PERFORMANCE CHARACTERIZATION OF INTEGRATED STATISTICAL PROCESS CONTROL SYSTEMS

A Dissertation by

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I have examined the final copy of this dissertation for form and content, and recommend that it be accepted in partial fulfillment of the requirement for the degree of Doctor of Philosophy with a major in Industrial Engineering.

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ABSTRACT

Business competition requires organizations to increase their products' quality and reduce cost at the same time. Statistical process control (SPC) techniques are important tools for monitoring process performance over time to detect special causes. Automatic process control (APC) systems, on the other hand, are utilized to regulate performance relative to a specified target. The literature indicates that combining APC and SPC systems result in integrated SPC (ISPC) systems offering an effective approach to process improvement.

The objective of this research was twofold. The first objective was aimed at identifying the various process factors likely to affect the long-term performance of ISPC systems. The process considered was one of discrete parts manufacturing characterized by the integrated moving average model IMA (1, 1). A simulation model was developed to represent system performance in terms of the mean squared error (MSE) of the resulting output and the average run length (ARL) of the SPC chart utilized. Simulated results were analyzed to identify influential factors likely to affect the system performance.

The second objective targeted the development of criteria for the economic performance of ISPC systems. Two mathematical cost models were developed utilizing Taguchi's quadratic loss function and accounted for key characteristics of the process and system design factors. These two models were used to derive criteria for the economic selection of the SPC chart design parameters. It is hoped that the proposed criteria will help practitioners select appropriate charting alternatives to minimize the total cost of operation.

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LIST OF ABBREVIATIONS

AMSD	Asymptotic Mean Squared Deviation
APC	Automatic Process Control
AR	Autoregressive
ARL	Average Run Length
ARIMA	Autoregressive Integrated Moving Average
ARMA	Autoregressive Moving Average
CUSUM	Cumulative Sum
I	Integral
IMA	Integrated Moving Average
ISPC	Integrated Statistical Process Control
EWMA	Exponentially Weighted Moving Average
MA	Moving Average
MMSE	Minimum Mean Squared Error
MSD	Mean Squared Deviation
MSE	Mean Squared Error
P	Proportional
PI	Proportional Integral
PID	Proportional Integral Derivative
SPC	Statistical Process Control

CHAPTER 1

INTRODUCTION

Statistical process control (SPC) is a procedure that focuses on process monitoring and control by separating common causes from assignable causes. Common causes are the sources of variation that are inherent in the process and cannot be eliminated when the process is in statistical control, while an assignable cause variation is unpredictable but can be easily detected and removed. The traditional tools of SPC are Shewhart control charts, which are based on assumptions that all processes are in control and observations are independent. Since the pioneering work of Shewhart in 1931, control charts have been successfully used to monitor process performance over time. However, because of the advanced measurement technology and shortened sampling interval, the independence of each observation is violated in many scenarios, especially in continuous process industries, e.g., chemical process. The lack of independence among samples always comes in the form of serial correlation, which can be either positively or negatively correlated. This behavior of process outputs significantly downgrades the performance of control charts. As a result, the control limits of control charts are narrower than what they should be and may signal false alarms more frequently. The consequence is unnecessary investigation, which consumes a considerable amount of time and money. Therefore, several authors point out that the traditional charts fail to control and improve the quality of correlated processes (Jiang et al., 2000; Loredo et al., 2002; Zhang, 1998).

Although there are a few methods which have been proposed to solve the correlation problem, the utilization of forecasting techniques is one of the most powerful solutions. Jiang et al. (2000) reported the success of the integrating SPC control charts and the autoregressive integrated moving average (ARIMA), which is a class of forecasting models for monitoring correlated observations. Loredo et al. (2002) contends that the ARIMA is a powerful tool for improving the ability of control charts to monitor correlated processes, since it is able to take the correlation structure into account. However, the method to select the most appropriate forecasting model and control charts to monitor the correlated data is still a widely discussed issue among many authors. Moreover, another critical issue after the integration is the continuous improvement of the monitoring method, since most process observations have no specific correlated pattern over time.

Another alternative is the utilization of automatic process control (APC). APC was developed to control processes, and it disregards the pattern of the observed data. For APC, frequent adjustment on the process can be done in order to keep the output on the desired target. Box and Kramer (1992) proposed the idea of using feedback control to compensate disturbances estimated by statistical forecasting, while SPC has been deployed to monitor the process output after the adjustment in order to detect an assignable cause which cannot be compensated by the controller. Montgomery et al. (1994) supported the claim that SPC can detect an assignable cause from the output rapidly, while APC can effectively keep a process on target. Some authors refer to systems, where SPC and APC are integrated, such as the integrated statistical process control (ISPC) system.

Chapter 2 is a review of the literature pertaining to the ISPC system. Chapter 3 provides a discussion leading to the research gap. Chapter 4 presents information on the research gap, together with objectives and research procedure. The analysis of statistical performance is provided in Chapter 5 in order to characterize the properties of the ISPC system. Economic cost models of the APC and the ISPC systems are presented in Chapter 6, and conclusions and future research are provided in Chapter 7.

CHAPTER 2

LITERATURE REVIEW

This chapter presents a review of publications in the area of the integrated statistical process control system and its implementation. This review is divided into six sections. The first section addresses a review of process models and disturbance models. The second section presents a definition of Statistical process control, underlying assumptions, and deterioration of its performance due to correlation. The third section reviews automatic process control, which includes different types of controllers. These are minimum mean squared error (MMSE), proportional integral (PI), and exponentially weighted moving average (EWMA) controller. The ISPC system, which focuses on the logic and advantages of the integration between control charts and automatic control, is discussed in the fourth section, followed by its economic models and Taguchi's method.

2.1 Process Models

For variable control charts, the quality characteristic is described by the measure of tendency and variability, and the process model under the normal condition can be expressed in the form of (Montgomery, 2001)

$$Y_t = \mu + e_t \tag{2.1}$$

where Y_t is the process output at time t, μ is the process mean, and e_t is the random error at time t. According to equation (2.1), the process data has a fixed mean, and the fluctuation is the result of white noise, which is independent, random, and normally distributed. This type of variability is called the stationary behavior, since the process data vary around a fixed mean in a predictable manner. The assumption underlying the traditional SPC is built from this process model.

Another model is used to represent the stationary behavior but correlated, and is shown in the form of

$$Y_t = \xi + \phi Y_{t-1} + e_t \tag{2.2}$$

where ξ is unknown constant, ϕ is the coefficient ranging from -1 to 1, and e_t is the random error at time t. This model is also known as first-order autoregressive. The observations from this model are dependent. A value above the mean tends to be followed by another value above the mean, whereas a value below the mean is usually followed by another such value. However, they still have a constant mean and variance.

The last process model is used to explain the non-stationary variation. This type of process always occurs in the chemical and process industries. The process is very unstable, and it drifts or wanders from the mean. Therefore, it does not have a fixed mean. The non-stationary model can be represented by using the first-order integrated moving average, IMA (1, 1), model or

$$Y_t = Y_{t-1} + e_t - \theta e_{t-1} \tag{2.3}$$

where y_t is the observation at time t, θ is the moving average parameter ranged from -1 to 1, (1-θ) is the drift rate, and e_t is the random error at time t.

The stationary and non-stationary behaviors of the process are the result of this disturbance. As a result, disturbances are integrated into the process model in order to explain the correlation structure and drift in the mean. If the model is identified correctly, the sequence of the output error represents white noises with zero mean and constant variance.

MacGregor (1998) signifies that disturbances can be differentiated into two categories: deterministic and stochastic. Stochastic disturbances occur in random and can be stationary or non-stationary, while deterministic disturbances might be in the form of sudden shift in the mean or ramp change and can be modeled by transfer functions and differential equations.

Box and Jenkins (1970) introduced the concept that stochastic disturbances can be modeled by stochastic difference equation models in the form of autoregressive integrated moving average (ARIMA) methodology. Different equations have been utilized in order to forecast one step ahead of disturbances, according to the characteristics of data (stationary or non-stationary). The stochastic difference equation can be categorized by following the stationarity of a data set. If the disturbance is stationary, then it can be represented by an autoregressive moving average (ARMA) or a mixed autoregressive moving average model. The autoregressive model of order p, AR (p), is expressed in the form of

$$\hat{Y}_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + ... + \phi_p Y_{t-p} + a_t \tag{2.4}$$

whereas, the autoregressive moving average, ARMA (p, q), is expressed as

$$\hat{Y}_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + ... + \phi_p Y_{t-p} - \theta_1 a_{t-1} - ... - \theta_q a_{t-q} + a_t \tag{2.5}$$

AR (p) is the regression model with lag values of dependent variables, while ARMA (p, q) is the mixed model between the autoregressive and moving average. Predicted disturbances depend on past and current values of disturbances as well as past and current values of errors. However, if the disturbance is not stationary, the Box and Jenkin's ARIMA model, ARIMA (p, d, q), is expressed in the form of

$$\hat{Y}_t = \phi_1 Y_{t-1} + .. + \phi_p Y_{t-P} + Y_{t-1} + (Y_{t-1} - Y_{t-2}) + ... + (Y_{t-d-1} - Y_{t-d}) - \theta_1 a_{t-1} - .. - \theta_q a_{t-q} + a_t \tag{2.6}$$

The ARIMA (p, d, q) model indicates p order of the autoregressive part, d the amount of differencing, and q the order of the moving average. The integrated moving average, IMA (1, 1), can be considered a special case of ARIMA (1, 1, 1) with $\phi = 0$.

MacGregor (1990) applied Deming's funnel experiment to explain that control policy can keep the process mean on target due to the reduction of uncontrollable disturbances. If the observations are correlated and there is drift from the mean, the output is a drifting process mean in the form of equation (2.2). The estimation of disturbances in the process can be modeled by the AR (1) model. MacGregor also showed that process adjustment can reduce the variation caused by disturbances. The variance regarding rule 2 of Deming's experiment is small compared to rule 1, when process variables are highly correlated ($\phi = 1$), so the adjustment can help to keep the process mean on target.

Box and Kramer (1992) suggested that disturbances can be modeled by relying on the stationarity of models. If the disturbance is stationary, the autoregressive processes should be appropriate to model the process. To the contrary, the IMA (1, 1) model is recommended for modeling non-stationary disturbances. This non-stationary model is utilized to explain drifting behavior of output from a fixed target when there is no adjustment in the process.

Montgomery et al. (1994) pointed out that the IMA (1, 1) model is robust to the misspecification of the disturbance model. Even when the stochastic disturbance model follows another ARIMA model, IMA (1, 1) with the appropriate θ is still an excellent choice of fitting model for the disturbance.

Nembhard and Mastrangelo (1998) noted that a shift in this process can occur as the result of assignable causes (e.g., machine shutdown, or changes in raw materials, equipment or products) and during the transient state (production startup), which results in abundant loss in the process. The transient phase is induced by the dynamic behavior of the process, which causes the output to lag behind input before reaching a steady state. The dynamic behavior of processes can be modeled by deploying continuous state space equations from the control theory. The integration between SPC and APC provides the opportunity to perform an adjustment, which can significantly decrease the transient-period length as well as the variation of processes.

Jiang et al. (2000) used a different approach to model disturbances for testing the performance of the special-cause chart (SCC) and exponentially weighted moving average for stationary (EWMAST) charts proposed by Zhang (1998), which are specially designed for autocorrelated data. Two different sets of sample data were selected and fitted to choose models for predicting disturbances. Results show that ARMA (3, 2) and ARMA (2, 1) represent the best fit and were utilized to assess the performance of the two charts.

Jiang (2004) noted that the selection of an appropriate disturbance model is an open issue for discussion among authors. Some suggest that the IMA, which is a non-stationary model, best represents disturbances in the process because of its flexibility. However, many others use a stationary model, such as ARMA, to model disturbances.

2.2 Statistical Process Control

According to MacCarthy and Wasusri (2002), statistical process control is a powerful tool to monitor and control processes and has been widely used in the manufacturing and non-manufacturing processes since it was first introduced by Walter A. Shewhart. The application of SPC control charts can be classified into four categories: process monitoring, planning, evaluating customer satisfaction, and forecasting. Among these categories, process monitoring is considered the traditional use of SPC tools in order to stabilize and improve the process capability. However, control charts can only work effectively when the sampled data follows all underlying assumptions (independent and normally distributed). These charts immediately lack robustness, when observations violate these conditions. The problem is that most industrial processes are continuous and correlated. When the data is highly correlated, traditional charts signal a high rate of false alarms.

Traditional control charts were developed in 1931 by Shewhart for process monitoring. They have been widely used to distinguish between assignable causes and chance causes of variation. Several definitions of control charts are found in the literature. Shewhart (1931) defined the control charts as follows: "The control chart may serve, first, to define the goal or standard for a process that management strives to attain; second, it may be used as an instrument for attaining that goal and third, it may be serve as a means of judging whether the goal has been reached." A control chart may also be viewed as a statistical tool as defined by Duncan (1956): "… is a statistical device principal used for the study and control of repetitive processes." Moreover, Feigenbaum (1983) defined a control chart as: "… a graphical comparison of the actual product-characteristics with limits reflecting the ability to produce as shown by past experience on the product characteristics."

Therefore, a control chart is a graphical display used to monitor a process. It usually consists of a horizontal centerline corresponding to the in-control value of the parameter that is monitored, and lower and upper control limits. The control limits are neither determined arbitrarily, but rather based on statistical criteria, nor related to specification limits. If the sample point falls within the control limits, the process is deemed to be in control, or free from any assignable causes. Points beyond the control limits indicate an out-of-control process, i.e., assignable causes are likely present. This signals the need for corrective action in order to find and remove the assignable causes. The assignable causes, also called special causes, are the portion of the variability in a set of observations that can be traced to specific causes such as operators, materials, or equipment. On the other hand, chance causes, also called common causes, are the portion of the variability in a set of observations that is due to only random forces and cannot be traced to specific sources, such as, operators, materials, or equipment. The average run length (ARL) is used to evaluate the performance of control charts. The ARL can be calculated from

$$ARL_0 = \frac{1}{\alpha} \tag{2.7}$$

where α is the probability that any point exceeds the control limits. For a Shewhart $\overline{X}$ chart with 3σ limits, $\alpha = 0.0027$ is the probability that a single point falls outside the limits when the process is in control. Therefore, the ARL of the $\overline{X}$ chart when the process is in control is called ARL_0 or

$$ARL_0 = \frac{1}{\alpha} = \frac{1}{0.0027} = 370$$

Even if the process remains in control, an out-of-control signal will be generated on the average of every 370 samples. Moreover, the expected number of samples taken before the shift is detected is called ARL_1 or

$$ARL_1 = \frac{1}{1-\beta} \tag{2.8}$$

where β is the probability of points falling within the control limits after causing a shift. Therefore, the probability a shift will be detected on the first subsequent sample is $1-\beta$ (Montgomery, 2001).

Since the serial correlation in the output generates the dramatic disturbances which violate the underlying assumption of the Shewhart's control chart, different mathematical models are used to represent the disturbances (Montgomery and Mastrangelo, 1991). As a result, control charts, which are based on the disturbances model, have been developed in order to monitor the highly correlated observations.

2.2.1 Exponentially Weighted Moving Average Control Charts

The exponentially weight moving average (EWMA) technique has been widely used in order to monitor the process mean, since it weighs the average of all past and present observations. Therefore, this approach is deployed to keep track of the time series model, which is highly correlated. EWMA is defined as

$$Z_i = \lambda X_i + (1-\lambda) Z_{i-1} \tag{2.9}$$

where λ is the constant factor ($0 < \lambda < 1$), Z_i is the predicted value of process mean at time i, and X_i is the observed value of process mean at time i. The EWMA statistic has been extensively utilized to construct control charts with control limits or

$$\mu_0 + L\sigma\sqrt{\frac{\lambda}{2-\lambda}} \tag{2.10}$$

Lucas and Saccucci (1990) studied the performance of EWMA control charts by considering average run lengths at different λ and L. The selection of EWMA parameters and control charts can be done by identifying ARL_0 and ARL_1; corresponding values of λ and L are chosen later to achieve the desired average run length.

Montgomery and Mastrangelo (1991) suggested that EWMA charts are appropriate for monitoring correlated data, since the EWMA statistic is powerful in predicting the one-step-ahead value of the data, especially when the data structure follows ARIMA (0, 1, 1) or IMA (1, 1). EWMA control charts also work well for other ARIMA family types of data, e.g., AR (1), if the appropriate choice of λ is selected.

Montgomery (2001) suggested that the optimal range of λ should lie between 0.05 and 0.25, so the values of λ are always set at 0.05, 0.1, or 0.2. The rule of thumb in selecting λ is that the small value of λ should be used to detect small shifts, and this value is varied due to the size of the shift. For the width of the control limit, Montgomery recommended setting L at 3, since it works well in detecting a shift in many situations.

Box and Luceno (1997) suggested the use of the EWMA approach to forecast and keep track of the process mean, since it has been proved to be successful in estimating various time series. Even though it is not the perfect estimating technique, EWMA is efficient in predicting non-stationary time series. Moreover, if the disturbances follow IMA model, EWMA is the optimal estimator, because it provides the lowest minimum mean squared error (MMSE). This can be proved by considering the mean squared error

$$MSE = \sigma_y^2 - (\mu - T)^2 \tag{2.11}$$

If the EWMA equation is arranged in the form of IMA (1, 1) and the error is a white noise process, then the MSE of EWMA would be optimal since the white noise has a zero mean with the smallest variance, or

$$Z_{t+1} - Z_t = a_{t+1} - \theta a_t \tag{2.12}$$

Moreover, since the EWMA represents a recursive process, it can adapt itself to the system dynamics. Using the EWMA to exponentially weight the previous data makes sense and proves that EWMA is optimal (causing the lowest minimum mean square error) if the disturbance in the process follows the IMA (1, 1).

EWMA charts are more robust to the violation of the normality assumption than traditional control charts (Borror et al., 1999). If the normality holds, the in-control average run length (ARL_0) would be located at 370.4, because it corresponds to the 3-sigma control limits.

For EWMA charts, the value of L (width of the control limit) and lambda (weight factor) are selected in order to maintain the desired ARL_0 (370.4). Instead of using the normally distributed data, the observations are generated from t and Gamma distributions. The purpose of having different types of distribution is to observe the detection ability of control charts when the data is not normally distributed. When observations follow the Gamma distribution, the in-control ARL from the EWMA is much larger than Shewhart or moving range (MR) charts, especially when the weight factor is small (0.05). In this case, the result shows that the capability of the Shewhart chart is severely deteriorated when the data is based on the Gamma distribution. However, if a mean shift occurs in the Gamma distributed data, EWMA charts would have smaller out-of-control ARL (ARL_1) than MR charts, when the shift size is smaller than 1.5 standard deviations (mean and standard deviation of the process are known). The MR charts seem to have a better performance in the case of shifts at 2.5 and 3.0 standard deviations.

Another scenario is that the data is t-distributed at various degrees of freedom (from 1 to 50). If the process is in control, the ARL_0 of EWMA charts is close to 370.4, especially when the number of degrees of freedom is high (the shape of the t-distribution is more like the normal distribution). On the contrary, the MR charts still have higher rates of signaling false alarms than EWMA charts. When a shift occurs in the t-distributed data, the performances of both EWMA and MR charts are similar to the Gamma-distribution case, i.e., the MR chart is more sensitive to a large shift size, while the EWMA chart is more appropriate in detecting small shifts.

Performances of the traditional Shewhart charts and EWMA charts, when the process is correlated, were compared in an experiment by Lu and Reynolds (1999). In this experiment, the weighting factor of control chart was set at 0.2. The experimental data was simulated by following the autoregressive model ARMA (1, 1), since Box Jenkin’s ARIMA model has been proved to be successful in fitting correlated processes. Two types of EWMA charts (residual and observation) were utilized in order to monitor this data. Results show that residual EWMA charts are appropriate when detecting small shifts, while the observation chart is for larger shifts. Moreover, the residual chart is more sensitive to detect a shift after a few sample points. A second experiment compared the performance of both Shewhart and EWMA charts when the process is highly correlated and there is a shift. After the Shewhart and MR charts were applied to the data, the control limits were too tight and signaled many false alarms, even when there was no shift. To the contrary, the EWMA chart relaxed the tightness of the control limits and signaled less false alarms, but the ARL was still lower than the desired level. Lu and Reynolds (1999) concluded that the ARL value could be improved if the parameters of the EWMA are based on large initial data, since the accuracy of the time series estimation depends on the sample size.

