Name ________________

5.8 Trig Review

Date ________________

1. *Solving trig equations*
Use your knowledge of inverse trig functions, special angles and the unit circle to solve the following equations. Give exact answers (those that involve π or $\sqrt{2}$) whenever possible.

a. $\sin(\beta) = -.7568,\ 0 \le \beta < 2\pi$.

b. $\cos 2\theta = -1/2,\ -2\pi \le \theta < 2\pi$.

c. $\tan(x) = -1$ on $[-4\pi, 4\pi]$

d. $\sec(x^2) = 2$ on $[0, 2\pi]$

e. $\left(\csc(x)\right)^2 - 2 = 0$ on $(-\infty, \infty)$

f. $4\cos(x) + 2\cot(x) = 2\sec(x)$ on $\left[-2\pi, 2\pi\right]$

2. Starting from the fact that $\cos(\alpha+\beta)=\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)$, find an identity for $\sin(\frac{\theta}{2})$. In other words, derive the "half-angle" identity for sine. Hint: $\theta=\frac{\theta}{2}+\frac{\theta}{2}$

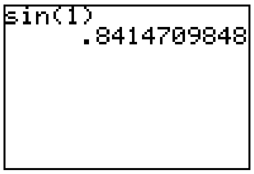
3. Either prove the statement or use a counter-example to show it is false. Hint: don't waste time proving things that are not true.

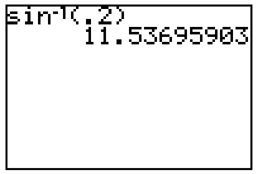
a. $\dfrac{\cos x}{1+\sin x}-\dfrac{\cos x}{1-\sin x}=-2\cot x$

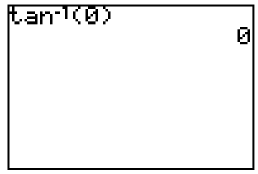
b. $\csc^2(\theta)+\sec^2(\theta)=\csc^2(\theta)\sec^2(\theta)$

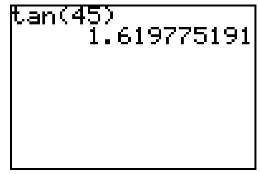
4. