

Date

Lesson 1: Chance Experiments, Sample Spaces, and Events

Exit Ticket

1. For the chance experiment described in Scenario Card 1, why is the probability of the event "spinning an odd number and randomly selecting a blue card" not the same as the probability of the event "spinning an even number and randomly selecting a blue card"? Which event would have the greater probability of occurring, and why?

2. Why is the probability of the event "spinning an odd number from Spinner 1 *and* randomly selecting a blue card" not equal to the probability of "spinning an odd number from Spinner 1 *or* randomly selecting a blue card"?

3. If one of the red cards is changed to a blue card, what is the probability of the event "spinning an odd number from Spinner 1 and randomly selecting a red card from the card bag"?







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Rules of the game for Scenario Card 1 described in the lesson:

- The scenario cards are shuffled, and one is selected.
- Each player reads the description of the chance experiment and the description of the five events described on the scenario card.
- Players independently assign the numbers 1–5 (no repeats) to the five events described on the scenario card based on how likely they think the event is to occur, with 5 being most likely and 1 being least likely.
- Once players have made their assignments, the chance experiment described on the scenario card is
 performed. Points are then awarded based on the outcome of the chance experiment. If the event described
 on the scenario card has occurred, the player earns the number of points corresponding to the number that
 player assigned to that event (1–5 points). If an event occurs that is not described on the scenario card, then
 no points are awarded for that event.
- If an outcome is described by two or more events on the scenario card, the player selects the higher point value.
- The chance experiment is repeated four more times with points being awarded each time the chance experiment is performed.
- The player with the largest number of points at the end of the game is the winner.



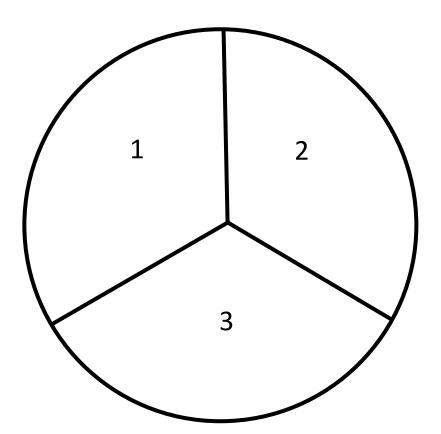








Spinner 1





Lesson 1:

Chance Experiments, Sample Spaces, and Events





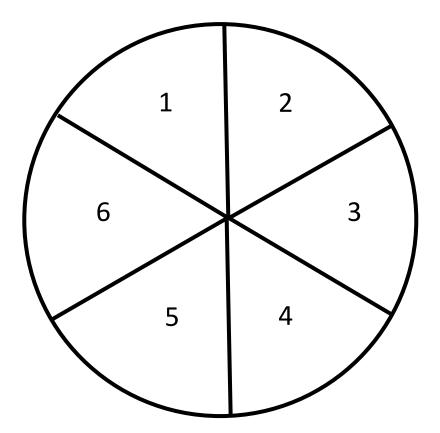
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ALGEBRA II

Spinner 2





Lesson 1:

Chance Experiments, Sample Spaces, and Events





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Lesson 2: Calculating Probabilities of Events Using Two-Way

Tables

Exit Ticket

Did male and female voters respond similarly to the survey question about building a new high school? Recall the original summary of the data.

	9	Should Our	Town Bui	ld a New H	igh School	?
	Y	es	N	lo	No Ai	nswer
Age (in years)	Male	Female	Male	Female	Male	Female
18–25	29	32	8	6	0	0
26–40	53 60 30 36		40	44	2	4
41–65			44	35	2	2
66 and Older	7	26	24	29	2	0

1. Complete the following two-way frequency table:

	Yes	No	No Answer	Total
Male	119		6	
Female				
Total		230	12	515

- 2. Use the above two-way frequency table to answer the following questions:
 - a. If a randomly selected eligible voter is female, what is the probability she will vote to build a new high school?
 - b. If a randomly selected eligible voter is male, what is the probability he will vote to build a new high school?









- 3. An automobile company has two factories assembling its luxury cars. The company is interested in whether consumers rate cars produced at one factory more highly than cars produced at the other factory. Factory A assembles 60% of the cars. A recent survey indicated that 70% of the cars made by this company (both factories combined) were highly rated. This same survey indicated that 10% of all cars made by this company were both made at Factory B *and* were *not* highly rated.
 - a. Create a hypothetical 1000 two-way table based on the results of this survey by filling in the table below.

	Car Was Highly Rated by Consumers	Car Was Not Highly Rated by Consumers	Total
Factory A			
Factory B			
Total			

b. A randomly selected car was assembled in Factory B. What is the probability this car is highly rated?









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Lesson 3: Calculating Conditional Probabilities and Evaluating Independence Using Two-Way Tables

Exit Ticket

A state nonprofit organization wanted to encourage its members to consider the State of New York as a vacation destination. They are investigating whether their online ad campaign influenced its members to plan a vacation in New York within the next year. The organization surveyed its members and found that 75% of them have seen the online ad. 40% of its members indicated they are planning to vacation in New York within the next year, and 15% of its members did not see the ad and do not plan to vacation in New York within the next year.

1. Complete the following hypothetical 1000 two-way frequency table:

	Plan to Vacation in New York Within the Next Year	Do Not Plan to Vacation In New York Within the Next Year	Total
Watched the Online Ad			
Did Not Watch the Online Ad			
Total			

2. Based on the two-way table, describe two conditional probabilities you could calculate to help decide if members who saw the online ad are more likely to plan a vacation in New York within the next year than those who did not see the ad.







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Calculate the probabilities you described in Problem 2. 3.

4. Based on the probabilities calculated in Problem 3, do you think the ad campaign is effective in encouraging people to vacation in New York? Explain your answer.



Calculating Conditional Probabilities and Evaluating Independence Using Two-Way Tables







Date

Lesson 4: Calculating Conditional Probabilities and Evaluating Independence Using Two-Way Tables

Exit Ticket

1. The following hypothetical 1000 two-way table was introduced in the previous lesson:

	Plan to Vacation in New York Within the Next Year	Do Not Plan to Vacation in New York Within the Next Year	Total
Watched the Online Ad	300	450	750
Did Not Watch the Online Ad	100	150	250
Total	400	600	1,000

Are the events "a randomly selected person watched the online ad" and "a randomly selected person plans to vacation in New York within the next year" independent or not independent? Justify your answer using probabilities calculated from information in the table.

A survey conducted at a local high school indicated that 30% of students have a job during the school year.
 If having a job and being in the eleventh grade are not independent, what do you know about the probability that a randomly selected student who is in the eleventh grade would have a job? Justify your answer.









Eighty percent of the dogs at a local kennel are in good health. If the events "a randomly selected dog at this kennel 3. is in good health" and "a randomly selected dog at this kennel weighs more than 30 pounds" are independent, what do you know about the probability that a randomly selected dog that weighs more than 30 pounds will be in good health? Justify your answer.



