Exponential Notation, Expanded Form and Bases

Definitions

1. EXPONENTIAL NOTATION: will be used to do expanded form (below). When multiplying a number times itself many times, exponential notation is quicker.
   Ex1. $8*8*8*8*8 = 8^5$

2. BASE: the number which is being multiplied times itself.
   Ex1.(Revisited) In 8^5, 8 is the base.

3. POWER / EXPONENT: the number of times you multiply the base. It is the superscript in exponential notation.
   Ex1.(Revisited) In 8^5, 5 is the power, or the exponent.

4. HINDU-ARABIC / BASE 10: our usual way of writing numbers. In this system, we use 10 different symbols (0,1, 2, 3, 4, 5, 6, 7, 8, 9), and each digit has a PLACE VALUE that is a power of ten.
   Ex2.
   $$\begin{array}{ccccc} 4 & 0 & 3 & 6 & 7 \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 10^4 & 10^3 & 10^2 & 10^1 & 10^0 \end{array}$$

5. BASE X (x can be any number): another way of writing numbers. In this system, we use X different symbols (01,2,3, …, X-1), and each digit has a PLACE VALUE that is a power of X.
   Ex3. Base 5 uses the symbols 0, 1, 2, 3, 4, and the place values are powers of 5.
   $$\begin{array}{ccccc} 1 & 0 & 0 & 3 & 4_{\text{five}} \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 5^4 & 5^3 & 5^2 & 5^1 & 5^0 \end{array}$$

6. BASE 10-EXPANDED FORM: combines definitions 1. and 4.
   Ex2.(Revisited)
   $40367 = (4*10^4) + (0*10^3) + (3*10^2) + (6*10^1) + (7*10^0)$
   or
   $40367 = (4*10^4) + (3*10^2) + (6*10^1) + (7*10^0)$ (without zeros),

7. BASE X-EXPANDED FORM: combines definitions 1. and 5.
   Ex3.(Revisited) In Base 5,
   $10034_{\text{five}} = (1*5^4) + (0*5^3) + (0*5^2) + (3*5^1) + (4*5^0)$
   or
   $10034_{\text{five}} = (1*5^4) + (3*5^1) + (4*5^0)$ (without zeros).

__Exponential Notation__ is a type of short-hand. Instead of writing 8*8*8*8*8, it is quicker and shorter to use exponential notation:

$$8^5$$

__The base__ *__The exponent, or the power__*

RULES

1. Anything raised to the zero power equals one:
   $1^0 = 1$ $2^0 = 1$ $13^0 = 1$ $\text{anything}^0 = 1$
2. Anything raised to the one power equals itself:
   $1^1 = 1$ $5^1 = 5$ $23^1 = 23$ $\text{anything}^1 = \text{itself}$

Special Case.

Something funny happens with tens:

$$10^0 = 1$$
$$10^1 = 10$$
$$10^2 = 100$$
$$10^3 = 1000$$
$$10^4 = 10000$$
$$10^5 = 100000$$
$$\ldots$$

For powers of ten only, the exponent equals the number of zeros.

- *Calculator Note:* Find out where the exponent button is on your calculator (sometimes has the symbol ^), so that instead of typing “8x8x8x8x8” you can just type “8^5”

__Base Ten, or Hindu Arabic Numbers__ is the usual way we write numbers, using digits 0 thru 9. Each Hindu Arabic/Base Ten number has an expanded form, which is obtained by multiplying each digit times its place value, then adding them all up.

Example:

This digit has a *__place value__* of 10^0, or 1.

10^4 10^3 10^2 10^1

$$4\ 0\ 3\ 6\ 7 = \underbrace{(4 * 10^4) + (0 * 10^3) + (3 * 10^2) + (6 * 10^1) + (7 * 10^0)}_{\textit{Expanded Form}}$$

Or, we can also use a slightly shorter expanded form: $(4 * 10^4) + (3 * 10^2) + (6 * 10) + (7 * 1)$.

What’s different?

$10^1 = 10$ $10^0 = 1$

Omit $(0 * 10^3)$, since it equals zero.

Base Two, or Binary is a way of writing number using only 0's and 1's. Each Base Two number has a Base Two Expanded Form, obtained in a similar way as above, but now with powers of two instead of powers of ten.

Example:

This digit has a *place value* of 2^0, or 1.

$2^3 \quad 2^2 \quad 2^1$

$$1\ 0\ 1\ 1_{\text{two}} \quad = \quad \underbrace{(1 * 2^3) + (0 * 2^2) + (1 * 2^1) + (1 * 2^0)}_{\textit{Base Two Expanded Form}}$$

Or, the slightly shorter expanded form looks like $(1 * 2^3) + (1 * 2) + (1 * 1)$.

Base Five is a way of writing number using only 0 through 4. Each Base Five number has a Base Five Expanded Form, obtained in a similar way as above, but now with powers of five.

Example

This digit has a *place value* of 5^0, or 1.