Another comparative study has been conducted by English et al. (2004) to compare the performance of traditional control charts (X bar) and EWMA charts for the autocorrelated processes. Shewhart control charts are appropriate only when the process output is independent and in statistical quality control, but not in the environment in which the data is correlated. According this experimental study, process output was simulated by generating numbers based on the autoregressive process ARMA (1, 1), and these two control charts were utilized to monitor the output. The average run lengths of each control chart were measured when there were different shift sizes occurred in the process (0, 0.5, 1, 1.5, 2, 2.5, 3 sigma: standard deviation of the error from the underlying process mean). Results show that the ARL of the $\overline{X}$ charts was much lower than that of the EWMA charts, especially when the output is highly correlated. When there is a mean shift, $\overline{X}$ charts need twice as many sample sizes as EWMA charts in order to detect the shift.

Chen and Elsayed (2002) addressed the benefit of using EWMA as a technique to filter the noise and estimate the true mean of disturbance in a system. If the disturbance follows IMA (1, 1), the EWMA is the optimal mean estimator of the process. They chose the optimal EWMA parameters (weight constant) by minimizing the expected difference between the estimated and actual mean of the disturbance. Moreover, the step change or mean shift in the disturbance can be detected by considering the dramatic change in the autocorrelation coefficients.

Triantafyllopoulos et al. (2005) addressed the fact that the traditional EWMA's capability to forecast would be downgraded if the process is not modeled correctly, especially when there is a sudden shift in the process. In this case, the EWMA lags behind the shift, and the adjustment for error might not be effective. For this reason, they proposed the application of a local-level model (a form of state-space model) in order to compensate for the variability and shift, since this method is based on the recursive Kalman gain factor which can adapt the predicted value to the changed output. Except for the traditional EWMA control chart, the following authors proposed special types of EWMA.

Mastrangelo and Montgomery (1991) introduced the moving centerline exponentially weighted moving average (MCEWMA) statistic, which is adaptive to the autocorrelated structure of the data. For MCEWMA, the EWMA statistic is used to calculate the process mean and mean error at a given time, or

$$\hat{M}_t = \alpha \, | e_t | + (1-\alpha)\hat{M}_{t-1} \tag{2.13}$$

The control limits for MCEWMA are shown as

$$(LCL, UCL) = \hat{Y}_t \pm (1.25K)\hat{M}_t \tag{2.14}$$

where K is the number of standard deviations, $\hat{Y}_t$ is the predicted process mean at time t,and M_t is the mean error at time t. The standard deviation of the error for MCEWMA is calculated from

$$\hat{\sigma}_e(t) \approx 1.25\hat{M}_t \tag{2.15}$$

Shaughnessy and Haugh (2002) studied the adaptive EWMA method when the disturbance is correlated, and they identified the fact that the appropriate value of λ depends on the underlying time series model of disturbances. In order to reduce the prediction error, the approach introduced by Cox (1961) has been utilized to achieve the optimal value of λ. If the noise process follows AR (1), the weight factor λ is

$$\lambda = 1 - \frac{1}{2}(\frac{1-\phi}{\phi}); \frac{1}{3} < \phi < 1 \tag{2.16}$$

However, when the disturbances can be modeled by IMA (1, 1), the optimal λ is equal to 1-θ.

Guo and Chen (2002) proposed a dynamic tuning EWMA (DT-EWMA) that can estimate the process mean and adapt it to the change from the random shifts and linear drifts. The mean estimator follows the standard EWMA equation as

$$\hat{\mu}_t = WY_t + (1-W)\hat{\mu}_{t-1} \tag{2.17}$$

where W is the EWMA control parameter. According to traditional EWMA methodology, the value of W is fixed, and it is not adaptive to any change in the process. For this reason, the ability to detect a shift or drift would be downgraded. Guo and Chen provided the algorithm to choose the optimal W, which starts by utilizing the control chart to detect a shift. If a shift is detected at time t = t*, then the optimal W at time t is equal to 1 / (t – t* + 1), since it is proved to minimize the mean squared deviation (MSD) or

$$MSD = E\{(\hat{\mu}_t - \mu_t)^2\} \tag{2.18}$$

When there is a shift, the value of W would increase (provide more weight to the forecast value instead of the previous mean). However, if there is no shift, W would decrease until it is equal to zero. This situation can prevent the tuning algorithm to counter with the linear drift, so the minimum value of W has been set. In this study, the process mean estimation performance was assessed by using normalized mean squared estimation error (MSEE / σ^2) or

$$MSEE / \sigma^2 = \{\frac{1}{n}\sum_{j=1}^{n}(\hat{\mu}_{t^*+j-1} - \mu_{t^*+j-1})^2\} / \sigma^2 \qquad (2.19)$$

where σ is the standard deviation of the process mean.

Another performance measurement is the deviation from the target, which can be formulated in the form of normalized mean squared error (MSE / σ^2) as

$$MSE / \sigma^2 = \{\frac{1}{n}\sum_{t=1}^{n}(Y_t - T)^2\} / \sigma^2 \qquad (2.20)$$

The result of the experiment shows that the DT-EWMA algorithm is easy to implement and is effective to adjust the process affected by random shifts and linear drifts.

2.2.2 Cumulative Sum Control Charts

Since the Shewhart control chart uses only information from the last observation, it is not sensitive to small shifts in the process. For this reason, a cumulative sum (CUSUM) chart is utilized as an alternative when the shift size is small. A CUSUM chart may be constructed for individual observations, which always occur in practice (Montgomery, 2001). The CUSUM statistic can be computed as

$$C_i^+ = \max[0, x_i - (\mu_0 + K) + C_{i-1}^+] \qquad (2.21)$$

$$C_i^- = \max[0, (\mu_0 - K) - x_i + C_{i-1}^-] \qquad (2.22)$$

where μ_0 is the target value, K is the reference value (often chosen about halfway between the target: μ_0 and out-of-control value of the mean: μ_1), and C_i^+ and C_i^- are the one-sided upper and lower CUSUMS. If either C_i^+ or C_i^- exceeds the decision interval (H), the process is considered to be out of control. Using H = 4 or H =5 and K = 0.5 always provide a CUSUM that has good

ARL properties against a shift of about 1σ in the process mean. The average run length of the CUSUM chart can be calculated by utilizing Siegmund's approximation as

$$\left.\begin{aligned} ARL &= \frac{\exp(-2\Delta b) + 2\Delta b - 1}{2\Delta^2} \\ &= \frac{\exp(-2*(\delta - K)*(H + 1.166) + 2*(\delta - K)*(H + 1.166) - 1)}{2*(\delta - K)^2} \end{aligned}\right\} \quad (2.23)$$

where δ is the shift magnitude.

2.3 Automatic Process Control

Automatic process control and statistical process control are two strategies that are utilized in different industries. While SPC originates from the parts industry, APC is widely used in the process industry. However, these two strategies share the same objective – quality improvement of the product. The assumption underlying SPC is that observations are independent and completely stationary, while APC focuses on keeping the process mean on target. For this reason, APC disregards the correlation of data and adjusts the process with the frequency that ensures the lowest deviation from the target (Montgomery et al., 1994).

2.3.1 Minimum Mean Squared Error Controllers

There are many types of controllers (feedforward, feedback, and adaptive), and each one is appropriate for specific scenarios. However, the minimum mean squared error controller is one of the industrial standards (Montgomery et al., 1994). The MMSE controller is the application of the one-step-ahead forecasting statistic to compensate for disturbance. The objective is to minimize the mean squared error of the output response. Mean squared error is the average of the squared deviation from the target or the summation between the output variance (σ_y^2) and the square of the target deviation $(\mu\text{-T})^2$ or

$$MSE = \sigma_y^2 + (\mu - T)^2 \quad (2.24)$$

Intuitively, the process mean (μ) should be set on the target in order to reduce the MSE. Box and Luceno (1997) proved that the EWMA statistic is the best forecast to providing the lowest MSE, or

$$\hat{Z}_{t+1} = \lambda Z_t + \theta \hat{Z}_t ; \lambda = 1 - \theta \qquad (2.25)$$

where θ is a smoothing constant. For this reason, the EWMA is also called the MMSE statistic. The nature of this statistic is a recursive process that automatically updates the forecast value when the new observation is obtained. Therefore, the estimated value is based on observation in the past.

MacGregor et al. (1984) pointed out that the MMSE controller uses the MMSE statistic to predict and compensate for the stochastic disturbances in the process (whitening the autocorrelated data). If the disturbance model is correctly specified, the residual after the adjustment would be uncorrelated. Moreover, another important property of the MMSE controller is the ability to optimally reduce the error variance.

Box et al. (1994) proposed that the optimal control action can be derived in the form of the minimum mean squared error as

$$X_t = \frac{\theta - \phi}{1 - \phi B} a_t \qquad (2.26)$$

when the disturbance model follows the ARMA (1, 1) as

$$D_t = \frac{1 - \theta B}{1 - \phi B} a_t \qquad (2.27)$$

However, if the disturbance model follows the ARMA (1, 1, 1) model or

$$D_t = \frac{1 - \theta B}{(1 - \phi B)(1 - B)} a_t \qquad (2.28)$$

then the adjustment signal from the MMSE controller is

$$X_t = \frac{\theta - \phi - 1 + \phi B}{(1 - \phi B)(1 - B)} a_t \tag{2.29}$$

where θ is the moving average coefficient, ϕ is the autoregressive coefficient, and a_t is the random error. The disturbance model IMA (1, 1) is considered a special case of ARMA (1, 1, 1) where ϕ = 0.

2.3.2 Exponentially Weighted Moving Average Controllers

EWMA controller is a run-by-run (RbR) controller, which operates by using an output feedback to regulate the process. The operation mode of the controller can be categorized into two modes, rapid and gradual. In the rapid mode, the controller compensates for a disturbance only when the large disturbance is detected. Otherwise, the controller is in the gradual mode, which continuously responds to the gradual drift in the process (Ingolfsson and Sachs, 1993).

Ingolfsson and Sachs (1993) investigated the stability and sensitivity of the EWMA controller when the process is in statistical quality control (gradual mode) by applying the EWMA statistic in the form of

$$a_t = \alpha + \beta x_t \tag{2.30}$$

with three types of processes: a deterministic first-order process as

$$Y_t = \alpha + \beta x_t \tag{2.31}$$

a deterministic second-order process as

$$Y_t = \alpha + \beta x_t + \delta x_t^2 \tag{2.32}$$

and a drifting process with noise as

$$Y_t = \alpha + \beta x_t + k\sigma t + e_t \tag{2.33}$$

where Y_t is the output response at time t, X_t is the control signal at time t, α and β are the regression coefficients, e_t is the random error at time t, and σ is the variance of e_t. The asymptotic mean squared deviation (AMSD) was used as the criteria to measure deviation from the target of the output response, or

$$MSD = E\{(Y_{\infty} - T)^2\} \tag{2.34}$$

The general solution for α (ratio of convergence) and γ in each type of process was derived in order to obtain the operating region when the output response (Y_t) swiftly converges to the target value (T).

The EWMA controller is widely used in the semiconductor industry and batch-oriented (run-to-run) processes, which need frequent adjustment since there are always disturbances in their quality characteristics. The EWMA statistic is powerful in forecasting one-step-ahead value of the process response (Castillo, 2001).

Castillo (2002) pointed that the EWMA statistic can be utilized to manipulate the control signal and compensate for disturbances, since it is easy to tune and robust to the non-normality of disturbances. Moreover, the EWMA controller is equivalent to the widely used integral (I) controller. This type of controller is extensively used in the industry, especially as a flow controller in order to control flow processes, since it is an efficiency tool for forecasting one-step-ahead disturbance and compensating dead time (Venkatesan, 2001). Dead time is caused by process disturbances, and reduces the response speed of the output due to input adjustments. For the integral controller, the output response converges to the target in a short period if the sampling period is short because of the large control signal. Moreover, it provides the minimum mean squared error if the drift disturbance is IMA (1, 1).

Castillo (2002) investigated the performance of the EWMA controller when there are IMA (1, 1) disturbances, a deterministic trend (DT) and a random walk with drift (RWD):

$$N_t = N_{t-1} - \theta\varepsilon_{t-1} + \varepsilon_t \tag{2.35}$$

$$N_t = \delta t + \varepsilon_t \tag{2.36}$$

$$N_t = N_{t-1} + \delta + \varepsilon_t \tag{2.37}$$

The minimum AMSD of processes with each type of disturbance was derived. The EWMA controller guarantees the lowest AMSD if the process disturbances follow IMA (1, 1). If a DT or RWD represent the process, then the value of λ directly depends on the drift size in the process in order to reduce the AMSD.

2.3.3 Proportional-Integral Controllers

The most widely used device in the process control industry is the three-term controller, which is the linear combination of three modes of control action (Box et al., 1994). The equation for this controller is in the form of

$$X_t = k_o + k_D \frac{de_t}{dt} + k_P e_t + k_I \int e_t dt \tag{2.38}$$

where X_t is the control action from the device, e_t is the output error at time t, k_0 is a constant, k_D is the derivative gain, k_P is the proportional gain, and k_I is the integral gain. If the value of k_D is set at zero, the proportional-integral-derivative (PID) controller will become a proportional-integral (PI) controller, which is proportional to the last error and the sum of all previous errors, or

$$X_t = k_P e_t + k_I \sum_{k=0}^{t} e_k \tag{2.39}$$

The PI controller is the most popular control method among all other control schemes, because of its robust property to disturbance in the model. This is not limited to stationary disturbances alone, but non-stationary ones as well. Moreover, it has a simple structure and is easy to implement in the process (Tsung et al., 1998). Venkatesan (2001) also confirmed that the PI controller has been widely used for a long time in the industry in order to control various variables, e.g., pressure, speed, torque, velocity, etc.

Box and Luceno (1997) derived the constrained-input PI controller adjustment x_t,which depends on only the last two observations Y_t and Y_{t-1}. The objective of this design is to reduce variance of the output response and control signal. Therefore, the controller is designed to minimize these variances by following the constraint as

$$\min_{G,P}\{\frac{Var(Y_1)}{\sigma_\varepsilon^2}+\alpha\frac{\text{var}(\nabla X_t)}{\sigma_\varepsilon^2}\} \tag{2.40}$$

Then, in order to achieve the best control signal, the optimal value of a linear coefficient combination is obtained as

$$\nabla X_t = -G(Y_t + P\nabla Y_{t-1}) = c_1 Y_t + c_2 Y_{t-1} \tag{2.41}$$

Jiang et al. (2002) used the signal-to-noise (SN) ratio (process mean after the shift/standard deviation of the error) as the criteria to choose the optimal parameters of the PI controller with ARMA (1, 1) disturbances.

2.4 Integrated Statistical Process Control Systems

Montgomery et al. (1994) used Shewhart charts along with EWMA and cumulative sum (CUSUM) charts to monitor the simulated data with different shift sizes. The MMSE controller was used to compensate for disturbances. Both the average run length and the average squared deviation from the target were used to measure performance. Based on a simulated study, they concluded that the combination of SPC and APC results in a smaller average squared deviation than running APC solely. In addition, Shewhart control charts outperformed EWMA and CUSUM charts in terms of detecting a sudden shift, especially when the shift size is large. However, when there is a trend, EWMA and CUSUM charts indicated better performance than

Shewhart control charts. For the average run length, CUSUM has a smaller ARL than that of EWMA and Shewhart charts.

Tsung et al. (1998) introduced a performance measurement, σ^2_e / σ^2_a (representing the variance of white noise and output, respectively), which is utilized to derive the optimal values of ARMA (1, 1) and ARIMA (1, 1, 1) respectively. From a simulation, results showed that the PI controller is more robust than the P, I, and MMSE controllers since it can be operated to achieve $\sigma^2_e / \sigma^2_a = 1$ in a broader operation area (different values of ϕ and θ), although disturbances are estimated by ARMA or ARIMA. Moreover, Jiang and Tsui (2002) reported that the PI controller can still work well when the model of disturbances is misidentified or gain values of the process are changed, since the transient and signal-to-noise ratios of the control action for the PI controller is higher than the MMSE controller. For the MMSE controller, they noted that monitoring the output signal is more efficient than control action, when ϕ is positive and shift size is large, since its transient ratio is high and ARL is small. The control signal monitoring seems to be a better choice when the shift is small (less than $3\sigma_e$), since the ARL of the output signal is larger than the control signal. However, when the value of ϕ is negative, observing the output signal is more efficient than the control signal, since it has a higher signal-to-noise ratio and lower ARL. For the PI controller, if a chart cannot detect a shift in the transient state, then observing a control action would be more likely because of the low ARL.

Zhang (1998) introduced the exponential weight moving average charts for stationary process in order to monitor the autocorrelated data. Since the autocorrelated structure has been accounted for the calculation of the control limit, the range of the limit is wider than the typical EWMA or MCEWMA charts proposed by Montgomery and Mastrangelo (1991). The proposed EWMA charts were tested with data simulated from AR (1) and ARMA (1, 1). Results show that

EWMAST has a better performance than EWMA or MCEWMA charts, when the autocorrelation is not positively strong and the shift size is small to medium (0-2 standard deviation of the process error).

Nembhard (1998) used a simulating dynamics system (SIMULINK) to model first- and second-order dynamic processes, with disturbances modeled using the ARMA model. Dynamic processes were modeled by continuous state-space equations, which are widely used in control engineering. SPC and APC were integrated in the form of a PI controller, Shewhart charts and EWMA charts in order to control and monitor the process. Different performance measures were investigated, including the squared error of the output process, number of adjustments, average adjustment, and number of alarms. It was shown again that for the first-order dynamic process, the PI-controller and Shewhart charts have better performance than using only the PI controller or no control policy in the system.

Janakiram and Keats (1998) studied the integration between SPC and APC in a powder-loading operation for an automobile air-bag initiator. In this integration, Shewhart charts ($\overline{X}$ and R charts) were utilized to monitor the weight of explosive powder, while disturbances (humidity) were modeled using the autoregressive model, ARMA (0, 1, 1) or IMA (1, 1). The control action is manipulated by the MMSE controller in order to cancel predicted stochastic disturbance in the form of

$$X_t = -\frac{\lambda}{g}\sum_{j=1}^{t} e_j \tag{2.42}$$

$$e_j = Y_j - T \tag{2.43}$$

where g is the gain of the MMSE controller, λ is the gain of the EWMA parameter, Y_j is the process output at time j, and T is the target. The correlogram of residual from the disturbance model shows that all autocorrelation coefficients at every lag are not significantly different from zero, which means that IMA is the appropriate model to estimate disturbances. The performance evaluation shows that the integration between SPC and APC can monitor and control stochastic disturbances effectively.

Capilla et al. (1999) deployed the integration of SPC and APC to monitor and control polymer viscosity by considering the temperature of the reactor as a controllable factor. In this case study, an MMSE controller was used to adjust the process, while disturbances were modeled using AR (1) and ARMA (1, 1). Two types of charts, EWMA and CUSUM, were utilized in order to detect assignable causes. Performance evaluation based on simulated data showed that utilizing SPC and APC can reduce the variability of viscosity, while significant disturbances can be detected or compensated rapidly.

Jiang et al. (2000) proposed the ARMA chart to monitor disturbances in an APC-controlled system. When a shift size was small (less than 0.75 σ_a; σ_a being the standard deviation of an independent, identically distributed process, a_1, a_2,…), the ARMA charts detected a mean shift more efficiently than EWMA charts.

Jiang and Tsui (2000) introduced a signal-to-noise ratio in transient state and steady state to perform an error measurement on the ISPC system as

$$R_T^Z = \frac{\mu_T^Z}{\sigma_Z}, R_S^Z = \frac{\mu_S^Z}{\sigma_Z} \tag{2.44}$$

where R_T^Z is the signal-to-noise ratio in the transient state, R_S^Z is the signal-to-noise ratio in the steady state, μ_T is the process mean in the transient state, μ_S is the process mean in the steady state, and σ_z is the standard deviation of the chart statistic. A shift in the mean was detected rapidly when the transient ratio was high (4 or 5). However, the authors noticed that the detection was delayed if the transient ratio was less than 3 and the steady-state ratio measured the shift detection capability during the steady state.

Nembhard et al. (2001) studied the consequences of integrating ARIMA-modeled disturbances and the first-order dynamic system by using a SIMULINK program. In this scenario, since the dynamic process was included in the model, the startup or transient-state- and steady-state- behavior were taken into consideration. Output of the process was monitored by SPC (EWMA charts), APC, and integration between SPC and APC in order to measure the performance of each approach. Results of the simulation show that deploying the APC policy solely can reduce the sum squared error significantly, but the integration between SPC and APC can greatly decrease the number and magnitude of adjustments. Moreover, using SPC and APC together might outperform deploying SPC or APC individually.

