Math 1301 - College Algebra
Final Exam Review Sheet

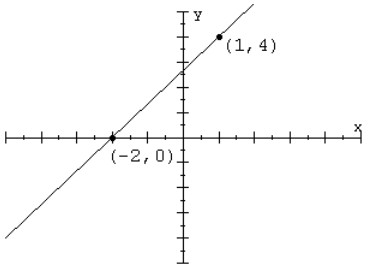
This review, while fairly comprehensive, should not be the only material used to study for the final exam. It should not be considered a preview of the final exam. It does not substitute for studying previous tests, quizzes, homework, class notes, text discussions, etc. There may be questions on the final exam unlike questions on this review, and vice versa. No question on this review will be exactly duplicated on the final exam. This review is longer than the final exam.

You may find the following formulas helpful:

$$x = \frac{-b}{2a} \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad m = \frac{\textit{rise}}{\textit{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

PART I. Multiple choice.

1. Find the slope of the line shown here.



a. -4 b. $-\frac{3}{4}$ c. $\frac{4}{3}$ d. 4 e. none of these

2. Find the slope of the line with equation $3(x + y) = 7x + 5$. (*Hint*: Put in the explicit form $y = mx + b$.)

a. $\frac{5}{3}$ b. 3 c. 4 d. $\frac{4}{3}$ e. none of these

3. The following function relates the profit P (hundreds of dollars) to the number of hundreds of Itchy-Palm Hand-Held Computers x that are manufactured.

$$P = -5(x^2 - 8x + 15)$$

Find the *smallest* break-even point for these computers (i.e. What number of computers makes the profit zero?)

a. 15 b. 300 c. $5,000$ d. -5 e. none of these

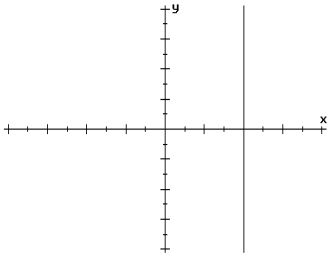
4. *Analytically* (by hand) find the root inputs of the function $f(x) = x^{-2} - 36$.

a. $x = \frac{1}{18},\ x = -\frac{1}{18}$ b. $x = \frac{1}{6},\ x = -\frac{1}{6}$ c. $x = 6,\ x = -6$ d. $x = 18,\ x = -18$

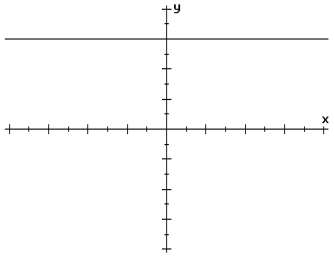
e. none of these

5. Which of the lines graphed here has *negative* slope?

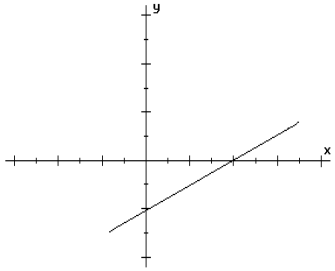
a.



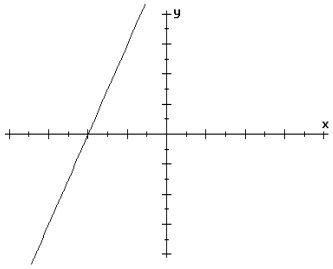
b.



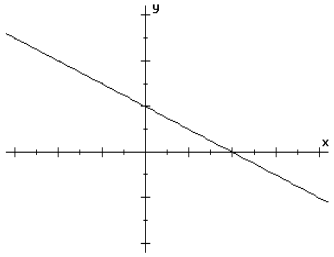
c.



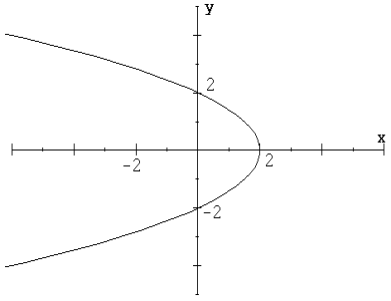
d.



e.



6. Which of the following statements is *true* about the graph shown here? (There is only one correct answer.)



a. The y-intercept is -4. b. The graph is a polynomial of degree 3. c. The graph cannot be used as a function.
d. The x-intercept is -2. e. The graph can be used as a function.

