GETTING SMARTER THROUGH ALGEBRAIC REASONING
Lesson 2.1 Actual and perceived notions of math ability

2.1 OPENER

Complete the survey. For each statement, circle the number that best represents your experiences and beliefs.

	False	More false than true	More true than false	True
1. Compared to others my age I am good at mathematics.	1	2	3	4
2. I learn things quickly in mathematics classes.	1	2	3	4
3. I have always done well in mathematics classes.	1	2	3	4
4. I am hopeless when it comes to mathematics classes.	1	2	3	4
5. Work in mathematics classes is easy for me.	1	2	3	4
6. I get good grades in mathematics classes.	1	2	3	4
7. It is important to me to do well in mathematics classes.	1	2	3	4
8. I am satisfied with how well I do in mathematics classes.	1	2	3	4
9. When I get frustrated with a math problem, I give up.	1	2	3	4
10. When I can't solve a math problem with one strategy, I will try another way to solve the problem.	1	2	3	4
11. Doing well in math is based more on ability than effort.	1	2	3	4

Excerpted and adapted from *Academic Self Description Questionnaire II*, Herb Marsh, SELF Research Centre, University of Western Sydney, Sydney, Australia.

After you complete the survey, reflect on the following questions:

- How did these questions help you to think about yourself as a learner?
- What patterns, if any, did you notice in your answers?
- What conclusions did you come to about yourself as a math learner?

2.1 CORE ACTIVITY

1. In the next few lessons, you will investigate how to become a better learner and a better mathematical problem solver. Answer the following questions to reflect on how you already think about **intelligence** or smartness. Then discuss your answers with your partner.
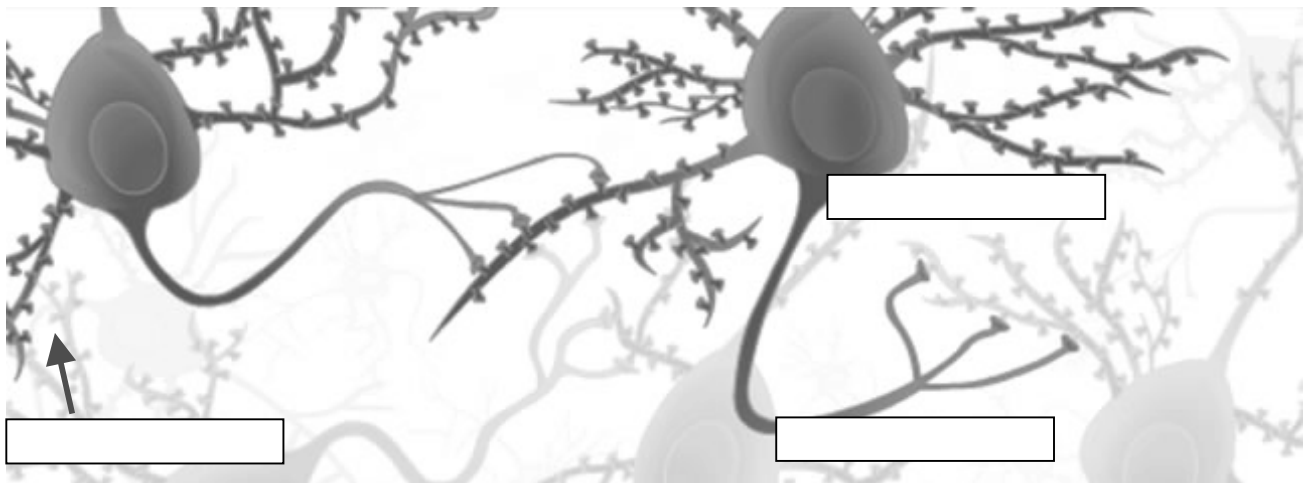
    a.   How would you define **intelligence** or smartness?

    b.   Do you think people are born with a certain level or amount of intelligence?

    c.   Do you think intelligence changes over your lifetime or do you think it stays the same?