Jiang and Tsui (2002) analyzed both the process output (e) and control action (X) , which are under the minimum mean squared error and proportional-integral control policies. If ARMA (1, 1) is the chosen model to estimate disturbances, then signal-to-noise ratios for the MMSE controller are

$$R_T^e = \mu, R_S^e = \mu\left|\frac{1-\phi}{1-\theta}\right| \tag{2.45}$$

$$R_T^X = \mu\sqrt{1-\phi^2}, R_S^X = \mu\frac{\sqrt{1-\phi^2}}{1-\theta} \tag{2.46}$$

where μ is the mean shift pattern. The transient and steady-state ratios depend on the value of ARMA parameters. The transient ratio cannot be tuned too high since it reduces the steady-state ratio. These two signal-to-noise ratios were deployed as criteria in the comparison of chart performance for detecting the shift. Jiang and Tsui utilized SN ratios to predict the performance of Shewhart and EWMA charts.

Gultekin et al. (2002) proposed the idea of integrating SPC and APC to minimize the output variation of continuously stirred tank reactors. In this case, the PI controller was selected to make an adjustment in the process. The integration between these two techniques was categorized into four different phases: disturbances detection using SPC, estimation of time of shift, determination of type of disturbance, and estimation of magnitude of disturbance and process adjustment. The process model included three types of disturbances: determinisitic (shift in the process), random input, and random output. In the detection phase, Shewhart and CUSUM charts were utilized to signal the out-of-control status when there was a shift in the process mean. The estimate of disturbance magnitude was obtained by deploying the maximum likelihood estimator to minimize the mean squared error for disturbance identification. By using a simulation, the integration showed that it can reduce the mean squared error by 81 percent, compared to utilization of the PI controller alone.

2.5 Economic Model of ISPC Systems

Elsayed and Chen (1994) proposed an economic model for $\overline{X}$ control chart using the quadratic loss function. The cost model consisted of four parts: inspection cost, cost of investigating false alarms, cost of finding an assignable cause, and quality cost during the in-control and out-of control period (due to Taguchi's loss function). Their model was based on Duncan's economic design of the $\overline{X}$ chart.

Jiang and Tsui (2000) introduced the economic model for the integration between SPC and APC. They assumed that the production process can move between two states, in control and out of control, while the duration of time in the in-control state follows the geometric distribution and the time interval from the beginning of the in-control state to the adjustment of the out-of-control state is considered as a production cycle. Moreover, it was also assumed that the shift occurs once every cycle, and no other shift occurs before the previous shift is detected and removed. The identification of any assignable causes was perfect, and there was no adjustment for any false alarm signals. The in-control cost is the sum of the quadratic loss of the in-control items and the expected diagnosis cost for false alarms, while the out-of-control cost is the sum of the expected quadratic loss of the out-of-control items, the diagnosis cost (cost of identifying assignable cause from the signal), and the adjustment cost (cost of correcting assignable causes). Another assumption was that the subgroup size is one, and the sampling frequency is fixed. When the shift size is a step mean shift, the diagnosis cost is zero. Different types of SPC (individual Shewhart and EWMA charts) could be applied with this model. The economic model is equal to

$$L_A = (\sigma^2 + \mu^2) - \frac{\mu^2 / p}{1/p + ARL_1} \qquad (2.47)$$

where L_A is the average quality cost, σ^2 is the variance of the white noise, μ is the size of the mean shift, p is the probability that an assignable cause occurs, and ARL_1 is the out-of-control average run length.

Wang and Yue (2001) proposed an economic model for the APC. Their assumption was that the disturbance on the quality characteristic follows IMA (1, 1) and the adjustment by automatic control aims to bring the quality characteristic back to the target. However, it cannot eliminate the effect of the previous wear and tear, which is considered in this model. Two types of APC adjustment were considered: minor and a major. A minor adjustment brings the quality back to the target but cannot eliminate the effect of wear and tear, while a major adjustment is one that not only keeps the characteristic on target but also removes the effect of wear and tear from the previous operations. As a result, the total expected quality cost consists of the cost of major adjustment, the cost of minor adjustment and the off-target cost. Then, the unit quality cost (UQC) of a product is derived as

$$UQC = \frac{C_{major}}{m.n} + \frac{C_{\min or}}{m} + \frac{\sum_{i=0}^{n-1} k[\sigma_i^2 + (1-\theta)^2 \sigma_i^2 \frac{m-1}{2}]}{n} \tag{2.48}$$

$$k = A / \Delta^2 \tag{2.49}$$

where C_{major} is the cost of major adjustment, C_{minor} is the cost of minor adjustment, m is the number of units produced between two minor adjustments, n is the number of minor adjustments in a production cycle, k is the proportionality constant, A is the loss because of a non-conforming item produce, Δ is the tolerance of the product characteristic, θ is the moving average parameter, and σ_i^2 is the variance of random error in the IMA (1, 1) model. Another important assumption is that a major adjustment is needed after n minor adjustments have been done.

Ben-Daya and Duffuaa (2002) incorporated Taguchi's loss function with the economic design of the control chart in order to construct a new economic model. The assumption underlying their model is that the production starts in the in-control sate (mean μ and standard deviation σ). However, an assignable cause with magnitude δ would result in an average shift in the process mean from μ to μ ± δ. The $\overline{X}$ control chart with centerline μ and control limits $\mu \pm k\sigma / \sqrt{n}$ was utilized to monitor the process. Once the process is out of control, the search for an assignable cause is initiated. After it has been eliminated, the process is back to the state of in control. It was assumed that the occurrence of an assignable cause follows a Poisson process, while the time interval between occurrences is exponentially distributed with mean 1/λ hours. The assumption of this model does not hold for the out-of-control shift due to tool wear and operation fatigue. As a result, the expected cost per production cycle E(C) was derived as

$$E(C) = \frac{(a_1 + a_2 n)E(T)}{h} + a_3'\frac{\alpha}{\lambda h} + a_3 + \frac{1}{\lambda}PL_{in}(n,k) + [E(T) - \frac{1}{\lambda}]PL_{out}(n,k) \qquad (2.50)$$

$$L_{in}(n,k) = \frac{A}{\Delta^2}\int_{\mu - k\sigma/\sqrt{n}}^{\mu + k\sigma/\sqrt{n}} (y-\mu)^2 f(y)dy \qquad (2.51)$$

$$L_{out}(n,k) = \frac{A}{\Delta^2}\int_{\mu-\Delta}^{\mu - k\sigma\sqrt{n}} (y'-\mu)^2 f(y')dy' + \int_{\mu + k\sigma\sqrt{n}}^{\mu+\Delta} (y'-\mu)^2 f(y')dy' \qquad (2.52)$$

$$E(T) = \frac{1}{\lambda} + \frac{h}{1-\beta} - \tau + gn + D \qquad (2.53)$$

where a_1 is the fixed sampling cost, a_2 is the variable sampling cost, a_3 is the cost of finding an assignable cause, a_3' is the cost of investigating a false alarm, a_4 is the expected out-of-control cost, n is the sample size, h is the length of the sampling interval, k is the control limit coefficient, μ is the process mean, δ is the shift magnitude from the mean, λ is the parameter of the exponential distribution governing the control period, Δ is the acceptable deviation of the quality characteristic from its target value, E(T) is the expected length of a production cycle, β is the P(not exceeding control limit-process out of control), α is the P(exceeding control limit-process in control), h / (1-β) - τ is the expected length of the out-of-control period, gn is the number of sample size taken until an assignable cause is detected, and D is the time to search for the assignable cause.

Duffuaa et al. (2004) utilized Taguchi's loss function to decide whether the APC should be performed or the process should be left as-is. For the integration, the adjustment cost was compared to the loss due to variability. The cost of an action was calculated based on the manipulation of control variables or the APC process. The deviation from the target was detected by SPC, and the loss due to the variability was derived. They proposed two different economical models when SPC and APC are integrated. For the first model, the cost of APC is diagnosis cost and adjustment cost per product or

$$Cost(APC) = \frac{B}{n} + \frac{C}{\overline{u}} \tag{2.54}$$

where B is the cost per measurement of product characteristics, C is the cost per adjustment, n is the measurement interval, and $\overline{u}$ is the predicted average number of products between successive adjustments. The loss per product comes from variation when the production is in control, variation when the production is out of control, time lag, and the measurement error. If the production is in control during the diagnosis, the mean squared deviation from the target is approximated by $D^2/3$ (the variance of the uniform distribution is $[(m+D)-(m-D)]^2/12$). Moreover, the variation during the out-of-control period is proportional to the number of defective units between successive diagnoses and the time lag. The cost of SPC is based on the quadratic loss function and is shown as

$$Cost(SPC) = \frac{A}{\Delta^2}\left[\frac{D^2}{3} + \frac{D^2}{\overline{u}}\left(\frac{n+1}{2} + l\right) + \sigma_m^2\right] \tag{2.55}$$

where A is the reworking/scrapping cost, D is the adjustment or control limit, Δ is tolerance of product characteristics, $\frac{n+1}{2}$ is the average number of defective units between successive diagnoses, σ_m^2 is the measurement error, and *l* is time lag (in units).

According to the second model, the cost of APC is equal to the cost of quality loss when the sample mean is plotted outside the control limit $m \pm D$. The other model was derived from the variance of the process as

$$C = \frac{A}{\Delta^2}\nu^2 \tag{2.56}$$

$$\nu^2 = (\frac{n+1}{2} + l)\frac{D^2}{\bar{u}} \tag{2.57}$$

where C is the adjustment cost, ν^2 is the mean squared deviation, A is the rework/scrap cost, and Δ is the half tolerance of product characteristics.

Yang and Sheu (2007) extended Lorenzen and Vance (1986)'s model to construct a quality cost model for the integration between SPC and APC. Their assumption is that a production cycle is the time from the beginning of the in-control state until the adjustment of the out-of-control state. A signal is triggered when there is an occurrence of an assignable cause. The time between each occurrence of a special cause is assumed to be exponentially distributed. The shift occurs only once in a production cycle, and another shift does not occur before the previous shift is detected and removed. Another assumption is that the process returns to the initial state of statistical control after the elimination of the assignable cause (renewal-reward process).

$$TC = \frac{C_0}{\theta} + C_1 + (-\varphi + nE + h * ARL_1 + \gamma_1 T_1 + \gamma_2 T_2) + \frac{SY}{ARL_0} + W + (\frac{a+bn}{h}) * (\frac{1}{\theta} - \varphi + nE + h * ARL_1 + \gamma_1 T_1 + \gamma_2 T_2) \tag{2.58}$$

$$\varphi = \frac{1-(1+\theta h)e^{-\theta h}}{\theta(1-e^{-\theta h})} \tag{2.59}$$

where TC is the expected total cost per cycle, C_0 is the quality cost per hour while producing in control, C_1 is the quality cost per hour while producing out of control, h-φ is the expected time between a shift and the next sample, E is the time to sample and chart one item, γ_1 is 1 if production continues during the repair of the process and 0 otherwise, T_1 is the expected time to discover the assignable cause, T_2 is the expected time to repair the assignable cause, $S = e^{-\theta h}/(1-e^{-\theta h})$ is the expected number of samples while in control, Y is the cost to investigate false alarms, W is the cost of locating and repairing an assignable cause, a is the fixed cost per item, and b is the cost per unit sampled.

2.6 Taguchi's Model

Taguchi (1986) defined quality loss as the loss to society caused by the product after it is shipped out. He proposed a quadratic loss function to estimate the cost due to the deviation from the target of the product characteristic. The expected loss E (L) is proportional to the squared deviation from the target value and the process standard deviation, or

$$E(L) = \frac{A}{\Delta^2}[\sigma^2 + (\mu - T)^2] \tag{2.60}$$

where E(L) is the expected loss per unit, A is the per unit cost of repair or rework, Δ is one half the tolerance spread, σ is the process standard deviation, μ is the process average, and T is the target value. It is interesting to note that the value of product is reflected in the repair and rework cost per unit, since the cost of product should be proportional to the cost due to deviation.

CHAPTER 3

DISCUSSION

The reduction of process variability is one of the most important aspects of improving process performance. Two statistically based approaches were introduced to solve this problem. The first approach is the statistical process control. This approach has the ability to effectively reduce variability by detecting special causes in the process. Once an assignable cause is removed, the process is in the state of statistical control and the variability is reduced. According to the literature, SPC has been successfully used to monitor discrete parts manufacturing.

However, when SPC is used with the continuous processes, its performance deteriorates. The assumption underlying SPC is that there is the possibility of bringing any processes back to the state of statistical control. The in-control state means that the process has only random variation around the fixed target so that the frequent adjustment is not required. Deming (1986) introduced the funnel experiment to demonstrate how adjustment affects processes in different scenarios. In that experiment, a funnel was mounted over a target, or the bull's eye, and a number of marbles were dropped through the funnel with the objective of hitting the target. Based on this analogy, Deming evaluated four adjustment rules. Rule 1: there is no adjustment for the position of the funnel. Rule 2: deviation is measured from the target and the point that the marble last hits. The funnel is moved in the opposite direction with an equal distance from its current position. Rule 3: the funnel is moved from the target to the position that is opposite to the dropping position of a marble. Rule 4: the position of the funnel is relocated to the latest dropping position of the marble.

Rule 1 is equivalent to the process that is in statistical control, and SPC is a powerful technique to monitor this type of process. For rules 2 and 3, there is an adjustment every time there is deviation from the target. If there is no adjustment in the process, output from the process will be stationary with highly correlated data or non-stationary due to disturbances. This might cause the process mean to wander from the desired target. The difference between these two rules is the control action (both rules uses different approaches to set the control action), which is used to compensate the deviation. Rule 4 is similar to rules 2 and 3, but the adjustment makes the process behave like a random walk model. Control actions in rules 2, 3, and 4 are equivalent to the utilization of APC for controlling a process mean within the target when there is a deviation in the process (process is not in the state of statistical control).

Montgomery (1992) suggested that SPC applies well to rule 1 of the funnel experiment, which requires no adjustment to the funnel (the process is still in control). Rule 2 adjusts the position of the funnel to the opposite direction every time the marble misses the target. This rule has a similar process model as that of APC. Box and Kramer (1992) proposed the idea of using feedback control to compensate for disturbances estimated by statistical forecasting. SPC has been deployed to monitor process output after the adjustment in order to detect an assignable cause, which cannot be compensated for by the controller. Montgomery et al. (1994) supported the idea that SPC can detect an assignable cause from the output rapidly, while APC can effectively keep a process on target.

Under the statistical framework, SPC works under the assumptions that the process is in control and that observations are normally and independently distributed as shown by equation (2.1). For this reason, when all assumptions have been satisfied, the control charts are used to draw conclusions regarding the process's state of statistical control. The standard Shewhart chart, which is the traditional SPC tool, is widely used to monitor most industrial processes, and it is powerful in identifying the special cause which results in a shift in the process mean. Moreover, if the control limits are set appropriately, the average run length or false alarm rate can be easily obtained. Another advantage is that control charts still work well, even when the normality assumption is slightly violated. However, many authors, including Alwan and Roberts (1998), Alwan (1992), Harris and Ross (1991), Montgomery and Mastrangelo (1991), and Maragah and Woodall (1992), pointing out the disadvantage of using SPC in the scenario when observations are correlated (even at a low level of correlation). The impact of correlation on SPC includes the increasing frequency of signaling false alarms and erroneous conclusions regarding the state of the process. Even though EWMA and CUSUM have been developed to monitor the correlated process, they require more statistical and mathematical background than the traditional Shewhart charts.

On the other hand, APC is developed by focusing on the process and assumes that there is another variable to handle the correlation. When the process mean tends to drift or shift from the target, a manipulated process variable is adjusted to correct the process output. A series of adjustments can be done in order to keep the process mean as close to the target as possible. Different techniques have been used to control the process, but these approaches work on the same assumption that the relationship between the input and output processes can be expressed by a specific dynamic model. If the model is correctly specified, APC is the most effective tool to reduce variation in the process.

Box and Jenkin's autoregressive integrated moving average, ARIMA (0, 1, 1) or IMA (1, 1), is usually used as the dynamic model to represent stochastic disturbances, since this type of time series model is non-stationary and flexible enough to fit most disturbances. However, the exact dynamic model of each process is unknown, and the misspecification always leads to an error in the adjustment.

Another critical issue is assignable causes in the process, since controllers make an adjustment when the process mean is off target. It is inexpensive to compensate for random variations, because the magnitude of fluctuation is small. It is not economical to adjust the process when there is a shift, especially a large shift size, since the cost of taking control may significantly increase.

As a result, APC has been integrated with SPC, since these two approaches perform different functions but share the same objective of focusing on variability reduction. APC estimates the disturbances and makes frequent adjustments until the process mean is on target, while SPC detects assignable causes. Many authors (Box and Kramer, 1992; Montgomery et al., 1994; Box et al., 1994; Box and Luceno, 1997; and Castillo, 2002) support the idea of integrating these control policies together in order to improve the process quality. Disadvantages of each technique are eliminated when these two approaches are integrated.

One of the most critical issues regarding the advantage of the ISPC system is the response to process variability. SPC takes action only when assignable causes occur, while APC adjusts processes when they are off target (Montgomery et al., 1994). Crowder et al. (1997) showed that it is normal for the process to be affected by common causes, and rapid control by APC can reduce the process variability effectively. However, the manipulation of APC to compensate for abrupt changes in the process mean or variance due to a special cause is not appropriate because of the large size of shift. In this environment, control charts can be complimentary to APC, since it is a powerful tool to detect and remove a shift from the process.

Another important issue, which supports integration, is the attempt to improve the process quality by combining the advantage of each technique. SPC originated in the discrete parts industry and focuses on manufacturing items with the smallest variation. On the other hand, APC has been widely used in the process industry, where adjustment cost is negligible, and the process mean can be frequently adjusted.

However, Gultekin et al. (2002) reported that a large number of discrete part manufacturers have started to use the automatic control in their processes. For example, the impending chatter is analyzed by using the acoustic data before the milling machine is operated, so the process is automatically adjusted to anticipate the acceptable quality of surface finish.

The performance of the ISPC depends on process variability. According to the literature, the IMA (1, 1) parameter λ is always assumed to be constant. For this reason, performance studies of integration are based on simulated data from only one disturbance model. However, the range of λ in the IMA (1, 1) model can be varied from 0 to 1, and the observations generated wander more for larger values of λ (Vander Weil, 1996). Therefore, a specific value of λ generates a unique disturbance model, and this is useful for conducting the simulation to select the best ISPC system.

CHAPTER 4

RESEARCH PROCEDURE

Two different methodologies, SPC and APC, have been used in parallel for a long time, and they share a common objective. Recently, a number of research works support the integration of these two approaches. However, this research is primarily based on a specific scenario or uses only one disturbance model to simulate data for the analysis. Research that addresses the implementation boundaries and utilization in practice is limited.

There are no specific guidelines for when the combination should be utilized. A comprehensive review of the literature has shown that practitioners in the industry should have guidelines for choosing the appropriate control policy. If quality practitioners know the moving average coefficient of the disturbance model and the shift size, they should be able to accurately select the best control approach.

4.1 Research Gap

Stability in the manufacturing process must be maintained in order to improve and reduce variability. SPC is an effective tool for reducing variability in the process. However, a control chart is not the best method for reducing variability around the target in correlated observations. On the other hand, the APC, which is based on the idea of disturbance prediction and the manipulation of the control variable, has been effectively used, especially when the process is not stationary. APC responds to regulate the output response when there is a deviation from the target, but it does not remove the assignable causes and does not provide information concerning the process. Therefore, the integration between APC and SPC could lead to substantial improvement in process quality, since they compliment each other.

A limited amount of researches has been conducted to evaluate the statistical performance of the ISPC system, since it is typically based on one specific disturbance model or type of controller and control chart. For this reason, results may not be applied to different scenarios. Moreover, there is no clear evidence to indicate the boundaries for economic integration (when these two approaches should be integrated) or the economic consequence after integration.

Another interesting issue is the signal to be monitored by SPC. Montgomery (2001) pointed out whether the control chart should be used to monitor the control error or the control signal is still in question. Practitioners need guidelines in order to monitor the right signal for achieving the highest performance of integrating SPC and APC. Figure 4.1 shows the application of control charts to monitor the residual (error) and control signals.

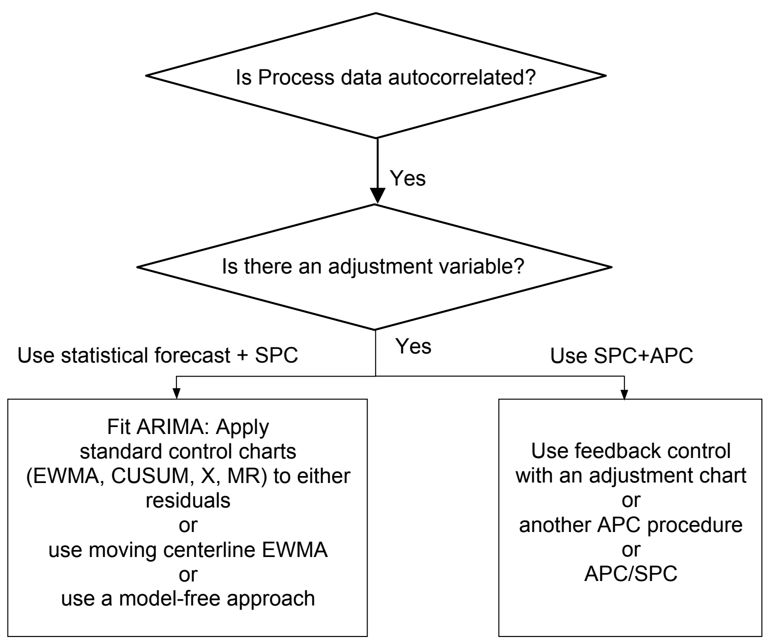

Figure 4.1. Application of SPC to monitor the error and control signals (Montgomery, 2001).

For parts industry, the cost regarding the adjustment and frequency of monitoring is considered high, and SPC originated in this industry. By contrast, APC, which is widely used in the process industry, considers the cost of being off target as a critical issue, while the adjustment cost is negligible. Since these two approaches are different economically, the optimal economic scheme for the integration is derived.

4.2 Research Objectives

The objective of this research was twofold. The first was to characterize the statistical performance of ISPC systems under varying levels of selected factors. These include model coefficient, shift magnitude, type of controller, and type of chart. Efforts were made to study the effect of monitoring the error signal as opposed to the correction signal. The system performance was characterized in terms of the mean squared error and the average run length. While the first is used to evaluate the performance of APC systems, the second is typically used to evaluate the performance of SPC systems. This helps identify scenarios where the integration would result in an improved performance of the system.

The second objective of this research was to evaluate the economic performance of such ISPC systems. Research efforts included consideration of the expected benefits of monitoring the performance of the APC controller using a SPC technique. A mathematical cost function was developed and utilized to study the tradeoff between the sensitivity of the APC controller and that of the SPC charting technique. This helped identify scenarios where the economic benefits of the integration can be maximized for the specified process model.

4.3 Stages of Research Procedure

In order to achieve the above objectives, the following research procedures were done. The first stage, focusing on the statistical performance of the ISPC system under varying disturbance models, included the following:

1. Sets of simulated data were generated to represent disturbances. The disturbances were monitored and controlled by both APC and SPC.
2. Different disturbance models were considered by adjusting the parameter λ in the IMA model. The effect of parameter change λ in the IMA (1, 1) model was analyzed. Different shift sizes in the form of step signal were added to the process to observe the performance of each control approach.
3. The response variables, the mean squared error and average run length, were used as measurement criterion for evaluating each control policy. The MSE is a measurement of the average deviation from the target, while the ARL is the average number of points plotted before a point indicates an out-of-control status.
4. Changes in the disturbance models associated with changes in shift sizes, types of control charts, choices of controllers, and signals to be monitored were analyzed using a statistically designed experiment.

The second stage of this research was devoted to the economic feasibility of utilizing the integrated approach. By extending the quadratic loss function proposed by Taguchi et al. (1989), the loss due to variability in the process was calculated. Also, the cost of performing APC action was accumulated with the quadratic loss, and this determined the economic consequence of using the ISPC system under different levels of model parameters.

CHAPTER 5

STATISTICAL PERFORMANCE

In this chapter, the experimental design is utilized for evaluating the statistical performance of the ISPC systems. For the experiment, process disturbances, which are simulated at specified levels of wandering λ from 0.1 to 1, and a step signal, which causes a shift ranging from 0.5 to 2.5 standard deviation, were added to the process model to generate process observations. These observations were compensated to meet the target by using controllers (MMSE and PI), while the error and control signal after the adjustment were monitored by control charts (Shewhart and EWMA). The performance of the integration between controllers and control charts was evaluated by considering MSE and ARL.

5.1 Simulation Modeling

A simulation was conducted using @Risk, Version 4.5, an add-on software package for Microsoft Excel, to generate a set of random numbers with mean = 0 and variance = 1. This simulated data was used as random errors for the integrated moving average, IMA (1, 1), which represents the process disturbance

$$\hat{N}_t = \hat{N}_{t-1} + a_t - (1-\lambda)a_{t-1}\,;\; 0 < \lambda < 1$$

The value of λ was varied from 0 to 1, since Vander Weil (1996) showed that the simulated IMA data wanders (drifts) more for larger values of λ. For each λ, the disturbances were simulated from period 1 to 100 (100 observations), while the selected process model was the non-stationary model in equation (2.2). Different levels of step shifts (0.5 and 2.5 units) were added to the process model at period 51 in order to assess the robustness of the ISPC system to shifts. Two types of controllers, MMSE and PI, were utilized in order to compensate for the disturbances and shifts in the process. The assumption for these controllers was the gain of the manipulated variable (X) is equal to 1, and the lag period between control signal and action is 1. For the MMSE controller, the optimal adjustment for IMA (1, 1): $X_t = X_{t-1} + \lambda a_t$ was used to control disturbances. For the constrained-input PI controller, the optimal control signal for IMA (1, 1) was shown in equation (2.38). The optimal value of C_1 and C_2 for each λ was calculated by using the solver function in EXCEL. The deviation from the target after the adjustment (Y_t-T) and the control signal (X_t) from both controllers was monitored by control charts, the Shewhart control charts for individual measurements and EWMA charts. Due to individual charts, the observations from period 1 to 50 were used to construct phase 1 control charts. For the EWMA, the value of λ was set at 0.4, and the width of the limits (L) was set at 3.054 (Montgomery, 2001). As shown in Figure 5.1, a number of simulated runs of N = 10,000 was selected to generate disturbances in order to obtain the MSE and ARL_1 for each ISPC system.

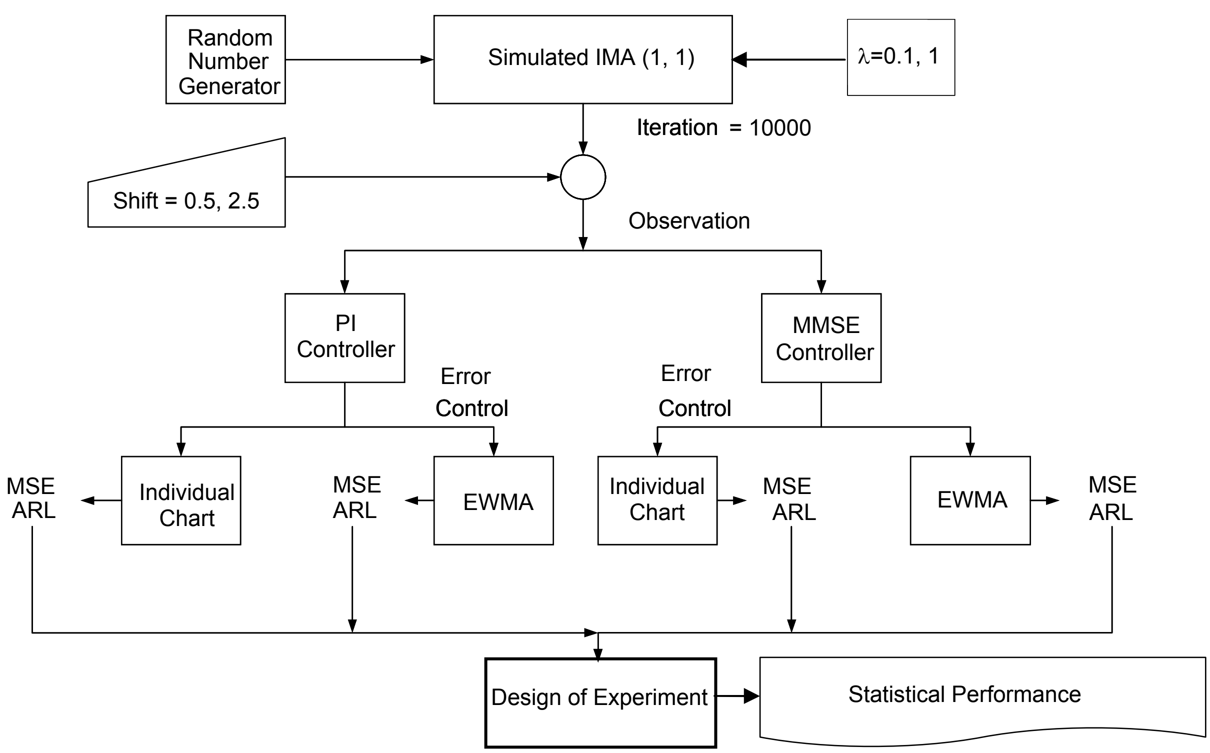

Figure 5.1. Simulation model for statistical performance.

5.2 Model Validation

According to Law and Kelton (1982), the validity of a simulation model depends on how accurate it can resemble the output data expected from the actual system. Tsung et al. (1998) pointed out that the ISPC system should be able to keep track of the process output and reduce variation from the target. For the MSE response, Box et al. (1994) claimed that the output standard deviation corresponding to the controller is $\sigma_a*(1+\lambda)^{1/2}$, when σ_a is the random error of the disturbance, which follows IMA (1, 1). Since the simulation model is based on the correlated data IMA (1, 1), which is generated from the random number ~N (0, 1), the error (deviation from the target) after the adjustment should have a constant mean at zero and constant variance in the range specified by Box et al. Table 5.1 shows the mean, variance, and 95 percent confidence intervals of the output error after the adjustment under different conditions, when lambda is set at 0.1. Since all confidence intervals include $\sigma_a*(1+\lambda)^{1/2} = 1*(1+0.1)^{1/2} = 1.004809$, it can be concluded that the simulation model is accurate enough to represent the actual system.

TABLE 5.1

MEAN AND VARIANCE OF THE OUTPUT ERROR AFTER THE ADJUSTMENT

SPC	APC	Shift	$\sigma_a*(1+\lambda)^{1/2}$	Mean	Variance	95% C.I. of Variance
EWMA	MMSE	0.5	1.004809	0	1.00396	(0.713116, 1.294804)
		3.5	1.004809	0	1.15893	(0.734198, 1.583662)
	PI	0.5	1.004809	0	1.00582	(0.711075, 1.300565)
		3.5	1.004809	0	1.18052	(0.731072, 1.629968)
X	MMSE	0.5	1.004809	0	1.00631	(0.713114, 1.299506)
		3.5	1.004809	0	1.18509	(0.818315, 1.551865)
	PI	0.5	1.004809	0	1.00575	(0.70636, 1.30514)
		3.5	1.004809	0	1.18412	(0.772108, 1.596132)

For the ARL response, the validation was performed by benchmarking the ability of the developed simulation model with the one from Huang and Lin (2002). Huang and Lin used IMA (1, 1) model with $\lambda = 0.5$ to represent the disturbance, and the shift magnitudes were set at 1, 5, and 10. The process output was monitored and controlled by the ISPC system. In the experiment, disturbance was added into the system at time t =150, and the detection was performed until time t = 400 (maximum ARL =250). As shown in Table 5.2, the result of the experiment simulation (ARL) from Huang and Lin's work was compared with the one from the simulation model, which was run under the same conditions. According to the comparison, all 95 percent confidence intervals of the ARL from the proposed simulation model include the values from Huang and Lin's model.

TABLE 5.2

ARL COMPARISON BETWEEN TWO SIMULATION MODELS

Shift	Huang and Lin's Model		Proposed Simulation Model			
	X-Chart	EWMA-Chart (W=0.9,L=3)	X-Chart		EWMA-Chart (W=0.9, L=3)	
			ARL	95% C.I.	ARL	95% C.I.
1	176.18	221.13	178.32	(175.99, 180.64)	219.43	(217.28, 221.58)
5	3.96	8.01	3.29	(2.47, 4.09)	7.7	(7.18, 8.21)
10	1	1	0.66	(0.239, 1.07)	0.89	(0.68, 1.09)

5.3 Data Analysis

The MSE and ARL from the ISPC system under different scenarios were analyzed using Design-Expert, Version 7. The factors which have a highly significant effect on these responses were identified with quantified levels of effect. The 2^5 experiment was conducted in order to study the effect of potential factors on the responses, which are mean squared error and average run length. Input factors are listed in Table 5.3, and experimental data for both responses is shown in Appendix A.

TABLE 5.3

LIST OF FACTORS: MSE AND ARL

Factor	Low	High
A (Lambda)	0.1	1
B (Shift magnitude)	0.5	2.5
C (Type of controller)	MMSE	PI
D (Type of chart)	Individual X	EWMA
E (signal)	Control (X)	Error (e)

5.3.1 Analysis of Mean Squared Error

As shown in Figure 5.2, the half-normal plot points out that the interaction ABDE is significant, while the effect list indicates that shift magnitude (B) contributes as much as 81.57 percent on the response MSE. The second highest effect comes from the interaction between A and E, which accounts for 6.27 percent. Therefore, further investigation of the effect on shift size of the MSE might be required. The analysis of variance (ANOVA) in Table 5.4 shows that the shift size and interaction between λ and types of controllers are statistically significant with p-value < 0.0001.

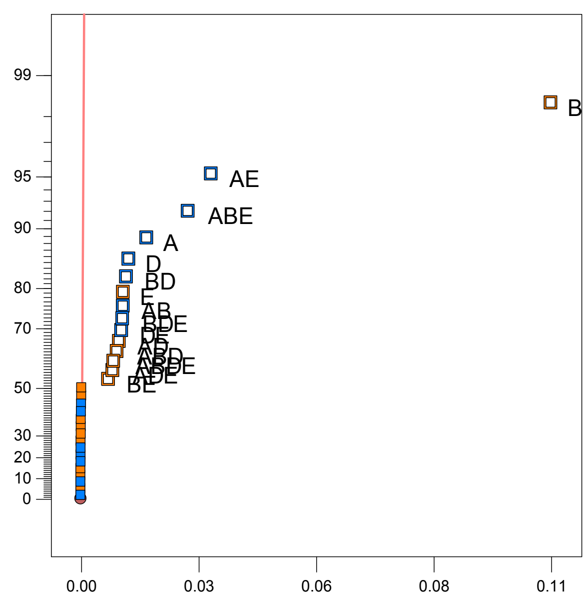

Figure 5.2. Half-Normal plot: MSE.

TABLE 5.4

ANALYSIS OF VARIANCE: MSE

Source	Sum of Squares	df	Mean Square	F Value	p-value
Model	0.12098	15	0.008065	182394.9	< 0.0001
A-Lambda	0.00194	1	0.00194	43875.6	< 0.0001
B-Shift	0.098691	1	0.098691	2231871	< 0.0001
D-SPC	0.001026	1	0.001026	23208.12	< 0.0001
E-Output	0.000805	1	0.000805	18205.74	< 0.0001
AB	0.0008	1	0.0008	18097.92	< 0.0001
AD	0.000661	1	0.000661	14950.77	< 0.0001
AE	0.007597	1	0.007597	171814.9	< 0.0001
BD	0.00093	1	0.00093	21042.98	< 0.0001
BE	0.000342	1	0.000342	7732.685	< 0.0001
DE	0.000744	1	0.000744	16816.35	< 0.0001
ABD	0.000588	1	0.000588	13293.13	< 0.0001
ABE	0.005137	1	0.005137	116176.6	< 0.0001
ADE	0.000456	1	0.000456	10307.49	< 0.0001
BDE	0.000779	1	0.000779	17623.81	< 0.0001
ABDE	0.000482	1	0.000482	10906.57	< 0.0001
Residual	7.08E-07	16	4.42E-08		
Total	0.12098	31			

According to Figure 5.3, cube plots represent the interaction among λ, type of controllers, and signal at different values of shift magnitudes (0.5 and 2.5). At the high level of shift magnitude, the minimum average MSE (1.081) is obtained at the high level of drift rate, when the EWMA chart is used to monitor the error signal. Similar results (1.0035) are obtained at the low level of shift. However, a maximum average MSE (1.2044) is obtained at the low level of drift rate, when the Shewhart chart is used to monitor the error signal. Similar results (1.00306) are obtained at the low level of shift.

5.3.2 Analysis of Average Run Length

The half-normal plot in Figure 5.4 points out that the interaction of ABDE is significant, while the effect list indicates that type of signal contributes as much as 67.99 percent on the ARL followed by the shift magnitude and lambda in which the percent contribute are 9.16 percent and 3.13 percent respectively. Therefore, further investigation of the effect of type of signal and shift magnitude on ARL might be required.

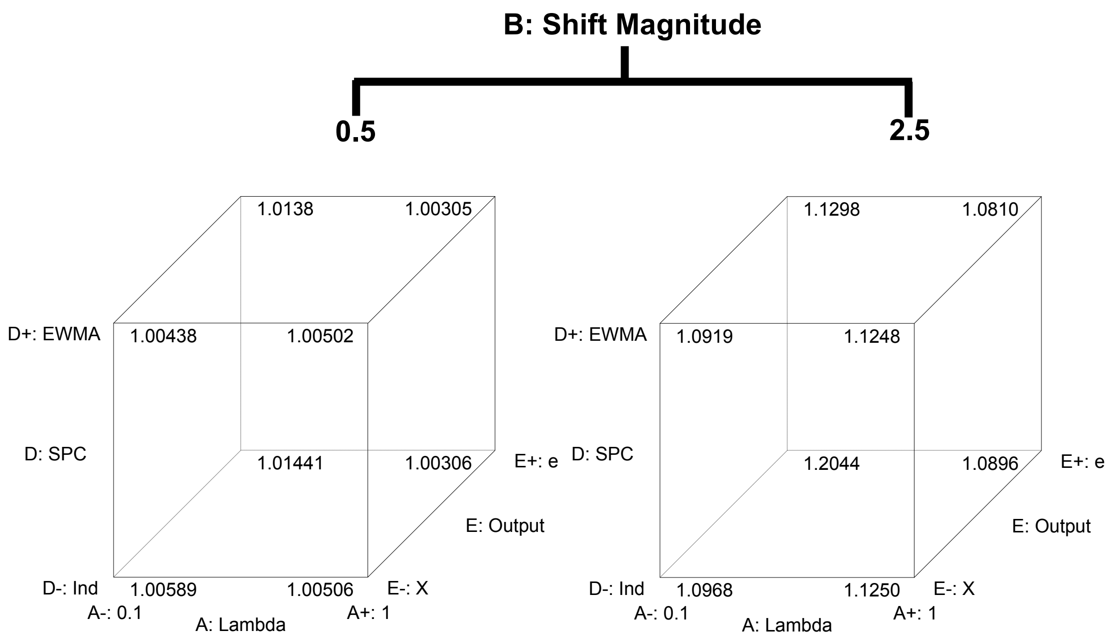

Figure 5.3. Cube plots of the ABDE interaction (MSE).

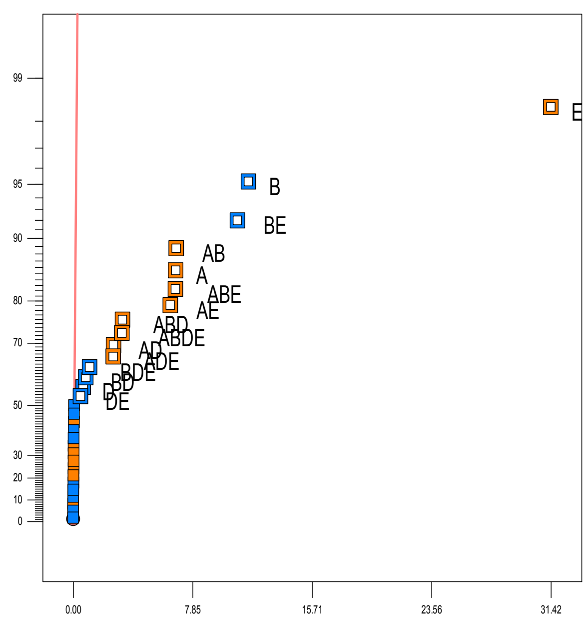

Figure 5.4. Half-Normal plot: ARL.

As shown in Table 5.5, the analysis of variance (ANOVA) shows that the type of signal and the interaction ABDE are statistically significant with p-value < 0.0001.

TABLE 5.5

ANALYSIS OF VARIANCE: ARL

Source	Sum of Squares	df	Mean Square	F Value	p-value
Model	11681.36	15	778.7575	81596.69	< 0.0001
A-Lambda	377.6164	1	377.6164	39565.91	< 0.0001
B-Shift	1087.642	1	1087.642	113961.1	< 0.0001
D-SPC	3.555111	1	3.555111	372.4976	< 0.0001
E-Output	7960.311	1	7960.311	834066	< 0.0001
AB	381.4902	1	381.4902	39971.8	< 0.0001
AD	56.82047	1	56.82047	5953.538	< 0.0001
AE	313.7838	1	313.7838	32877.66	< 0.0001
BD	5.621975	1	5.621975	589.0596	< 0.0001
BE	912.7705	1	912.7705	95638.32	< 0.0001
DE	1.839937	1	1.839937	192.7851	< 0.0001
ABD	84.60989	1	84.60989	8865.259	< 0.0001
ABE	348.7414	1	348.7414	36540.45	< 0.0001
ADE	55.27052	1	55.27052	5791.138	< 0.0001
BDE	9.282879	1	9.282879	972.642	< 0.0001
ABDE	82.00514	1	82.00514	8592.339	< 0.0001
Residual	0.152704	16	0.009544		
Total	11681.51	31			

Cube plots shown in Figure 5.5 represent the interaction among λ, shift magnitude, and type of controller for different types of signal (control and error signal). When the control signal was monitored, the ARL was significantly lower than when the error signal was monitored. Therefore, the control signal should be monitored rather than the error signal. When the EWMA chart was used to monitor the control signal, the shift rate was high while the drift rate was slow. This scenario caused a minimum ARL of 1.54, and similar results (2.15) were obtained when the EWMA chart was used to monitor the error signal. However, a maximum ARL of 3.36 was obtained when the shift size is small and drift rate is fast, and the Shewhart chart was used to monitor the control signal. Similar results (46.37) were obtained when the error signal was monitored by the EWMA chart.

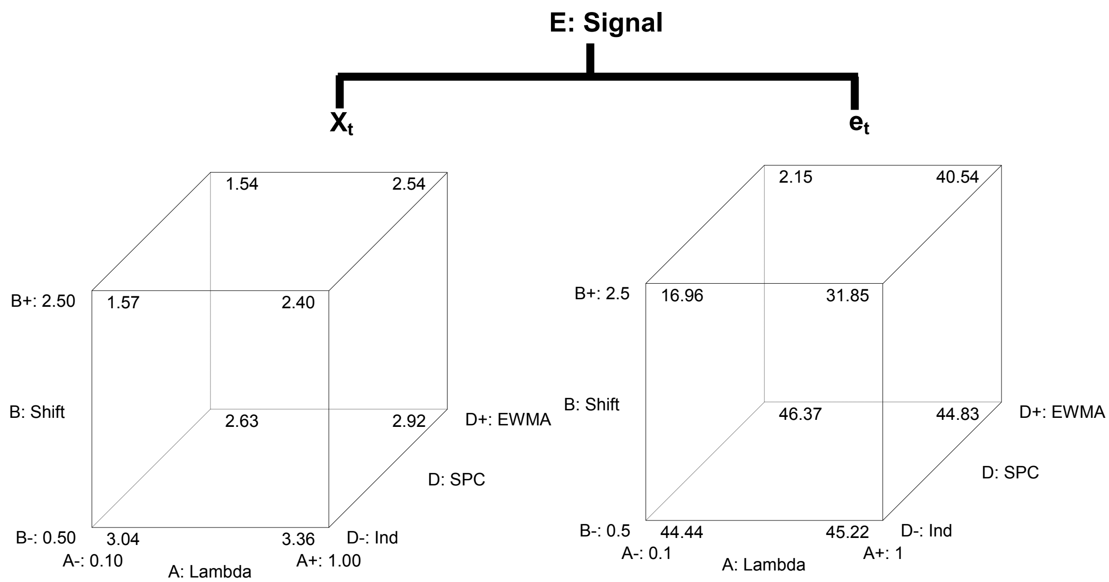

Figure 5.5. Cube plots of the ABDE interaction (ARL).

After characterization of the integration system under different conditions was performed, the ISPC system was justified by developing economic models for both APC and ISPC systems in order to derive the economic boundaries of the integration. Chapter 6 discusses this justification.

CHAPTER 6

ECONOMIC COST MODELS

This chapter explains how mathematical models for estimating the loss from utilizing an APC system and an ISPC system were formulated. It was assumed that the process follows an IMA (1, 1), subjected to a shift in input. Cost elements were due to the loss incurred when the process output deviated from the target value. The common cost for both systems was the cost during the steady state and transient state. In the transient state, the APC system compensated for the shift until the process output was back within the acceptable range. However, in the ISPC system, the SPC acted to detect the shift and signal the out-of-control condition. For this reason, economic loss during the steady state was different for the two systems. Moreover, the ISPC system had to deal with the cost of false alarms. Therefore, a tradeoff was considered in order to select the most economical system to monitor and control the process. After the individual cost models for each control system were derived, the justification was done to recommend the most appropriate control chart to integrate with the APC system. The assumptions in this modeling include the following: (1) the process is the discrete manufacturing process, and (2) the control policy follows the post-process measurement technique, which takes a measurement of the final product and then makes an adjustment in the process (for the next item) after the information has been received.

6.1 Notation

Notations in the economic cost models are as follows:

θ Moving average parameter

A Rework or scrap cost ($ per unit)

Δ Half the tolerance of the product characteristic

μ Process average

σ Process variance

δ Shift magnitude

R Production rate (unit per hour)

δ_0 Range of the controller

λ Occurrence rate of shift (per hour)

l Lag time

T Adjustment time

ε Sensitivity

F False alarm cost ($ per occurrence)

α Probability of type 1 error

ARL_0 Average run length when the process is in control

β Probability of type 2 error

ARL_1 Average run length when the process is out of control

n Subgroup size

h Sampling interval

k Width of the control limits (Individual measurement chart)

H Decision interval (CUSUM chart)

K Reference interval (CUSUM chart)

L Width of the control limits (EWMA chart)

W Weight factor (EWMA chart)

6.2 Economic Cost Model of APC System

In the beginning, the process started with a state of control (steady state), and there was some noise in the process, which was compensated for by the controller. If the occurrence of an assignable cause follows the Poisson distribution with mean λ, then the average time until the occurrence of the assignable causes is $1/\lambda$. At time t_0, there was a shift occurring at the input; this was the starting point of the transient period. After a shift occurred, the controller reacted to compensate for the shift. However, there was a lag between the acknowledgement of shift occurring and the reaction of the actuator. As a result, the controller experienced a time lag (l) to compensate for the shift. The period of the time lag lasted from time t_0 until t_1. Moreover, because of the dynamic behavior of the controller and the process, there was a gap before the error was fully compensated. It is interesting to note that the range of adjustment (δ_0) from the controller can be either smaller or longer than the shift magnitude ($\delta < \delta_0$ or $\delta > \delta_0$). Since the process adjustment followed the post-process technique, the process adjustment time equaled the production of one item and the schematic of the cycle time for the APC system, as shown in Figure 6.1.

According to Figure 6.1, two schematics represent the process output (y_t) and compensation or control action (X_t) from the controller. The centerline of process output schematics is the target that the process wants to achieve, while the centerline of the control action is zero. The control action was approximately normally distributed with a constant variance (σ^2). Since there were two periods of adjustment, the analysis of total loss is presented in the following two sections – loss during steady state and loss during transient state.

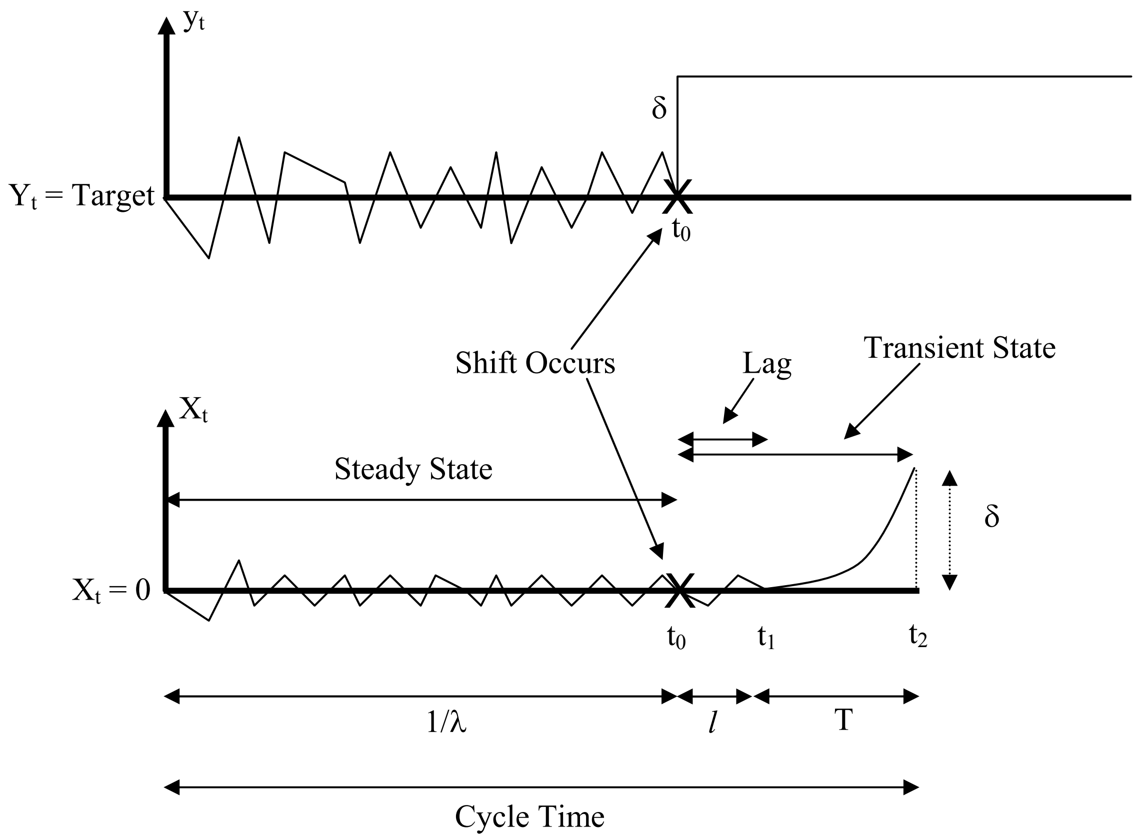

Figure 6.1. Schematics representation of cycle time (APC system).

6.2.1 Loss During Steady State

Steady state covers the period before the occurrence of the shift ($t < t_0$). The expected value and variance of the control action at time t (X_t) during the steady state period are

$$E\,(X_t)=0,\ Var\,(X_t)=\sigma^2;\ 0<t<t_0$$

The loss per unit in the steady-state period can be expressed in terms of the quadratic loss function proposed by Taguchi et al. (1989) as

$$E(L_S) = (\frac{A}{\Delta^2})(0+\sigma^2) = (\frac{A}{\Delta^2})\sigma^2 \qquad (6.1)$$

If R is the production rate, then the expected number of items produced during the steady-state period is

$$E(N_S) = R(\frac{1}{\lambda}) = \frac{R}{\lambda} \tag{6.2}$$

As a result, the expected cost ($) during the steady state is

$$E(C_S) = E(L_S) * E(N_S)$$

$$= (\frac{R}{\lambda})(\frac{A}{\Delta^2}\sigma^2) \tag{6.3}$$

Therefore, the expected cost during this period depends on the variance of the control action and the frequency of the shift occurrence.

6.2.2 Loss During Transient State

The transient state covers the lag period (*l*) and the adjustment period (T). The loss analysis during the transient state can be categorized based on the shift magnitude (δ) and the range of the controller (δ_0). Two different cases are considered depending on the relationship between the shift magnitude (δ) and the range of the controller (δ_0).

Case#1: $\delta < \delta_0$

After the shift occurs in the process, one characteristic of the controller is the lag period (*l*), which causes delay in the response time to compensate for the shift. If the shift affects only the expected value of the control action and not the variance, then the expected quadratic loss per unit can be obtained as

$$E(L_l) = (\frac{A}{\Delta^2})\sigma^2 \tag{6.4}$$

If the expected number of items produced during the lag period is $E(N_l) = Rl$, then the expected cost ($) in this period is

$$E(C_l) = (\frac{A}{\Delta^2})\sigma^2 Rl \tag{6.5}$$

According to Jiang and Tsui (2002), the expected control action of the MMSE controller during the transient period (not include lag time), when a shift of size δ occurs at time t_1, is equal to

$$E(X_t) = (1-\theta^{t-t_1+1})\delta \text{ ; } t_1 \le t \le t_2 \tag{6.6}$$

where θ is the moving average parameter of the process model (IMA (1, 1)) considered, and X_t is the control action at time t. After the lag period is surpassed, the controller is expected to fully compensate for the shift at time t_2 in one adjustment cycle. However, because of the sensitivity of the controller (rate of change in the output of a controller with respect to changes in the inputs), the output error is ε (small positive value) in which the value of y_t is the closest to the target.

$$\frac{(\theta^{t_2-t_1} - \theta^{t_2-t_1+1})\delta}{1-\theta} = \varepsilon$$

i.e.,

$$\frac{\theta^{t_2-t_1}(1-\theta)\delta}{1-\theta} = \varepsilon$$

and,

$$\theta^{t_2-t_1} = \frac{\varepsilon}{\delta} \tag{6.7}$$

Therefore, the adjustment period can be expressed as

$$T = t_2 - t_1 = \frac{\ln(\varepsilon/\delta)}{\ln\theta} \tag{6.8}$$

From equation (6.8), if $\varepsilon = \delta$, there is no adjustment period (T = 0). However, if $\delta > \varepsilon$, the steady-state period (T) is greater than zero. Therefore, the average control action during the adjustment period is given by

$$E(X_A) = (1-\theta^{t_2-t_1+1})\delta = (1-\theta^{T+1})\delta \tag{6.9}$$

Consequently, the expected loss per unit during the adjustment period can be obtained as

$$E(L_A) = (\frac{A}{\Delta^2})[E^2(X_t) + \sigma^2]$$

$$= (\frac{A}{\Delta^2})[\{(1-\theta^{T+1})\delta\}^2 + \sigma^2]$$

$$= (\frac{A}{\Delta^2})[\delta^2(1-\theta^{T+1})^2 + \sigma^2] \quad (6.10)$$

The expected number of items produced during the adjustment period is $E(N_A) = RT$, and the expected cost ($) during the adjustment period is

$$\mathrm{E(C_A)} = E(L_A) * E(N_A)$$

$$= (\frac{A}{\Delta^2})[\delta^2(1-\theta^{T+1})^2 + \sigma^2]RT \quad (6.11)$$

Therefore, the expected cost ($) during the transient state ($\delta < \delta_0$) is the summation of equations (6.5) and (6.11), i.e.,

$$\mathrm{E(C_T)} = E(C_l) + E(C_A)$$

$$= (\frac{A}{\Delta^2})\sigma^2 Rl + (\frac{A}{\Delta^2})[\delta^2(1-\theta^{T+1})^2 + \sigma^2]RT$$

$$= (\frac{A}{\Delta^2})R\{\sigma^2 l + (\delta^2(1-\theta^{T+1})^2 + \sigma^2)T\} \quad (6.12)$$

Obviously, when the drift rate of process model is high (fast drift) and the adjustment time is long, this scenario will cause the maximum expected cost during the transient state. The opposite result is obtained when the drift rate is small (slow drift).

Case#2: $\delta > \delta_0$

In this case, the controller is expected to compensate for the shift step by step (each step or cycle of the compensation is equal to its range δ_0) until control action is within the range $\delta \pm \varepsilon$.

From Figure 6.2., the relation between the control action and the shift magnitude can be approximated in the form of a linear function; therefore, $X_t = \delta$, and the expected of these two values are

$$E(X_t) = E(\delta) \tag{6.13}$$

If the shift magnitude is approximated by a triangular distribution with E (δ) = $2\delta/3$, then

$$E(X_t) = \frac{2\delta}{3}$$

$$= \frac{2\delta}{3} \tag{6.14}$$

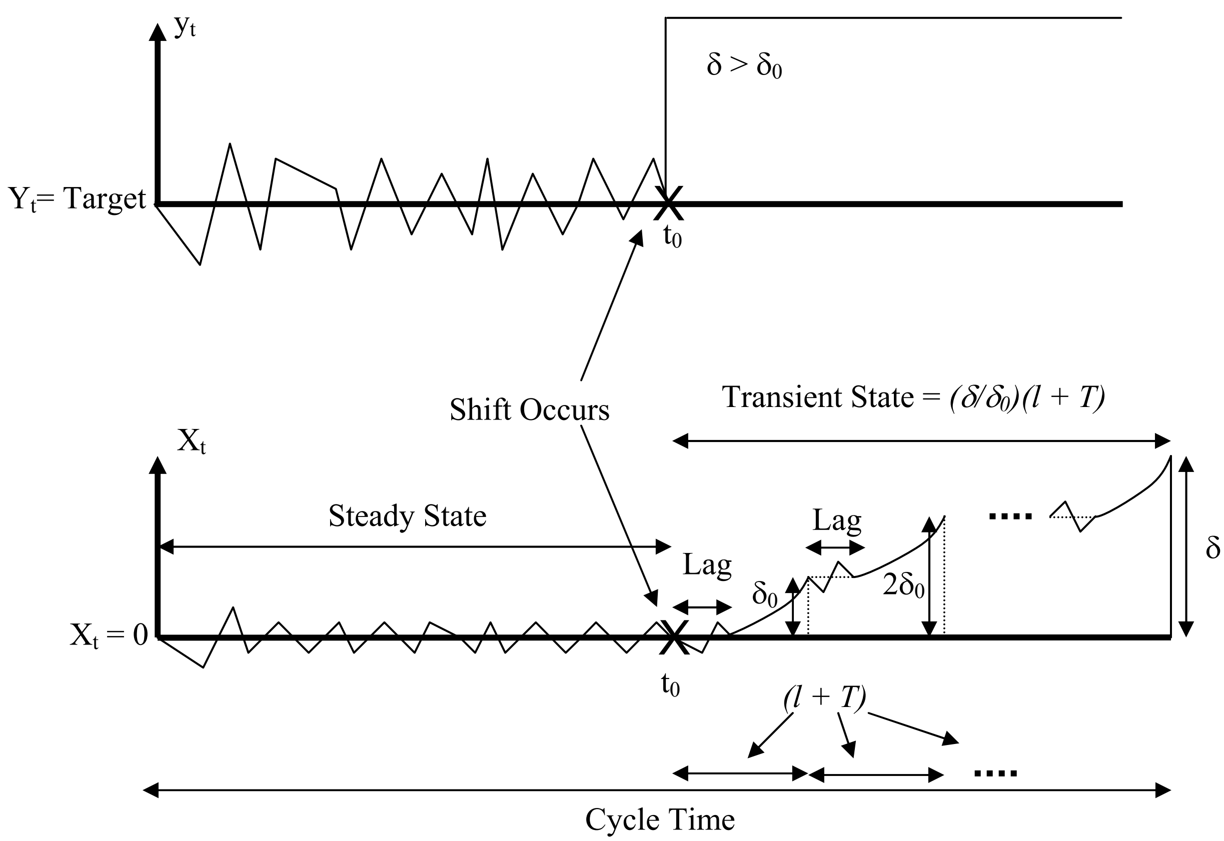

Figure 6.2. Schematic representation of cycle time (APC System, $\delta > \delta_0$).