7. What is the abstract domain of the function $f(x) = \dfrac{7}{3x-6}$?

a. all real numbers except -6 b. all real numbers except 2 c. all real numbers
d. all real numbers except 7 e. none of these

8. Given that $f(x) = 2x + 5$, find $f(f(0))$.

a. 5 b. 30 c. 0 d. 15 e. none of these

9. Given that $f(x) = 2x + 5$, find $f(h + 2)$.

a. $2h + 7$ b. $h + 7$ c. $2h + 9$
d. $2h + 5$ e. none of these

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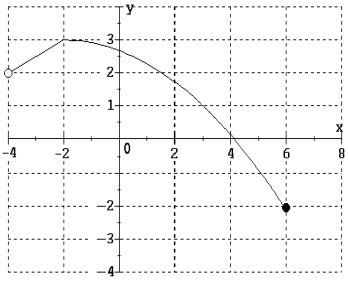
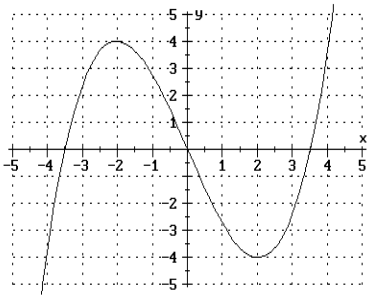
10. Find the *range* of the function graphed here.



a. $[-2,\ 2)$ b. $(-4,\ 6]$ c. $(-\infty,\ \infty)$ d. $[-2,\ 3]$ e. none of these

11. For the function f graphed here, determine $f(2)$.



a. 4 b. 2 c. -4 d. -8 e. none of these

12. If we assume that the function f graphed in Problem 11 is a polynomial function, what is the *smallest possible degree* the polynomial could have?

a. 5 b. 3 c. 2 d. 4 e. none of these

13. Which of the following numbers is *not* a root input of the function f graphed in Problem 11?

a. 0 b. 3 c. 3.5 d. -3.5 e. none of these

14. What is the set of lower inputs of the function f graphed in Problem 11?

a. $(-\infty, 0)$ b. $(-\infty, -3.5) \cup (0, 3.5)$ c. $(0, 3.5)$ d. $(-3.5, 0) \cup (3.5, \infty)$ e. none of these

15. The function f graphed in Problem 11 is decreasing over which set?

a. $(-3.5, 0)$ b. $(-\infty, -2) \cup (2, \infty)$ c. $(4, -4)$ d. $(-2, 2)$ e. none of these

16. Which of the following points is a turning point of the function f graphed in Problem 11?

a. $(-4, -5)$ b. $(3.5, 0)$ c. $(-2, 4)$ d. $(0, 0)$ e. none of these

17. Evaluate $F(x-1)$ given that $F(x) = x^2 - x$.

a. $x^2 - 2x + 1$ b. $x^2 - x - 1$ c. $x^2 - 3x - 1$ d. $x^2 - 3x + 2$ e. none of these

18. *Analytically* (by hand) find the root inputs of the function $f(x) = 2x^2 - 5x + 2$.

a. $x = \frac{1}{3},\ x = -2$ b. $x = \frac{1}{3},\ x = -\frac{1}{2}$ c. $x = 3,\ x = -2$ d. $x = \frac{1}{2},\ x = 2$

e. none of these

19. Evaluate $f^{-1}(4)$ if $f(x) = 5x - 6$. (*Hint*: You must set up and solve an equation.)

a. 2 b. $\frac{1}{4}$ c. -4 d. $\frac{1}{2}$

e. none of these

20. Consider the set $S = \{x \mid x > -5 \text{ and } x < 5\}$. Which number below is *not* an element of S ? (There is only one correct answer.)

a. $\frac{2}{3}$ b. 0 c. -1 d. $16^{1/2}$ e. 6

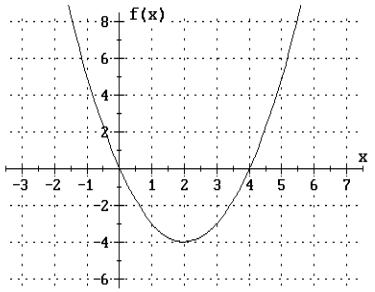
21. An antique chair is expected to increase in value by \$30 each year from 2000, when it was worth \$300. If we let v be the value of the chair and t be the elapsed time in years from 2000, which function shown here can be used to predict the value of the chair in the future?

a. $v = 300t$ b. $v = 300t + 30$ c. $v = 10t - 300$ d. $v = 30t + 300$ e. none of these

22. Find the y-intercept of the exponential function $y = 3(1.5)^x$.

a. 3 b. 0 c. 4.5 d.1.5 e. none of these

23. The graph of the function $f(x) = x^2 - 4x$ is shown here.



Use this graph to find the solution set of the inequality $x^2 - 4x < 0$. (Answers are given in interval notation.)

