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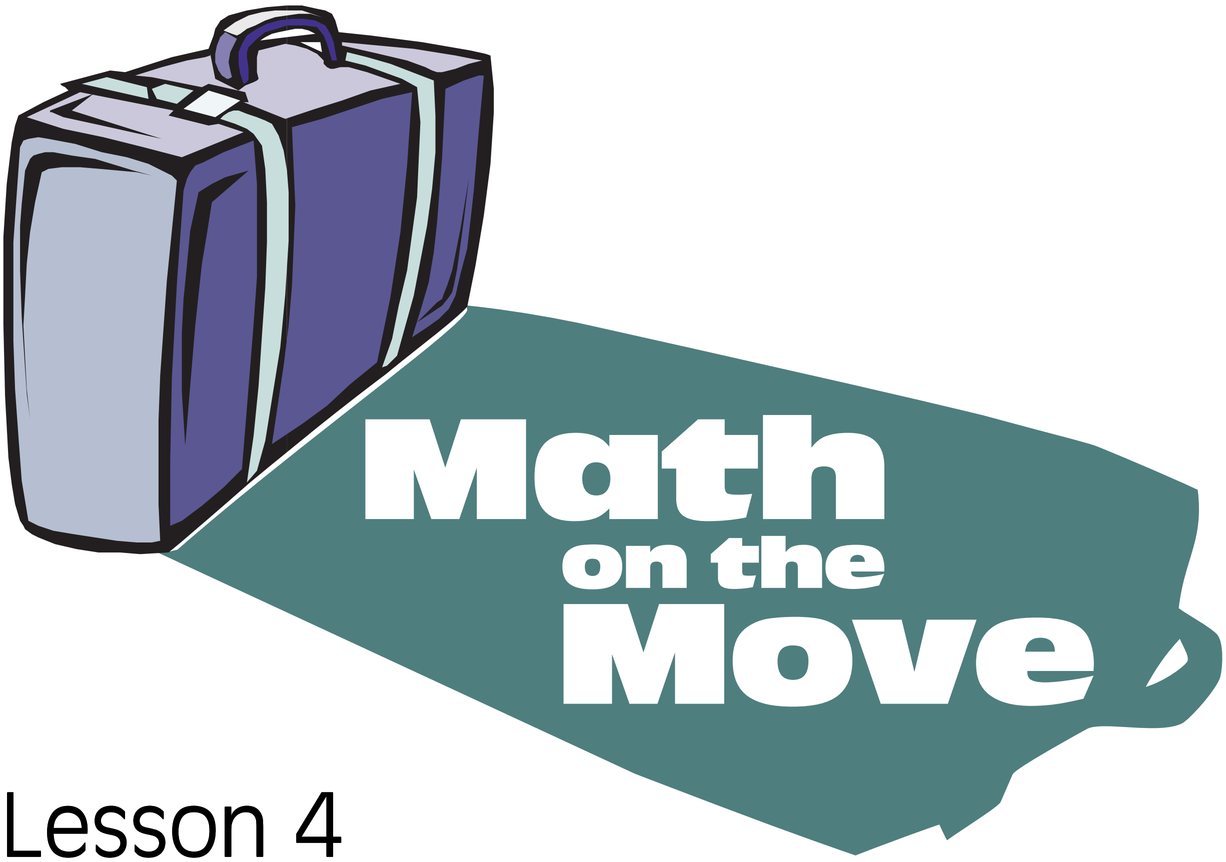

Lesson 4

Factors and Multiples

Objectives

- Understand what factors and multiples are
- Write a number as a product of its prime factors
- Find the greatest common factor and least common multiple of two numbers
- Use the correct order of operations

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One day, you and a group of your friends decide to play basketball. There are 12 people all together, and you are trying to decide how to separate the teams. Your friend Ramón suggests that you play a tournament with several teams. How many different teams can be made using 12 people?

We could think of this problem using multiplication.

You and your friends can be considered as 1 group of 12 people,

$$1 \times 12 = 12$$

you could separate into 2 teams of 6 people,

$$2 \times 6 = 12$$

or 3 teams of 4 people.

$$3 \times 4 = 12$$

You could also make 4 teams of 3, 6 teams of 2, or even 12 teams of 1.

$$4 \times 3 = 6 \times 2 = 12 \times 1 = 12$$

If we arrange all the whole numbers that were used to multiply to 12, they are:

1, 2, 3, 4, 6, 12

Notice that 12 is divisible by all of these numbers.

They are all of the **factors** of 12. Read the following definition very carefully.

- When whole numbers, other than zero, are multiplied together, each number is a **factor** of the product. Similarly, if a whole number divides evenly into a number, then the divisor and quotient are **factors** of that number. For example, 2 and 7 are factors of 14 because $2 \times 7 = 14$, and also because $14 \div 7 = 2$.

In the basketball problem, we said two different ways of grouping were

3×4 and 4×3

When we list factors of a number, we need to only count each factor once, so do not write the same factor twice. Thus, 3 and 4 are listed only once as factors of 12.

1. List all the factors of the following numbers:
   a) 24

   b) 10

   c) 36

Still thinking about 12, we said that two of its factors are 2 and 6.

Notice that 2 has no factors other than 1 and itself, 2. Because of this fact, 2 is defined to be a **prime** number.

- A number is **prime** if its <u>only</u> factors are 1 and itself.
  For example, 5 is prime because no numbers divide into it evenly except 1 and 5.

6, on the other hand, has more factors. All the numbers that divide evenly into 6 are

1, 2, 3, 6

Because there are factors of 6 in addition to 1 and 6, we say that 6 is a **composite** number. In other words, it is a <u>composition</u> of many factors.

- A **composite** number is a whole number greater than 1 that has factors in addition to 1 and itself. For example, 4 is composite because it has a factor of 2.

As we said, 6 is composite, and we may write it as a product of two of its factors, say

$$6 = 2 \times 3$$

Before, we said that

$$12 = \boxed{6} \times 2$$

Now, we may say that

$$12 = \boxed{2 \times 3} \times 2$$

We just substituted something equal to 6 where the number 6 used to be.

Now, the number 12, written as

$$12 = 2 \times 3 \times 2$$

has two factors of 2, and a factor of 3.

Written this way, 12 is a product of only prime factors!

FACT

The number 1 is neither prime nor composite!

FACT

Every whole number can be written as a product of prime factors! This is a very special property called the ***Fundamental Theorem of Arithmetic!***

One useful way to factor a number into primes is by creating a factor tree diagram.

Example

Write 72 as a product of its prime factors.

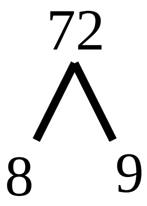
Solution

We will solve this with the factor tree method.

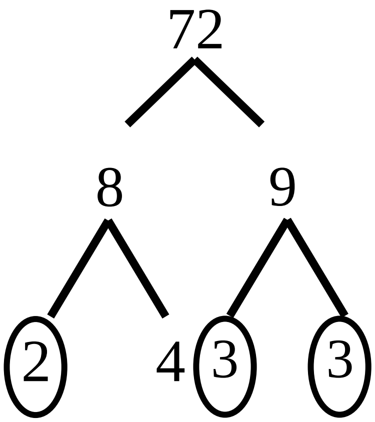
Step 1: Write the number you want to factor.

72

Step 2: Draw two “branches” off that number, with factors that multiply to the number above them. Never use the factor 1.