2. Label the parts of the neuron.

Using the terms provided, complete sentences 3-6 to show your understanding of the parts of the brain.

neurons	axons	synapse	dendrites

3. The main job of _____ is to transmit information or messages.

4. Information is sent from one neuron and flows across a gap, called a _____, to other neurons.

5. The _____ carry information away from the cell body.

6. The _____ bring information to the cell body.

Using the terms provided, complete sentences 7-10 to show what you have learned about the brain and getting smarter.

easier	harder	less	changes	stays the same	Middle age	Adolescence

7. The brain physically _____ when you learn.

8. _____ is a prime time for learning.

9. Stronger and more complex connections make information transmission _____.

10. New and stronger connections are made when you work hard at something new. When you work _____, you get smarter.

2.1 CONSOLIDATION ACTIVITY

Part I: Individual reflection and writing

Answer the following prompts:

1. What do you think mathematical problem solving is?

2. How were you problem solving when you worked through the Bike and Skateboard Problems?

3. How can observing patterns be a strategy in solving a problem?

Part II: Pair-share

- With your partner, take turns sharing your thoughts and what you wrote above about problem solving.

- Be prepared to share your ideas about problem solving as your teacher leads the class in a discussion.

Part III: Compare and contrast

As you watch the animations in class and see different definitions of mathematical problem solving, consider these questions. Record some notes and be prepared for a discussion after the animation.

4. Tony, Vijay, Leticia and Trey each have thoughts about what mathematical problem solving is. How is their thinking similar to what you and your partner talked about?


5. How is their thinking different?

HOMEWORK 2.1

Notes or additional instructions based on whole-class discussion of homework assignment:

**Myths and the brain:** Have you ever heard people say that

- adults do not grow new brain cells—all brain development takes place in childhood?

- humans use only 10 percent of their brains?

Both of these statements are myths. Scientists have found that the brain continues to grow and change well into adulthood. Scientists have also shown—with advanced MRI imaging—that humans use most of their brains, not just 10 percent.
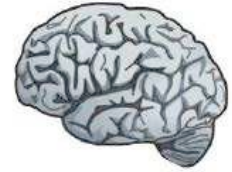
In the past decades, results from brain research have changed ideas about intelligence. For example, scientists have learned that a person's mindset is very important to his or her mental success. Rather than focusing on obstacles or worrying about whether they can do something, people who are mentally successful focus on what is necessary for solving a problem or meeting a challenge.

1. Write complete sentences in response to each of the following questions:

    a. What are some myths about intelligence that you have already uncovered in this course?

    b. What are some ways to increase the number of dendrites you have and thus become smarter?

2. Apply the skills and strategies you learned in the Bike and Skateboard Problems earlier in the unit to the following scenario. Rectangular tables are arranged in rows with chairs around each row, as shown in the picture.

    1 table   6 chairs

    2 tables   10 chairs

    3 tables   14 chairs
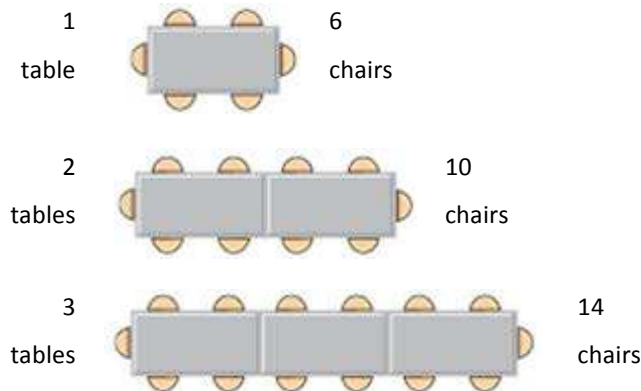
    There is a relationship between the number of tables and chairs in each row. With this in mind, answer the following questions and provide evidence for your answers.

    If the pattern continues…

    a. …How many chairs will fit around a row of 5 tables?

    b. …How many chairs will fit around a row of 11 tables?

    c. …How many tables are needed to seat 30 chairs?

