

PRINT NAME _____ (_____)
 Last Name, First Name MI (What you wish to be called)

ID # _____ EXAM DATE Thursday, May 3, 2012, 8am

I swear and/or affirm that all of the work presented on this exam is my own and that I have neither given nor received any help during the exam.

ST1	43			ST2	6
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SIGNATURE _____

DATE _____

INSTRUCTIONS Besides this cover page, there are 22 pages of questions and problems on this exam. **MAKE SURE YOU HAVE ALL THE PAGES.** If a page is missing, you will receive a grade of zero for that page. Read through the entire exam. If you cannot read anything, raise your hand and I will come to you. Place your I.D. on your desk during the exam. Your I.D., this exam, and a straight edge are all that you may have on your desk during the exam. **NO CALCULATORS!** If you need scratch paper, I will provide it. You may remove the staple if you wish. Print your name on all sheets. Pages 1-22 are Fill-in-the-Blank/Multiple Choice questions. Expect no part credit on these pages. For these questions write your answer in the blank provided. Next find your answer from the list of possible answers. Then write the corresponding letter or letters for your answer in the blank provided. Finally, circle this letter or letters. There are no free response pages on this exam. To insure credit on all pages, you should write complete solutions and explain your solutions fully and carefully. Your entire solution may be graded, not just your final answer. **SHOW YOUR WORK!** If your solution does not justify your answer, you may be given a grade of zero. Every thought you have should be expressed in your best mathematics. Partial credit may be given as deemed appropriate. Proof-read your solutions and check your computations as time allows.

GOOD LUCK!!!!!!!!!!!!!!

Scores
 page points score
 score

1	5	
2	12	
3	8	
4	5	
5	5	
6	8	

Scores
 page points score

7	8	
8	14	
9	18	
10	10	
11	8	
12	8	

Scores
 page points

13	8	
14	8	
15	8	
16	13	
17	9	
18	14	

Scores
 page points score

19	8	
20	8	
21	9	
22	6	
23		
24		
25		
26		
27		
ST4	31	
ST1	43	
ST2	66	
ST3	60	
ST4	31	
Tot.	200	

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True or False. Matrix Algebra. Circle True or False, but not both. If I cannot read your answer, it is wrong.

1. (1 pt.) A) True or B) False Matrix addition is associative.
2. (1 pt.) A) True or B) False Multiplication of square matrices is associative.
3. (1 pt.) A) True or B) False If A is an invertible square matrix, then $(A^T)^{-1}$ exists, but $(A^T)^{-1} \neq (A^{-1})^T$.
4. (1 pt.) A) True or B) False If A and B are invertible square matrices, then $(AB)^{-1}$ exists, but $(AB)^{-1} \neq A^{-1} B^{-1}$.
5. (1 pt.) A) True or B) False If A and B are square matrices, then $(AB)^T$ exists, but $(AB)^T \neq A^T B^T$.

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions

$x_1 + x_2 + x_3 + x_4 = 3$ Use Gauss elimination to solve this system of linear algebraic equations.
 $x_1 + 2x_2 + x_3 = 2$ Circle the letter or letters that correspond to your answer from the
 $x_3 + x_4 = 1$ possibilities below.
 $x_2 + 2x_3 + 2x_4 = 2$

6.(3 pts.) $x_1 =$ _____. _____ A B C D E 7.(3 pts.) $x_2 =$ _____. _____ A B C D E

8.(3 pts.) $x_3 =$ _____. _____ A B C D E 9.(3 pts.) $x_4 =$ _____. _____ A B C D E

Possible answers this page.

A) 0 B) 1 C) 2 D) 3 E) 4 AB) 5 AC) 6 AD) 7 AE) 8 BC) 9 BD) 10 BE) -1 CD) -2
 CE) -3 DE) -4 ABC) -5 ABD) -6 ABE) -7 BCD) -8 BCE) -9 BDE) -10
 ABCD)None of the above.

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Let

$$A = \begin{bmatrix} 1 & -i \\ -i & -1 \end{bmatrix}, \quad \vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \text{and} \quad \vec{b} = \begin{bmatrix} -1 \\ i \end{bmatrix}. \quad \text{Solve } \underset{2 \times 2}{A} \underset{2 \times 1}{\vec{x}} = \underset{2 \times 1}{\vec{b}}.$$

Write your answer according to the directions given in class (attendance is mandatory).

10. (4 pts.) If $[A|\vec{b}]$ is reduced to $[U|\vec{c}]$ using Gauss elimination, then

$$[U|\vec{c}] = \text{_____}. \quad \text{_____} \quad \text{A B C D E}$$

11. (4 pts.) The general solution of $\underset{2 \times 2}{A} \underset{2 \times 1}{\vec{x}} = \underset{2 \times 1}{\vec{b}}$ can be written

$$\text{as } \begin{bmatrix} x \\ y \end{bmatrix} = \text{_____}. \quad \text{_____} \quad \text{A B C D E}$$