a. $(0, 4)$ b. $(-\infty, 0) \cup (4, \infty)$ c. $(-4, \infty)$

d. $(-\infty, -1) \cup (5, \infty)$ e. none of these

24. What is the graphing window used in Problem 23?

a. $[-\infty, \infty] \times [-\infty, \infty]$ b. $[7, -3] \times [8, -6]$ c. $[-3, 7] \times [-6, 8]$

d. $[-3.5, 7.5] \times [-6, 8]$ e. none of these

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25. What is the minimum output of the function f graphed in Problem 23?

a. none b. -6 c. -4 d. 0 e. none of these

26. Show that $\frac{2}{3}$ is in the range the given function by finding a matching input: $f(x) = \frac{7}{3x+6}$.

a. $x = \frac{3}{2}$ b. $x = -1$ c. $x = 2$ d. $x = -\frac{2}{3}$

27. Which of the following numbers is an integer (i.e. whole number)?

a. $2/10$ b. $27^{1/3}$ c. $18^{1/2}$ d. 5^{-1} e. none of these

28. Which of the following numbers is *not* a real number?

a. $\sqrt{2}$ b. $\sqrt[3]{-8}$ c. $\sqrt[4]{-4}$ d. 5^{-1} e. none of these

29. Evaluate $log_3 81$.

a. 27 b. 1 c. -4 d. 4 e. none of these

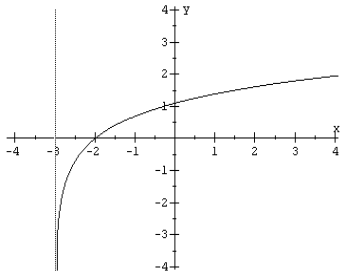
30. Evaluate $log\ \frac{1}{100}$.

a. 2 b. 5 c. -2 d. 10 e. none of these

31. Evaluate $log_{125} 5$.

a. -2 b. 6 c. -4 d. 2 e. none of these

32. Find the *domain* of the function graphed here.



a. $(-\infty,\ 2)$ b. $[-3, 4]$ c. $(-\infty,\ \infty)$ d. $(-3,\ \infty)$ e. none of these

33. For the exponential function $y = 3(1.5)^x$, evaluate $y(-1)$.

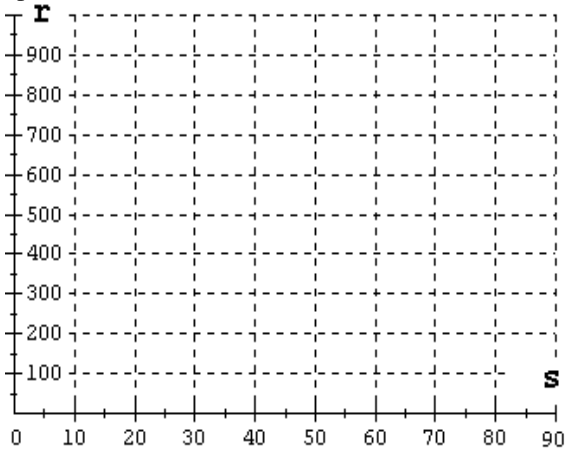
a. -4.5 b. 0.22 c. 1.5 d. 2 e. none of these

PART II. Show your work as appropriate.

34. Gail's Gourmet Bagels (GGB) is benefiting from the recent bagel craze by expanding its number of franchises. The following table contains data relating the number of GGB shops to the company's average weekly sales revenue (in thousands of dollars).

Number of Shops (s)	10	17	17	23	32	45	47	53	60	75	85 (current number)
Average Weekly Revenue (r)	115	180	190	250	360	470	462	535	625	755	865

34a. Plot this table on the set of axes provided.



34b. Graph the straight line containing the points $(10, 115)$ and $(85, 865)$. Does this line appear to "fit" this set of data points? Yes ☐ No ☐

34c. *Use the line* drawn in part 34b to predict the average weekly sales revenue for GGB six months in the future, provided GGB is planning to open 5 more stores (above the current number) during this time period. $\$$ ☐

34d. Find the slope of the line drawn in part 34b. *slope* $=$ ☐

34e. Interpret the slope of the line drawn in part 34b using a complete sentence.

☐

34f. *Analytically* find the y-intercept of the line drawn in part 34b. (Do not estimate from the graph.) y-int. $=$ ☐

34g. Let r be the output variable and let s be the input variable. Write the function symbol rule for the line drawn in part 34b.

☐

34h. Express in function notation the average weekly sales revenue for GGB, provided 100 is the number of stores. *You do not need to evaluate.* ☐

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34i. Can the original table of data be used as a function? Yes ☐ No ☐ Explain.