Calculating Conditional Probabilities and Evaluating Independence Using **Two-Way Tables**





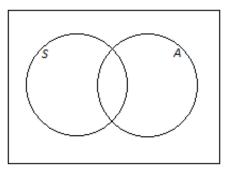


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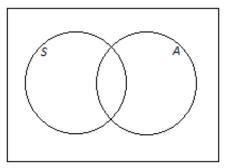
Lesson 5: Events and Venn Diagrams

Exit Ticket

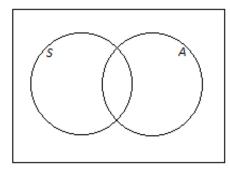
- 1. At a high school, some students take Spanish, and some do not. Also, some students take an arts subject, and some do not. Let *S* be the set of students who take Spanish and *A* be the set of students who take an arts subject. On the Venn diagrams given, shade the region representing the following instances:
 - a. Students who take Spanish and an arts subject



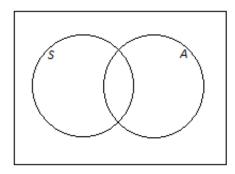
c. Students who take Spanish but do not take an arts subject



b. Students who take Spanish or an arts subject



d. Students who do not take an arts subject



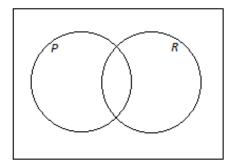








2. When a player is selected at random from a high school boys' baseball team, the probability that he is a pitcher is 0.35, the probability that he is right-handed is 0.79, and the probability that he is a right-handed pitcher is 0.26. Let *P* be the event that a player is a pitcher, and let *R* be the event that a player is right-handed. A Venn diagram is provided below.



Use the Venn diagram to calculate the probability that a randomly selected player is each of the following. Explain how you used the Venn diagram to determine your answer.

- a. Right-handed but not a pitcher
- b. A pitcher but not right-handed
- c. Neither right-handed nor a pitcher









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Lesson 6: Probability Rules

Exit Ticket

- 1. Of the light bulbs available at a store, 42% are fluorescent, 23% are labeled as long life, and 12% are fluorescent *and* long life.
 - a. A light bulb will be selected at random from the light bulbs at this store. Rounding your answer to the nearest thousandth where necessary, find the probability that
 - i. The selected light bulb is not fluorescent.
 - ii. The selected light bulb is fluorescent given that it is labeled as long life.
 - b. Are the events "fluorescent" and "long life" independent? Explain.

- 2. When a person is selected at random from a very large population, the probability that the selected person is righthanded is 0.82. If three people are selected at random, what is the probability that
 - a. They are all right-handed?
 - b. None of them is right-handed?









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Lesson 7: Probability Rules

Exit Ticket

- 1. When a call is received at an airline's call center, the probability that it comes from abroad is 0.32, and the probability that it is to make a change to an existing reservation is 0.38.
 - a. Suppose that you are told that the probability that a call is both from abroad and is to make a change to an existing reservation is 0.15. Calculate the probability that a randomly selected call is either from abroad or is to make a change to an existing reservation.

b. Suppose now that you are *not* given the information in part (a), but you are told that the events "the call is from abroad" and "the call is to make a change to an existing reservation" are independent. What is the probability that a randomly selected call is either from abroad or is to make a change to an existing reservation?

2. A golfer will play two holes of a course. Suppose that on each hole the player will score 3, 4, 5, 6, or 7, with these five scores being equally likely. Find the probability, and explain how the answer was determined that the player's total score for the two holes will be

a. 14.

b. 12.







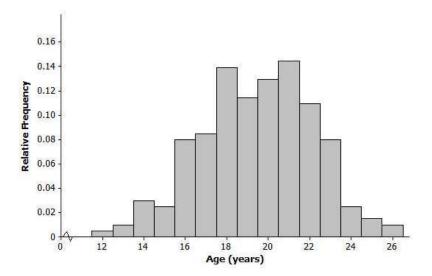


Date

Lesson 8: Distributions—Center, Shape, and Spread

Exit Ticket

A local utility company wanted to gather data on the age of air conditioners that people have in their homes. The company took a random sample of 200 residents of a large city and asked if the residents had an air conditioner, and if they did, how old it was. Below is the distribution in the reported ages of the air conditioners.



- 1. Would you describe this distribution of air conditioner ages as approximately symmetric or as skewed? Explain your answer.
- 2. Is the mean of the age distribution closer to 15, 20, or 25 years? Explain your answer.
- 3. Is the standard deviation of the age distribution closer to 3, 6, or 9 years? Explain your answer.







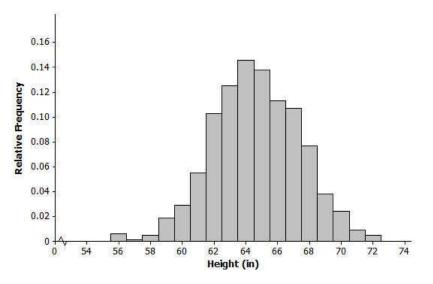
Lesson 9 M4

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Lesson 9: Using a Curve to Model a Data Distribution

Exit Ticket

The histogram below shows the distribution of heights (to the nearest inch) of 1,000 young women.



1. What does the width of each bar represent? What does the height of each bar represent?

2. The mean of the distribution of women's heights is 64.6 in., and the standard deviation is 2.75 in. Interpret the mean and standard deviation in this context.









3. Mark the mean on the graph, and mark *one* deviation above and below the mean. Approximately what proportion of the values in this data set are within one standard deviation of the mean?

4. Draw a smooth curve that comes reasonably close to passing through the midpoints of the tops of the bars in the histogram. Describe the shape of the distribution.

5. Shade the area of the histogram that represents the proportion of heights that are within one standard deviation of the mean.







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Lesson 10: Normal Distributions

Exit Ticket

The weights of cars passing over a bridge have a mean of 3,550 pounds and standard deviation of 870 pounds. Assume that the weights of the cars passing over the bridge are normally distributed. Determine the probability of each instance, and explain how you found each answer.

a. The weight of a randomly selected car is more than 4,000 pounds.

The weight of a randomly selected car is less than 3,000 pounds. b.

c. The weight of a randomly selected car is between 2,800 and 4,500 pounds.

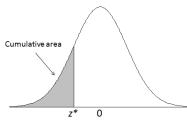






M4

Standard Normal Curve Areas



Lesson 10

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.6	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.5	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.4	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
-3.3	0.0005	0.0005	0.0005	0.0003	0.0003	0.0003	0.0003	0.0003	0.0004	0.0002
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0012	0.0012	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0160	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0599
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641







ALGEBRA II

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999







Date _____

Lesson 11: Normal Distributions

Exit Ticket

- 1. SAT scores were originally scaled so that the scores for each section were approximately normally distributed with a mean of 500 and a standard deviation of 100. Assuming that this scaling still applies, use a table of standard normal curve areas to find the probability that a randomly selected SAT student scores
 - a. More than 700.

b. Between 440 and 560.

- 2. In 2012, the mean SAT math score was 514, and the standard deviation was 117. For the purposes of this question, assume that the scores were normally distributed. Using a graphing calculator, and without using *z*-scores, find the probability (rounded to the nearest thousandth), and explain how the answer was determined that a randomly selected SAT math student in 2012 scored
 - a. Between 400 and 480.

b. Less than 350.









Standard Normal Curve Areas

								```````````````````````````````````````		
							Z*	0		
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0160	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0599
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Cumulative area







Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999







	NYS COMMON CORE MATHEMATICS CURRICULUM	Mid-Module Assessment Task	M4
		ALGEBRA	. //
I	Name	Date	

- 1. On his way to work every day, Frank passes through two intersections with traffic signals. Sometimes the lights are green when he arrives at the light; sometimes they are red, and he must stop. The probability that he must stop at the first signal is P(first) = 0.4. The probability that he must stop at the second signal is P(second) = 0.5. The probability that he must stop at both lights is 0.3. Suppose we randomly select one morning that he travels to work, and we look at the outcomes of the two lights.
  - a. Describe the event "first, not second" in words.

b. List all of the outcomes in the sample space.

c. Calculate *P*(first or second), and interpret your result in context.







ALGEBRA II

d. Calculate the probability of the event described in part (a).

- e. Are the events "stopping at first light" and "stopping at second light" independent? Explain your answer.
- f. Assuming the probability of stopping at the first signal does not change from day to day, how surprising would it be for Frank to have to stop at the first light five days in a row? If he does have to stop at the first light five days in a row, would you question the model that assigns a probability of 0.5 to the first light each day?







- 2. An online bookstore sells both print books and e-books (books in an electronic format). Customers can pay with either a gift card or a credit card.
  - a. Suppose that the probability of the event "print book is purchased" is 0.6 and that the probability of the event "customer pays using gift card" is 0.2. If these two events are independent, what is the probability that a randomly selected book purchase is a print book paid for using a gift card?

b. Suppose that the probability of the event "e-book is purchased" is 0.4; the probability of the event "customer pays using gift card" is 0.2; and the probability of the event "e-book is purchased and customer pays using a gift card" is 0.1. Are the two events "e-book is purchased" and "customer pays using a gift card" independent? Explain why or why not.

3. Airlines post the estimated arrival times for all of their flights. However, sometimes the flights arrive later than expected. The following data report the number of flights that were "on time" or "late" for two different airlines in November 2012 for all flights to Houston, Chicago, and Los Angeles:

	Houst	on	Chica	go	Los Angeles				
	On Time	On Time Late On Tir		Late	On Time	Late			
Airline A	7,318	1,017	466	135	544	145			
Airline B	598	70	8,330	1,755	2,707	566			

a. Use the data to estimate the probability that a randomly selected flight arriving in Houston will be on time.







Consider only the flights to Chicago and Los Angeles for these two airlines combined.

	Chicago	Los Angeles
On Time	8,796	3,251
Late	1,890	711

b. Explain what it means to say that the events "arriving on time" and "Chicago" are independent.

c. Do the events "arriving on time" and "Chicago" appear to be approximately independent? Explain your answer.

- 4. The average height of the 140 million U.S. males is 5 ft. and 10 in. Some males from the U.S. become professional basketball players. The average height of the 350–450 professional basketball players in the NBA (National Basketball Association) is about 6 ft. and 7 in. Which of the following probabilities should be larger? Or would they be similar?
  - The probability that a U.S. male over 6 ft. tall is a professional basketball player
  - The probability that a professional basketball player is over 6 ft. tall

Explain your reasoning.







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- 5. A researcher gathers data on how long teenagers spend on individual cell phone calls (in number of minutes). Suppose the research determines that these calls have a mean 10 minutes and standard deviation 7 minutes.
  - a. Suppose the researcher also claims that the distribution of the call lengths follows a normal distribution. Sketch a graph displaying this distribution. Be sure to add a scale and to label your horizontal axis.







b. Using your graph, shade the area that represents the probability that a randomly selected call lasts more than 12 minutes. Is this probability closer to 0.50 or to 0.05?

c. After looking at the above mean and standard deviation of the call length data, a second researcher indicates that she does not think that a normal distribution is an appropriate model for the call length distribution. Which researcher (the first or the second) do you think is correct? Justify your choice.









Date

# Lesson 12: Types of Statistical Studies

#### **Exit Ticket**

Is the following an observational study or an experiment? Explain your answer.

Also, if it is an experiment, then identify the treatment variable and the response variable in the context of the problem. If it is an observational study, identify the population of interest.

A study is done to see how high soda will erupt when mint candies are dropped into two-liter bottles of soda. You
want to compare using one mint candy, five mint candies, and 10 mint candies. You design a cylindrical mechanism,
which drops the desired number of mint candies all at once. You have 15 bottles of soda to use. You randomly
assign five bottles into which you drop one candy, five into which you drop five candies, and five into which you
drop 10 candies. For each bottle, you record the height of the eruption created after the candies are dropped into
it.

2. You want to see if fifth-grade boys or fifth-grade girls are faster at solving Ken-Ken puzzles. You randomly select twenty fifth-grade boys and twenty fifth-grade girls from fifth graders in your school district. You time and record how long it takes each student to solve the same Ken-Ken puzzle correctly.









# Lesson 13: Using Sample Data to Estimate a Population Characteristic

#### **Exit Ticket**

Indicate whether each of the following is a summary measure from a population or from a sample. Choose the one that is more realistically the case. If it is from a population, identify the population characteristic. If it is from a sample, identify the sample statistic. Explain your reasoning.

a. 88% of the more than 300 million automobile tires discarded per year are recycled or used for fuel.

b. The mean number of words that contain the letter *e* in the Gettysburg Address

c. 64% of respondents in a recent poll indicated that they favored building a proposed highway in their town.









