NAME ______________________ DATE ____________ CLASS ______________

Circular Motion and Gravitation

Problem D

PERIOD AND SPEED OF AN ORBITING OBJECT

PROBLEM

The Galilean moon Io orbits Jupiter at a mean distance of 4.22×10^5 km. Io's orbital period is 1.77 Earth days. Use this data to calculate the mass of Jupiter.

SOLUTION

1. DEFINE

Given: $T = 1.77 \text{ days} \times 24 \text{ h/day} \times 60 \text{ min/h} \times 60 \text{ s/min} = 1.53 \times 10^5 \text{ s}$

$r = 4.22 \times 10^5 \text{ km} = 4.22 \times 10^8 \text{ m}$

Unknown: $m = ?$

2. PLAN

Choose an equation or situation: Use the equation for the period of an object in circular orbit, and rearrange the equation to solve for m.

$$T = 2\pi\sqrt{\frac{r^3}{Gm}}$$

$$T^2 = 4\pi^2 \frac{r^3}{Gm}$$

$$m = 4\pi^2 \frac{r^3}{GT^2}$$

3. CALCULATE

Substitute the values into the equation(s) and solve:

$$m = 4\pi^2 \frac{(4.22 \times 10^8 \text{ m})^3}{\left(6.673 \times 10^{-11} \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right)(1.53 \times 10^5 \text{ s})^2} = \boxed{1.90 \times 10^{27} \text{ kg}}$$

4. EVALUATE

The mass of Jupiter is 1.90×10^{27} kg.

ADDITIONAL PRACTICE

1. Earth's moon orbits Earth at a mean distance of 3.84×10^8 m and has an orbital period of 27.4 days. Use this data to calculate Earth's mass.
2. Saturn's moon Titan orbits Saturn at a mean distance of 1.22×10^6 km and has an orbital period of 15.9 Earth days. Use this data to calculate Saturn's mass.
3. The Galilean moon Europa orbits Jupiter with a period of 3.55 Earth days. Calculate the mean distance between Europa and Jupiter.
4. The asteroid Ceres orbits the sun with an orbital period of 4.61 Earth years. What is the mean radius of Ceres' orbit? ($m_s = 1.99 \times 10^{30}$ kg)
5. What is the orbital speed of the asteroid Ceres (see item 4 above)?

6. What is the orbital speed of a satellite that orbits Earth at an altitude of 30.0 km? ($r_E = 6.38 \times 10^6$ m; $m_E = 5.97 \times 10^{24}$ kg)

7. What is the orbital period of a satellite that orbits Earth at an altitude of 30.0 km? ($r_E = 6.38 \times 10^6$ m)

8. Mercury has the shortest orbital period of any planet in the solar system. Mercury's mean distance from the sun is 5.79×10^{10} m. Calculate Mercury's orbital period, and express your answer in Earth days. ($m_s = 1.99 \times 10^{30}$ kg)

Givens | *Solutions*

7. $m_1 = m_2 = 9.95 \times 10^{41}$ kg

$F_g = 1.83 \times 10^{29}$ N

$G = 6.673 \times 10^{-11} \frac{\text{N•m}^2}{\text{kg}^2}$

$$r = \sqrt{\frac{Gm_1m_2}{F_g}} = \sqrt{\frac{\left(6.673 \times 10^{-11} \frac{\text{N•m}^2}{\text{kg}^2}\right)(9.95 \times 10^{41}\ \text{kg})^2}{1.83 \times 10^{29}\ \text{N}}} = \boxed{1.90 \times 10^{22}\ \text{m}}$$

8. $m_1 = 1.00$ kg

$m_2 = 1.99 \times 10^{30}$ kg

$F_g = 274$ N

$G = 6.673 \times 10^{-11} \frac{\text{N•m}^2}{\text{kg}^2}$

$$r = \sqrt{\frac{Gm_1m_2}{F_g}} = \sqrt{\frac{\left(6.673 \times 10^{-11} \frac{\text{N•m}^2}{\text{kg}^2}\right)(1.00\ \text{kg})(1.99 \times 10^{30}\ \text{kg})}{274\ \text{N}}} = \boxed{6.96 \times 10^{8}\ \text{m}}$$

9. $m_1 = 1.00$ kg

$m_2 = 3.98 \times 10^{31}$ kg

$F_g = 2.19 \times 10^{-3}$ N

$G = 6.673 \times 10^{-11} \frac{\text{N•m}^2}{\text{kg}^2}$

$$r = \sqrt{\frac{Gm_1m_2}{F_g}} = \sqrt{\frac{\left(6.673 \times 10^{-11} \frac{\text{N•m}^2}{\text{kg}^2}\right)(1.00\ \text{kg})(3.98 \times 10^{31}\ \text{kg})}{2.19 \times 10^{-3}\ \text{N}}} = \boxed{1.10 \times 10^{12}\ \text{m}}$$

10. $F_g = 125$ N

$m_1 = 4.5 \times 10^{13}$ kg

$m_2 = 1.2 \times 10^{14}$ kg

$G = 6.673 \times 10^{-11} \frac{\text{N•m}^2}{\text{kg}^2}$

$$r = \sqrt{\frac{Gm_1m_2}{F_g}} = \sqrt{\frac{\left(6.673 \times 10^{-11} \frac{\text{N•m}^2}{\text{kg}^2}\right)(4.5 \times 10^{13}\ \text{kg})(1.2 \times 10^{14}\ \text{kg})}{125\ \text{N}}} = \boxed{5.4 \times 10^{7}\ \text{m}}$$

$$r = \sqrt{\frac{Gm_1m_2}{F_g}} = \sqrt{\frac{\left(6.673 \times 10^{-11} \frac{\text{N•m}^2}{\text{kg}^2}\right)(4.5 \times 10^{13}\ \text{kg})(1.2 \times 10^{14}\ \text{kg})}{125\ \text{N}}} = \boxed{5.4 \times 10^{7}\ \text{m}}$$

Additional Practice D

1. $r = 3.84 \times 10^{8}$ m

$T = 27.4\ \text{d} = 2.37 \times 10^{6}$ s

$$m = 4\pi^2 \frac{r^3}{GT^2} = 4\pi^2 \frac{(3.84 \times 10^{8}\ \text{m})^3}{\left(6.673 \times 10^{-11} \frac{\text{N•m}^2}{\text{kg}^2}\right)(2.37 \times 10^{6}\ \text{s})^2} = \boxed{5.96 \times 10^{24}\ \text{kg}}$$

2. $r = 1.22 \times 10^{6}\ \text{km} = 1.22 \times 10^{9}$ m

$T = 15.9\ \text{d} = 1.37 \times 10^{6}$ s

$$m = 4\pi^2 \frac{r^3}{GT^2} = 4\pi^2 \frac{(1.22 \times 10^{9}\ \text{m})^3}{\left(6.673 \times 10^{-11} \frac{\text{N•m}^2}{\text{kg}^2}\right)(1.37 \times 10^{6}\ \text{s})^2} = \boxed{5.72 \times 10^{26}\ \text{kg}}$$

3. $T = 3.55\ \text{d} = 3.07 \times 10^{5}$ s

$m = 1.90 \times 10^{27}$ kg

$$r = \sqrt[3]{\frac{GmT^2}{4\pi^2}} = \sqrt[3]{\frac{\left(6.673 \times 10^{-11} \frac{\text{N•m}^2}{\text{kg}^2}\right)(1.90 \times 10^{27}\ \text{kg})(3.07 \times 10^{5}\ \text{s})^2}{4\pi^2}}$$

$r = \boxed{6.71 \times 10^{8}\ \text{m}}$

4. $T = 4.61\ \text{y} = 1.45 \times 10^{8}$ s

$m = 1.99 \times 10^{30}$ kg

$$r = \sqrt[3]{\frac{GmT^2}{4\pi^2}} = \sqrt[3]{\frac{\left(6.673 \times 10^{-11} \frac{\text{N•m}^2}{\text{kg}^2}\right)(1.99 \times 10^{30}\ \text{kg})(1.45 \times 10^{8}\ \text{s})^2}{4\pi^2}}$$