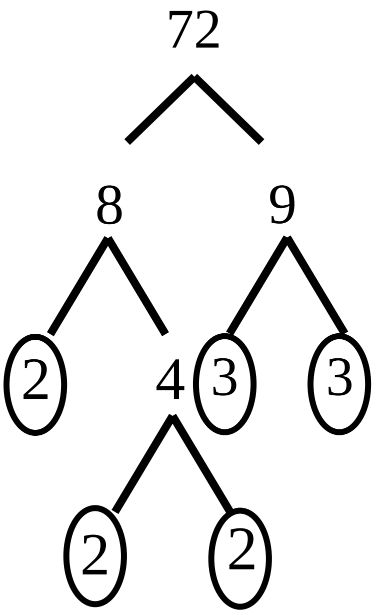


Step 3: Continue to draw branches off each factor, until you have reached a prime number. Circle the prime factors as they occur



Factor 8 and 9, and circle the prime factors.

Now we factor the 4, and circle its prime factors.



Weeping willow

Our factor tree factoring is now complete, but we are not yet finished with the problem!

Step 4: Rewrite as a product of primes.

We see that our final product is equal to

$$72 = 2 \times 3 \times 3 \times 2 \times 2 = 2 \times 2 \times 2 \times 3 \times 3$$

Here is step-by-step method for what we just did.

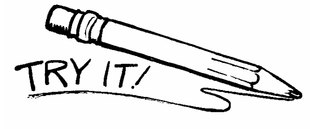
FACT

The order that we multiply numbers does not matter.

Algorithm

To factor a number:

1. Write the number you wish to factor at the top.
2. Below that number, draw two branches. The numbers at the end of the branches will be factors whose product is the number above (do not use the factors 1 and the number).
3. Once a prime number is reached, circle it, and continue to factor the composite numbers.
4. Write the number as a product of its prime factors.



2. Factor each number using a factor tree, then write the number as a product of prime factors.

   a) 64 b) 100 c) 36

The factor-tree method is very useful for finding the prime factors of a number, but we can also use it to find the **common factors** among two (or more) numbers.

When comparing two (or more) numbers:

- Factors that only one of the numbers has, are called **unique factors**.
- Factors that the numbers share are called **common factors**.
- The largest factor two (or more) numbers share is their **greatest common factor**, or their **GCF**. For instance, the GCF of 4 and 6 is 2.

Consider the following example.

Example

Find the greatest common factor of 90 and 135.

Solution

The most obvious way to solve this problem is to list the factors of each number, find their common factors, and then determine which factor is the largest.

Factors of 90: 1, 2, 3, 5, 9, 10, 18, 30, **45**, 90

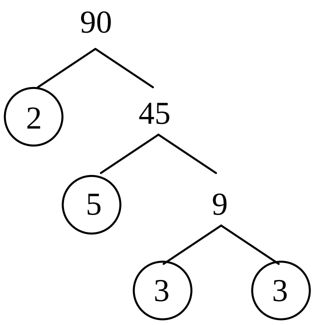
Factors of 135: 1, 3, 5, 9, 15, 27, **45**, 135

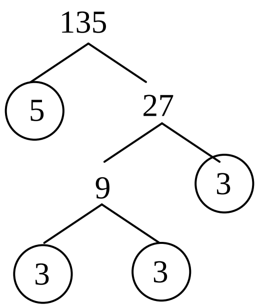
So we see that 45 is the GCF of 90 and 135.

However, it was not efficient to list every factor of 90 and 135. It is also easy to miss factors, and make mistakes using this method. Luckily, there is an easier way to solve this problem that uses factor trees and Venn diagrams.

The other method:

Step 1: Factor each number using a factor tree, and rewrite it as a product of prime factors.