$5^4 \ 5^3 \ 5^2 \ 5^1$

$$4\ 0\ 3\ 4\ 4_{\text{five}} \quad = \quad \underbrace{(4 * 5^4) + (3 * 5^2) + (4 * 5) + (4 * 1)}_{\textit{Base Five Expanded Form}}$$

Base Anything! Using similar ideas, we can do base 2, base 3, base 4, ……, base anything! Just change the place values.

Converting between Base 10 and Base X

TO Hindu Arabic/Base Ten

Example:
Convert the base eight number 27011_{eight} to Hindu Arabic/Base Ten.

1. Change to Base Eight Expanded Form: $= (2 * 8^4) + (7 * 8^3) + (1 * 8) + (1 * 1)$
2. Just multiply / add up part 1: $= (2 * 4096) + (7 * 512) + 8 + 1$
   $= 8192 + 3584 + 9$
   $= 11785$

In general, if you start with a number in base Y, and want to convert it to Hindu Arabic/Base Ten:

1. Change to Base Y Expanded Form.
2. Just multiply / add up what you found in part 1.

FROM Hindu Arabic/Base Ten

Example:
Convert the Hindu Arabic/Base Ten number 10 into base two/binary:

1. List powers of two until you reach 10:
   $2^0 = 1$ $\quad 2^2 = 4$
   $2^1 = 2$ $\quad 2^3 = 8$
   $\cancel{2^4 = 16}$ (is greater than 10. Too big, stop here!)
2. Start with the largest power from step 1, and start dividing:

$$2^3\overline{)10} = 8\overset{1}{\overline{)10}} \quad \text{remainder } 2$$

$$2^2\overline{)2} = 4\overset{0}{\overline{)2}} \quad \text{remainder } 2$$

$$2^1\overline{)2} = 2\overset{1}{\overline{)2}} \quad \text{remainder } 0$$

$$2^0\overline{)0} = 1\overset{0}{\overline{)0}} \quad \text{remainder } 0$$

Therefore $10 = 1010_{two}$ in base two.

Let's check our answer by converting 1010_{two} back into Hindu Arabic/Base Ten:
$1010_{two} = (1*2^3) + (1*2^1) = 8 + 2 = 10$ √

In general, if you start with a number in Hindu Arabic/Base Ten and you want to convert it to base Y:

1. List powers of Y until you reach the number.
2. Start with the largest power from step 1, and start dividing. Each time you divide, you obtain another digit of the base Y number.

Another example: Convert 6826 into base five:

2. List powers of five until you reach 6826:

$5^0 = 1$ $\quad$ $5^4 = 625$

$5^1 = 5$ $\quad$ $5^5 = 3125$

$5^2 = 25$ $\quad$ $\cancel{5^6 = 15625}$ (is greater than 6826. Too big, stop here!)

$5^3 = 125$

2. Start with the largest power from step 1, and start dividing:

$$5^5\overline{)6826} = 3125\overset{2}{\overline{)6826}} \quad \text{remainder } 576$$

$$5^4\overline{)576} = 625\overset{0}{\overline{)576}} \quad \text{remainder } 576$$

$$5^3\overline{)576} = 125\overset{4}{\overline{)576}} \quad \text{remainder } 76$$

$$5^2\overline{)76} = 25\overset{3}{\overline{)76}} \quad \text{remainder } 1$$

$$5^1\overline{)1} = 5\overset{0}{\overline{)1}} \quad \text{remainder } 1$$

$$5^0\overline{)1} = 1\overset{1}{\overline{)1}} \quad \text{remainder } 0$$

Therefore the Hindu Arabic/Base Ten number 6826, is 204301_{five} in base five.

Let's check our answer by converting 204301_{five} back into Hindu Arabic/Base Ten:

$204301_{\text{five}} = (2*5^5) + (4*5^3) + (3*5^2) + (1*5^0)$

$= (2*3125) + (4*125) + (3*25) + (1*1)$

$= 6250 + 500 + 75 + 1$

$= 6826$ √

Summary

in Base 10

Find digits of Base X-expanded form
by using division and remainders:
= . . . + $(?*X^2) + (?*X^1) + (?*X^0)$
Then write # in Base X: $??????_X$

You already know the digits of
Base X-expanded form.
Just add and multiply.

in Base X

279_{ten} in Base 10

Ex
Find digits of Base 5-expanded form.
$(?*5^3) + (?*5^2) + (?*5^1) + (?*5^0)$
Using division and remainders,

$279 \div 5^3 = \boxed{2}$ R 29
$29 \div 5^2 = \boxed{1}$ R 4
$4 \div 5^1 = \boxed{0}$ R 4
$4 \div 5^0 = \boxed{4}$

Therefore, expanded form in Base 5 is:
$(2*5^3) + (1*5^2) + (0*5^1) + (4*5^0)$
$= 2104_{five}$

You already know the digits of
Base 5-expanded form.
Just add and multiply.
$(2*5^3) + (1*5^2) + (0*5^1) + (4*5^0)$
$= 250 + 25 + 0 + 4$
$= 279_{ten}$

2104_{five} in Base 5