STAYING SHARP 2.1

Practicing algebra skills & concepts	**1.** Complete the square box problem. - 2 - 7	**2.** A **sequence** is a set of numbers that follow a pattern. A **term** is a value within that sequence of numbers. In the sequence below, 4 is the first term, -8 is the second term, and so on. What is the 6th term in the sequence below? 4, -8, 16, -32, … Answer: Evidence for Answer:
Preparing for upcoming lessons	**3.** Each type of ball represents a different digit (0-9) that remains consistent across the three equations. Figure out the number that matches each of the balls and explain your reasoning.	Answer: Evidence for Answer:
Reviewing pre-algebra ideas	**4.** You are traveling down the interstate averaging 50 mph. How much time has passed when you have traveled 120 miles?	**5.** Pay close attention to the order operations to find the value of: $$26 + (10 - 2^{3}) \div 4 + 3 \cdot 6$$ Show each step:

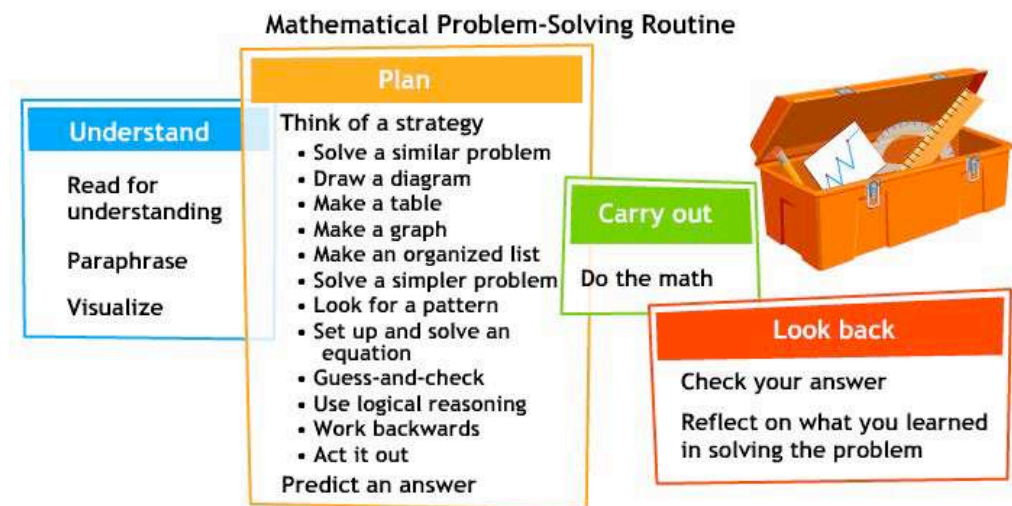# Lesson 2.2 A problem-solving routine

2.2 OPENER

Matchsticks are used to make the pattern shown. Thirteen matchsticks are used to make four squares.
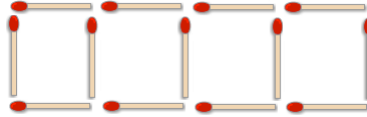
1. How many squares could you make using:

   a. 16 sticks?

   b. 22 sticks?

   c. 64 sticks?

2. Describe the process or processes you used to solve the problems.

2.2 CORE ACTIVITY

Use the Mathematical Problem-Solving Routine to solve the problem in this activity.

Mathematical Problem-Solving Routine

Understand
Read for understanding
Paraphrase
Visualize

Plan
Think of a strategy
- Solve a similar problem
- Draw a diagram
- Make a table
- Make a graph
- Make an organized list
- Solve a simpler problem
- Look for a pattern
- Set up and solve an equation
- Guess-and-check
- Use logical reasoning
- Work backwards
- Act it out
Predict an answer

Carry out
Do the math

Look back
Check your answer
Reflect on what you learned in solving the problem

Arcade Tokens Problem

James is playing an arcade game with tokens. In the first game, he hits the jackpot and doubles the number of tokens that he has. In the second game, James wins 12 more tokens. In his third and final game, he loses 4 tokens. James now has 170 tokens. How many tokens did he have before he started playing the game?

Understand

1. Try to visualize the situation.

2. State the problem in your own words.

3. What is the important information in the problem?

Plan

4. What strategy will you use to solve the problem? Why?

5. Predict an answer.

Carry out

6. Execute your plan. Show your work.

Look back

7.   Is your answer reasonable?

8. What did you learn from solving this problem?

2.2 CONSOLIDATION ACTIVITY

Compare the way you and your partner solved the Arcade Tokens Problem with the way another partner pair solved the problem.

