phase space, EEG, Epilepsy, Spikes.

Why Curvature in L-Curve: Combining Soft Constraints

Uram Anibal Sosa Aguirre, Martine Ceberio, and Vladik Kreinovich

Computational Sciences Program and Department of Computer Science
University of Texas, El Paso, TX 79968, USA
usosaaguirre@miners.utep.edu, {mceberio,vladik}@utep.edu

Abstract. In solving inverse problems, one of the successful methods of determining the appropriate value of the regularization parameter is the *L-curve method* of combining the corresponding soft constraints, when we plot the curve describing the dependence of the logarithm x of the mean square difference on the logarithm y of the mean square non-smoothness, and select a point on this curve at which the curvature is the largest. This method is empirically successful, but from the theoretical viewpoint, it is not clear why we should use curvature and not some other criterion. In this paper, we show that reasonable scale-invariance requirements lead to curvature and its generalizations.

Keywords: soft constraints, inverse problems, regularization, L-curve, curvature

1 Formulation of the Problem

Inverse problem: a brief reminder. In science and engineering, we are interested in the state of the world, i.e., in the values of different physical quantities that characterize this state. Some of these quantities we can directly measure, but many quantities are difficult or even impossible to measure directly.

For example, in geophysics, we are interested in the density and other properties of the material at different depths and different locations. In principle, it is possible to drill a borehole and directly measure these properties, but this is a very expensive procedure, and for larger depths, the drilling is not possible at all. To find the values of such difficult-to-measure quantities $q = (q_1, \ldots, q_n)$, we measure the values of the auxiliary quantities $a = (a_1, \ldots, a_m)$ that are related to q_i by a known dependence $a_i = f_i(q_1, \ldots, q_n)$, and then reconstruct the values q_j from these measurement results.

In the idealized situation when measurements are absolutely accurate, we can then reconstruct the desired values q_j from the system of m equations $a_1 = f_1(q_1, \ldots, q_n)$, $\ldots$, $a_m = f_m(q_1, \ldots, q_n)$. In real life, measurements are never 100% accurate, so the measured values a_i are only approximately equal to $f_i(q_1, \ldots, q_n)$. Usually, it is assumed that the measurement errors $a_i - f_i(q_1, \ldots, q_n)$ are independent normally distributed random variables with 0 means and the same variance; see, e.g. [3]. In this case, the constraint that the

values q_j are consistent with the observations a_i can be described as a constraint $s \le s_0$ on the sum $s \stackrel{\text{def}}{=} \sum_{i=1}^{m} (a_i - f_i(q_1, \ldots, q_n))^2$. The value s_0 depends on the confidence level: the larger s_0, the more confident we are that this constraint will be satisfied. For each value x_0, the constraint $x \le x_0$ is a *soft constraint*: there is a certain probability that this constraint will be violated.

Often, this constraint is described in a logarithmic scale, as $x \le x_0$, where $x \stackrel{\text{def}}{=} \ln(s)$.

Regularization: how to take into account additional constraints. Often, there are additional constraints on q_j. Usually, the values q_j are more regular than randomly selected values. Methods for taking these additional regularity constraints into account are known as *regularization methods*; see, e.g., [4].

For example, in geophysics, the density values at nearby locations are usually close to each other. In other words, the differences $q_j - q_{j'}$ corresponding to nearby locations should be small.

This constraint can also be described in statistical terms: that there is a prior distribution on the set of all the tuples, in which all the differences $q_j - q_{j'}$ are independent and normally distributed with 0 mean and the same variance. In this case, the constraint that the values q_j are consistent with this prior distribution can be also described as a constraint $t \le t_0$ on the sum $t \stackrel{\text{def}}{=} \sum_{(j,j')} (q_j - q_{j'})^2$. This constraint is also often described in a logarithmic space, as $y \le y_0$, where $y \stackrel{\text{def}}{=} \ln(t)$.

We can combine the two constraints, e.g., by using the Bayesian statistics to combine the prior distribution (describing the regularity of the actual values) and the distribution corresponding to measurement uncertainty. For the resulting posterior distribution, the Maximum Likelihood method of determining the optimal values of the quantities q_j is then equivalent to minimizing the sum $s + \lambda \cdot t$, for some coefficient λ depending on the variance of the prior distribution.

There are also other more complex regularization techniques; see [4].