The expected loss per unit during the transient period in this case is

$$E(L_T) = (\frac{A}{\Delta^2})\{(\frac{2\delta}{3})^2 + \sigma^2\}$$

$$= (\frac{A}{\Delta^2})(\frac{4\delta^2}{9} + \sigma^2) \quad (6.15)$$

Since there is always a lag before each adjustment, the cycle time of an adjustment is *l*+T.. As a result, the average number of items produced in one cycle time of adjustment is R(*l*+T), and the expected number of items produced during the whole transient period is

$$E(N_T) = \frac{\delta}{\delta_0} R(l + T) \quad (6.16)$$

The expected cost ($) during the transient period when $\delta > \delta_0$ can be obtained as

$$E(C_T) = (\frac{A}{\Delta^2})(\frac{4\delta^2}{9} + \sigma^2)\frac{\delta}{\delta_0} R(l + T) \quad (6.17)$$

If p and q are the probability of $\delta < \delta_0$ and $\delta > \delta_0$ such that p + q =1, then the expected cost ($) during the transient period is

$$E(C_T) = (\frac{A}{\Delta^2})R\{[\sigma^2 l + (\delta^2(1-\theta^{T+1})^2 + \sigma^2)T]p + (\frac{4\delta^2}{9} + \sigma^2)\frac{\delta}{\delta_0}(l + T)q\} \quad (6.18)$$

Therefore, the expected total cost of the APC system is

$$E(TC)_{APC} = E(C_S) + E(C_T)$$

Substituting the expected values from equations (6.3) and (6.18),

$$= (\frac{A}{\Delta^2})R\{\frac{\sigma^2}{\lambda} + [\sigma^2 l + (\delta^2(1-\theta^{T+1})^2 + \sigma^2)T]p + (\frac{4\delta^2}{9} + \sigma^2)\frac{\delta}{\delta_0}(l + T)q\} \quad (6.19)$$

It is interesting to note that the value of the expected cost can vary depending on lag time, adjustment time, and probability of shift magnitude, which can be estimated based on the historical data of the occurrence of the shift.

6.3 Economic Cost Model of ISPC System

As shown in Figure 6.3, before the occurrence of the shift in the process, the controller is active to keep the output y_t on the target. This is similar to the APC case. However, the difference is that an SPC chart has been integrated to work in conjunction with the controller. Therefore, during the steady state period (in-control period), there are a number of false alarms from the control chart (the characterization part suggested that the control signal should be monitored).

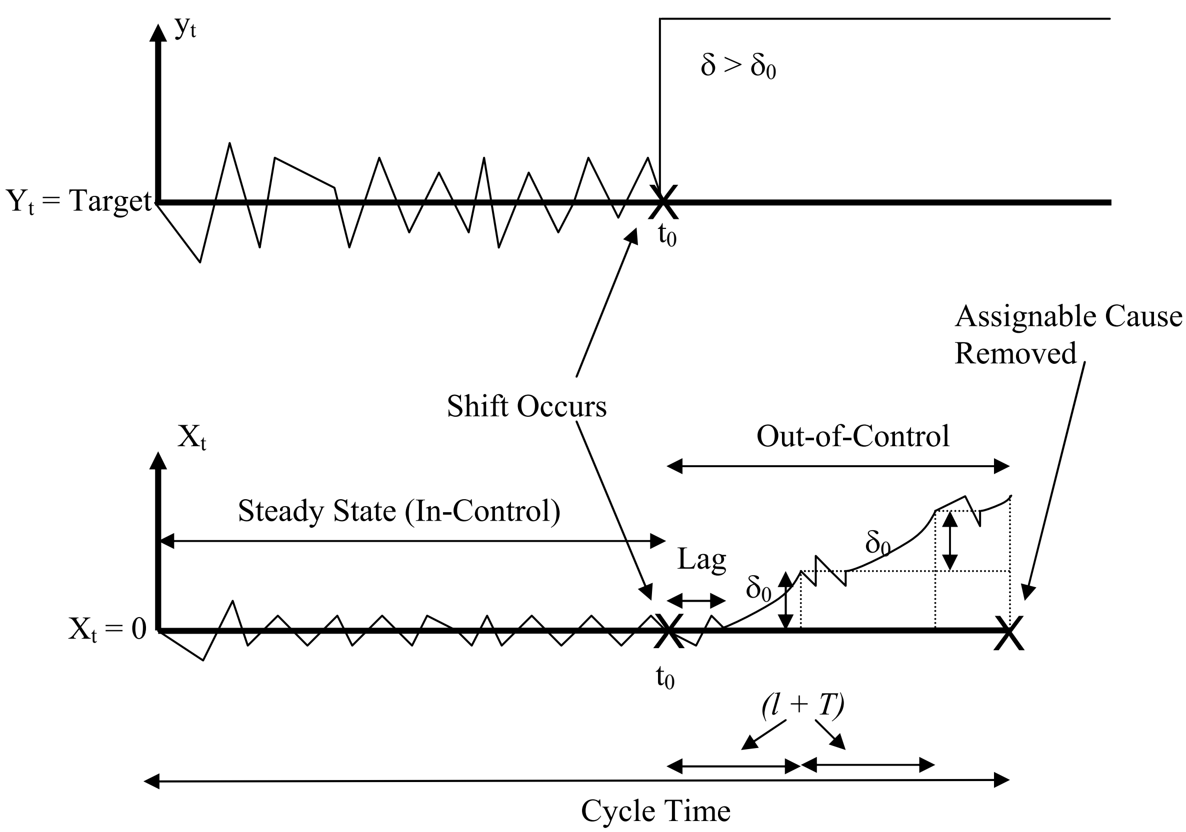

Figure 6.3. Schematic representation of cycle time (ISPC system).

When there is a shift occurring in the output (the beginning of the out-of-control period), the controller is still active in order to compensate for that shift. Afterwards, the SPC signals the occurrence of a shift when the out of control signal is plotted outside the control limits and there is a search for an assignable cause, followed by its removal. For this reason, the loss cost due to the loss is categorized into two elements. Namely, the cost before the occurrence of the shift (steady state or in-control period) and cost after the shift has occurred (out-of-control period).

6.3.1 Loss During In-Control Period

The cost elements in this period come from the loss due to the control action's variance (same value as E (C_S) for the APC system) and false alarms, which incur an additional cost to search for the assignable causes when none exist. The expected number of false alarms during the steady state period is given by

$$E(N_F) = \frac{E(N_S)}{1/\alpha} = \frac{R}{\lambda}\alpha \tag{6.20}$$

The expected cost ($) due to false alarms during an in-control period equals

$$E(C_F) = \frac{R}{\lambda}\alpha F \tag{6.21}$$

It is interesting to note that there are two cases regarding the cost of false alarm investigation. The first is that the operation must be stopped during the search. Another is that the search and operation can be done in parallel so that the cost regarding a false alarm in this case is lower than the one from the first case.

6.3.2 Loss During Out-of-Control Period

When a shift occurs in the process, the SPC chart takes time to detect. The period of time from the occurrence until the chart signals a shift is expressed in term of the average run length (ARL_1). Similar to the APC case, the adjust property depends on δ and δ_0. When $\delta > \delta_0$, this scenario for the ISPC system is similar to the APC case. As a result, the expected loss per unit during the out-of-control period is

$$E(L_o) = (\frac{A}{\Delta^2})[\frac{4\delta^2}{9} + \sigma^2] \tag{6.22}$$

and the expected cost ($) in this period is equal to

$$E(C_o) = (\frac{A}{\Delta^2})[\frac{4\delta^2}{9} + \sigma^2]ARL_1 \tag{6.23}$$

Therefore, the expected total cost ($) for the ISPC system is

$$E(TC)_I = E(C_S) + E(C_F) + E(C_o)$$

Substituting the expected values from equations (6.3), (6.21), and (6.23),

$$= \frac{R}{\lambda}((\frac{A}{\Delta^2})\sigma^2 + \alpha F) + (\frac{A}{\Delta^2})[\sigma^2 l + (\delta^2(1-\theta^{T+1})^2 + \sigma^2)T]p + (\frac{4\delta^2}{9} + \sigma^2)ARL_1 q\} \tag{6.24}$$

The difference between the expected cost of the APC and ISPC systems is the false alarm cost and the cost during the out-of-control period.

6.4 Economics of ISPC System

According to the above economic model of the controller and the ISPC system, the only cost to the APC system is quadratic loss when the controller cannot adjust the process mean within the target. After the SPC is added to the system, there would be an additional cost of false alarms. However, the tradeoff is that SPC helps to detect a shift so that the loss in the out-of – control period might be less than the one controlled by the APC system. For this reason, economic integration would require that the total cost of the APC is greater than that of the ISPC system. This scenario can be justified by using the net savings expected, E (NS), when the ISPC system is implemented, or

$$\mathrm{E(NS)} = E(TC)_{APC} - E(TC)_I \tag{6.25}$$

If the cost savings is positive, the ISPC system would be preferred. Therefore, the criterion for economic integration can be expressed as

$$E(TC)_{APC} - E(TC)_I \geq 0$$

Substituting the expected values from equations (6.19) and (6.24),

$$(\frac{A}{\Delta^2})(\frac{4\delta^2}{9} + \sigma^2)\frac{\delta}{\delta_0}R(l+T)q - \{\frac{R}{\lambda}\alpha F + (\frac{A}{\Delta^2})(\frac{4\delta^2}{9} + \sigma^2)ARL_1 q\} \geq 0$$

$$(\frac{A}{\Delta^2})(\frac{4\delta^2}{9} + \sigma^2)q\{\frac{\delta}{\delta_0}R(l+T) - ARL_1\} - \frac{R}{\lambda}\alpha F \geq 0 \qquad (6.26)$$

Equation (6.26) is true only if these two criteria are satisfied.

Criterion 1:

Integrating SPC in the system is useful only if there are net savings in the transient period. On the other hand, the value of (δ / δ_0)*R*(l+T), which is the transient period of the APC system, has to be greater than the average run length (ARL_1) of the SPC chart. i.e.,

$$\frac{\delta}{\delta_0}R(l+T) > ARL_1 \qquad (6.27)$$

According to the first criterion, equation (6.27) indicates that the maximum average run length (ARL_1) that the control chart might have is $(\delta / \delta_0) * R(l+T)$, for given δ / δ_0, R, l, and T; therefore, that the ISPC system would be selected. It is worth noting here that if the design parameters of the controller were selected such that $(\delta / \delta_0) * R(l+T) \leq 1.0$, there would be no economic justification for the ISPC system. Otherwise, integration would be feasible, subjected to the cost of investigating false alarms. Furthermore, equation (6.27) offers a possible tradeoff between investing in a high-end controller, with high range, small lag, and transient times, and a low-end controller to be integrated with an SPC chart.

Criterion 2:

Given that the first criterion has been satisfied (the control chart's ARL_1 is less than the maximum ARL_1 allowed), the expected cost of false alarms must be examined. If $(\delta/\delta_0) * R(l+T) - ARL_1 = \Delta_I$ such that $\Delta_I > 0$, then equation (6.26) can be expressed as

$$(\frac{A}{\Delta^2})(\frac{4\delta^2}{9}+\sigma^2)q\Delta_I - \frac{R}{\lambda}\alpha F \geq 0 \qquad (6.28)$$

Expressing α in term of the average run length during the in-control state (ARL_0), the second criterion can be expressed as

$$\frac{RF}{\lambda(\frac{A}{\Delta^2})(\frac{4\delta^2}{9}+\sigma^2)q\Delta_I} \leq ARL_0 \qquad (6.29)$$

Equation (6.29) indicates the minimum ARL_0 of the control chart required for economic integration. The parameter F represents the cost of investigating a false signal. This cost depends to a great extent on the out-of-control action plan adopted. The value of F is expected to be high, should the process be stopped while investigating the chart signal. However, if the signal can be investigated during operation, F is expected to be relatively low. High production rates coupled with high values of F would impose serious limitations on the economic feasibility of the integration. Unfortunately, the evaluation of equation (6.29) depends on the value of Δ_I and the type of control chart selected for integration. The following section represents an application of the model for the selection of an appropriate control chart as an illustration.

6.5 Sample Computation

To illustrate the above procedure, consider the following example. A manufacturing process and its control system have the following characteristics and cost coefficients:

$\delta_0 = 4\sigma$ $\lambda = 1.5$ occurrence per hour

R = 100 units per hour A = $ 50 per unit

$l = 0.005$ hour $\delta = 8\sigma$

T = 0.02 hour q = 0.7

F = $ 200 per occurrence $\Delta = 0.001$

$\sigma = 0.000167$ $\mu_0 = 0.00$

The process manager is interested in comparing three types of control charts for individual measurements. These include the Shewhart, CUSUM, and EWMA charts. According to equation (6.27), the ARL_1 of the control chart selected should not exceed the following:

$$\frac{\delta}{\delta_0} R(l+T) = 2 * 100 * (0.02 + 0.005) = 5$$

If the ARL_1 of the control chart is less than five, it would satisfy the first criterion. However, its ARL_0 has to be greater than the minimum ARL_0 obtained using equation (6.29). These values depend on the chart selected and its design parameters. The following charting alternatives are considered.

Shewhart Type Chart

Since subgroup size (n) =1 and sampling interval (h) =1/R, the average run length ARL_0 and ARL_1 need to be quantified at different values of control width factor (k). Table 6.1 shows the results of using equations (2.7) and (2.8).

TABLE 6.1

ARL OF INDIVIDUAL CHART

K	ARL_1	Δ_I	min ARL_0	ARL_0	NS
4	2	3	563.60	15787.19	$22.81
3.5	1.47	3.55	475.77	2149.34	$21.82
3	1.19	3.81	443.61	370.39	-$5.94
2.5	1.07	3.93	430.40	80.519	-$134.61
2	1.02	3.98	425.18	21.98	-$575.31

The maximum expected net saving is achieved at k = 4.

CUSUM Chart

The design of the CUSUM chart requires selection of the reference value (K) and the decision interval (H), both of which can be used to calculate the average run length utilizing equations (2.21) and (2.22). The results obtained at H = 4 are summarized in Table 6.2, while those obtained at H = 5 are summarized in Table 6.3. As shown in Tables 6.2 and 6.3, the maximum expected net saving is achieved at K = 1.0 for both values of H.

TABLE 6.2

ARL OF CUSUM CHART FOR H = 4

K	ARL_1	Δ_I	min ARL_0	ARL_0	NS
1.5	1.99	3.01	560.98	1195314	$23.76
1.25	1.81	3.19	530.36	130031.4	$25.04
1	1.67	3.33	507.14	15344.06	$25.42
0.75	1.54	3.46	488.95	2053.78	$20.78
0.5	1.44	3.57	474.28	338.09	-$11.32
0.25	1.34	3.66	462.22	77.23	-$143.79

TABLE 6.3

ARL OF CUSUM CHART FOR H = 5

K	ARL_1	Δ_I	min ARL_0	ARL_0	NS
1.5	2.39	2.61	646.83	24008594	$20.61
1.25	2.18	2.82	598.73	1584156	$22.26
1	1.99	3.00	563.41	113413.3	$23.55
0.75	1.85	3.15	536.76	9230.15	$23.39
0.5	1.72	3.28	515.65	938.22	$11.65
0.25	1.61	3.39	498.57	141.93	-$67.20

EWMA Chart

This chart design requires specifying values of the control limit widths (L) and weight factor (W), both of which are used in calculating the average run length using the method of Lucas and Saccucci (1990). Calculated values of the expected net savings obtained at selected levels of L and W are shown in Table 6.4.

TABLE 6.4

ARL OF EWMA CHART

L	W	ARL_1	Δ_I	min ARL_0	ARL_0	NS
3.09	1	1.22	3.78	497.94	500	$3.14
3.087	0.75	1.23	3.77	498.42	500	$3.06
3.071	0.5	1.36	3.64	501.019	499	$1.98
3.054	0.4	1.46	3.54	503.81	498	$1.14
3.023	0.3	1.6	3.4	508.97	497	-$0.02
2.998	0.25	1.69	3.31	513.22	496	-$0.78
2.962	0.2	1.8	3.2	519.46	496	-$1.65
2.814	0.1	2.2	2.8	546.78	492	-$5.02
2.615	0.05	2.68	2.32	588.39	487	-$9.08
2.437	0.03	3.09	1.91	631.36	480	-$12.72

Table 6.4 indicates that the maximum net saving of $3.14 per hour is obtained at L = 3.09 and W = 1.0. Overall, the maximum expected net savings is obtained by integrating the CUSUM

chart with H = 4 and K =1. To gain better understanding of the model performance, a sensitivity analysis is performed in the following section.

6.6 Sensitivity Analysis

In this section, a sensitivity analysis was conducted to examine the effect of the various input parameters on the decision variables. Since the selection of the control system depends on two criteria regarding the boundary of the average run length, the responses, ARL_0 and ARL_1, are analyzed in the following two sections.

6.6.1 Sensitivity Analysis of ARL_1

For ARL_1, a factorial experiment was used to examine the effect of the APC's parameters on the selection of ARL_1, and there are four factors (δ / δ_0, R, l, and T) to be considered according to equation (6.27). The values of these parameters have been set by considering the work of Weheba and Nickerson (2005) and practical levels (Table 6.5). The selected design is a three-level factorial with a total of 81 runs. The design matrix of this experiment is shown in Appendix B.

TABLE 6.5

LIST OF FACTORS: MAX ARL_1

Factor	Low	Medium	High
δ / δ_0	1.5	2	2.5
R	70	100	130
l	0.0001	0.005	0.009
T	0.01	0.02	0.03

As a result, the analysis of variance for the reduced model, as shown in Table 6.6, indicates that there are significant effects from two factor interactions: δ / δ_0, R, l, T, $(\delta / \delta_0) * R$, $(\delta / \delta_0) * l$, $(\delta / \delta_0) * T$, $R * l$, $R * T$.

TABLE 6.6

ANALYSIS OF VARIANCE: MAX ARL_1

Source	Sum of Squares	df	Mean Square	F Value	p-value
Model	479.8325	9	53.31472	6493.943	< 0.0001
δ / δ_0	83.33264	1	83.33264	10150.24	< 0.0001
R	119.945	1	119.945	14609.77	< 0.0001
L	32.89447	1	32.89447	4006.676	< 0.0001
T	212.3753	1	212.3753	25868.15	< 0.0001
(δ / δ_0) * R	4.7961	1	4.7961	584.1839	< 0.0001
(δ / δ_0) * *l*	1.137825	1	1.137825	138.5916	< 0.0001
(δ / δ_0) * T	8.246469	1	8.246469	1004.452	< 0.0001
R * *l*	1.651225	1	1.651225	201.1257	< 0.0001
R * T	11.86803	1	11.86803	1445.572	< 0.0001
Residual	0.582904	71	0.00821		
Total	480.4154	80			

The normal probability plot (Figure 6.4) shows that the residuals are normally distributed, and there are no outliers.

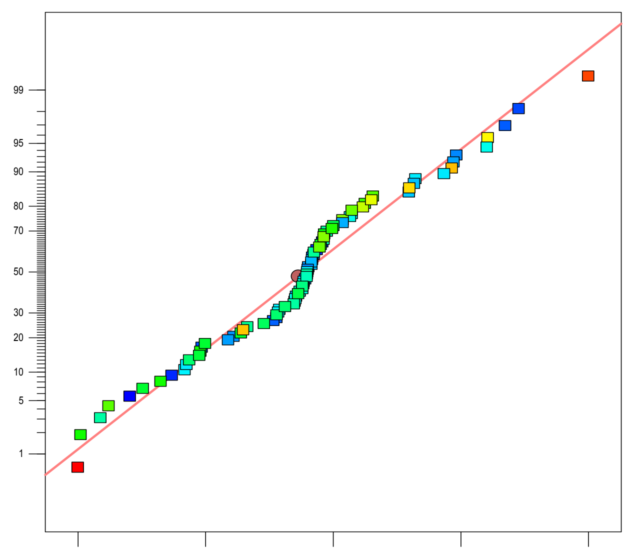

Figure 6.4. Normal probability plot of the residual: max ARL_1.

According to Figure 6.5, the maximum ARL_1 reaches the highest value of 8.2, when

δ / δ_0 and R are at their high levels (2.5 and 130). These conditions would favor the utilization of an ISPC system. However, the ARL_1 significantly drops to less than 4.87, when δ / δ_0 is reduced to only 1.5 at the high level of R. When both parameters are set at their low levels, the maximum ARL_1 decreases to about 2.6. If δ / δ_0 increases to 2.5 and R is kept at the low level, then the maximum ARL_1 is significantly increased to 4.38. This would represent a challenge for the ISPC system's performance. Therefore, the higher the level of δ / δ_0 and R, the more the opportunity for utilizing the ISPC system.

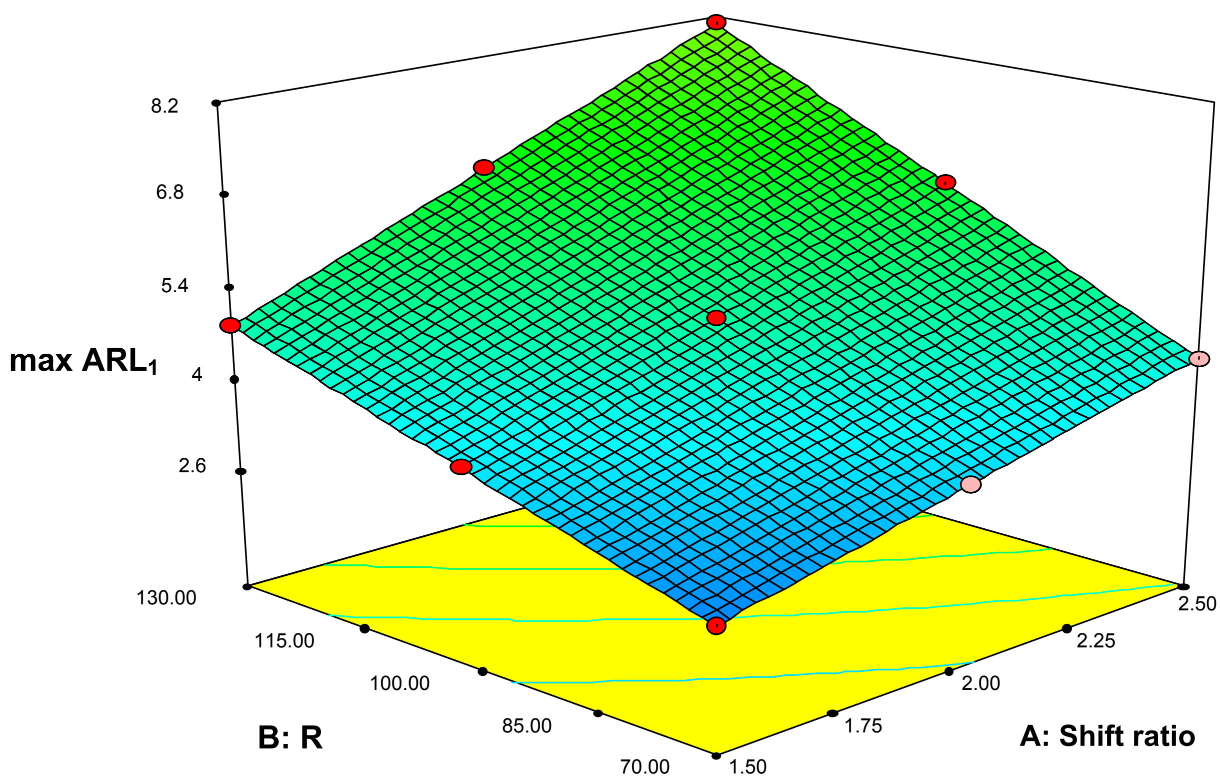

Figure 6.5. Response surface plot of the interaction $(\delta / \delta_0) * R$.

According to Figure 6.6, the highest value of maximum ARL_1 of 7.2 is obtained at the high level of δ / δ_0 and l (1.5 and 0.001). This scenario increases the chance that the ISPC system would be implemented. When δ / δ_0 is at the low level (1.5), the ARL_1 significantly drops to 4.35. On the other hand, if both parameters are set at the low level, then the maximum ARL_1 decreases to 3.14, which is the lowest value of the maximum ARL_1. Hence, a decrease in the values of both lag and shift ratio would reduce the opportunity for the ISPC system to be utilized because of its low maximum ARL_1.

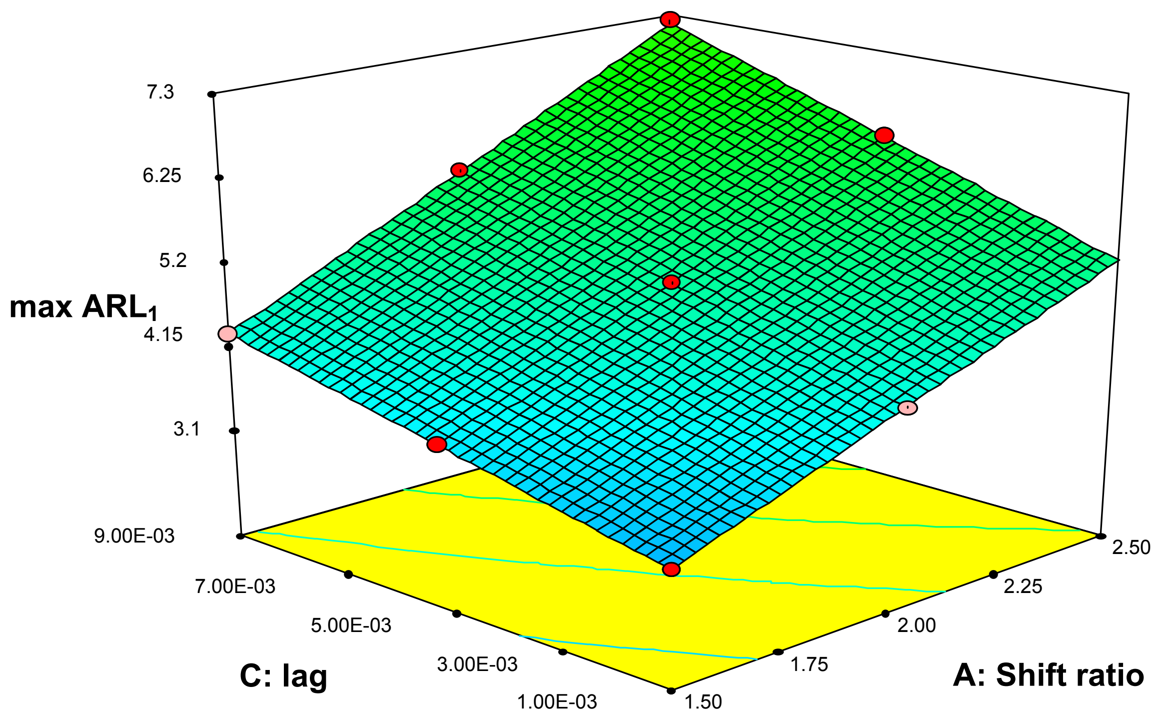

Figure 6.6. Response surface plot of the interaction $(\delta / \delta_0) * l$.

As shown in Figure 6.7, at the low levels of δ / δ_0 and T (1.50 and 0.01), the lowest value of maximum ARL_1 is achieved at 2.24. If one of these values is increased to the high level while the other is kept at the low level, the maximum ARL_1 significantly increases. The maximum ARL_1 reaches the highest point at 8.7, when δ / δ_0 and T are set at their high levels. As a result, if either δ / δ_0 or T is at the high level, the maximum allowed ARL_1 of the ISPC would be maximized. In this scenario, the adjustment time seems to have more effect on the value of the maximum ARL_1 than the shift ratio.

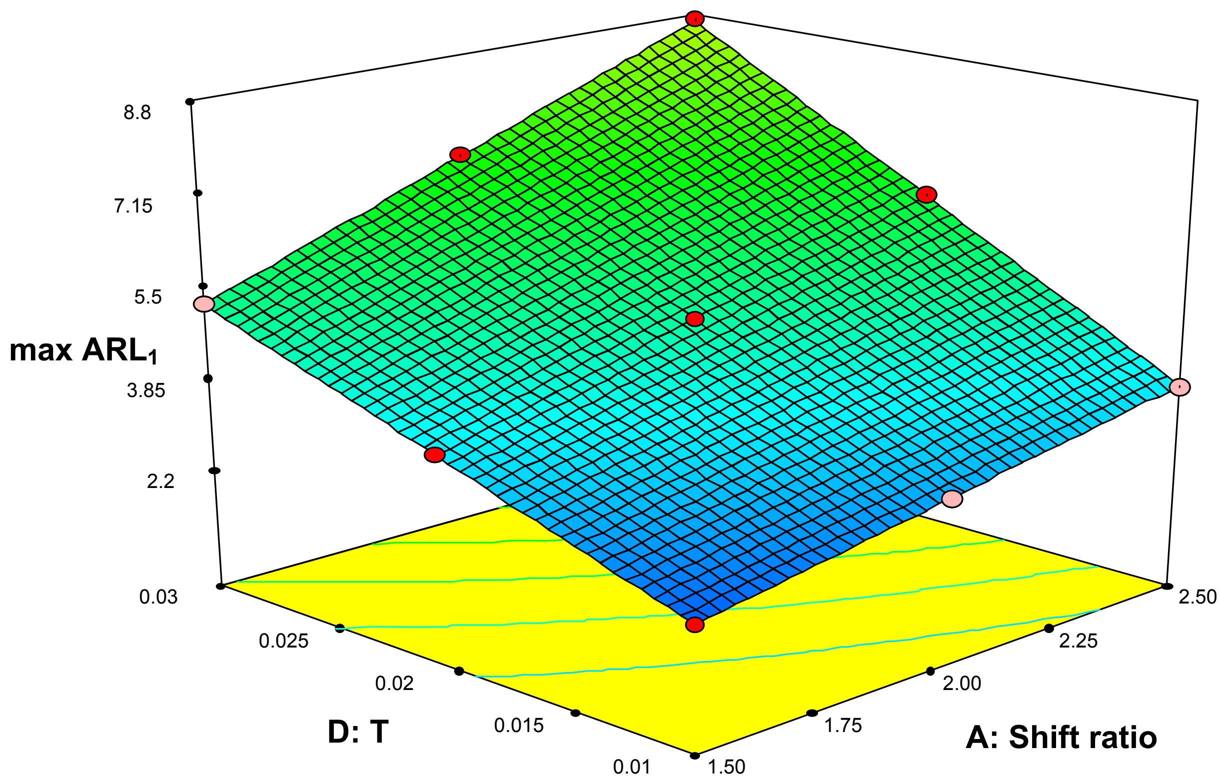

Figure 6.7. Response surface plot of the interaction (δ / δ_0) * T.

According to Figure 6.8, the maximum ARL_1 of 7.48 is at the lowest point, when R and l are at their low levels (70 and 0.001). The ARL_1 would significantly increase if R is set at the high level (130). At the high levels of R and l (130 and 0.009), the maximum ARL_1 is at the highest point or 7.5. It is interesting to note that lag time seems to have less effect on the maximum ARL_1 than production rate.

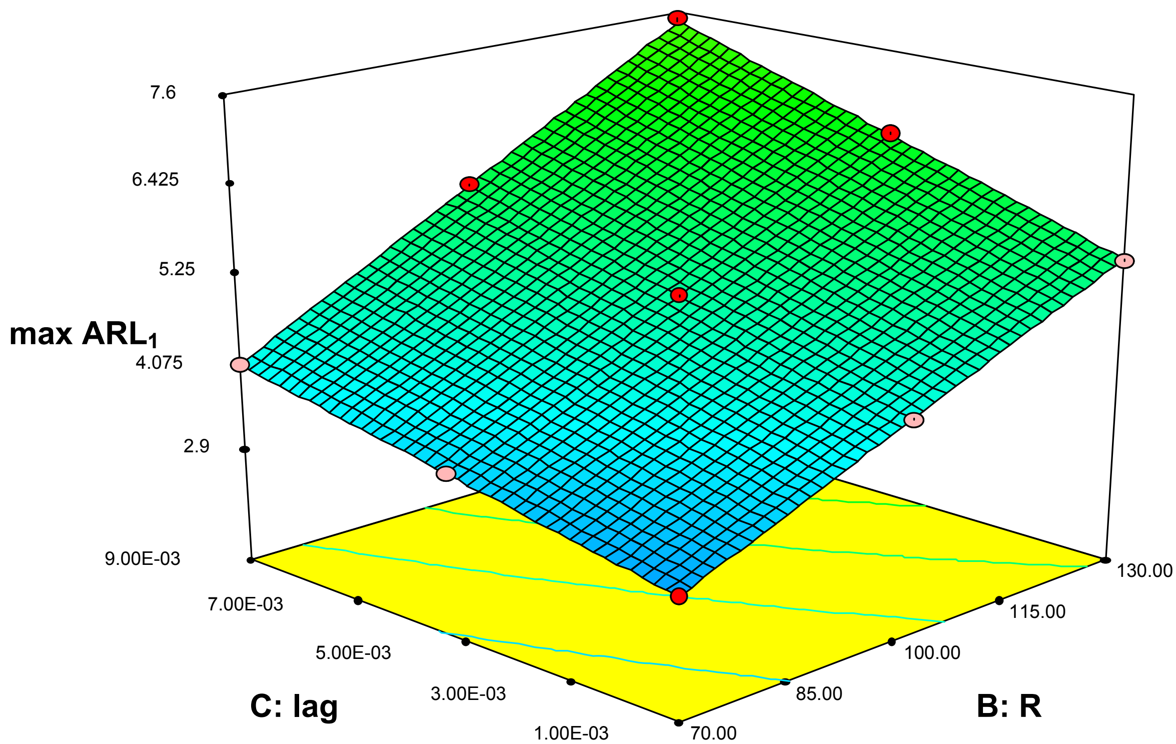

Figure 6.8. Response surface plot of the interaction R * *l*.

Figure 6.9 shows that the highest value of maximum ARL_1 of 9.1 is achieved when R and T are high (130 and 0.03). However, the ARL_1 significantly drops to 4.9 when R decreases to 70. If both parameters are set at the low levels, then the maximum ARL_1 decreases to 2.1. If R increases to 130 and T is kept at the low level, the maximum ARL_1 of 2.09 is obtained.

In conclusion, the value of the maximum ARL_1 depends on the two factors interaction between δ / δ_0 or R with *l* or T. The chance of implementing the ISPC systems would be increased if two of these three factors are set at their high levels.

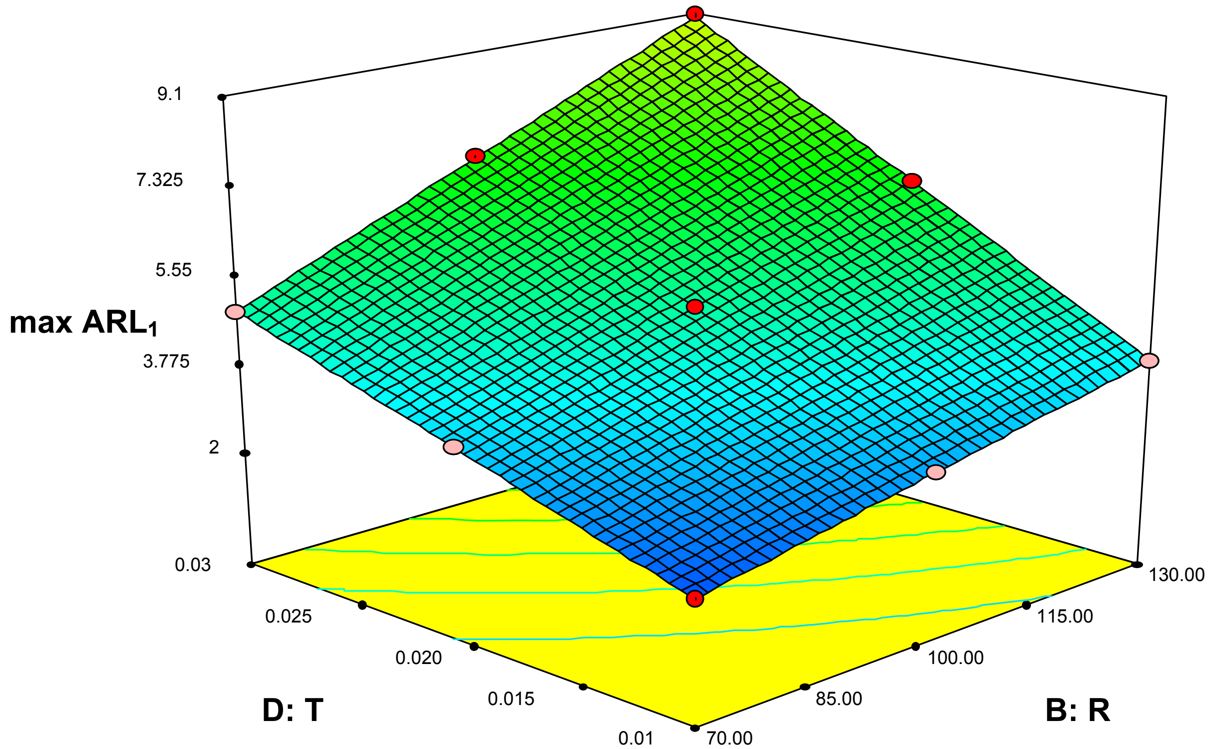

Figure 6.9. Response surface plot of the interaction R * T.

6.6.2 Sensitivity Analysis of ARL_0

Many factors affect the value of the minimum ARL_0 (equation (6.29)); therefore, an experiment was conducted to study the effect of each parameter on the minimum ARL_0. A three-level factorial experiment would require a total of 3^8 runs. However, in practice, F and λ can be expressed as functions of R, while A, δ, and σ are expressed as functions of Δ. Consequently, the design can be reduced to a three-level factorial with a total of 81 runs. The list of factors and their respective levels are shown in Table 6.7, and the experimental data is shown in Appendix C.

TABLE 6.7

LIST OF FACTORS: MIN ARL_0

Factor	Low	Medium	High
R	70	100	130
Δ	0.0005	0.001	0.0015
q	0.1	0.55	1
Δ_I	1	5.5	10

The analysis of variance (Table 6.8) shows that there are significant effects from two factor interactions and quadratic terms (R, q, Δ_I, R^2, q^2, and Δ_I^2), while the normal probability plot in Figure 6.10 indicates that the error is normally distributed.

TABLE 6.8

ANALYSIS OF VARIANCE: MIN ARL_0

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Model	202.0182	13	15.53986	42.87348	< 0.0001
R	38.32515	1	38.32515	105.7366	< 0.0001
q	37.75482	1	37.75482	104.1631	< 0.0001
Δ_I	33.09641	1	33.09641	91.31084	< 0.0001
R^2	6.155265	1	6.155265	16.98198	0.0001
q^2	6.237698	1	6.237698	17.2094	< 0.0001
Δ_I^2	3.310066	1	3.310066	9.132256	0.0036
Residual	24.28473	67	0.362459		
Total	226.303	80			

According to Figure 6.11, the minimum ARL_0 reaches the maximum value of 509.01 when the production rate is set at 130 units per hour. However, when the production rate is at its low level of 70 units per hour, the minimum ARL_0 is significantly reduced to 272.88. This would increase the chance for implementing the ISPC system in the process.

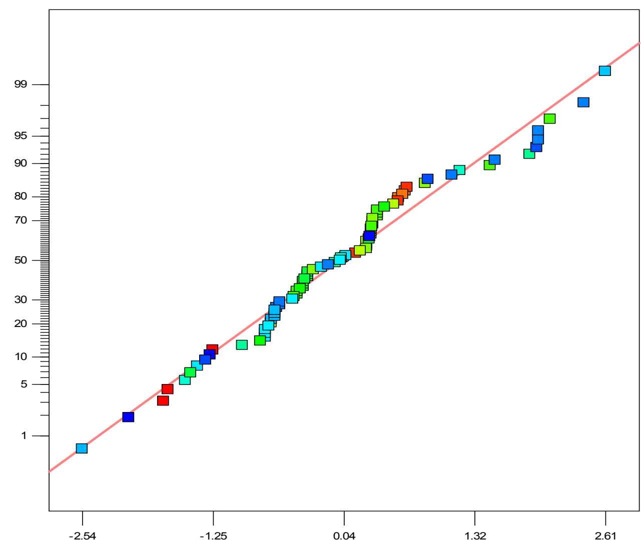

Figure 6.10. Normal probability plot of the residual: min ARL_0.

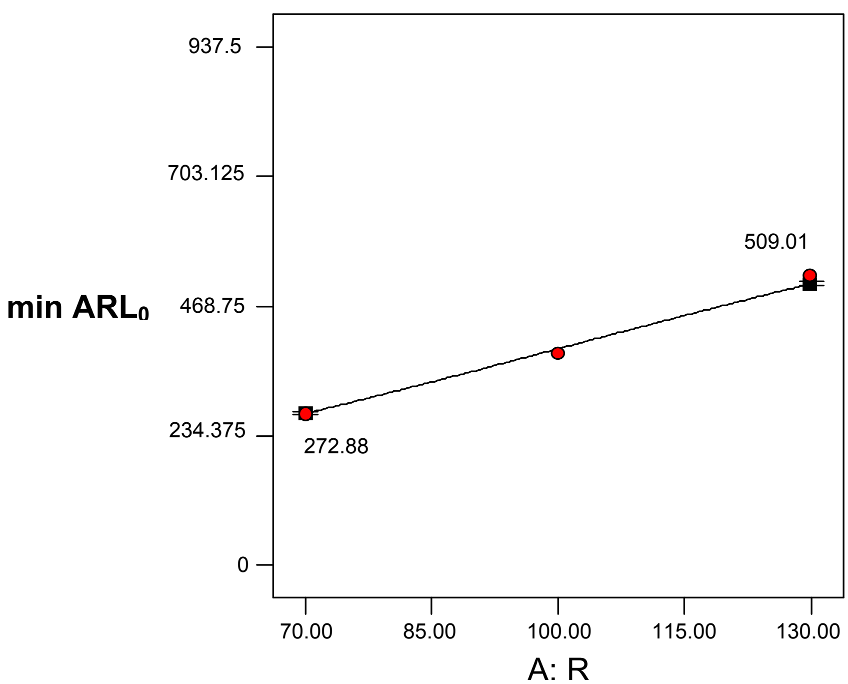

Figure 6.11. Main effect plot of R.