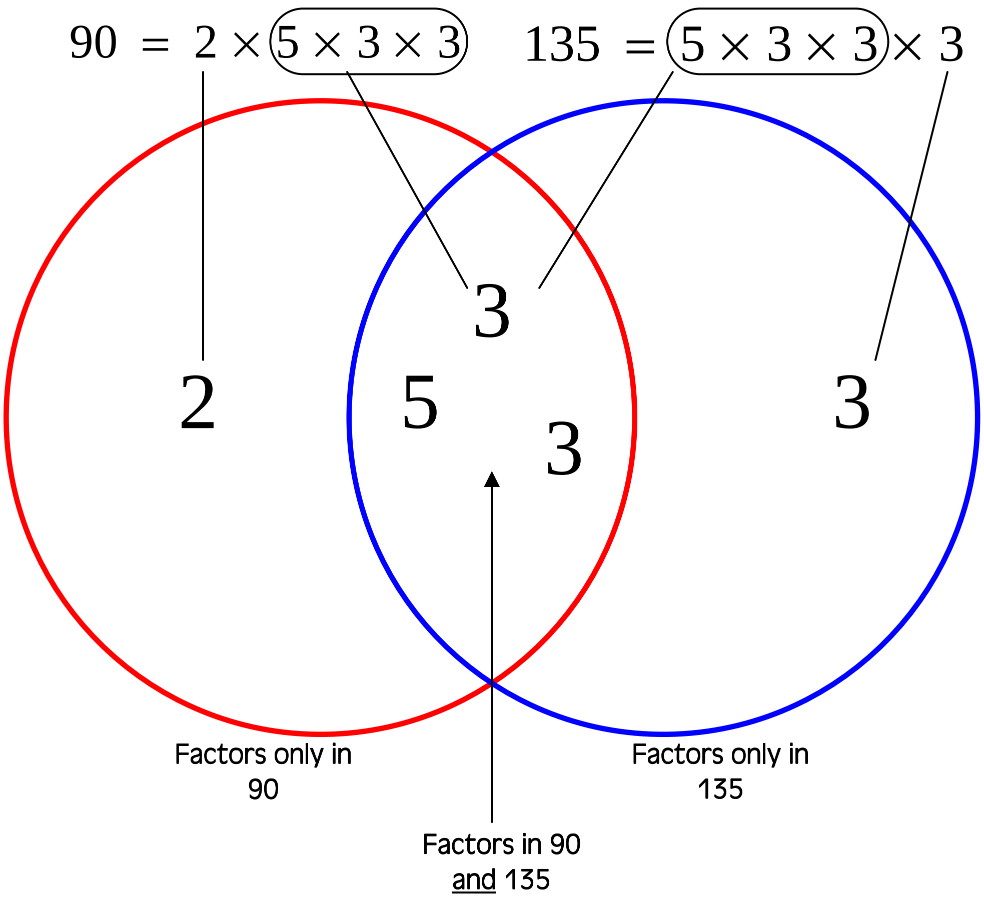




$90 = 2 \times 5 \times 3 \times 3$

$135 = 5 \times 3 \times 3 \times 3$

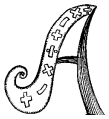
Step 2: Sort using a Venn diagram



Step 3: From the first method, we found that the GCF was 45. Look at the common prime factors of 90 and 135. They are 5 and two 3s. Notice that

$$5 \times 3 \times 3 = 45$$

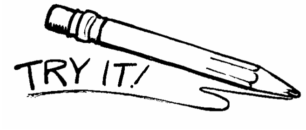
This is the same answer as the first method, and it is a good way to avoid forgetting factors!



Algorithm

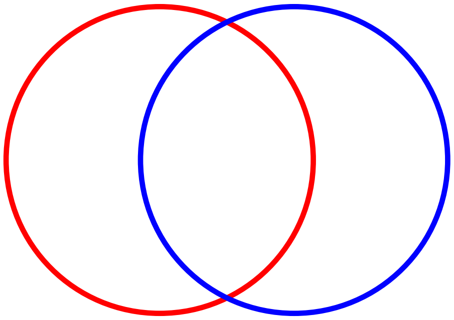
To find the greatest common factor (GCF):

1. Factor each number, and rewrite it as a product of prime factors.
2. Organize the factors of each number using a Venn diagram.
3. Multiply all of the numbers in the center section of the Venn diagram together. This is the GCF.

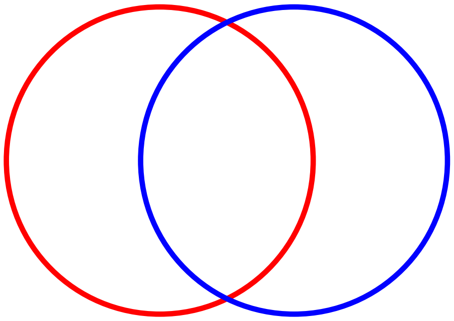


3. Find the greatest common factor for each pair of numbers.

a) 72 and 108



b) 70 and 315



This will be a good preparation for when we work with fractions.

After you and your friends are done playing basketball, you decide to invite everyone to your house for a barbecue. On your way home, you stop at the store to pick up hot dogs and hot dog buns.

You notice that hot dogs come in packs of 6, and the hot dog buns come in packs of 8. You realize that this is not good, because you want to have the same number of hot dogs as buns. Your friend Ramón suggests that you buy multiple packs of hot dogs and buns so you will have the same number of hot dogs as buns. How do you determine the number of packs of hot dogs and hot dog buns you need to buy?

You think to yourself how many hot dogs you would get if you buy multiple packs. You realize that this is a multiplication problem. The number of packs you buy, times 6, gives you the number of hot dogs you get. The numbers of hot dogs you can get are

6, 12, 18, 24, 30, 36...

You then think of how many buns you would get, if you bought multiple packs. The number of packs, times 8, gives you the number of buns you get. The numbers of buns you can get are

8, 16, 24, 32, 40, 48...

Hot dogs come in **multiples** of 6, and buns come in **multiples** of 8.

- A **multiple** of a number is the product of that number and any whole number besides zero. For example, 20 is a multiple of 4 $(4 \times 5 = 20)$

As we can see, 6 and 8 have some multiples in common

Multiples of 6: 6, 12, 18, **24**, 30, 36, 42, **48**...
Multiples of 8: 8, 16, **24**, 32, 40, **48**, 56, 72...

We see that 6 and 8 both have the multiples 24 and 48. These are common multiples. In our problem, common multiples will happen when we have the same number of hot dogs as buns. The smallest multiple these numbers share is the **least common multiple**.

- The smallest multiple two numbers both have is called the **least common multiple**, or **LCM**.

So, you can get 24 hot dogs and buns. More specifically, if you buy 4 packs of hot dogs and 3 packs of buns, you will get 24 of each. This is because

$$4 \times 6 = 3 \times 8 = 24 .$$

Example

Find the LCM of 12 and 20

Solution

Using the most obvious method, we can just list multiples of each number.

Multiples of 12 are: 12, 24, 48, **60**, 72, ...

Multiples of 20 are: 20, 40, **60**, 80, 100, ...

Our least common multiple is 60.

FACT *The LCM of two numbers is the product of those two numbers divided by their GCF.*

Let's check to see if this fact works.

Factors of 12: 1, 2, 3, **4**, 6, 12

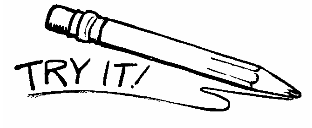
Factors of 20: 1, 2, **4**, 5, 10, 20.

4 is the GCF.

$$12 \times 20 = 240$$

$$240 \div 4 = 60$$

It works! You can use this fact to check your answer when finding the LCM of two numbers.



4. Find the least common multiple for each pair of numbers.
   a) 8 and 16

   b) 24 and 84

   c) 13 and 17

The LCM will be very useful for fractions.

When planning your barbecue, you realize that your friends are really hungry. Each person who comes will want to eat 3 hot dogs. Since there were 12 people playing, you realize that 24 hot dogs will not be enough. If each person eats 3 hot dogs, then you will need at least 36 hot dogs. Remembering that you want to have the same number of hot dogs as buns, you decide to get 48 hot dogs and buns. How many hot dogs will be left over if each person eats 3 hot dogs?