How to determine a regularization parameter. As we have mentioned, the actual value of the regularization parameter depends on the prior distribution and is, therefore, reasonably subjective. It is therefore desirable to find the value of this parameter based on the data.

For each value of the parameter λ, we can find the corresponding solution $q_j(\lambda)$, and, based on this solution, compute the values $x(\lambda)$ and $y(\lambda)$ of the quantities x and y. These two values represent a point on a plane. Points corresponding to different values λ form a curve. In these terms, the question of which value λ to choose can be reformulated as which point on the curve should we choose?

In practice, often, this curve has a clear turning point, a point that is distinct from others – as a point at which the curve "curves" the most. In such cases,

when we have an L-shaped curve, it is reasonable to select the turning point as the point corresponding to the solution. This idea often leads to a good solution; see, e.g., [1, 2].

In line with the above description, the desired point is selected as a point at which the absolute value $|C|$ of the curvature $C = \dfrac{x'' \cdot y' - y'' \cdot x'}{((x')^2 + (y')^2)^{3/2}}$ takes the largest possible value; here, as usual, x' denotes the derivative $\dfrac{dx}{d\lambda}$, and x'' denotes the second derivative of x with respect to the parameter λ.

Remaining open problem. Empirically, the method of selecting a point with the largest curvature works well. It is therefore desirable to come up with a theoretical justification for the use of curvature function – or at least for a class containing the curvature function.

What we do in this paper. We provide such a justification: specifically, we show that reasonable properties select a class of functions that include curvature.

2 Analysis of the Problem

Let us first analyze the invariance properties of curvature.

Scale-invariance. The numerical values of each quantity depend on the selection of a measuring unit. For example, if instead of meters, we use centimeters, then all numerical values get multiplied by 100. In general, if we select a new measuring unit which is c times smaller than the previous one, then all numerical values get multiplied by c.

If we change a measuring unit for a to a new one which is c_a time smaller, then the numerical values of a_i and $a_i - f_i(q_1, \ldots, q_n)$ get multiplied by c_a. As a result, the sum $s = \sum\limits_{i=1}^{n} (a_i - f_i(q_1, \ldots, q_n))^2$ gets multiplied by c_a^2, and the original value $x = \ln(s)$ changes to $x + \Delta_x$, where we denoted $\Delta_x \stackrel{\text{def}}{=} \ln(c_a^2)$.

Similarly, if we change a measuring unit for q to a new one which is c_q time smaller, then the numerical values of q_j and $q_j - q_{j'}$ get multiplied by c_q. As a result, the sum $t = \sum (q_j - q_{j'})^2$ gets multiplied by c_q^2, and the original value $y = \ln(t)$ changes to $y + \Delta_y$, where we denoted $\Delta_y \stackrel{\text{def}}{=} \ln(c_q^2)$.

Under these changes $x(\lambda) \to x(\lambda) + \Delta_x$ and $y(\lambda) \to y(\lambda) + \Delta_y$, the derivatives do not change – since Δ_x and Δ_y are constants – and thus, the curvature does not change. Thus, the curvature is invariant under these scale transformations.

Invariance under re-scaling of parameters. Instead of the original parameter λ, we can use a new parameter μ for which $\lambda = g(\mu)$. This re-scaling of a parameter does not change the curve itself and thus, does not change its curvature. So, the curvature is invariant under these scale transformations.

Our idea. Our main idea is to describe all the functions which are invariant with respect to both types of re-scalings.

3 Main Result

Definition. *By a* parameter selection criterion *(or simply* criterion*, for short), we mean a function $F(x, y, x', y', x'', y'')$ of six variables. We say that the parameter selection criterion $F(x, y, x', y', x'', y'')$ is:*

- scale-invariant *if for all possible values Δ_x and Δ_y, we have*

$$F(x + \Delta_x, y + \Delta_y, x', y', x'', y'') = F(x, y, x', y', x'', y'');$$

- invariant w.r.t. parameter re-scaling *if for every function $g(z)$ and for the functions $\widetilde{x}(\mu) = x(g(\mu))$ and $\widetilde{y}(\mu) = y(g(\mu))$, we have*

$$F(\widetilde{x}, \widetilde{y}, \widetilde{x}', \widetilde{y}', \widetilde{x}'', \widetilde{y}'') = F(x, y, x', y', x'', y'').$$

Notation. By $C(x, y, x', y', x'', y'')$, we denote the parameter selection criterion corresponding to curvature.