Figure 6.12 indicates the quadratic effect of the probability q. The minimum ARL_0 is significantly higher than the one at the high level of q (1.0). As shown, a ten-fold increase in the estimated value of q results in a reduction of 1926.06 in the minimum ARL_0. Therefore, the opportunity that the ISPC system is utilized would be maximized if q is increased.

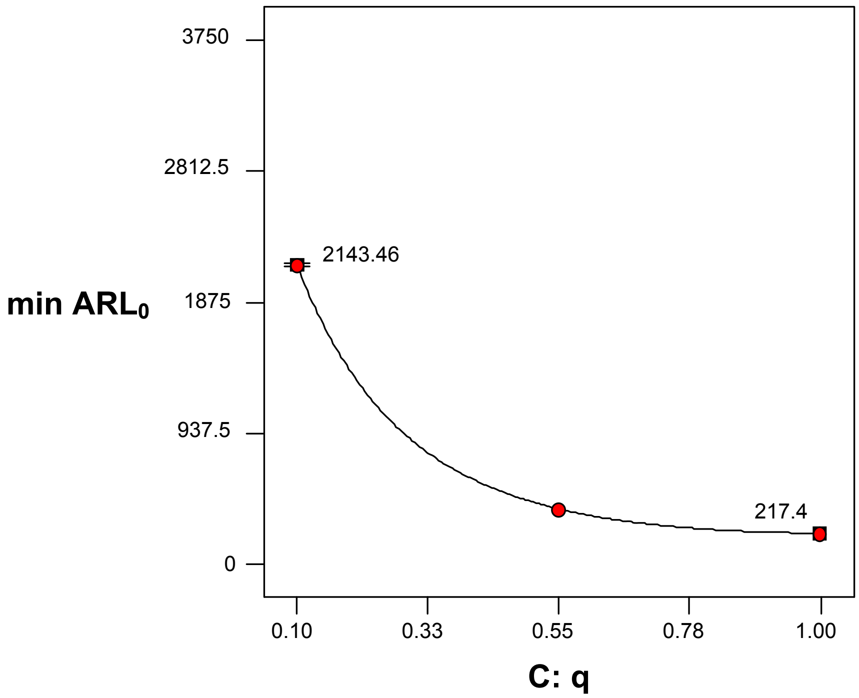

Figure 6.12. Main effect plot of q.

As shown in Figure 6.13, when the value of Δ_I is increased from 1 to 10, the minimum ARL_0 drops from 2141.38 to 217.29. Hence, if Δ_I is at the high level, the greater the chance that the ISPC system would be preferred.

In conclusion, the minimum ARL_0 would be maximized if either q or Δ_I are high while the production rate is set at the low level. This scenario increases the chance that the control chart could satisfy the second criterion. However, when one of these factors is set at the opposite level to the one mentioned above, the chance of utilizing the ISPC system would be minimized.

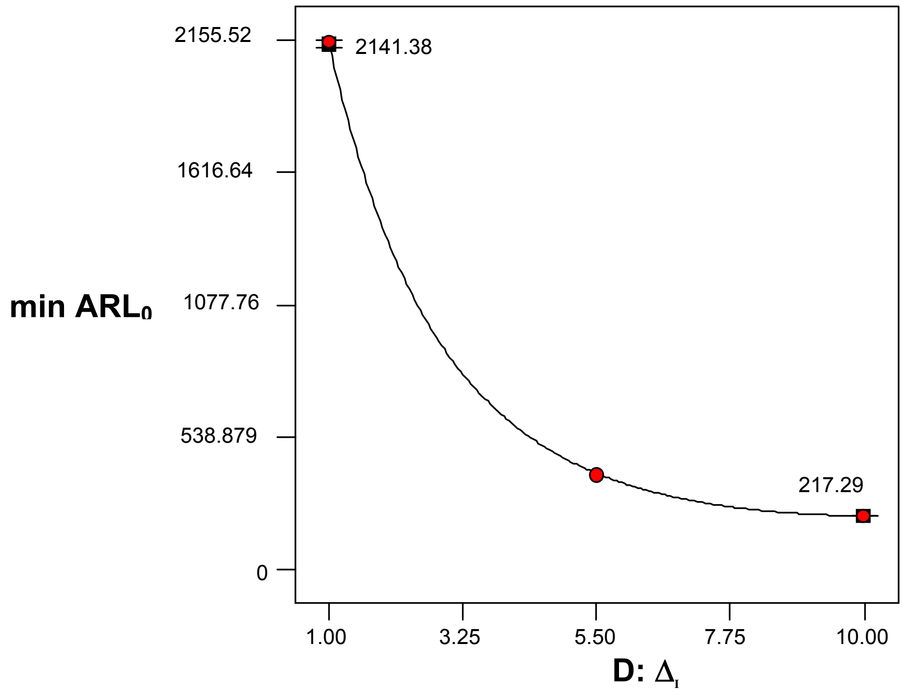

Figure 6.13. Main effect plot of Δ_I.

CHAPTER 7

CONCLUSIONS AND FUTURE RESEARCH

7.1 Conclusions

The first objective of this research was to characterize the statistical performance of the ISPC system under varying levels of selected factors. These included model coefficient, shift magnitude, type of controller, and type of control chart. In order to achieve this objective, a simulation model was developed for the purpose of studying the impact of these factors on the long-term performance of the system. The process model considered is one of discrete manufacturing characterized by the integrated moving average IMA (1, 1) model with different levels of drift rate. It was assumed that the process mean is subjected to a random shift of a specified magnitude following the Poisson process. Combinations involving the minimum mean squared error and proportional integral controllers were considered with the Shewhart individual measurement and exponentially weighted moving average (EWMA) control charts. These control charts were used to monitor both the control signal and output errors under varied levels of process factors. Observed values of the mean squared error and average run length were analyzed using a two-level factorial design. Results indicated that it is more effective to monitor the control signal than the output error, regardless of the type of the controllers used. This offers an answer to the question posed by Montgomery (2001) about whether to monitor the control signal or the output error for the best performance of ISPC systems. In addition, results identified a number of influential factors to be considered in estimating the total cost of operation.

The second objective of this research was to evaluate the economic performance of the ISPC systems. In meeting this objective, two mathematical models were developed, with due consideration of the quadratic loss concept proposed by Taguchi (1986). The first represents the expected loss of utilizing an APC system alone and accounts for the cost of operation during the steady and transient states of the production cycle. The expected total loss was expressed in terms of the controller's range, lag time, adjustment time, and variance. Also, the model accounts for the rate of production, product value, design tolerance, shift magnitude, and rate of occurrence. It was shown that total cost depends on the shift magnitude relative to the range of the controller and its associated probability of occurrence.

The second cost function was developed to represent the expected loss of utilizing an ISPC system. In contrast to the former, this cost model included the additional cost of false alarms during the steady state and the cost of operation during the out-of-control state. Both were expressed in terms of average run length of the control chart utilized.

The difference between these two cost models was used to obtain the net savings expected from the integration. Two criteria were derived to indicate boundaries for economic integration in terms of the required average run length during in-control and out-of-control stages. As such, this would help select among alternative SPC charting techniques to maximize the expected net savings. To the best of my knowledge, this is the first attempt made to offer the economic justification of utilizing the ISPC system. Furthermore, the model can be easily modified to represent other process models including the autoregressive AR (1) and the autoregressive moving average ARMA (1, 1) models. This can be accomplished by modifying the adjustment time during the transient state to accommodate differences in their dynamic behaviors.

An application of the economic criteria was presented for the selection among three charting alternatives to monitor the control signal. For a specified set of process characteristics and cost coefficients, it was shown that the use of a cumulative sum chart with H = 4 and K =1 would maximize the net expected savings as a result of the integration. To investigate the model performance, a sensitivity analysis was performed utilizing a three-level factorial design involving a wide range of process characteristics and cost coefficients. Results indicate that the adjustment time (T) has the highest effect on the maximum allowable average run length during the out-of-control state, followed by the production rate (R) and shift ratio (δ/δ_0). On the other hand, the average run length during the in-control state was found to be sensitive to changes in the production rate followed by the probability q and Δ_I. A procedure for obtaining the reliable estimate of the probability p based on the historical data has been proposed.

7.2 Future Research

Advantages of the model developed in this research suggest that it might be utilized in justifying the use of ISPC systems. An extension of the research for other process models could easily be made. Moreover, assumptions made to maintain the generality of the model might be examined. In addition, the following are proposed areas for future research.

Effect of Measurement Errors

An implicit assumption was made that the measurements are accurate and error free. In any measurement system, as noted by Shewhart, there are two sources of error: bias and imprecision. If the error is negligible, the assumption made in this research would hold true. Otherwise, the overall performance of the ISPC systems would be affected. The significance of research in this area would be to investigate the effect of measurement error on the expected net savings and provide methods for compensating their effects.

Multivariate process control

In this research, only one output variable was considered; therefore, an extension to the multivariate scenario would be required when the controller can adjust for more than one variable at the same time. Situations exist where these variables may be correlated, leading to an increased rate of false alarms. Research in this area would help identify appropriate charting alternatives for economic integration.

Accounting for system degradation

The assumption was that the APC system would maintain its original performance characteristic over time. Like most systems, controllers are subject to wear and tear and may experience an increasing failure rate during their service life. Therefore, integration may become economically feasible as the performance of the controller deteriorates over time. Future research could identify the appropriate time for SPC integration as an alternative to total replacement of the APC system.

Incorporating the value of knowledge gained

While this research focused on the cost of ISPC systems, it is anticipated that the statistical monitoring of the control signal would result in gaining additional process-specific knowledge. This knowledge could be used to improve the quality of the process or product. Hence, the model may be expanded to include the value of the knowledge gained and consequently offer more economic justification of the integration.

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APPENDICES

APPENDIX A

DESIGN MATRIX FOR MSE AND ARL

Standard Order	Drift Rate	Shift	Controller	Chart	Signal	MSE	ARL
1	0.1	0.5	MMSE	Ind	X	1.00592	2.0362
2	1	0.5	MMSE	Ind	X	1.00504	2.3758
3	0.1	2.5	MMSE	Ind	X	1.0965	1.05794
4	1	2.5	MMSE	Ind	X	1.12506	1.3815
5	0.1	0.5	PI	Ind	X	1.00585	2.052
6	1	0.5	PI	Ind	X	1.00507	2.3491
7	0.1	2.5	PI	Ind	X	1.0971	1.0569
8	1	2.5	PI	Ind	X	1.125006	1.414
9	0.1	0.5	MMSE	EWMA	X	1.00469	1.4899
10	1	0.5	MMSE	EWMA	X	1.00508	1.8339
11	0.1	2.5	MMSE	EWMA	X	1.0916	1.0782
12	1	2.5	MMSE	EWMA	X	1.124614	1.3798
13	0.1	0.5	PI	EWMA	X	1.00407	1.7741
14	1	0.5	PI	EWMA	X	1.00496	2.0114
15	0.1	2.5	PI	EWMA	X	1.0921	1.05998
16	1	2.5	PI	EWMA	X	1.124952	1.6935
17	0.1	0.5	MMSE	Ind	e	1.01438	43.4212
18	1	0.5	MMSE	Ind	e	1.00305	44.2674
19	0.1	2.5	MMSE	Ind	e	1.2045	16.0418
20	1	2.5	MMSE	Ind	e	1.089507	30.8625
21	0.1	0.5	PI	Ind	e	1.01444	43.4563
22	1	0.5	PI	Ind	e	1.00307	44.181
23	0.1	2.5	PI	Ind	e	1.2042	15.8751
24	1	2.5	PI	Ind	e	1.089779	30.8422
25	0.1	0.5	MMSE	EWMA	e	1.01371	45.4457
26	1	0.5	MMSE	EWMA	e	1.00307	43.7739
27	0.1	2.5	MMSE	EWMA	e	1.129688	1.157
28	1	2.5	MMSE	EWMA	e	1.080866	39.5636
29	0.1	0.5	PI	EWMA	e	1.01388	45.2909
30	1	0.5	PI	EWMA	e	1.00303	43.8926
31	0.1	2.5	PI	EWMA	e	1.129819	1.1419
32	1	2.5	PI	EWMA	e	1.081126	39.5123

APPENDIX B

DESIGN MATRIX FOR MAX (ARL_1)

Standard Order	Shift Ratio	Production Rate	Lag	Tolerance	Max ARL1
1	1.5	70	0.001	0.01	1.155
2	2	70	0.001	0.01	1.54
3	2.5	70	0.001	0.01	1.925
4	1.5	100	0.001	0.01	1.65
5	2	100	0.001	0.01	2.2
6	2.5	100	0.001	0.01	2.75
7	1.5	130	0.001	0.01	2.145
8	2	130	0.001	0.01	2.86
9	2.5	130	0.001	0.01	3.78
10	1.5	70	0.005	0.01	1.575
11	2	70	0.005	0.01	2.1
12	2.5	70	0.005	0.01	2.625
13	1.5	100	0.005	0.01	2.25
14	2	100	0.005	0.01	3
15	2.5	100	0.005	0.01	3.75
16	1.5	130	0.005	0.01	2.925
17	2	130	0.005	0.01	3.9
18	2.5	130	0.005	0.01	4.875
19	1.5	70	0.009	0.01	1.995
20	2	70	0.009	0.01	2.66
21	2.5	70	0.009	0.01	3.325
22	1.5	100	0.009	0.01	2.85
23	2	100	0.009	0.01	3.68
24	2.5	100	0.009	0.01	4.75
25	1.5	130	0.009	0.01	3.705
26	2	130	0.009	0.01	4.94
27	2.5	130	0.009	0.01	6.175
28	1.5	70	0.001	0.02	2.205
29	2	70	0.001	0.02	2.94
30	2.5	70	0.001	0.02	3.675
31	1.5	100	0.001	0.02	3.15
32	2	100	0.001	0.02	4.2
33	1.5	100	0.001	0.02	3.15
34	1.5	130	0.001	0.02	4.095
35	2	130	0.001	0.02	5.46
36	2.5	130	0.001	0.02	6.825
37	1.5	70	0.005	0.02	2.625
38	2	70	0.005	0.02	3.5

Standard Order	Shift Ratio	Production Rate	Lag	Tolerance	Max ARL1
39	2.5	70	0.005	0.02	4.375
40	1.5	100	0.005	0.02	3.75
41	2	100	0.005	0.02	5
42	2.5	100	0.005	0.02	6.25
43	1.5	130	0.005	0.02	4.875
44	2	130	0.005	0.02	6.5
45	2.5	130	0.005	0.02	8.125
46	1.5	70	0.009	0.02	3.045
47	2	70	0.009	0.02	4.06
48	2.5	70	0.009	0.02	5.075
49	1.5	100	0.009	0.02	4.35
50	2	100	0.009	0.02	5.8
51	2.5	100	0.009	0.02	7.25
52	1.5	130	0.009	0.02	5.655
53	2	130	0.009	0.02	7.54
54	2.5	130	0.009	0.02	9.425
55	1.5	70	0.001	0.03	3.255
56	2	70	0.001	0.03	4.34
57	2.5	70	0.001	0.03	5.425
58	1.5	100	0.001	0.03	4.65
59	2	100	0.001	0.03	6.1
60	2.5	100	0.001	0.03	7.75
61	1.5	130	0.001	0.03	6.045
62	2	130	0.001	0.03	8.06
63	2.5	130	0.001	0.03	10.075
64	1.5	70	0.005	0.03	3.675
65	2	70	0.005	0.03	4.9
66	2.5	70	0.005	0.03	6.125
67	1.5	100	0.005	0.03	5.25
68	2	100	0.005	0.03	7
69	2.5	100	0.005	0.03	8.75
70	1.5	130	0.005	0.03	6.825
71	2	130	0.005	0.03	9.1
72	2.5	130	0.005	0.03	11.375
73	1.5	70	0.009	0.03	4.095
74	2	70	0.009	0.03	5.46
75	2.5	70	0.009	0.03	6.825
76	1.5	100	0.009	0.03	5.85
77	2	100	0.009	0.03	7.8
78	2.5	100	0.009	0.03	9.75
79	1.5	130	0.009	0.03	7.605
80	2	130	0.009	0.03	9.98
81	2.5	130	0.009	0.03	12.11

APPENDIX C

DESIGN MATRIX FOR MIN (ARL_0)

Standard Order	Production Rate	Δ	q	Δ_I	Min ARL_0
1	70	0.0005	0.1	1	8287.88
2	100	0.0005	0.1	1	11760.6
3	130	0.0005	0.1	1	14980.3
4	70	0.001	0.1	1	8279.4
5	100	0.001	0.1	1	11851.6
6	130	0.001	0.1	1	14880.3
7	70	0.0015	0.1	1	8284.91
8	100	0.0015	0.1	1	11835.62
9	130	0.0015	0.1	1	14870.2
10	70	0.0005	0.55	1	1506.35
11	100	0.0005	0.55	1	2151.77
12	130	0.0005	0.55	1	2797.51
13	70	0.001	0.55	1	1506.35
14	100	0.001	0.55	1	2151.12
15	130	0.001	0.55	1	2798.89
16	70	0.0015	0.55	1	1506.35
17	100	0.0015	0.55	1	2151.78
18	130	0.0015	0.55	1	2797.53
19	70	0.0005	1	1	828.49
20	100	0.0005	1	1	1183.56
21	130	0.0005	1	1	1583.63
22	70	0.001	1	1	828.49
23	100	0.001	1	1	1183.56
24	130	0.001	1	1	1538.63
25	70	0.0015	1	1	828.49
26	100	0.0015	1	1	1183.56
27	130	0.0015	1	1	1538.63
28	70	0.0005	0.1	5.5	1506.35
29	100	0.0005	0.1	5.5	2170.93
30	130	0.0005	0.1	5.5	2811.51
31	70	0.001	0.1	5.5	1506.35
32	100	0.001	0.1	5.5	2134.79
33	130	0.001	0.1	5.5	2760.51
34	70	0.0015	0.1	5.5	1506.35
35	100	0.0015	0.1	5.5	2151.12
36	130	0.0015	0.1	5.5	2797.51
37	70	0.0005	0.55	5.5	267.88
38	100	0.0005	0.55	5.5	398.26

Standard Order	Production Rate	Δ	q	Δ_I	Min ARL_0
39	130	0.0005	0.55	5.5	501.64
40	70	0.001	0.55	5.5	272.88
41	100	0.001	0.55	5.5	382.26
42	130	0.001	0.55	5.5	523.64
43	70	0.0015	0.55	5.5	273.88
44	100	0.0015	0.55	5.5	391.26
45	130	0.0015	0.55	5.5	508.64
46	70	0.0005	1	5.5	154.64
47	100	0.0005	1	5.5	226.19
48	130	0.0005	1	5.5	279.75
49	70	0.001	1	5.5	155.64
50	100	0.001	1	5.5	209.19
51	130	0.001	1	5.5	279.75
52	70	0.0015	1	5.5	150.64
53	100	0.0015	1	5.5	215.19
54	130	0.0015	1	5.5	279.75
55	70	0.0005	0.1	10	836.49
56	100	0.0005	0.1	10	1183.56
57	130	0.0005	0.1	10	1538.68
58	70	0.001	0.1	10	828.49
59	100	0.001	0.1	10	1198.56
60	130	0.001	0.1	10	1556.36
61	70	0.0015	0.1	10	814.49
62	100	0.0015	0.1	10	1176.56
63	130	0.0015	0.1	10	1598.63
64	70	0.0005	0.55	10	157.64
65	100	0.0005	0.55	10	215.19
66	130	0.0005	0.55	10	282.75
67	70	0.001	0.55	10	150.64
68	100	0.001	0.55	10	215.19
69	130	0.001	0.55	10	280.75
70	70	0.0015	0.55	10	150.64
71	100	0.0015	0.55	10	215.19
72	130	0.0015	0.55	10	279.75
73	70	0.0005	1	10	82.85
74	100	0.0005	1	10	124.36
75	130	0.0005	1	10	161.86
76	70	0.001	1	10	84.85
77	100	0.001	1	10	122.38
78	130	0.001	1	10	161.86
79	70	0.0015	1	10	81.85
80	100	0.0015	1	10	118.31
81	130	0.0015	1	10	156.86