1. Describe or list strategies you and your partner used that were similar to those used by the other partner pair.

2. Describe or list strategies you and your partner used that were different from the ones the other partner pair used.

3. With your partner, recall strategies that you used on earlier problems in the course and record them on your Personal Record Sheet. You will add new problems to the sheet and record strategies you used to solve those problems, including a homework problem for this lesson, the Card Table Problem.

Personal Record Sheet: Problem-Solving Strategies

Mark a ✓ for strategies that you used to work on the problem or problem type.

Problem or Problem Type	Strategies Used											
	Solve a similar problem	Draw a diagram	Make a table	Make a graph	Make an organized list	Solve a simpler problem	Look for a pattern	Set up and solve an equation	Guess-and-check	Use logical reasoning	Work backwards	Act it out
Bike and Skateboard Problem												
Extension of Bike and Skateboard Problem												
Shape Equations												
Square Box Problems												
Arcade Tokens Problem												

Personal Record Sheet: Problem-Solving Strategies

Mark a ✓ for strategies that you used to work on the problem or problem type.

Problem or Problem Type	Strategies Used											
	Solve a similar problem	Draw a diagram	Make a table	Make a graph	Make an organized list	Solve a simpler problem	Look for a pattern	Set up and solve an equation	Guess-and-check	Use logical reasoning	Work backwards	Act it out

HOMEWORK 2.2

Notes or additional instructions based on whole-class discussion of homework assignment:

Ms. Francis is planning a birthday party for her daughter. There will be 22 children at the party, and in order to seat them all she needs to rent square card tables. Only 1 child can sit at each side of a card table.

Ms. Francis wants to arrange the tables in a rectangular shape so they look like one large table. The room is large enough that the rectangle can be made with more than one row of tables.

Part A: How many different arrangements can Ms. Francis make to seat all 22 children?

Part B: What is the smallest number of tables that Ms. Francis needs to rent?

DIRECTIONS: To solve the problem, apply the steps of the Mathematical Problem-Solving Routine.

Understand

1. Try to visualize the situation. Consider drawing a diagram.

2. State the problem in your own words.

3. What is the important information in the problem?

Plan

4. What strategy will you use to solve the problem? Why?

Predict an answer.

Carry out

5. Execute your plan. A grid is provided to help you solve the problem. You can use the grid to draw table arrangements. Report your answers to Part A and Part B, along with evidence for your answer.
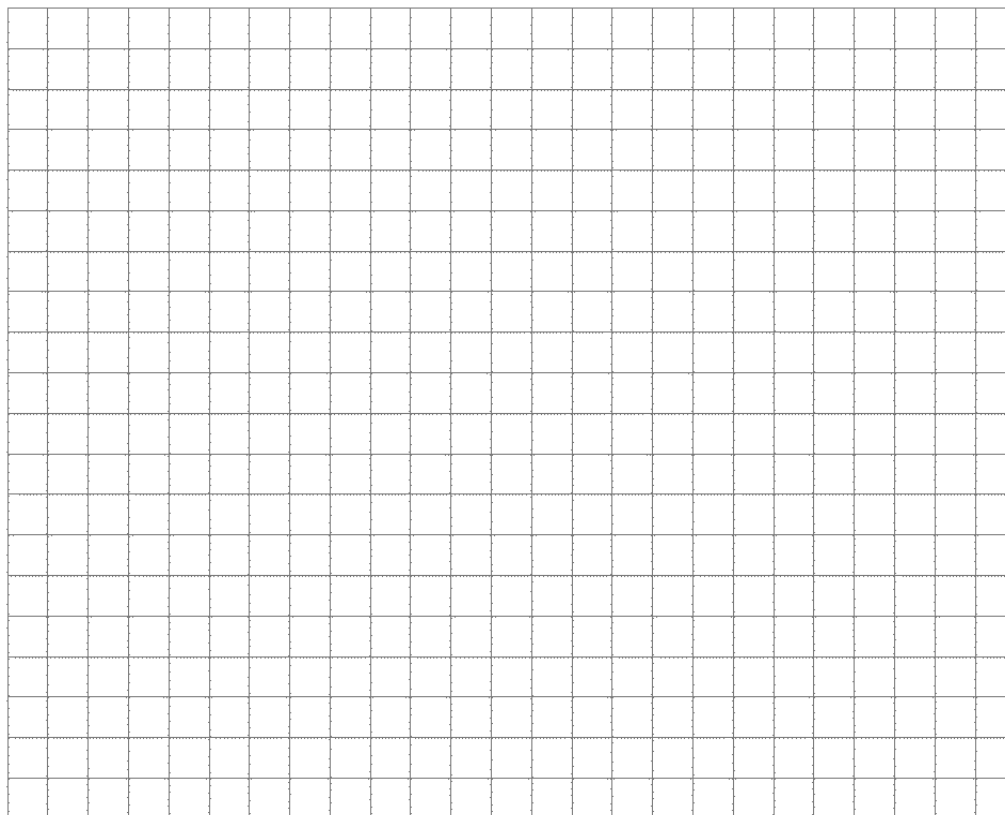
Look back