This is a simple subtraction problem. There are 48 hot dogs altogether, and 36 of them will be eaten. So, the number of hot dogs left over is

$$48 - 36 = 12.$$

Even though this problem looks simple, what we don't realize is that the original problem looked like this:

$$48 - 12 \times 3.$$

This is because we started with 48 hot dogs, and there were 12 people who wanted 3 hot dogs each. When we look at this problem, our initial instinct is to perform the operations from left to right. If we do that, we get the following answer:

$$\underbrace{48 - 12} \times 3$$

$$36 \times 3 = 108$$

However, we know this is not correct because our answer must be less than the number of hot dogs we started with, which was 48. This shows that we must have some rules for the order in which we do things in math. Thus, we will learn the correct order to do operations.

One rule mathematicians decided was that multiplication comes before addition. But what about all the other operations, like division and subtraction?

Mathematicians have agreed on the following order of operations:

Algorithm

Order of Operations

1. Simplify Expressions in *Parentheses*.
2. Evaluate *Exponents*.
3. Perform all *Multiplication* and/or *Division* working from left to right.
4. Perform all *Addition* and/or *Subtraction* working from left to right.

One way we can remember the right order to calculate expressions is by using one of the following phrases:

"Please Excuse My Dear Aunt Sally," or, if you prefer, "PEMDAS."

Please	**P**arenthesis
Excuse	**E**xponents
My **D**ear	**M**ultiplication or **D**ivision
Aunt **S**ally	**A**ddition or **S**ubtraction

Example

Simplify $4\times(6+2)$

Solution

First, we see that this expression involves multiplication and addition, and also contains a set of parentheses. The correct order of operations we should use is:

$4\times(6+2)=4\times8$ (Addition inside Parentheses)

$=32$ (Multiplication)

Example

Simplify $9+5\times3$

Solution

$9+5\times3=9+15$ (Multiplication)

$=24$ (Addition)

FACT

Even though there is no "$\times$" sign, a number next to parentheses means you should multiply.

Example

Simplify $5(3^2-6)$

Solution

$5(3^2-6)=5(9-6)$ (Exponent inside Parentheses)

$=5(3)$ (Subtraction inside Parentheses)

=15 (Multiplication)

Example

Simplify $4+3\times5-(6-2)^2\div2$

Solution

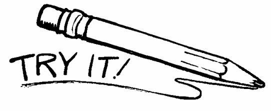
$4+3\times5-(6-2)^2\div2$

$=4+3\times5-4^2\div2$ (Parentheses)

$=4+3\times5-16\div2$ (Exponents)

$=4+15-8$ (Multiplication and Division)

$=11$ (Addition and Subtraction)



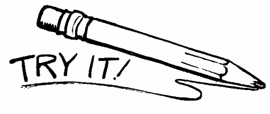
Simplify each expression.

5. $(4+5)^2$

6. $3^2+2\times7+3$

We now understand that parentheses are a way to group terms. They tell us to evaluate what is inside of them first. Parentheses can look different from what you are used to. Some common ways to write parentheses ( ) are with braces { }, and brackets [ ]. Even though each set of parentheses looks different, they mean the same thing. We sometimes use different looking parentheses sometimes, to make number phrases easier to read. For instance, $((2+3)\times(3-1))^2$ looks better when we write it as $[(2+3)\times(3-1)]^2$.

Sometimes, parentheses are being used, but we cannot see them. This happens whenever we see a fraction. For instance, the expression $\frac{7+1}{9-5}$ really means $(7+1)\div(9-5)$. We know how to simplify expressions in this form. Notice that a fraction is another way to show division. We will explore this later.



Re-write the fraction using parentheses ( ) and the ÷ sign. Then simplify the expression.