Possible answers this page

$$\text{A)} \begin{bmatrix} 1 & i | 1 \\ 0 & 0 | 0 \end{bmatrix} \quad \text{B)} \begin{bmatrix} 1 & i | -1 \\ 0 & 0 | 0 \end{bmatrix} \quad \text{C)} \begin{bmatrix} 1 & -i | 1 \\ 0 & 0 | 0 \end{bmatrix} \quad \text{D)} \begin{bmatrix} 1 & -i | -1 \\ 0 & 0 | 0 \end{bmatrix} \quad \text{E)} \begin{bmatrix} 1 & i | 1 \\ 0 & 0 | 1 \end{bmatrix} \quad \text{AB)} \begin{bmatrix} 0 & 0 | 0 \\ 0 & 0 | 0 \end{bmatrix} \quad \text{AC)} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\text{AD)} \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad \text{AE)} y \begin{bmatrix} -i \\ 1 \end{bmatrix} \quad \text{BC)} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y \begin{bmatrix} i \\ 1 \end{bmatrix} \quad \text{BD)} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y \begin{bmatrix} -i \\ 1 \end{bmatrix} \quad \text{BE)} \begin{bmatrix} -1 \\ 0 \end{bmatrix} + y \begin{bmatrix} i \\ 1 \end{bmatrix} \quad \text{CD)} \begin{bmatrix} -1 \\ 0 \end{bmatrix} + y \begin{bmatrix} -i \\ 1 \end{bmatrix}$$

CE) No solution DE) Exactly one solution, but none of the above is correct

ABC) More than one solution, but a finite number and none of the above is correct.

ABD) An infinite number of solutions, but none of the above is a correct description

ABCDE) None of the above correctly describes the answer to the question

Total points this page = 8. TOTAL POINTS EARNED THIS PAGE _____

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Prof. Moseley

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True or false. Vector Space Theory. Let $S = \{\vec{x}_1, \dots, \vec{x}_k\} \subseteq W \subseteq V$ where W is a subspace of a vector space V over the scalars \mathbf{K} . Answer the following true false questions.

12. (1 pt.) A)True or B)False S is not a spanning set for W if $\forall \vec{X} \in W, \exists c_1, c_2, \dots, c_k$ in \mathbf{K} such that $\vec{X} = c_1\vec{x}_1 + \dots + c_k\vec{x}_k$.

13. (1 pt.) A)True or B)False S is not a linearly independent set if the only solution to $c_1\vec{x}_1 + \dots + c_k\vec{x}_k = \vec{0}$ is $c_1 = c_2 = \dots = c_k = 0$.

14. (1 pt.) A)True or B)False S is a basis for W if it is linearly independent and spans W .

15. (1 pt.) A)True or B)False A basis set for W is unique.

16. (1 pt.) A)True or B)False The dimension of \mathbf{R}^n is n .

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE _____

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

Let $A = \begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 4 \\ 3 & 0 & -9 & 3 \end{bmatrix}$. Compute the determinant of A.

17 (5 pts.) The numerical value of $\det(A)$ is $\det(A) =$ _____. _____ A B C D E

Possible answers this page.

A) 0 B) 1 C) 2 D) 3 E) 4 AB) 5 AC) 6 AD) 7 AE) 8 BC) 9 BD) 10 BE) 11
CD) 12 CE) 13 DE) 14 ABC) 15 ABD) 20 ABE) 21 ACD) -1 ACE) -2 ADE) -3
BCD) -4 BCE) -5 BDE) -6 CDE) -7 ABCD) -8 ABCE) -9 ABDE) -10 ACDE) -20

ABCDE) None of the above

Possible points this page = 5. POINTS EARNED THIS PAGE = _____

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions

Let $S = \{ \vec{x}_1, \vec{x}_2 \}$ and $\vec{0} = [0,0,0]^T$. Determine Directly Using the Definition (DUD) if the following sets of vectors are linearly independent. As explained in class, circle the appropriate answer that gives an appropriate method to prove that your results are correct (attendance is mandatory). Be careful, if you get the definitions wrong, you may miss them both.

18. (4 pts.) If $\vec{x}_1 = [2, 4, 8]^T$, $\vec{x}_2 = [3, 6, 12]^T$, then S is

_____. **A B C D E**

19. (4 pts.) If $\vec{x}_1 = [2, 2, 6]^T$, $\vec{x}_2 = [3, 3, 9]^T$, then S is

_____. **A B C D E**

Possible answers this page

A) linearly independent as $c_1 \vec{x}_1 + c_2 \vec{x}_2 = \vec{0}$ implies $c_1 = 0$ and $c_2 = 0$.

B) linearly independent as $3 \vec{x}_1 + (-2) \vec{x}_2 = \vec{0}$.

C) linearly dependent as $c_1 \vec{x}_1 + c_2 \vec{x}_2 = \vec{0}$ implies $c_1 = 0$ and $c_2 = 0$.

D) linearly dependent as $3 \vec{x}_1 + (-2) \vec{x}_2 = \vec{0}$.

E) linearly independent as it contains the zero vector.

AB) linearly dependent as it contains the zero vector.

AC) neither linearly independent or linearly dependent as the definition does not apply.

ABCDE) None of the above statements is true.

Total points this page = 8. TOTAL POINTS EARNED THIS PAGE _____

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions

You are to find an equation for a plane and an equation for a sphere. Recall that these equations are not unique. To get the equations given in the answers below, you should use the procedures illustrated in class (attendance is mandatory).