$r = \boxed{4.14 \times 10^{11}\ \text{m}}$

Givens	*Solutions*
5. $T = 4.61\ y = 1.45 \times 10^8$ s $m = 1.99 \times 10^{30}$ kg	$r = \sqrt[3]{\frac{GmT^2}{4\pi^2}} = \sqrt[3]{\frac{\left(6.673 \times 10^{-11}\ \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right)(1.99 \times 10^{30}\ \text{kg})(1.45 \times 10^8\ \text{s})^2}{4\pi^2}}$ $r = 4.14 \times 10^{11}$ m $v_t = \sqrt{G\frac{m}{r}} = \sqrt{\left(6.673 \times 10^{-11}\ \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right)\frac{(1.99 \times 10^{30}\ \text{kg})}{(4.14 \times 10^{11}\ \text{m})}} = \boxed{1.79 \times 10^4\ \text{m/s}}$
6. $r_1 = 30.0$ km $= 3.00 \times 10^4$ m $r_2 = 6.38 \times 10^6$ m $m = 5.97 \times 10^{24}$ kg	$r = r_1 + r_2 = (3.00 \times 10^4\ \text{m}) + (6.38 \times 10^6\ \text{m}) = 6.41 \times 10^6$ m $v_t = \sqrt{G\frac{m}{r}} = \sqrt{\left(6.673 \times 10^{-11}\ \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right)\frac{(5.97 \times 10^{24}\ \text{kg})}{(6.41 \times 10^6\ \text{m})}} = \boxed{7.88 \times 10^3\ \text{m/s}}$
7. $r_1 = 30.0$ km $= 3.00 \times 10^4$ m $r_2 = 6.38 \times 10^6$ m $m = 5.97 \times 10^{24}$ kg	$r = r_1 + r_2 = (3.00 \times 10^4\ \text{m}) + (6.38 \times 10^6\ \text{m}) = 6.41 \times 10^6$ m $T = 2\pi\sqrt{\frac{r^3}{Gm}} = 2\pi\sqrt{\frac{(6.41 \times 10^6\ \text{m})^3}{\left(6.673 \times 10^{-11}\ \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right)(5.97 \times 10^{24}\ \text{kg})}} = \boxed{5.11 \times 10^3\ \text{s}}$
8. $r = 5.79 \times 10^{10}$ m $m = 1.99 \times 10^{30}$ kg	$T = 2\pi\sqrt{\frac{r^3}{Gm}} = 2\pi\sqrt{\frac{(5.79 \times 10^{10}\ \text{m})^3}{\left(6.673 \times 10^{-11}\ \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right)(1.99 \times 10^{30}\ \text{kg})}} = 7.60 \times 10^6$ s $T = 7.60 \times 10^6\ \text{s} \times \frac{1\ \text{min}}{60\ \text{s}} \times \frac{1\ \text{h}}{60\ \text{min}} \times \frac{1\ \text{d}}{24\ \text{h}} = \boxed{88.0\ \text{d}}$

Additional Practice E

1. $d = 1.60$ m $\tau = 4.00 \times 10^2$ N • m $\theta = 80.0°$	$F = \frac{\tau}{d(\sin\theta)} = \frac{4.00 \times 10^2\ \text{N}\bullet\text{m}}{(1.60\ \text{m})(\sin 80.0°)}$ $F = \boxed{254\ \text{N}}$
2. $\tau_{net} = 14.0$ N • m $d' = 0.200$ m $\theta' = 80.0°$ $\tau = 4.00 \times 10^2$ N • m	$\tau_{net} = \tau - \tau'$ $\tau' = F_g d'(\sin\theta') = \tau - \tau_{net}$ $F_g = \frac{\tau - \tau_{net}}{d'(\sin\theta')} = \frac{4.00 \times 10^2\ \text{N}\bullet\text{m} - 14.\ \text{N}\bullet\text{m}}{(0.200\ \text{m})(\sin 80.0°)}$ $F_g = \frac{386\ \text{N}\bullet\text{m}}{(0.200\ \text{m})(\sin 80.0°)} = \boxed{1.96 \times 10^3\ \text{N}}$
3. $d = 2.44$ m $\tau = 50.0$ N • m $\theta = 90°$	$F = \frac{\tau}{d(\sin\theta)} = \frac{50.0\ \text{N}\bullet\text{m}}{(2.44\ \text{m})(\sin 90°)}$ $F = \boxed{20.5\ \text{N}}$