ALGEBRA II

### **Table of Random Digits**

Row																				
1	6	6	7	2	8	0	0	8	4	0	0	4	6	0	3	2	2	4	6	8
2	8	0	3	1	1	1	1	2	7	0	1	9	1	2	7	1	3	3	5	3
3	5	3	5	7	3	6	3	1	7	2	5	5	1	4	7	1	6	5	6	5
4	9	1	1	9	2	8	3	0	3	6	7	7	4	7	5	9	8	1	8	3
5	9	0	2	9	9	7	4	6	3	6	6	3	7	4	2	7	0	0	1	9
6	8	1	4	6	4	6	8	2	8	9	5	5	2	9	6	2	5	3	0	3
7	4	1	1	9	7	0	7	2	9	0	9	7	0	4	6	2	3	1	0	9
8	9	9	2	7	1	3	2	9	0	3	9	0	7	5	6	7	1	7	8	7
9	3	4	2	2	9	1	9	0	7	8	1	6	2	5	3	9	0	9	1	0
10	2	7	3	9	5	9	9	3	2	9	3	9	1	9	0	5	5	1	4	2
11	0	2	5	4	0	8	1	7	0	7	1	3	0	4	3	0	6	4	4	4
12	8	6	0	5	4	8	8	2	7	7	0	1	0	1	7	1	3	5	3	4
13	4	2	6	4	5	2	4	2	6	1	7	5	6	6	4	0	8	4	1	2
14	4	4	9	8	7	3	4	3	8	2	9	1	5	3	5	9	8	9	2	9
15	6	4	8	0	0	0	4	2	3	8	1	8	4	0	9	5	0	9	0	4
16	3	2	3	8	4	8	8	6	2	9	1	0	1	9	9	3	0	7	3	5
17	6	6	7	2	8	0	0	8	4	0	0	4	6	0	3	2	2	4	6	8
18	8	0	3	1	1	1	1	2	7	0	1	9	1	2	7	1	3	3	5	3
19	5	3	5	7	3	6	3	1	7	2	5	5	1	4	7	1	6	5	6	5
20	9	1	1	9	2	8	3	0	3	6	7	7	4	7	5	9	8	1	8	3
21	9	0	2	9	9	7	4	6	3	6	6	3	7	4	2	7	0	0	1	9
22	8	1	4	6	4	6	8	2	8	9	5	5	2	9	6	2	5	3	0	3
23	4	1	1	9	7	0	7	2	9	0	9	7	0	4	6	2	3	1	0	9
24	9	9	2	7	1	3	2	9	0	3	9	0	7	5	6	7	1	7	8	7
25	3	4	2	2	9	1	9	0	7	8	1	6	2	5	3	9	0	9	1	0
26	2	7	3	9	5	9	9	3	2	9	3	9	1	9	0	5	5	1	4	2
27	0	2	5	4	0	8	1	7	0	7	1	3	0	4	3	0	6	4	4	4
28	8	6	0	5	4	8	8	2	7	7	0	1	0	1	7	1	3	5	3	4
29	4	2	6	4	5	2	4	2	6	1	7	5	6	6	4	0	8	4	1	2
30	4	4	9	8	7	3	4	3	8	2	9	1	5	3	5	9	8	9	2	9
31	6	4	8	0	0	0	4	2	3	8	1	8	4	0	9	5	0	9	0	4
32	3	2	3	8	4	8	8	6	2	9	1	0	1	9	9	3	0	7	3	5
33	6	6	7	2	8	0	0	8	4	0	0	4	6	0	3	2	2	4	6	8
34	8	0	3	1	1	1	1	2	7	0	1	9	1	2	7	1	3	3	5	3
35	5	3	5	7	3	6	3	1	7	2	5	5	1	4	7	1	6	5	6	5
36	9	1	1	9	2	8	3	0	3	6	7	7	4	7	5	9 7	8	1	8	3
37	9	0	2	9	9	7	4	6	3	6	6 5	3 5	7	4	2	7	0	0	1	9
38 20	8	1	4	6	4	6	8	2	8	9	5	5	2	9	6	2	5	3	0	3
39	4	1	1	9 7	7	0	7	2	9	0	9	7	0	4 5	6	2	3	1	0	9 7
40	9	9	2	7	1	3	2	9	0	3	9	0	7	5	6	7	1	7	8	7



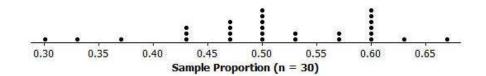


NYS COMMON CORE MATHEMATICS CURRICULUM	Lesson 14 M4
	ALGEBRA II
Name	Date

## Lesson 14: Sampling Variability in the Sample Proportion

### **Exit Ticket**

A group of eleventh graders wanted to estimate the population proportion of students in their high school who drink at least one soda per day. Each student selected a different random sample of 30 students and calculated the proportion that drink at least one soda per day. The dot plot below shows the sampling distribution. This distribution has a mean of 0.51 and a standard deviation of 0.09.



1. Describe the shape of the distribution.

2. What is your estimate for the proportion of *all* students who would report that they drink at least one soda per day?

3. If, instead of taking random samples of 30 students in the high school, the eleventh graders randomly selected samples of size 60, describe what will happen to the standard deviation of the sampling distribution of the sample proportions.







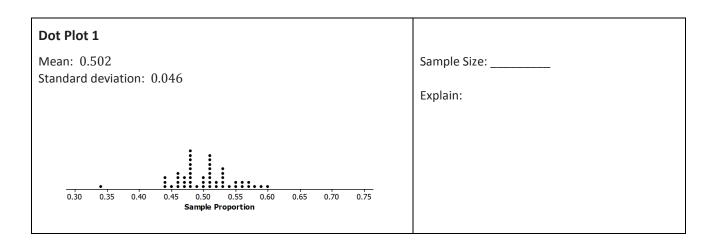


Date _____

## Lesson 15: Sampling Variability in the Sample Proportion

### **Exit Ticket**

Below are three dot plots of the proportion of tails in 20, 60, or 120 simulated flips of a coin. The mean and standard deviation of the sample proportions are also shown for each of the three dot plots. Match each dot plot with the appropriate number of flips. Clearly explain how you matched the plots with the number of simulated flips.



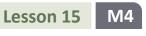
Dot Plot 2	
Mean: 0.518	Sample Size:
Standard deviation: 0.064	Explain:
0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 Sample Proportion	



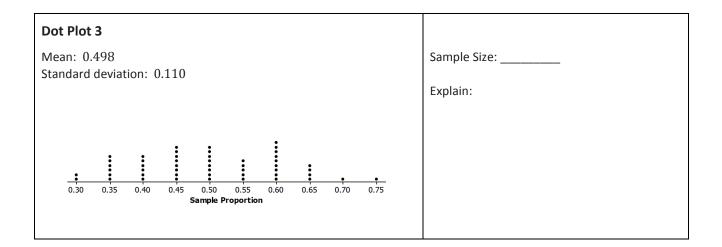








ALGEBRA II











Name___

Date _____

# Lesson 16: Margin of Error When Estimating a Population **Proportion**

#### **Exit Ticket**

1. Suppose you drew a sample of 12 red chips in a sample of 30 from a mystery bag. Describe how you would find plausible population proportions using the simulated sampling distributions we generated from populations with known proportions of red chips.

2. What would happen to the interval containing plausible population proportions if you changed the sample size to 60?









Name ____

Date _____

# Lesson 17: Margin of Error When Estimating a Population **Proportion**

### **Exit Ticket**

1. Find the estimated margin of error when estimating the proportion of red chips in a mystery bag if 18 red chips were drawn from the bag in a random sample of 50 chips.

Explain what your answer to Problem 1 tells you about the number of red chips in the mystery bag. 2.

3. How could you decrease your margin of error? Explain why this works.









Date _____

# Lesson 18: Sampling Variability in the Sample Mean

#### **Exit Ticket**

Describe what a simulated distribution of sample means is and what the standard deviation of the distribution indicates. You may want to refer to the segment lengths in your answer.









#### **Exercises 1–7: Random Segments**

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	7					_		8						9					
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	68				69		00				70				07				
	00		71		05			72			70				73				
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Date _____

# Lesson 19: Sampling Variability in the Sample Mean

### **Exit Ticket**

1. Describe the difference between a population distribution, a sample distribution, and a simulated sampling distribution, and make clear how they are different.

2. Use the standard deviation and mean of the sampling distribution to describe an interval that includes most of the sample means.









Date

### Lesson 20: Margin of Error When Estimating a Population Mean

#### **Exit Ticket**

At the beginning of the school year, school districts implemented a new physical fitness program. A student project involves monitoring how long it takes eleventh graders to run a mile. The following data were taken midyear.

What is the estimate of the population mean time it currently takes eleventh graders to run a mile based on a. the following data (minutes) from a random sample of ten students?

6.5, 8.4, 8.1, 6.8, 8.4, 7.7, 9.1, 7.1, 9.4, 7.5

The students doing the project collected 50 random samples of 10 students each and calculated the sample b. means. The standard deviation of their distribution of 50 sample means was 0.6 minutes. Based on this standard deviation, what is the margin of error for their sample mean estimate? Explain your answer.

c. Interpret the margin of error you found in part (b) in the context of this problem.





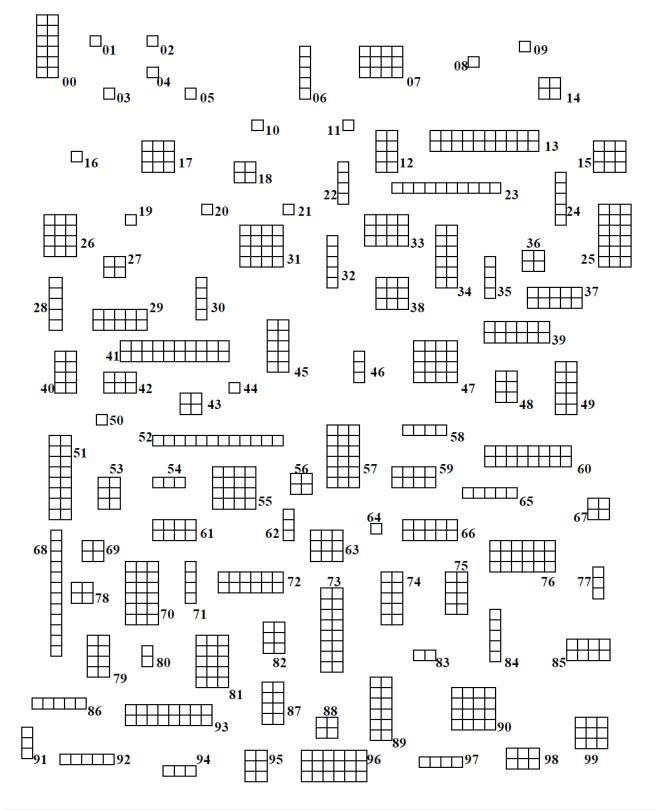


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Lesson 20 M4

ALGEBRA II

#### **Example 1: Describing a Population of Numerical Data**



EUREKA АТН

Lesson 20: Margin of Error When Estimating a Population Mean





Date

## Lesson 21: Margin of Error When Estimating a Population Mean

#### **Exit Ticket**

A Health Group study recommends that the total weight of a male student's backpack should not be more than 15% of his body weight. For example, if a student weighs 170 pounds, his backpack should not weigh more than 25.5 pounds. Suppose that ten randomly selected eleventh-grade boys produced the following data:

Body Weight	155	136	197	174	165	165	150	142	176	157
Backpack Weight	29.8	27.2	32.5	34.8	31.8	28.8	31.1	26.0	28.3	31.4

- a. For each student, calculate backpack weight as a percentage of body weight (round to one decimal place).
- b. Based on the data in part (a), estimate the mean percentage of body weight that eleventh-grade boys carry in their backpacks.
- c. Find the margin of error for your estimate of part (b). Round your answer to three decimal places. Explain how you determined your answer.
- d. Comment on the amount of weight eleventh-grade boys at this school are carrying in their backpacks compared to the recommendation by the Health Group.







M4

Lesson 21

#### Exercises 6–13: The Gettysburg Address

001 Four	045 any	089 nation	133 our	177 they	221 full	265 perish
002 score	046 nation,	090 might	134 poor	178 who	222 measure	266 from
003 and	047 so	091 live.	135 power	179 fought	223 of	267 the
004 seven	048 conceived	092 It	136 to	180 here	224 devotion,	268 earth.
005 years	049 and	093 is	137 add	181 have	225 that	
006 ago,	050 so	094 altogether	138 or	182 thus	226 we	
007 our	051 dedicated,	095 fitting	139 detract.	183 far	227 here	
008 fathers	052 can	096 and	140 The	184 so	228 highly	
009 brought	053 long	097 proper	141 world	185 nobly	229 resolve	
010 forth	054 endure.	098 that	142 will	186 advanced.	230 that	
011 upon	055 We	099 we	143 little	187 lt	231 these	
012 this	056 are	100 should	144 note,	188 is	232 dead	
013 continent	057 met	101 do	145 nor	189 rather	233 shall	
014 a	058 on	102 this.	146 long	190 for	234 not	
015 new	059 a	103 But,	147 remember,	191 us	235 have	
016 nation;	060 great	104 in	148 what	192 to	236 died	
017 conceived	061 battlefield	105 a	149 we	193 be	237 in	
018 in	062 of	106 larger	150 say	194 here	238 vain,	
019 liberty,	063 that	107 sense,	151 here,	195 dedicated	239 that	
020 and	064 war.	108 we	152 but	196 to	240 this	
021 dedicated	065 We	109 cannot	153 it	197 the	241 nation,	
022 to	066 have	110 dedicate,	154 can	198 great	242 under	
023 the	067 come	111 we	155 never	199 task	243 God,	
024 proposition	068 to	112 cannot	156 forget	200 remaining	244 shall	
025 that	069 dedicate	113 consecrate,	157 what	201 before	245 have	
026 all	070 a	114 we	158 they	202 us,	246 a	
027 men	071 portion	115 cannot	159 did	203 that	247 new	
028 are	072 of	116 hallow	160 here.	204 from	248 birth	
029 created	073 that	117 this	161 lt	205 these	249 of	
030 equal.	074 field	118 ground.	162 is	206 honored	250 freedom,	
031 Now	075 as	119 The	163 for	207 dead	251 and	
032 we	076 a	120 brave	164 us	208 we	252 that	
033 are	077 final	121 men,	165 the	209 take	253 government	
034 engaged	078 resting	122 living	166 living,	210 increased	254 of	
035 in	079 place	123 and	167 rather,	211 devotion	255 the	
036 a	080 for	124 dead,	168 to	212 to	256 people,	
037 great	081 those	125 who	169 be	213 that	257 by	
038 civil	082 who	126 struggled	170 dedicated	214 cause	258 the	
039 war,	083 here	127 here	171 here	215 for	259 people,	
040 testing	084 gave	128 have	172 to	216 which	260 for	
041 whether	085 their	129 consecrated	173 the	217 they	261 the	
042 that	086 lives	130 it,	174 unfinished	218 gave	262 people,	
043 nation,	087 that	131 far	175 work	219 the	263 shall	
044 or	088 that	132 above	176 which	220 last	264 not	







Date

### Lesson 22: Evaluating Reports Based on Data from a Sample

#### **Exit Ticket**

The Gallup organization published the following results from a poll that it conducted.

As health experts increasingly focus on the medical benefits of a healthy lifestyle and preventative healthcare, Americans say their doctor does commonly discuss the benefits of healthy habits with them. Specifically, 71% say their doctor usually discusses the benefits of engaging in regular physical exercise, and 66% say their doctor usually discusses the benefits of eating a healthy diet. Fewer Americans, 50%, say their doctor usually discusses the benefits of not smoking, although that number jumps to 79% among smokers.

#### **Survey Methods**

Results for this Gallup poll are based on telephone interviews conducted July 10–14, 2013, with a random sample of 2,027 adults, aged 18 and older, living in all 50 U.S. states and the District of Columbia.

For results based on the total sample of national adults, one can say with 95% confidence that the margin of sampling error is  $\pm 3$  percentage points.

Source: <u>http://www.gallup.com/poll/163772/americans-say-doctors-advise-health-habits.aspx</u>

- 1. The headline of the article is "Smokers Much More Likely Than Nonsmokers to Say Doctor Discusses Not Smoking." Do you agree with this headline? Explain your answer.
- 2. Using the data "71% say their doctor usually discusses the benefits of engaging in regular physical exercise," calculate the margin of error. Show your work.
- 3. How do your results compare with the margin of error stated in the article?
- 4. Interpret the margin of error in this context.







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Date _____

### Lesson 23: Experiments and the Role of Random Assignment

#### **Exit Ticket**

Runners who suffered from shin splints were randomly assigned to one of two stretching routines. One of the routines involved a series of pre-run and post-run dynamic stretches that last approximately 5 minutes before and after the run. The other routine involved a 1-minute hamstring stretch pre-run and no stretching post-run. After a 45-minute run, each runner will be assessed for shin splints.

a. Explain why this is an experiment.

b. Identify the subjects.

c. Identify the treatments.

- d. Identify the response variable.
- e. Why are the runners randomly assigned to one of two stretching routines?









Name ____

Date _____

### Lesson 24: Differences Due to Random Assignment Alone

**Exit Ticket** 

When a single group is randomly divided into two groups, why do the two group means tend to be different?









ALGEBRA II

### **Appendix A**

65	40	55	90
70	75	80	75
55	70	50	60
65	65	95	70
110	65	60	70



Differences Due to Random Assignment Alone





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Date

### Lesson 25: Ruling Out Chance

**Exit Ticket** 

Six ping-pong balls are labeled as follows: 0, 3, 6, 9, 12, 18. Three ping-pong balls will be randomly assigned to Group A; the rest will be assigned to Group B. Diff =  $\bar{x}_A - \bar{x}_B$ 

1. Calculate Diff, the difference between the mean of the numbers, on the balls assigned to Group A and the mean of the numbers on the balls assigned to Group B (i.e.,  $\bar{x}_A - \bar{x}_B$ ) when the 3 ping-pong balls selected for Group A are 3, 6, and 12.

2. Calculate Diff, the difference between the mean of the numbers, on the balls assigned to Group A and the mean of the numbers on the balls assigned to Group B (i.e.,  $\bar{x}_A - \bar{x}_B$ ) when the 3 ping-pong balls selected for Group A are 3, 12, and 18.

3. What is the greatest possible value of Diff, and what selection of ping-pong balls for Group A corresponds to that value?









4. What is the smallest (most negative) possible value of Diff, and what selection of ping-pong balls for Group A corresponds to that value?

5. If these 6 observations represent the burn times of 6 candles (in minutes), explain what a Diff value of 6 means in terms of (a) which group (A or B) has the longer average burn time and (b) the amount of time by which that group's mean exceeds the other group's mean.









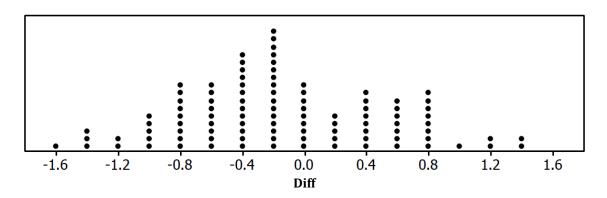


Date

### Lesson 26: Ruling Out Chance

#### **Exit Ticket**

Medical patients who are in physical pain are often asked to communicate their level of pain on a scale of 0 to 10 where 0 means no pain and 10 means worst pain. (Note: Sometimes a visual device with pain faces is used to accommodate the reporting of the pain score.) Due to the structure of the scale, a patient would desire a lower value on this scale after treatment for pain.



Imagine that 20 subjects participate in a clinical experiment and that a variable of "ChangeinScore" is recorded for each subject as the subject's pain score after treatment minus the subject's pain score before treatment. Since the expectation is that the treatment would lower a patient's pain score, you would desire a *negative* value for "ChangeinScore." For example, a "ChangeinScore" value of -2 would mean that the patient was in less pain (for example, now at a 6, formerly at an 8).

Recall that Diff =  $\bar{x}_A - \bar{x}_B$ . Although the 20 "ChangeinScore" values for the 20 patients are not shown here, below is a randomization distribution of the value Diff based on 100 random assignments of these 20 observations into two groups of 10 (Group A and Group B).

1. From the distribution above, what is the probability of obtaining a Diff value of -1 or less?









2. With regard to this distribution, would you consider a Diff value of -0.4 to be statistically significant? Explain.

3.

a. With regard to how Diff is calculated, if Group A represented a group of patients in your experiment who received a new pain relief treatment and Group B received a pill with no medicine (called a *placebo*), how would you interpret a Diff value of -1.4 pain scale units in context?

b. Given the distribution above, if you obtained a Diff value such as -1.4 from your experiment, would you consider that to be significant evidence of the new treatment being effective on average in relieving pain? Explain.









Date

### Lesson 27: Ruling Out Chance

### **Exit Ticket**

In the Exit Ticket of a previous lesson, an experiment with 20 subjects investigating a new pain reliever was introduced. The subjects were asked to communicate their level of pain on a scale of 0 to 10 where 0 means no pain and 10 means worst pain. Due to the structure of the scale, a patient would desire a lower value on this scale after treatment for pain. The value "ChangeinScore" was recorded as the subject's pain score after treatment minus the subject's pain score before treatment. Since the expectation is that the treatment would lower a patient's pain score, a *negative* value would be desired for "ChangeinScore." For example, a "ChangeinScore" value of -2 would mean that the patient was in less pain (for example, now at a 6, formerly at an 8).