20. (4 pts.) Let P be the plane through the point (3,0,0) and parallel to the plane with equation $2x + 4y = 3z + 10$. An equation for P

is _____. _____ A B C D E

21. (4 pts.) Let S be the sphere of radius 3 with center at (3,-3,0). An equation for S

is _____. _____ A B C D E

Possible answers for this page.

A) $2x + 4y + 3z = 0$ B) $2x + 4y - 3z = 0$ C) $2x + 4y + 3z = 2$ D) $2x + 4y - 3z = 2$ E) $2x + 4y + 3z = 4$

AB) $2x + 4y - 3z = 4$ AC) $2x + 4y + 3z = 6$ AD) $2x + 4y - 3z = 6$ AE) $2x + 4y + 3z = 8$ BC) $2x + 4y - 3z = 8$

BD) $4x + 3y + 3z = 10$ BE) $4x + 2y - 3z = 10$ CD) $4x + 2y + 3z = 12$ CE) $4x + 2y - 3z = 12$ DE) $4x + 2y + 3z = -1$

ABC) $4x + 2y - 3z = -1$ ABD) $(x+1)^2 + (y+3)^2 + z^2 = 2$ ABE) $(x-1)^2 + (y+3)^2 + z^2 = 4$ ACD) $(x-2)^2 + (y+3)^2 + z^2 = 2$

ACE) $(x-2)^2+(y+3)^2+z^2=4$ ADE) $(x+3)^2+(y+3)^2+z^2=2$ BCD) $(x-3)^2+(y+3)^2+z^2=4$
 BDE) $(x-4)^2+(y+3)^2+z^2=2$ ABCD) $(x-4)^2+(y+3)^2+z^2=4$ ABCE)None of the above
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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice

Let L_1 and L_2 be the two intersecting lines (check $t = 0$) whose parametric equations are given by:

$$L_1: x = 3 + t, \quad y = 0, \quad z = 1 - 3t \quad L_2: x = 3, \quad y = t, \quad z = 1 + 3t$$

where $t \in \mathbf{R}$. You are to find an equation of the plane P that contains L_1 and L_2 . Recall that this equation is not unique. To get the answers given below, you should recall the comments made in class (attendance is mandatory).

22. (2 pts.) The point where the lines intersect is _____ . _____ A B C D E
23. (2 pts.) A vector in the direction of L_1 is $\vec{a}_1 =$ _____ . _____ A B C D E
 (Recall that such a vector is not unique.)
24. (2 pts.) A vector in the direction of L_2 is $\vec{a}_2 =$ _____ . _____ A B C D E
 (Recall that such a vector is not unique.)
25. (4 pts.) A normal to the plane P is _____ . _____ A B C D E
 (Recall that a normal vector to a plane is not unique.)
26. (4 pts.) An equation for the plane P is _____ . _____ A B C D E
 (Recall that an equation for the plane is not unique.)

Possible answers this page.

A)(1,0,1) B)(2,0,1) C)(3,0,1) D)(4,0,1) E)(0,0,0) AB)(1,1,-1) AC) $\hat{i} - \hat{k}$ AD) $\hat{i} - 2\hat{k}$ AE) $\hat{i} - 3\hat{k}$
 BC) $\hat{i} - 4\hat{k}$ BD) $\hat{j} + \hat{k}$ BE) $\hat{j} + 2\hat{k}$ CD) $\hat{j} + 3\hat{k}$ CE) $\hat{j} + 4\hat{k}$ DE) $\hat{i} - \hat{j} + \hat{k}$ ABC) $2\hat{i} - 2\hat{j} + \hat{k}$

ABD) $3\hat{i} - 3\hat{j} + \hat{k}$ ABE) $4\hat{i} - 4\hat{j} + \hat{k}$ ACD) $-\hat{i} + 4\hat{j} + 2\hat{k}$ ACE) $-\hat{i} + 4\hat{j} - 2\hat{k}$ ADE) $-\hat{i} - 4\hat{j} + 2\hat{k}$
BCD) $-\hat{i} - 4\hat{j} - 2\hat{k}$ BCE) $2\hat{i} - \hat{j} - 2\hat{k}$ BDE) $x - y + z = 2$ CDE) $2x - 2y + z = 5$ ABCD) $3x - 3y + z = 10$
ABCE) $4x - 4y + z = 17$ ABDE) $x + 4y + 2z = 2$ ABCDE) $x + 4y - 2z = 2$ ABCDE) None of the above.
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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

Suppose a point mass M traces out a curve C and has the position vector as a function of time t with $t \in \mathbf{R} = (-\infty, \infty)$ given by $\vec{r} = 9t \hat{i} + 6 \cos(2t) \hat{j} - 6 \sin(2t) \hat{k}$. (Be careful, if you make a mistake, the rest is wrong.)