6. What can you learn from this problem? What strategy did you use and why did it work for this problem? For what type of problem would this strategy work in general?

7. Go back to the Personal Record Sheet: Problem-Solving Strategies. Add the Card Table Problem to one of the lines in the sheet. Then record the strategy or strategies you used to solve the problem.
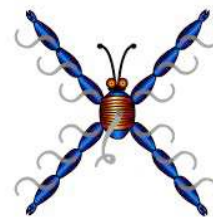
STAYING SHARP 2.2

Practicing algebra skills & concepts

1. Write the numbers of dots in the next three steps of this pattern:

1   3   6   10   15

2. What is the 7th term in the sequence below?

$$3, 12, 48, 192, \dots$$

Answer:

Evidence for Answer:

Preparing for upcoming lessons

3. Use the set of clues in the Shape Equations below to find the value of each shape.

● + ■ + ▲ = 14

2 × ▲ = 12

■ − ● = 2

Answer: ● = _____ ■ = _____ ▲ = _____

Evidence for Answer:

4. Extend the pattern in this table.

x	y
1	2
2	5
3	8
4	11
5	
6	
7	

Describe your pattern:

Based on your pattern, what would **y** be when **x** = 13?

Reviewing pre-algebra ideas

5. If your car gets 34 miles per gallon of gas and you have traveled 153 miles, how many gallons of gas have you used?

6. What is the **first** step in simplifying the expression

$$(9 - 7 \times 3 + 4)^2 ?$$

[A] square 4
[B] subtract 7 from 9
[C] add 3 and 4
[D] multiply 7 by 3

Explanation of your answer:

Lesson 2.3 Collaborating around a mid-unit assessment

2.3 OPENER

Zorkian Ziders always grow a single hair from each body segment. The pictures show the Zider at one month, two months, and three months of age. Assume the Ziders continue to grow in the same pattern. How old would a Zider be that had 73 hairs?

Understand	Plan
Read for understanding	**Think of a strategy**
Paraphrase	• Solve a similar problem
Visualize	• Draw a diagram
	• Make a table
	• Make a graph
	• Make an organized list
	• Solve a simpler problem
	• Look for a pattern
	• Set up and Solve an equation
	• Guess-and-check
	• Use logical reasoning
	• Work backwards
	• Act it out
	Predict the answer

List the Zorkian Ziders Problem on your Personal Record Sheet and select strategies that would help you solve the problem. Use this space, as needed, to write or sketch anything that will help you decide on which strategies you will use to solve the problem.

2.3 MID-UNIT ASSESSMENT

Today you will take a mid-unit assessment. For this assessment, you and your partner will solve a mathematical problem and describe how you used the Mathematical Problem-Solving Routine to do so.

2.3 REVIEW MID-UNIT ASSESSMENT

You and your partner will analyze the mid-unit assessment prior to submitting your product for the teacher to score.