In the experiment, the null hypothesis would be that the treatment had no effect. The average change in pain score for the treatment group would be the same as the average change in pain score for the placebo (control) group.

- 1. The alternative hypothesis would be that the treatment was effective. Using this context, which mathematical relationship below would match this alternative hypothesis? Choose one.
  - a. The average change in pain score (the average "ChangeinScore") for the treatment group would be less than the average change in pain score for the placebo group (supported by  $\bar{x}_{\text{Treatment}} < \bar{x}_{\text{Control}}$ , or  $\bar{x}_{\text{Treatment}} \bar{x}_{\text{Control}} < 0$ ).
  - b. The average change in pain score (the average "ChangeinScore") for the treatment group would be greater than the average change in pain score for the placebo group (supported by  $\bar{x}_{\text{Treatment}} > \bar{x}_{\text{Control}}$ , or  $\bar{x}_{\text{Treatment}} \bar{x}_{\text{Control}} > 0$ ).









2. Imagine that the 20 "ChangeinScore" observations below represent the change in pain levels of the 20 subjects (chronic pain sufferers) who participated in the clinical experiment. The 10 individuals in Group A (the treatment group) received a new medicine for their pain while the 10 individuals in Group B received the pill with no medicine (placebo). Assume for now that the 20 individuals have similar initial pain levels and medical conditions. Calculate the value of Diff =  $\bar{x}_A - \bar{x}_B = \bar{x}_{\text{Treatment}} - \bar{x}_{\text{Control}}$ . This is the result from the experiment.

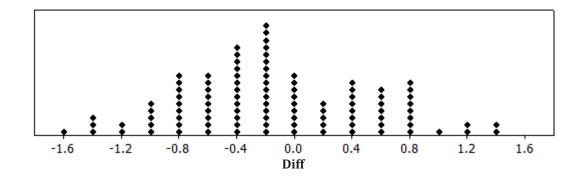
Group	ChangeinScore
А	0
А	0
А	-1
А	-1
А	-2
А	-2
А	-3
А	-3
А	-3
А	-4
В	0
В	0
В	0
В	0
В	0
В	0
В	-1
В	-1
В	-1
В	-2







3. Below is a randomization distribution of the value  $\text{Diff} = \bar{x}_A - \bar{x}_B$  based on 100 random assignments of these 20 observations into two groups of 10 (shown in a previous lesson).



With reference to the randomization distribution above and the inequality in your alternative hypothesis, compute the probability of getting a Diff value as extreme as or more extreme than the Diff value you obtained in the experiment.

- 4. Based on your probability value from Problem 3 and the randomization distribution above, *choose one* of the following conclusions:
  - a. Due to the small chance of obtaining a Diff value as extreme as or more extreme than the Diff value obtained in the experiment, we believe that the observed difference did not happen by chance alone, and we support the claim that the treatment is effective.
  - b. Because the chance of obtaining a Diff value as extreme as or more extreme than the Diff value obtained in the experiment is not small, it is possible that the observed difference may have happened by chance alone, and we cannot support the claim that the treatment is effective.



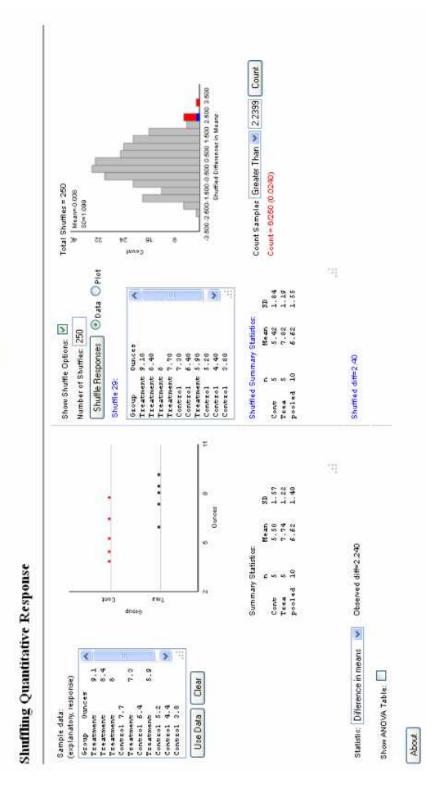






#### **Appendix: Screenshot of Applet**

Rossman/Chance Applet Collection









Date _____

# Lesson 28: Drawing a Conclusion from an Experiment

#### **Exit Ticket**

Explain why you constructed a randomization distribution in order to decide if wing length has an effect on flight time.





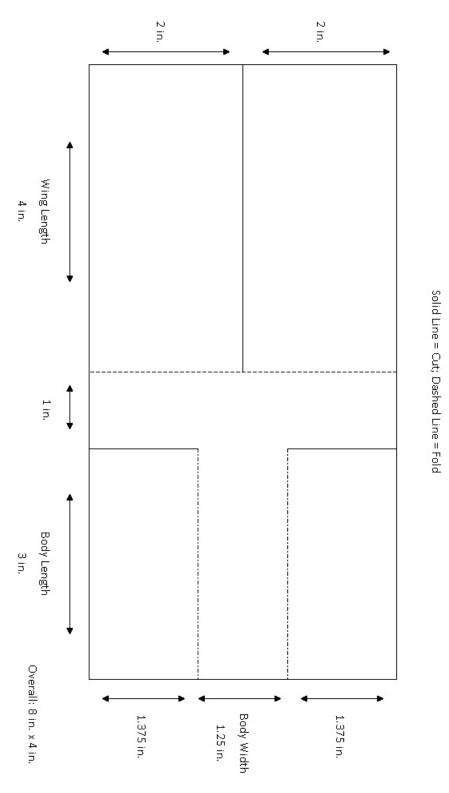






ALGEBRA II

#### **Appendix: Blueprint**





Lesson 28: Drawing a Conclusion from an Experiment

engage^{ny}

© 2015 Great Minds eureka-math.org ALG II-M4-SAP-1.3.0-09.2015 Lesson 29:

**EUREKA** 

MATH

Rubric for the Judging of Statistics Posters

Score	1/1	9	m		-
Overall Impact of the Display Use of pace, dimensions of question, readolity, namess, poster design aspects)	Poster is nearly constructed, including good use fours, pictures, and extras. The overall display is eye-catching but retains statistical substance. Good use of space for graphical presentation. Addresses multiple dimensions of the question or problem.	Addresses multiple dimensions of a question. Good use of space. Forth could be larger but do not really detarct from the message. Could be a liftle neater but really does not detract from the message.	Good use of space. Addresses multiple dimensions of a question. Readability or neatness detract from the overall appeal of the poster.	Serious problems with meatness or organization prevent the poster from being eye-catching and understandable. Multiple dimensions of the question addressed. Could use space better.	The poster is unidimensional Poor use of space for graphics. Major nearness or readability issues.
Technical Aspects (Speiling, Grammar, Consistency of colors or patterns)	Poster uses colors and patterns well. Correct grammar and spelling are used.	Bettar use of color or patterns would help the presentation, but in general the poster grabs the attention of the viewer. Correct grammar and spelling are used.	Use of more or different colors, would vasify improve the appeal of the poster. Minor grammar and/or spelling mistakes.	Sertous problems colors or patterns prevent the poster from being eye-catching and understandable. OR Multiple mistakes in OR Multiple mistakes in pratmant or spelling prevent the poster from being eye-catching and understandable.	The poster is has multiple spelling or grammar/spelling errors AND isn't consistent with colors or patterns so much so that it severely distracts from the moster
Clarity of the Message (How well is a story told?)	Question or purpose is clearly stated, and the presentation leads to the conclusion on a path that is easy to follow. The results of the study are immediately obvious to the viewer.	At least one link in the chain from the question through the results to the conclusion is difficult to follow.	The progression from question to conclusion can be followed, at least in part, but only with considerable effort, and the information on the back may be needed to confirm.	The information on the back is required in order for any relationships in the poster to be understood.	The poster is virtually incomprehensible.
Appropriateness of the Graphs for the Data (Statistical Appropriateness)	Graphs are appropriate for the question and data, and they are correctly constructed.	Errors or inaccuracies are present in at least one graph. More appropriate display(s) would improve the presentation.	Significant gap exists in the demonstration of understanding of the graphics, or how the graphics relate to the purpose of the poster.	Although some part of the graphs is correct, substantial errors lead to invalid or inappropriate conclusions.	The displays are inappropriate and incorrect for the research question and data types. The question is badly misunderstood and the results are nonsensical.
Creativity (Data collection methods, sample size issues, who cares factor)	Overall question is interesting, phrasing of titles, captions, and question are creative. Shows creative thought in topic, graph design, or dain collection. Collects data appropriately. Answers an important topic.	Overall question is interesting. Some creativity in design or data collection. Collects appropriate data.	Some creativity. Data could be better but it doesn't distract.	Creativity and topic are of some interest. Data collection could be improved with larger samples.	The poster appears to have been constructed with very little or no creativity or with improper data collection methods.

Drawing a Conclusion from an Experiment

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### To Be Customized by the Instructor:

Note: The rubric was taken from "Poster Judging Rubric" at the "Poster Competition and Project Competition" page of the American Statistical Association, <u>www.amstat.org/education/posterprojects/pdfs/PosterJudgingRubric.pdf</u>.



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Date _____

# Lesson 30: Evaluating Reports Based on Data from an Experiment

#### **Exit Ticket**

What are the aspects of a well-designed experiment that show a causal relationship?







NYS COMMON CORE MATHEMATICS CURRICULUM

End-of-Module Assessment Task M4

ALGEBRA II

Name

Date _____

- 1. Suppose you wanted to determine whether students who close their eyes are better able to estimate when 30 seconds have passed than students who do not close their eyes. (You ask students to tell you when to stop a stopwatch after they think 30 seconds have passed.) You find the first 50 students arriving at school one day. For those 50, you flip a coin to decide whether or not they will close their eyes during the test. Then, you compare the amounts by which each group overestimated or underestimated.
  - a. Did this study use *random sampling*? Explain your answer by describing what purpose random sampling serves in such a study.

b. Did this study make use of *random assignment*? Explain your answer by describing what purpose random assignment serves in such a study.

c. Would the study described above be an *observational study* or an *experimental study*? Explain how you are deciding.

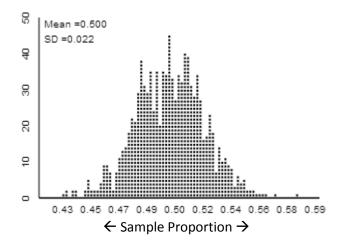






2. A Gallup poll conducted July 10–14, 2013, asked a random sample of U.S. adults: "How much attention do you pay to the nutritional information that is printed on restaurant menus or posted in restaurants, including calories and sugar and fat content?" The sample results were that 43% of the respondents said they pay a "fair amount" or a "great deal" of attention. Suppose there had been 500 people in the study.

The following graph displays the results from 1,000 random samples (each with sample size 500) from a very large population where 50% of respondents "pay some attention" and 50% "pay little or no attention."



a. Based on the simulation results above, are the sample data (43% responding "pay some attention") consistent with the simulation? In other words, do these results cause you to question whether the population is 50/50 on this issue? Explain.

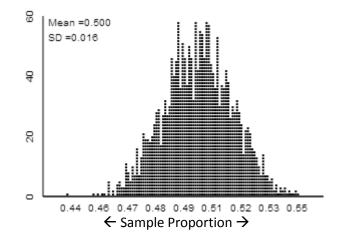
Do you believe it is reasonable to generalize the results from this study to all U.S. adults? Explain.







Suppose Gallup plans to conduct a new poll of a random sample of 1,000 U.S. adults on an issue where the population is evenly split between two responses. The following graph displays the results from 2,000 random samples (each with sample size 1,000) from such a population.



b. Based on these simulation results, estimate the expected margin of error for the Gallup poll. Explain how you developed your estimate.

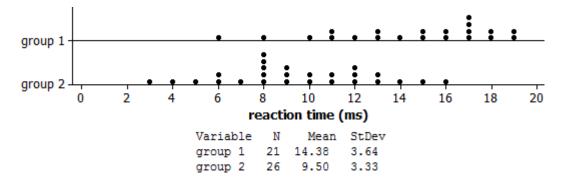
c. Suppose the study used a sample size of 2,000 instead of 1,000. Would you expect the margin of error to be larger or smaller?







3. A randomized experiment compared the reaction time (in milliseconds) for subjects who had been sleep deprived (group 1) and subjects who had not (group 2).



- a. Based on the above output, for which group would it be more reasonable to use a normal curve to model the reaction time distribution? Justify your choice.
- b. The difference in means is 14.38 9.50 = 4.88. One of the researchers claims that the reaction time if you are sleep deprived is 5 ms greater than the reaction time if you are not sleep deprived. Explain one reason why this claim is potentially misleading.

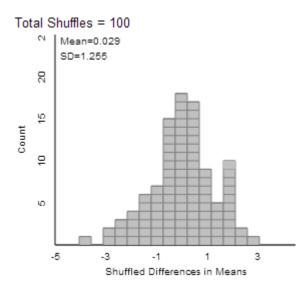
c. Describe how to carry out a simulation analysis to determine whether the mean reaction time for group 1 is significantly larger than the mean reaction time for group 2.







d. The graph below displays the results of 100 repetitions of a simulation to investigate the difference in sample means when there is no real difference in the treatment means. Use this graph to determine whether the observed mean reaction time for group 1 is significantly larger than the observed mean reaction time for group 2. Explain your reasoning.









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