35a. Does the line with the symbol rule $3x - 2y = 7$ contain the point $(3, 2)$? Yes ☐ No ☐ Explain.

35b. Is the symbol rule for the line shown in part 35a written in *explicit form*? Yes ☐ No ☐ Explain.

36. Solve the inequality $3(x - 2) \leq 7(x + 1)$ *analytically*.

37. Water usage in Gotham City has been increasing by 4% per year since 1998, a trend that is expected to continue until 2010. In 1998, water usage was 300 million gallons. Let's assume the function $W(t)$ outputs the water usage in Gotham City (in millions of gallons) during year t, where t is the elapsed number of years since 1998.

37a. Find how much water was used in Gotham City in 1999; then express this information in function notation.

37b. Why is it appropriate to assume that W is an exponential function? Briefly explain.

37c. Find the initial value c and growth factor (or base) b for the function W.

$c =$ ☐ $b =$ ☐

37d. Write the symbol rule for W. $W(t) =$ ☐

37e. When the water usage in Gotham City reaches 700 million gallons, the city will need to switch from water wells to surface water. Use your answer from part 37d to write the equation that can be solved to predict the year when water usage will reach 700 million gallons. ☐ *Do not solve the equation.*

37f. Plot the *application domain* for the function W on a number line.

38. A quadratic function has its vertex at $(3, 4)$, its y-intercept is at -6, and its two x-intercepts are at approximately -1 and 5. Choose an appropriate graphing window for the graph. (*Hint*: Choose the graphing window so that the important features of the function will be visible.)

$$\left[\_\_\_\_, \_\_\_\_\right] \times \left[\_\_\_\_, \_\_\_\_\right]$$

39. Consider the function $h(t) = \frac{5t-2}{8}$; is the value 1 in the *range* of this function? Yes ☐ No ☐ Explain.

40. Consider the function $h(t) = (t-1)^{1/2}$; is the value 3 in the *range* of this function? Yes ☐ No ☐ Explain.

41. Without using a calculator or computer, match each of the following functions with its graph i)-iv).

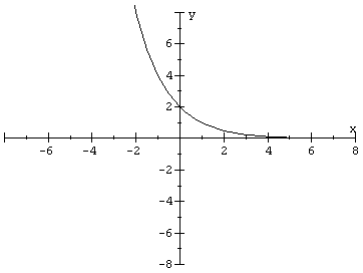
41a. $y = 2(1.5)^x$ ☐

41b. $y = 2 + 1.5x$ ☐

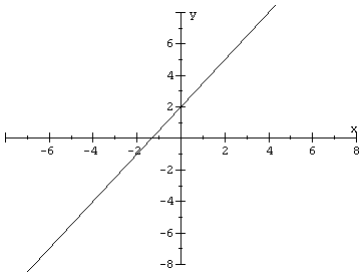
41c. $y = 2(0.5)^x$ ☐

41d. $y = x^3 - 1.5x^2 + 0.5x + 2$ ☐

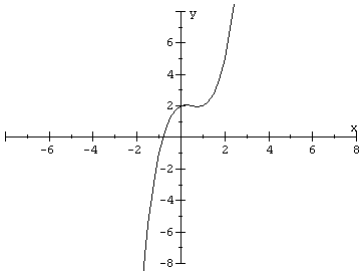
i)



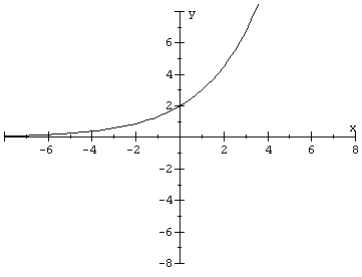
ii)



iii)



iv)



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42. The monthly revenue function for Barry's Wool Blankets is:

$$R(x) = 350xe^{-0.001x}$$

In this formula, $R(x)$ represents the monthly revenue in dollars when x blankets are sold. The function R has been tabulated in the table shown here. What is the maximum revenue Barry can generate in a month from the sale of these blankets? How many blankets must he sell to get this amount of revenue?

maximum revenue = \$ [ ] *number of blankets* = [ ]

x	$R(x)$
0	0.00
176	51658.85
352	86644.10
529	109089.20
705	121921.20
882	127787.60
1058	128549.30
1235	125713.20
1411	120449.60

x	$R(x)$
1588	113568.70
1764	105796.30
1941	97527.68
2117	89204.64
2294	80982.25
2470	73123.60
2647	65651.46
2823	58717.23
3000	52276.41

43. Find the y-intercept of the function described in Problem 3. y-int. = [ ] Interpret this value in the context of the problem.