7. $\frac{4^2-4\times 2}{3+1}$

8. $\frac{(12-6)^2-10\times 3}{3\times 2}$

☑ Review

1. Highlight the following definitions:
    a. factors
    b. prime
    c. composite
    d. unique factors
    e. common factors
    f. Greatest Common Factor (GCF)
    g. multiples
    h. Least Common Multiple (LCM)

2. Highlight the “Algorithm” boxes.

3. Write one question you would like to ask your mentor, or one new thing you learned in this lesson.

Practice Problems

Math On the Move Lesson 4

Directions: Write your answers in your math journal. Label this exercise Math On the Move – Lesson 4, Set A and Set B.

Set A

1. *True* or *False*: All the factors of 24 are prime.

2. Write the <u>prime factorization</u> of the following numbers (Hint: factor trees are useful tools.)
    a) 55 b) 63 c) 144 d) 210

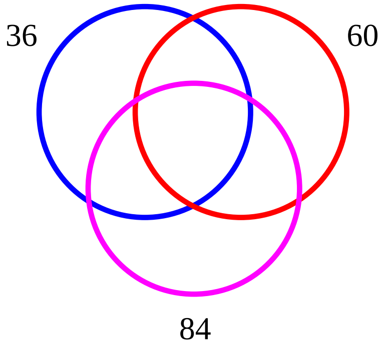
3. Find the GCF and LCM of the following pairs
    a) 3 and 5 b) 66 and 165 c) 130 and 182 d) 322 and 1150
    e) 13 and 24 f) 41 and 42 g) 98 and 100 h) 12 and 120

Set B

1. Does the greatest common factor of 3 and 13 exist? Explain why or why not.

2. Find the GCF and LCM for 36, 60, and 84.

(*Hint*: Use the following Venn diagram. The product of all the numbers within the Venn diagram is the LCM)



3. How many multiples does the number 7 have? How do you know?

1. a) 1, 2, 3, 4, 6, 8, 12, 24

   b) 1, 2, 5, 10

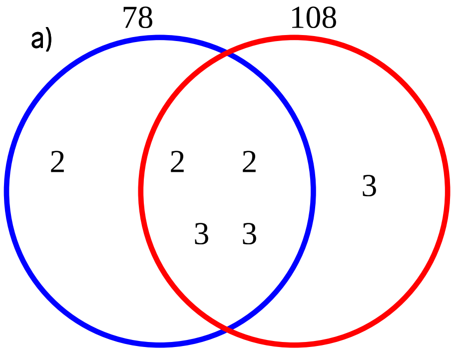
   c) 1, 2, 3, 4, 6, 9, 12, 18, 36

2. a) $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

   b) $2 \times 2 \times 5 \times 5 = 100$

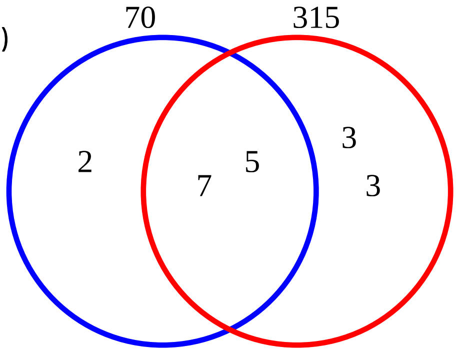
   c) $2 \times 2 \times 3 \times 3 = 36$

3. a)



$2 \times 2 \times 3 \times 3 = 36$

b)



$7 \times 5 = 35$

4. a) 16 b) 168 c) 221

5. $(4+5)^2 = 9^2 = \boxed{81}$

6. $3^2 + 2 \times 7 + 3 = 9 + 14 + 3 = \boxed{26}$

7. $\dfrac{4^2 - 4 \times 2}{3+1} = \left(4^2 - 4 \times 2\right) \div (3+1) = (16-8) \div 4 = 8 \div 4 = \boxed{2}$

8. $\dfrac{(12-6)^2 - 10 \times 3}{3 \times 2} = \left[(12-6)^2 - 10 \times 3\right] \div (3 \times 2)$

$$\left[6^2 - 30\right] \div 6$$

$$6 \div 6 = \boxed{1}$$

End of Lesson 4