NUMB3RS Activity: Prime Time
Episode: "Prime Suspect"

Topic: Prime numbers **Grade Level:** 9 - 12
Objective: Students will explore an unproven conjecture and find prime number sums.
Time: 15 - 20 minutes

Introduction

When a mathematician's daughter is kidnapped for her father's knowledge, the FBI and Charlie get involved. The kidnappers want the mathematician's proof of the Riemann hypothesis in order to break all Internet security codes and gain access to certain financial information.

The Riemann hypothesis focuses on special prime factorization of very large numbers (which are too large for this activity). To have students gain an appreciation for the hypothesis, the activity will focus on a similar idea known as Goldbach's conjecture. Goldbach's conjecture states that any even number greater than 4 can be written as the sum of two odd prime numbers (for example, 12 = 5 + 7). These even integers are called *Goldbach numbers*.

Since it was first proposed in 1742, it has remained an unproved conjecture (like the Riemann hypothesis). At one time, a $1,000,000 prize was offered to anyone who could prove Goldbach's conjecture – this prize went unclaimed.

Discuss with Students

Note that many Goldbach numbers can be written using different pairs of odd prime numbers. For example, 24 can be written as 5 + 19, 7 + 17, or 11 + 13. Because we are interested only in which two prime numbers give the sum, make sure students realize that, as may be expected, the order of the addends does not matter.

To save time, you may want to divide the class into groups, and assign different numbers from 4 to 60 to each group. There are typically more ways to write the sums for the larger numbers than the smaller numbers, so be sure to assign some small and some large numbers to each group.

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Student Page Answers:

1. *3 + 7 or 5 + 5* **2.** *5 + 13, 7 + 11* **3.** *See chart below*

6 3+3	**8** 3+5	**10** 3+7 5+5	**12** 5+7	**14** 3+11 7+7	**16** 3+13 5+11	**18** 5+13 7+11	**20** 3+17 7+13	**22** 3+19 5+17 11+11	**24** 5+19 7+17 11+13	**26** 3+23 7+19 13+13	**28** 5+23 11+17	**30** 7+23 11+19 13+17	**32** 3+29 13+19
34 3+31 5+29 11+23 17+17	**36** 5+31 7+29 13+23 17+19	**38** 7+31 19+19	**40** 3+37 11+29 17+23	**42** 5+37 11+31 13+29 19+23	**44** 3+41 7+37 13+31	**46** 3+43 5+41 17+29 23+23	**48** 5+43 7+41 11+37 17+31 19+29	**50** 3+47 7+43 13+37 19+31	**52** 5+47 11+41 23+29	**54** 7+47 11+43 13+41 17+37 23+31	**56** 3+53 13+43 19+37	**58** 5+53 11+47 17+41 29+29	**60** 7+53 13+47 17+43 19+41 23+37 29+31

4. *The larger Goldbach numbers seem to have more odd prime number sums than the smaller Goldbach numbers.* **5.** *yes* **6.** *find odd prime number sums for the Goldbach numbers 62 to 70* **7.** *Find a counterexample – that is, find an even number greater than 4 that cannot be written as the sum of two odd prime numbers.* **8.** *No. What makes the Goldbach conjecture interesting is that there is no "greatest" number, so that even though it has been shown to be true through some very large number x, there is still the possibility that x + 2 does not work, so it still needs to be tested.*

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Name: _____ Date: _____

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While the Riemann hypothesis is a bit difficult to explain, suffice it to say it deals with prime numbers. Imagine a very large number whose prime factorization involves many prime numbers, some of which are hundreds of digits long. If the Riemann hypothesis could be proved, then it could be used to develop an algorithm that would quite quickly find those prime numbers.

To gain an appreciation of this, this activity will focus on Goldbach's conjecture – another unproven conjecture involving prime numbers. Goldbach's conjecture states that any even number greater than 4 can be written as the sum of two odd prime numbers (for example, 12 = 5 + 7). These even integers are called *Goldbach numbers*.

1. Write an odd prime number sum for the Goldbach number 10.

2. Many Goldbach numbers can be written using more than one pair of odd prime numbers. For example, 16 can be written as 3 + 13 or 7 + 9. Find two different odd prime number sums for the Goldbach number 18.

3. For each of the following Goldbach numbers, write all possible sums of two odd prime numbers.

6	**8**	**10**	**12**	**14**	**16**	**18**
20	**22**	**24**	**26**	**28**	**30**	**32**
34	**36**	**38**	**40**	**42**	**44**	**46**
48	**50**	**52**	**54**	**56**	**58**	**60**

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4. As the Goldbach numbers get larger, what seems to happen to the number of odd prime number sums?

5. Has this proven that every even integer from 4 to 60 can be written as the sum of two odd primes?

6. How could you prove the Goldbach conjecture true for the numbers from 4 to 70?

7. How could the Goldbach conjecture be proved false?

While this conjecture seems to be true, it remains unproven since it was first proposed by Christian Goldbach to Leonard Euler in 1742. Recently, a computer was used to verify the conjecture is true for values up to 200,000,000,000,000,000. However, that is not a proof of the conjecture.

8. Suppose the Goldbach conjecture has been shown to be true for some very large number (like the one mentioned above). Does that mean it will work for all numbers?

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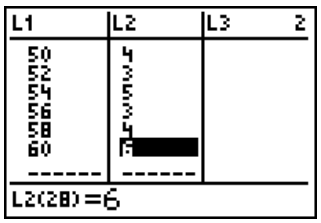
The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research

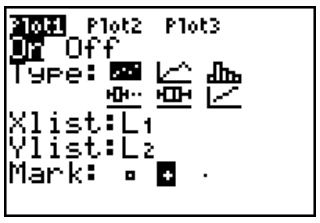
Extension
Activity: Goldbach Partitions

Introduction

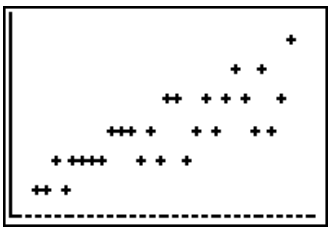
A Goldbach number is an even number greater than 4 that can be written as the sum of two odd prime numbers. A *Goldbach partition* is the number of different sums a Goldbach number has. For example, the Goldbach number 46 has four different sums (3 + 43, 5 + 41, 17 + 29, and 23 + 23). Therefore, its partition has a value of 4. To better display these results, use your calculator to create the scatter plot for the partitions and their values from 4 to 60.



Press [STAT], choose **1:Edit…**, and enter the partitions in L_1 and their values in L_2.



Press [2nd] [STAT PLOT] and match the settings shown above.



To view the graph, press [ZOOM] and choose **9:ZoomStat**.

Notice that in general, the partition values are becoming greater. This might suggest that as numbers become larger, there is a greater opportunity for them to be a Goldbach number.

Additional Resources

- For a calculator that lists the Goldbach pairs, go to **http://plus.maths.org/issue2/xfile/index.html**

- Visit the Web sites below to learn more about the Goldbach conjecture, as well as some informal ways to try to prove it is true.

  http://www.petrospec-technologies.com/Herkommer/goldbach.htm

  http://www.sciencenews.org/articles/20031011/mathtrek.asp

  http://mathworld.wolfram.com/GoldbachConjecture.html

- Join the Great Internet Mersenne Prime Search (GIMPS) by downloading the screen saver at **http://www.mersenne.org/freesoft.htm**. It uses your computer's downtime to perform mathematical calculations to search for prime numbers.

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