HOMEWORK 2.3

Notes or additional instructions based on whole-class discussion of homework assignment:

Use the Mathematical Problem-Solving Routine as you answer these questions. If you have trouble remembering the steps, refer to the lesson "A problem-solving routine" in this topic.
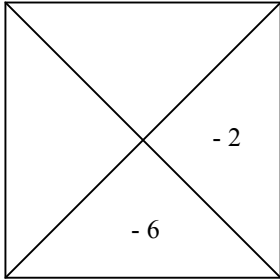
1. The Wildcats basketball team scored 18 points in the first half of a game. All of the points came from either 2- or 3-point baskets. If a total of 8 shots were made (some two-pointers and some three-pointers), how many of each type of shot were made?

    Provide evidence for your solution.

2. In the second half of the game, the Wildcats again scored 18 points. This time, however, we don't know how many total shots were made. List all of the possible combinations of 2- and 3-point shots that could total 18 points. Include a sentence explaining how you know that you found all of the possible answers.

STAYING SHARP 2.3

Practicing algebra skills & concepts

1. Complete the square box problem.

- 2

- 6

2. The following pattern is called Pascal's triangle. Complete Row 5 of the triangle.

```
Row 0            1
Row 1          1   1
Row 2        1   2   1
Row 3      1   3   3   1
Row 4    1   4   6   4   1
```

Preparing for upcoming lessons

3. How many different ways can you arrange the letters B, C, and D? Make a list. Explain how you know that you found all of the possible arrangements.

4. Search for patterns in the table. Then, use the patterns to complete the next three rows of the table.

Input	Output
- 3	15
- 2	12
-1	9
0	6

Describe the patterns you used:

Reviewing pre-algebra ideas

5. A survey showed that 30% of students bring their lunch to school. The survey polled 300 students. How many of the 300 students do **not** bring their lunch to school?

6. The value of 193 · 714 is 137802.

What is the value of 1.93 · 71.4?
(Place the decimal point in the correct position.)

1 3 7 8 0 2

Explain how you knew where to place the decimal point.

Lesson 2.4 Applying effort and growing your brain

2.4 OPENER

1. Write the next three terms for each pattern:

     a. 12, 8, 4, 0, ___, ___, ___

     b. $-9, -6, -3, 0,$ ___, ___, ___

2. Continue the pattern for the set of equations in each of the tables below.

a.

$5 \times 3 = 15$
$5 \times 2 = 10$
$5 \times 1 = 5$
$5 \times 0 = 0$
$5 \times -1 = $ ___
$5 \times -2 = $ ___
$5 \times -3 = $ ___

b.

$-5 \times 3 = -15$
$-5 \times 2 = -10$
$-5 \times 1 = -5$
$-5 \times 0 = 0$
$-5 \times -1 = $ ___
$-5 \times -2 = $ ___
$-5 \times -3 = $ ___

3. Continue the pattern for the set of equations in each of the tables below.

a.

$15 \div 5 = 3$
$10 \div 5 = 2$
$5 \div 5 = 1$
$0 \div 5 = 0$
$-5 \div 5 = $ ___
$-10 \div 5 = $ ___
$-15 \div 5 = $ ___

b.

$15 \div (-5) = -3$
$10 \div (-5) = -2$
$5 \div (-5) = -1$
$0 \div (-5) = 0$
$-5 \div (-5) = $ ___
$-10 \div (-5) = $ ___
$-15 \div (-5) = $ ___

2.4 CORE ACTIVITY

1. Practice, hard work, and dedication are three behaviors that make a better athlete or musician. Which of these three behaviors can help you become a better student or problem solver? Explain.

2. Work with your partner to answer these questions:

    a. What kinds of changes happen in your brain when you choose easy tasks, things you know you can solve?

    b. What kinds of changes happen in your brain when you choose hard tasks?

    c. Do you have to get the right answer for your brain to change? Explain.

    d. Explain how your brain changes when you are confused and having trouble with a problem.

    e. Can anyone become a famous athlete or a star at something just by practicing? Explain.
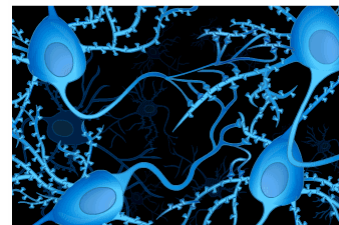
2.4 CONSOLIDATION ACTIVITY

In this activity, you will use a co-construction routine to work with your partner and the class to review multiplication and division of signed numbers.

Co-construction Routine

Step 1. Work with your partner.
Write observations and/or solutions on your whiteboard.

Step 2. Discuss your observations and/or solutions with the class.
Decide as a class which conclusions can accurately be drawn.

Step 3. Record the conclusions in your activity book.

One way to understand the rules for multiplying and dividing signed numbers is to study patterns. The table shows the completed patterns from the Opener. With your partner, discuss the patterns you see. Be prepared to discuss with the class.

$5 \times 3 = 15$	$-5 \times 3 = -15$	$15 \div 5 = 3$	$15 \div (-5) = -3$
$5 \times 2 = 10$	$-5 \times 2 = -10$	$10 \div 5 = 2$	$10 \div (-5) = -2$
$5 \times 1 = 5$	$-5 \times 1 = -5$	$5 \div 5 = 1$	$5 \div (-5) = -1$
$5 \times 0 = 0$	$-5 \times 0 = 0$	$0 \div 5 = 0$	$0 \div (-5) = 0$
$5 \times -1 = -5$	$-5 \times -1 = 5$	$-5 \div 5 = -1$	$-5 \div (-5) = 1$
$5 \times -2 = -10$	$-5 \times -2 = 10$	$-10 \div 5 = -2$	$-10 \div (-5) = 2$
$5 \times -3 = -15$	$-5 \times -3 = 15$	$-15 \div 5 = -3$	$-15 \div (-5) = 3$

1. Record the co-constructed conclusions about the patterns your class found.

   Patterns for multiplying and dividing signed numbers

2. Now, discuss the following question with your partner and record your answer below.

   How did your brain change as a result of your effort in this co-construction activity?

3. Solve the following problems. Use the conclusions/rules for multiplication and division of signed numbers.

   a. $-3 \times 11 =$

   b. $48 \div 4 =$

   c. $-1 \times -11 =$

   d. $9 \times -5 =$

   e. $10 \div -1 =$

   f. $-1 \times -4 =$

   g. $-7 \times 5 =$

   h. $-9 \times -4 =$

   i. $-35 \div 7 =$

   j. $-4 \div 1 =$

HOMEWORK 2.4

Notes or additional instructions based on whole-class discussion of homework assignment:

| |
| |

1. Solve the following problems. Use the patterns/rules for multiplying and dividing signed numbers.

   a.   -1   ×   9   =   **b.**   -55   ÷   -11   =

   c.   12   ÷   -6   =   **d.**   -7   ÷   1   =

   e.   -6   ÷   -6   =   **f.**   -6   ×   -4   =

   g.   80   ÷   -8   =   **h.**   6   ÷   -6   =

   i.   -8   ×   -12   =   **j.**   -5   ×   -10   =

2. Sometimes students mix up the rules for working with signed numbers. Decide whether each student's statement below is true or false. If it is false, write a correction for the statement.

Student	Statement	True or false?	If false, corrected statement
Allen:	"A negative plus a negative is a positive."	True False	
Beth:	"A negative times a positive is a negative."	True False	
Courtney:	"A negative minus a negative is a positive."	True False	
David:	"The answer to a positive plus a negative could be positive, negative, or zero. It depends on the numbers you're adding."	True False	

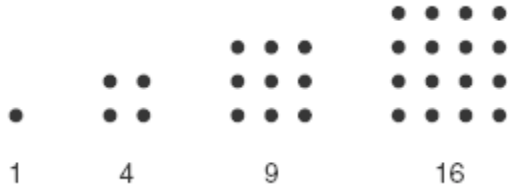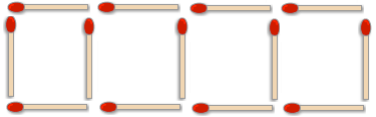3. Evaluate the following expressions for $a = 2$, $b = -3$, and $c = 4$.