27. (3 pts) The velocity $\vec{v}(t)$ of M at time $t = 0$ is $\vec{v}(0) =$ _____. _____ A B C D E

28. (3 pts) The acceleration $\vec{a}(t)$ of M at time $t = 0$ is $\vec{a}(0) =$ _____. _____ A B C D E

29. (4 pts) $\vec{v}(0) \times \vec{a}(0) =$ _____. _____ A B C D E

30. (2 pts) $\|\vec{v}(0) \times \vec{a}(0)\| =$ _____. _____ A B C D E

31. (2 pts) $\|\vec{v}(0)\| =$ _____. _____ A B C D E

32. (4 pts) The curvature of the curve C when $t = 0$ is _____. _____ A B C D E

Possible answers for this page.

A) $3\hat{i} - 4\hat{k}$ B) $2(3\hat{i} - 4\hat{k})$ C) $3(3\hat{i} - 4\hat{k})$ D) $4(3\hat{i} - 4\hat{k})$ E) $-8\hat{j}$ AB) $-16\hat{j}$ AC) $-24\hat{j}$ AD) $-32\hat{j}$
AE) $-8(4\hat{i} + 3\hat{j})$ BC) $-32(4\hat{j} + 3\hat{k})$ BD) $-72(4\hat{i} + 3\hat{k})$ BE) $-128(4\hat{i} + 3\hat{k})$ CD) $\sqrt{9t + 16}$

CE) $\sqrt{9t^2 + 16}$ DE) $\sqrt{9t^2 + 16\cos^2(t) + 16\sin^2(t)}$ ABC) 0 ABD) 1 ABE) 5 ACD) 10 ACE) 15
 ADE) 20 BCD) 40 BCE) 160 BDE) 360 CDE) 640 ABCD) $8/(25)$ ABCE) $4/(25)$
 ABDE) $8/(125)$ ACDE) $2/25$ ABCDE) None of the above.

Possible points this page = 18. POINTS EARNED THIS PAGE = _____

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

Let $w = f(x,y) = ax^2e^y \cos y$ where $x = g(t)$ and $y = h(t)$. Hence $w = f(g(t),h(t))$. Assume

$g(0) = 1$, $h(0) = 0$, $g'(0) = 2$, and $h'(0) = -5$. You are to compute $\left. \frac{dw}{dt} \right|_{t=0}$

33. (3 pts.) $\left. \frac{\partial w}{\partial x} \right|_{(x,y)=(1,0)} = \underline{\hspace{2cm}} . \underline{\hspace{2cm}}$ A B C D E

34. (3 pts.) $\left. \frac{\partial w}{\partial y} \right|_{(x,y)=(1,0)} = \underline{\hspace{2cm}} . \underline{\hspace{2cm}}$ A B C D E

35. (4 pts.) $\left. \frac{dw}{dt} \right|_{t=0} = \underline{\hspace{2cm}} . \underline{\hspace{2cm}}$ A B C D E

Possible answers this page.

A)0 B)1 C) 2 D)3 E) 4 AB) 5 AC) 6 AD) 7 AE) 8 BC) 9 BD) 10 BE) 11 CD) 12 CE)13
 DE) 14 ABC) 15 ABD) -1 ABE) -2 ACD) -3 ACE) -4 ADE) -5 BCD) -6 BDE) -7
 CDE) -8 ABCD) -9 ABCE)-10 ABDE) -11 ACDE) -12 ABCDE) None of the above
 Possible points this page = 10. POINTS EARNED THIS PAGE = _____
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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

Let $w = f(x,y) = 3e^{5x} \cos(y) + 5y$, P be the point $(0,0) \in \mathbf{R}^2$, and \hat{u} be a unit vector in the direction of $\bar{v} = 3\hat{i} - 4\hat{j}$. Compute the following.

36. (4 pts.) $\left. \nabla f \right|_{(x,y)=(0,0)} = \underline{\hspace{2cm}} \cdot \underline{\hspace{2cm}} \quad \text{A B C D E}$

37. (4 pts.) $\left. D_{\hat{u}} f(P) = D_{\hat{u}} f \right|_{(x,y)=(0,0)} = \underline{\hspace{2cm}} \cdot \underline{\hspace{2cm}} \quad \text{A B C D E}$

Possible answers.

A) $5\hat{i} + 5\hat{j} - \hat{k}$ B) $10\hat{i} + 5\hat{j} - \hat{k}$ C) $15\hat{i} + 5\hat{j} - \hat{k}$ D) $20\hat{i} + 5\hat{k} - \hat{k}$ E) $5\hat{i} + 5\hat{j}$ AB) $10\hat{i} + 5\hat{j}$

AC) $15\hat{i}+5\hat{j}$ AD) $20\hat{i}+5\hat{j}$ AE) $10\hat{i}+15\hat{j}$ BC) 0 BD) 1 BE) 2 CD) 3 CE) 4 DE) 5 ABC) 6
 ABD) 7 ABE) 8 ACD) 9 ACE) 10 ADE) -1 BCD) -2 BCE) -3 BDE) -4 CDE) -5
 ABCD) -6 ABCE) -7 ABDE) -8 ACDE) -9 BCDE) -10 ABCDE) None of the above
 Possible points this page = 8. POINTS EARNED THIS PAGE = _____
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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions

Let S be the surface which is defined by the graph of the function $z = f(x,y)$. Suppose using geometric notation that $\nabla f \Big|_{(x,y)=(1,1)} = 3\hat{i} - 4\hat{j}$. and that $f(1,1) = 6$. Let P be the point on S where $x=1$ and $y=1$.

38. (4 pts.) Using geometric notation, a normal to the surface S at the point on S where $x = 1$ and

$y = 1$ is _____ . _____ A B C D E

Recall that the representation of this vector is not unique. To get the vector given below, you should use the procedure illustrated in class (attendance is mandatory).

39. (4 pts.) The equation of the tangent plane to the surface S at the point on the surface where

$x = 1$ and $y = 1$ is _____ . _____ A B C D E

Recall that this equation is not unique. To get the equation given below, you should use the procedure illustrated in class (attendance is mandatory).

Possible answers.

A) $\hat{i} - 4\hat{j}$ B) $2\hat{i} - 4\hat{j}$ C) $3\hat{i} - 4\hat{j}$ D) $4\hat{i} - 4\hat{j}$ E) $\hat{i} - 4\hat{j} + \hat{k}$ AB) $2\hat{i} - 4\hat{j} + \hat{k}$ AC) $3\hat{i} - 4\hat{j} + \hat{k}$ AD) $4\hat{i} - 4\hat{j} + \hat{k}$
 AE) $\hat{i} - 4\hat{j} - \hat{k}$ BC) $2\hat{i} - 4\hat{j} - \hat{k}$ BD) $3\hat{i} - 4\hat{j} - \hat{k}$ BE) $4\hat{i} - 4\hat{j} - \hat{k}$ CD) $x - 4y + z = -3$ CE) $2x - 4y + z = -2$
 DE) $3x - 4y + z = -1$ ABC) $4x - 4y + z = 0$ ABD) $x - 4y - z = -3$ ABE) $2x - 4y - z = -2$ ACD) $3x - 4y - z = -1$
 ACE) $4x - 4y - z = 0$ ADE) $6x + 4y + z = 9$ BCD) $6x + 4y - z = 9$ BCE) $6x - 4y + z = 9$ BDE) $6x - 4y - z = 9$

CDE) $6x+4y+z=11$ ABCD) $6x+4y-z=11$ ABCE) $6x-4y+z=11$ ABDE) $6x-4y-z=11$

ACDE) $6x-5y-z=11$ BCDE) $6x+5y+z=11$ ABCDE) None of the above.

Possible points this page = 8. POINTS EARNED THIS PAGE = _____

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice

Consider the function $f: \mathbf{R}^2 \rightarrow \mathbf{R}$ defined by $z = f(x,y) = x^3 + (3/2)x^2 + y^2 + 6y + 8$.

40. (4 pts.) Using geometric notation, the formula for ∇f in the variables x and y is

$\nabla f =$ _____. _____ A B C D E

41. (4 pts.) The set of critical points of this function, call it C , is

$C =$ _____. _____ A B C D E

Possible answers this page.

A) $(3x^2+3x)\hat{i}+2y\hat{j}$ B) $(3x^2+3x)\hat{i}-2y\hat{j}$ C) $(3x^2-3x)\hat{i}+2y\hat{j}$ D) $(3x^2-3x)\hat{i}-2y\hat{j}$ E) $(3x^2+3x)\hat{i}+(2y+2)\hat{j}$
 AB) $(3x^2+3x)\hat{i}+(2y-2)\hat{j}$ AC) $(3x^2+3x)\hat{i}+(2y+4)\hat{j}$ AD) $(3x^2+3x)\hat{i}+(2y-4)\hat{j}$ AE) $(3x^2+3x)\hat{i}+(2y+6)\hat{j}$
 BC) $(3x^2+3x)\hat{i}+(2y-6)\hat{j}$ BD) $(3x^2+3x)\hat{i}+(2y+8)\hat{j}$ BE) $(3x^2+3x)\hat{i}+(2y-8)\hat{j}$ CD) $(3x^2+6)\hat{i}+(2y+2)\hat{j}$
 CE) \emptyset DE) \mathbf{R}^2 ABC) $\{(0,0),(1,0)\}$ ABD) $\{(0,-1),(-1,-1)\}$ ABE) $\{(0,-2),(-1,-2)\}$

$ACD)\{(0,-3),(-1,-3)\}$ $ACE)\{(0,-4),(-1,-4)\}$ $ADE)\{(0,1),(-1,0)\}$ $BCD)\{(0,1),(-1,-1)\}$
 $BCE)\{(0,-1),(1,1)\}$ $BDE)\{(-1,0)\}$ $CDE)\{(-1,-1)\}$ $ABCD)\{(1,-1)\}$ $ABCE)\{(0,-1)\}$
 $ABCDE)$ None of the above

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice

Let P be the point in \mathbf{R}^3 (i.e. 3-space) which has rectangular coordinates $(\sqrt{2}, \sqrt{2}, 0)_R$. Give the cylindrical and spherical coordinates of P. Begin by drawing a picture. Be sure to give the coordinates in the correct form.

42. (4 pts.) The cylindrical coordinates of P are _____ . _____ A B C D E

43. (4 pts.) The spherical coordinates of P are _____ . _____ A B C D E

Possible answers this page

A) $(\sqrt{2}, \pi/2, \sqrt{2})_C$ B) $(\sqrt{2}, 0, \sqrt{2})_C$ C) $(\sqrt{2}, \pi/4, 0)_C$ D) $(2, \pi/4, 2)_C$ E) $(\sqrt{2}, \pi/4, \pi/4)_C$

AB) $(2, \pi/6, 0)_C$ AC) $(2, \pi/3, 0)_C$ AD) $(2, \pi/3, 0)_C$ AE) $(2, \pi/2, 0)_C$ BC) $(1, \pi/4, 0)_C$
 BD) $(2, \pi/2, \pi/4)_S$ BE) $(2, 0, \pi/4)_S$ CD) $(2, \pi/4, \pi/2)_S$ CE) $(\sqrt{8}, \pi/4, \pi/4)_S$
 DE) $(2, \pi/6, \pi/2)_S$ ABC) $(2, \pi/4, \pi/2)_S$ ABD) $(2, \pi/3, \pi/2)_S$ ABE) $(2, \pi/2, \pi/2)_S$
 BCD) $(1, \pi/4, \pi/2)_S$ BCE) $(1, \pi/4, 0)_S$ CDE) None of the above.

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Evaluate the iterated integral $I = \int_0^2 \int_0^1 (6x^3y^2 + 6ye^x) dy dx$ in steps by first finding a single integral and

then the numerical value of the double integral.

44. (4pts.) Doing the first step in the evaluation of this double integral results in the single integral

$I =$ _____ . _____ A B C D E

45. (4pts.) The final numerical value of I is

$I =$ _____ . _____ A B C D E

Possible answers

A) $\int_0^2 (2x^3 + e^x) dx$ B) $\int_0^2 (2x^3 - e^x) dx$ C) $\int_0^2 (2x^3 + 2e^x) dx$ D) $\int_0^2 (2x^3 - 2e^x) dx$ E) $\int_0^2 (2x^3 + 3e^x) dx$

AB) $\int_0^2 (2x^3 - 3e^x) dx$ AC) $\int_0^2 (2x^3 + 4e^x) dx$ AD) $\int_0^2 (2x^3 - 4e^x) dx$ AE) $\int_0^2 (6x^3 + 4e^x) dx$ BC) $\int_0^2 (6x^2 + 2xe) dy$

BD) $\int_0^2 (2x^3 + e^x) dy$ BE) $\int_0^1 (2x^3 + 2e^x) dy$ CD) 0 CE) 1 DE) $4 + 4e^2$ ABC) $4 - 4e^2$ ABD) $5 + 5e^2$
 ABE) $5 - 5e^2$ ACD) $6 + 6e^2$ ACE) $6 - 6e^2$ ADE) $7 + e^2$ BCD) $7 - e^2$ BCE) $6 + 2e^2$
 BDE) $6 - 2e^2$ CDE) $5 + 3e^2$ ABCD) $5 - 3e^2$ ABCE) $4 + 3e^2$ ABDE) $4 - 3e^2$ ACDE) $3 + 2e^2$
 BCDE) $3 - 2e^2$ ABCDE) None of the above

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice

Let $A = \int_{\gamma}^{\delta} \int_{\alpha}^{\beta} g(x,y) dy dx$ be the area of the region in the first quadrant bounded by the curves

$y = 2x^2$, $4x + y = 16$, and $x = 0$. Determine $g(x,y)$, α , β , γ , and δ . Begin by drawing an appropriate sketch (careful with the scales). Do not evaluate the integral.

46. (1 pts.) $g(x,y) =$ _____. _____ A B C D E

47. (4 pts.) $\alpha =$ _____. _____ A B C D E

48. (4 pts.) $\beta =$ _____. _____ A B C D E

49. (2 pts.) $\gamma =$ _____. _____ A B C D E

50. (2 pts.) $\delta =$ _____. _____ A B C D E

Possible answers.

A) 0 B) 1 C) 2 D) 3 E) 4 AB) -1 AC) -2 AD) -3 AE) -4 BC) $(3/2)x^2$ BD) $2x^2$ BE) $3x^2$
 CD) $6x^2$ CE) y DE) $y/2$ ABC) y^2 ABD) $2y^2$ ABE) $12-3x$ ACD) $16-4x$ ACE) $24-6x$ ADE) $48-12x$
 BCD) $\sqrt{2x}$ BCE) $\sqrt{2y}$ BDE) \sqrt{x} CDE) \sqrt{y} ABDE) None of the above

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You are to evaluate the iterated integral $I = \int_0^1 \int_0^{2x} \int_0^y -30axy^2z \, dz dy dx$.

51. (3pts.) Doing the first step in the computation results in the double iterated integral

$I =$ _____ . _____ A B C D E

52. (3pts.) Doing the second step in the computation results in the single iterated integral

$I =$ _____ . _____ A B C D E

53. (3pts.) After the computation is complete, the numerical value of I is

$I =$ _____ . _____ A B C D E

Possible answers.

A) $\int_0^1 \int_0^{2x} 5x^2y^4 dy dx$ B) $\int_0^1 \int_0^{2x} -5x^2y^4 dy dx$ C) $\int_0^1 \int_0^{2x} 10x^2y^4 dy dx$ D) $\int_0^1 \int_0^{2x} -10x^2y^4 dy dx$ E) $\int_0^1 \int_0^{2x} 15x^2y^4 dy dx$ AB) $\int_0^1 \int_0^{2x} -15x^2y^4 dy dx$

$$\text{AC)} \int_0^1 \int_0^{2x} 20x^2 y^4 dy dx \quad \text{AD)} \int_0^1 \int_0^{2x} -20x^2 y^4 dy dx \quad \text{AE)} \int_0^1 \int_0^{2x} 5x^2 y^5 dy dx \quad \text{BC)} \int_0^1 \int_0^{2x} -5x^2 y^5 dy dx \quad \text{BD)} \int_0^1 32x^7 dx \quad \text{BE)} \int_0^1 -32x^7 dx$$

$$\text{CD)} \int_0^1 64x^7 dx \quad \text{CE)} \int_0^1 -64x^7 dx \quad \text{DE)} \int_0^1 96x^7 dx \quad \text{ABC)} \int_0^1 -96x^7 dx \quad \text{ABD)} \int_0^1 128x^7 dx \quad \text{ABE)} \int_0^2 -128x^7 dx \quad \text{ACD)} 0 \quad \text{ACE)} 4$$

$$\text{ADE)} 8 \quad \text{BCD)} 12 \quad \text{BCE)} 16 \quad \text{BDE)} -4 \quad \text{CDE)} -8 \quad \text{ABCD)} -12 \quad \text{ABCE)} -16 \quad \text{ABDE)} 8/3 \quad \text{ACDE)} 12/5$$

$$\text{BCDE)} 16/7 \quad \text{ABCDE)} \text{None of the above}$$

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Let $V = \int_{\mu}^{\nu} \int_{\gamma}^{\delta} \int_{\alpha}^{\beta} g(x, y, z) dz dy dx$ be the volume of the solid region R in the first octant that has the

origin on its surface and is bounded on the top by the sphere $x^2 + y^2 + z^2 = 36$, on its sides by the cylinder $x^2 + y^2 = 9$ and the planes $x = 0$ and $y = 0$, and on the bottom by the plane $z = 0$. Determine $g(x, y, z)$, α , β , γ , δ , μ , and ν (i.e. set up an iterated integral in rectangular coordinates which gives the volume V of R). Begin by drawing an appropriate sketch. DO NOT EVALUATE.

$$54. (1 \text{ pts.}) g(x, y, z) = \underline{\hspace{2cm}} . \underline{\hspace{2cm}} \quad \text{A B C D E}$$

$$55. (1 \text{ pts.}) \alpha = \underline{\hspace{2cm}} . \underline{\hspace{2cm}} \quad \text{A B C D E}$$

$$56. (4 \text{ pts.}) \beta = \underline{\hspace{2cm}} . \underline{\hspace{2cm}} \quad \text{A B C D E}$$

$$57. (1 \text{ pts.}) \gamma = \underline{\hspace{2cm}} . \underline{\hspace{2cm}} \quad \text{A B C D E}$$

$$58. (4 \text{ pts.}) \delta = \underline{\hspace{2cm}} . \underline{\hspace{2cm}} \quad \text{A B C D E}$$

$$59. (1 \text{ pts.}) \mu = \underline{\hspace{2cm}} . \underline{\hspace{2cm}} \quad \text{A B C D E}$$

$$60. (2 \text{ pts.}) \nu = \underline{\hspace{2cm}} . \underline{\hspace{2cm}} \quad \text{A B C D E}$$

Possible answers this page.

A)0 B)1 C)2 D)3 E)4 AB)5 AC) -1) AD) -2 AE)-3 BC)-4 BD)-5 BE) $2 - (2x/3) - (y/3)$
 CD) $\sqrt{4-x^2-y^2}$ CE) $\sqrt{16-x^2-y^2}$ DE) $\sqrt{36-x^2-y^2}$ ABC) $\sqrt{64-x^2-y^2}$ ABD) $4-x^2-y^2$
 ABE) $\sqrt{1-x^2}$ BCD) $\sqrt{4-x^2}$ BCE) $\sqrt{9-x^2}$ ABCD) $\sqrt{16-x^2}$ ABCE) $\sqrt{4-y^2}$ ABDE) $\sqrt{9-y^2}$
 ABCDE) None of the above

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple

Let C be the curve that is the path of a point mass M whose position vector is given by:

$\vec{r} = 5t^2 \hat{i} + 3t^3 \hat{j}$, $t \in [0, 1]$, (i.e. $0 \leq t \leq 1$). Also let $\vec{F}(x,y) = y \hat{i} - x \hat{j}$. You are to compute
 $I = \int_C \vec{F}(x,y) \cdot d\vec{r}$.

61. (1 pts.) A parametrization of the curve is _____. _____ A B C D E

62. (2 pts.) With the above parameterization, $\vec{F}(x(t),y(t))$ along the curve C is

$\vec{F}(x(t),y(t)) =$ _____. _____ A B C D E

63. (2 pts.) The velocity $\vec{v} = \frac{d\vec{r}}{dt}$ along the curve C is $\vec{v}(t) =$ _____. _____ A B C D E

64. (3 pts.) The numerical value for I is I = _____. _____ A B C D E

A) $x(t) = 5t^2, y(t) = t^3$ B) $x(t) = 5t^2, y(t) = 2t^3$ C) $x(t) = 5t^2, y(t) = 3t^3$ D) $x(t) = 5t^2, y(t) = 4t^3$
 E) $10t^3\hat{i} + 3t^2\hat{j}$ AB) $10t^3\hat{i} + 6t^2\hat{j}$ AC) $10t^3\hat{i} + 9t^2\hat{j}$ AD) $10t^3\hat{i} + 12t^2\hat{j}$ AE) $t^3\hat{i} - 5t^2\hat{j}$ BC) $2t^3\hat{i} - 5t^2\hat{j}$
 BD) $3t^3\hat{i} - 5t^2\hat{j}$ BE) $4t^3\hat{i} - 5t^2\hat{j}$ CD) 0 CE) 1 DE) 3 ABC) 4 ABD) 5 ABE) 6 ACD) 7 ACE) 8
 ADE) 9 BCD) -1 BCE) -2 BDE) -3 CDE) -4 ABCD) -5 ABCE) -6 ABDE) -7
 ACDE) -8 BCDE) -9 ABCDE) None of the above.

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Let $\vec{F}(x, y) = M(x, y)\hat{i} + N(x, y)\hat{j}$ be a vector field with $M(x, y) = 12xy^3 + 2x$ and $N(x, y) = 18x^2y^2$.

65. (1 pts.) $\frac{\partial M}{\partial x} =$ _____ . _____ A B C D E

66. (1 pts.) $\frac{\partial M}{\partial y} =$ _____ . _____ A B C D E

67. (1 pts.) $\frac{\partial N}{\partial x} =$ _____ . _____ A B C D E

68. (1 pts.) $\frac{\partial N}{\partial y} =$ _____ . _____ A B C D E

69. (1 pt.) True or False $\vec{F}(x, y)$ is a conservative vector field.

70. (3 pts.) A potential function for $\vec{F}(x, y)$ exists and is given by

$f(x, y) =$ _____ . _____ A B C D E

(The answer may not be unique. Do as instructed in class. Attendance is mandatory.)

Possible answers.

A) $12xy^2$ B) $24xy^2$ C) $36xy^2$ D) $48xy^2$ E) $4y^3+2$ AB) $8y^3+2$ AC) $12y^3+2$ AD) $16y^3+2$ AE) $12x^2y$
BC) $24x^2y$ BD) $36x^2y$ BE) $48x^2y$ CD) $2x^2y^3 + x^2$ CE) $4x^2y^3 + x^2$ ABC) $6x^2y^3 + x^2$ ABD) $8x^2y^3+x^2$
ABE) $\vec{F}(x,y)$ is conservative and conservative vector fields do not have potential functions.

ACD) $\vec{F}(x,y)$ is not conservative, and non-conservative vector fields do not have potential functions.

ACE) A potential function exists for $\vec{F}(x,y)$, but none of the above functions is a potential function for $\vec{F}(x,y)$.
ABCDE) None of the above

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Let $\vec{F}(x,y) = ax^2yz\hat{i} + xy^2z\hat{j} + xyz^2\hat{k}$ be a vector field . Compute the following:

71. (4 pts.) $\text{div } \vec{F} =$ _____. _____ A B C D E

72. (5 pts.) $\text{curl } \vec{F} =$ _____. _____ A B C D E

Possible answers this page.

A) $6xyz$ B) $8xyz$ C) $10xyz$ D) $12xyz$ E) $2yz + 2xz + 2xy$ AB) $4yz + 4xz + 4xy$

AC) $(xz^2 - xy^2)\hat{i} + (x^2y - yz^2)\hat{j} + (y^2z - x^2z)\hat{k}$ AD) $(xz^2 - xy^2)\hat{i} + (2x^2y - yz^2)\hat{j} + (y^2z - 2x^2z)\hat{k}$

AE) $(xz^2 - xy^2)\hat{i} + (3x^2y - yz^2)\hat{j} + (y^2z - 3x^2z)\hat{k}$ BC) $(xz^2 - xy^2)\hat{i} + (4x^2y - yz^2)\hat{j} + (y^2z - x^2z)\hat{k}$

BD) $(xz^2 - xy^2)\hat{i} + (xz^2 - xy^2)\hat{j} + (xz^2 - xy^2)\hat{k}$ ABCDE) None of the above.

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73. (3 pts.) The integral equality $\int_C \vec{F} \cdot d\vec{r} = \iint_S \text{curl } \vec{F} \cdot d\vec{S}$ appears in

_____. _____ A B C D E

74. (3 pts.) The integral equality $\int_C Pdx + Qdy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$ appears in

_____. _____ A B C D E

Possible answers this page

A) Green's Theorem B) Gauss's Divergence Theorem C) Stokes' Theorem
D) Fundamental Theorem of Calculus E) Fundamental Theorem of Linear Algebra
AB) Fundamental Theorem of Algebra AC) Fubini's Theorem
AD) Mean Value Theorem AE) Intermediate Value Theorem BC) None of the above.

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