[ ]

44. Suppose you are trying to find an approximate solution to the equation

$$x^4 - 6x^3 + 2x^2 = 35 - 30x$$

44a. Write an associated function for this equation. $f(x) =$ ☐

44b. An associated function f for the given equation has been tabulated below with two different table domains.
Which *one* of the tables is most useful to determine a solution to the given equation? Circle your answer.

i)

x	4.412	4.413	4.414	4.415	4.416
$f(x)$	-0.09	-0.049	-0.008	0.032	0.073

ii)

x	-1.000	-0.500	0	0.500	1.000
$f(x)$	-56	-48.68	-35	-20.18	-8

ii) From the table circled above, estimate a solution correct to two decimal places.

solution = ☐

45. For the polynomial function $g(z) = z^3 + 2z^2 - z - 2$:

45a. What is the degree of g? *degree* = ☐

45b. What is maximum *possible* number of x-intercepts of g? ☐

45c. What is the maximum *possible* number of turning points of g? ☐

45d. What is the leading coefficient of g? ☐

45e. What is the y-intercept of g? ☐

45f. Is the number -1 a root input of g? Yes ☐ No ☐ (*Hint*: Compute $g(-1)$.)

45g. Is the number 3 a lower input of g? Yes ☐ No ☐ (*Hint*: Compute $g(3)$.)

45h. Is the number 0 an upper input of g? Yes ☐ No ☐ (*Hint*: Compute $g(0)$.)

45i. Write the abstract domain of g. *domain* = ☐

46. Explain why each of the following functions is *not* a polynomial function.

46a. $p(x) = \frac{3}{2} - \sqrt{x+1}$

46b. $h(m) = \sqrt{3} - \frac{1}{m^2 - m}$

47. Which of the following equations and inequalities appear to be more easily solved by analytical methods (i.e. by hand)? Which appear to be good candidates for solution by numerical or graphical methods (i.e. through the use of an associated function)?

47a. $x + 9 = x(x+2)$ ☐

47b. $x + 9 \leq x(x+2)$ ☐

47c. $x + 9 < 3(x+2)$ ☐

47d. $x + 9 = \frac{3}{x+2}$ ☐

47e. $x + 9 > \frac{x+2}{3}$ ☐

47f. $x + 9 \geq \frac{3}{x+2}$ ☐

47g. $x^3 + 5x^2 - 14x + 4 = 0$ ☐

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47h. $x^3 + 5x^2 - 14x = 0$

48. Let y be the average number of children born to an adult American female of age x. Use mathematical notation to indicate that the average number of children born to an adult American female is a function of her age.

49. The monthly *demand* (i.e. quantity sold) of Barry's Wool Blankets is a function of the price per blanket. Let d be the demand for these blanket, and let p be the price per blanket in dollars.

49a. Interpret $d(173) = 705$ using a complete sentence.

49b. Interpret $d^{-1}(1764) = 60$ using a complete sentence.

49c. Is the expression $d(0)$ meaningful? Explain

50. The quantity y is *directly proportional* to the quantity x, and $y = 15$ when $x = 3$. Find the constant of proportionality k and write the symbol rule for a function that can be used to determine the value y based on the number x.

$y =$

51. The quantity y is *inversely proportional* to the quantity x, and $y = 15$ when $x = 3$. Find the constant of proportionality k and write the symbol rule for a function that can be used to determine the value y based on the number x.

$y =$

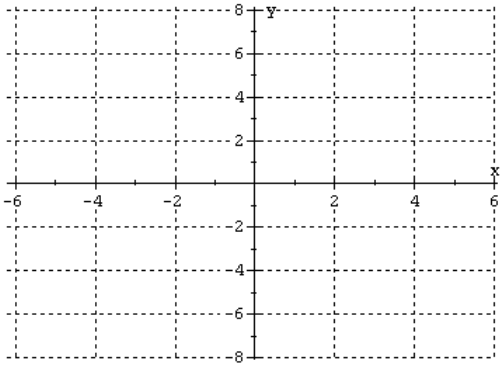
52. Consider the quadratic function $y = g(x) = x^2 + x - 6$.

52a. *Analytically* find the x-intercept(s) of this function. x-int(s). =

52b. *Analytically* find the coordinates of the vertex of this function. *vertex* =

52c. *Carefully* sketch a graph of the function g on the set of axes given below; a table has been provided for your convenience to compute coordinates of points on the parabola.

x						
y						



53. Estimate two solutions to the following equation by graphing an associated function: $6x^2 - 23x + 20 = 0$.

solutions =

54. Estimate two solutions to the following equation by graphing an associated function: $6x^2 - 23x \leq -20$.

solutions =