Comment. Once a criterion is selected, for each problem, we use the value λ for which the value $F(x(\lambda), y(\lambda), x'(\lambda), y'(\lambda), x''(\lambda), y''(\lambda))$ is the largest.

Main result. *A parameter selection criterion which is scale-invariant and invariant w.r.t. parameter re-scaling if and only if it has the form*

$$F(x, y, x', y', x'', y'') = f\left(C(x, y, x', y', x'', y''), \frac{x'}{y'}\right)$$

for some function $f(C, z)$.

Proof.

1°. For each tuple (x, y, x', y', x'', y''), by taking $\Delta_x = -x$ and $\Delta_y = -y$, we conclude that $F(x, y, x', y', x'', y'') = F(0, 0, x', y', x'', y'')$. Thus, we conclude that $F(x, y, x', y', x'', y'') = F_0(x', y', x'', y'')$, where we denoted $F_0(x', y', x'', y'') \stackrel{\text{def}}{=} F(0, 0, x', y', x'', y'')$, i.e., we conclude that the value of the parameter selection criterion does not depend on x and y at all.

In terms of the function F_0, invariance w.r.t. parameter re-scaling means that $F_0(\widetilde{x}', \widetilde{y}', \widetilde{x}'', \widetilde{y}'') = F_0(x', y', x'', y'')$.

2°. When we go from the original function $x(\lambda)$ to the new function $\widetilde{x}(\mu) = x(g(\mu))$, the chain rule for differentiation leads to $\widetilde{x}' = x' \cdot g'$ and thus, $\widetilde{x}'' = x'' \cdot (g')^2 + x' \cdot g''$. Similarly, $\widetilde{y}' = y' \cdot g'$ and $\widetilde{y}'' = y'' \cdot (g')^2 + y' \cdot g''$.

In particular, at the point where $g' = 1$, we have $\widetilde{x}' = x$, $\widetilde{x}'' = x'' + x' \cdot g''$, $\widetilde{y}' = y'$, and $\widetilde{y}'' = y'' + y' \cdot g''$, and thus, invariance w.r.t. parameter re-scaling means that $F_0(x', y', x'' + x' \cdot g'', y'' + y' \cdot g'') = F_0(x', y', x'', y'')$. This is true for

every possible values of g''. In particular, for $g'' = -\dfrac{y''}{y'}$, we have $y'' + y' \cdot g'' = 0$ and thus,

$$F_0(x', y', x'', y'') = F_0\left(x', y', x'' - x' \cdot \frac{y''}{y'}, 0\right).$$

Since

$$x'' - x' \cdot \frac{y''}{y'} = C \cdot \frac{((x')^2 + (y')^2)^{3/2}}{y'},$$

we thus conclude that

$$F_0(x', y', x'', y'') = h(C, x', y'),$$

where

$$h(C, x', y') \stackrel{\text{def}}{=} F_0\left(x', y', C \cdot \frac{((x')^2 + (y')^2)^{3/2}}{y'}, 0\right).$$

For the new function $h(C, x', y')$, since the curvature is invariant w.r.t. parameter re-scaling, invariance means that $h(C, \widetilde{x}', \widetilde{y}') = h(C, x', y')$. This means that

$$h(C, x', y') = h(C, x' \cdot g', y' \cdot g').$$

This is true for every possible values of g'. In particular, for $g' = \dfrac{1}{x'}$, we have $x' \cdot g' = 1$ and thus,

$$F(x, y, x', y', x'', y'') = F_0(x', y', x'', y'') = h(C, x', y') = h\left(C, 1, \frac{y'}{x'}\right),$$

i.e., $F(x, y, x', y', x'', y'') = f\left(C, \dfrac{y'}{x'}\right)$ for $f(C, z) \stackrel{\text{def}}{=} h(C, 1, z)$.

The statement is proven.

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CoProD
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Constraint programming techniques are important components of intelligent systems. They constitute a nice and easy way to represent many practical situations. They have been successfully applied to a number of fields, such as scheduling of air traffic, software engineering, networks security, chemistry. Despite their proven usefulness, they are still under-utilized in real-life projects. One reason might be the lack of effective communication of researchers with domain practitioners about constraints in general and their use in decision making in particular.

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