   a. $a^2 \cdot b$

   b. $\dfrac{6 \bullet (a - c)}{b}$

   c. $b^2 + ac$

   d. $-b^2 + a + c$

   e. $\dfrac{-2 \bullet (b + 2c)}{b}$

STAYING SHARP 2.4

Practicing algebra skills & concepts

1. Complete the next three terms in the pattern below.

1   4   9   16

2. Where do square numbers (i.e., 1, 4, 9, 16, …) get their name? [Hint: Use your work on Question 1.]

Explain:

Preparing for upcoming lessons

3. Jim picks a number. He doubles that number, then adds five, and gets 41 as his result.
What was the original number Jim picked?

Answer with supporting work:

4. Matchsticks are used to make the pattern shown. Four squares require 13 matchsticks. How many squares can be made with 40 matchsticks?

Answer with supporting work:

Reviewing pre-algebra ideas

5. If Mark got 6 questions out of 40 incorrect, what percent of the questions did he get **correct**?

6. a. What is the value of $3 \times 5 + 6 \times 2 + 1$?

b. Insert parentheses so that the expression has a value of 67:

$$3 \times 5 + 6 \times 2 + 1$$

c. Verify the value of your new expression:

Lesson 2.5 Checking for understanding with an online assessment

2.5 OPENER

Today, you will take an online assessment. When preparing for any assessment, it's a good idea to review learning goals. In this Opener, you'll learn where to locate these goals.

1. Where could you go to find the learning goals for the topics **Exploring problem-solving strategies** and **Getting smarter through algebraic reasoning**?

2. Discuss these learning goals with your partner. Which ones do you feel you know well?

3. Which learning goals do you still need to work on?

2.5 ONLINE ASSESSMENT

You will take an online assessment.

2.5 REVIEW ONLINE ASSESSMENT

You will work with your class to review the online assessment questions.

Problems we did well on:	Skills or concepts that are addressed in these problems:
Problems we did not do well on:	**Skills or concepts that are addressed in these problems:**

Addressing areas of incomplete understanding

Use this page and notebook paper to take notes and re-work particular online assessment problems that your class identifies.

Problem #_____	**Work for problem:**
Problem #_____	**Work for problem:**
Problem #_____	**Work for problem:**

HOMEWORK 2.5

Notes or additional instructions based on whole-class discussion of homework assignment:
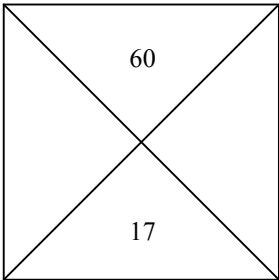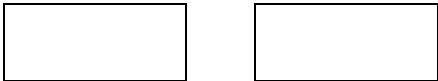
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For homework, you will take the online *More practice* in the topic **Getting smarter through algebraic reasoning**.

Homework Assignment

Part I: Complete the online *More practice* in the topic **Getting smarter through algebraic reasoning**. Note the skills and ideas for which you need more review.

Part II: Complete *Staying Sharp 2.5*.

STAYING SHARP 2.5

Practicing algebra skills & concepts

1. Complete the square box problem.

2. Each term of a sequence is 2 more than the previous term. If the **third** term is 8, find the first five terms of the sequence.

Answer with supporting work:

Preparing for upcoming lessons

3. What two numbers have:
A sum of 17 and a difference of 1?

A sum of 17 and a difference of 3?

A sum of 17 and a difference of 5?

Explain your process for finding these pairs of numbers.

4. Search for patterns in the table. Then, use the patterns to complete the next three rows of the table.

Input	Output
1	0
2	2
3	6
4	12

Describe the patterns you used:

Reviewing pre-algebra ideas

5. When Roberto got a puppy from the shelter, it weighed 11 pounds. The puppy gained 40% of its original weight in the first month that Roberto had it. How much did the puppy weigh after the first month?

6. Represent $\dfrac{1}{4}$ and $\dfrac{3}{4}$ in three ways:

By shading In the appropriate part of each rectangle:

As decimals:

As percentages: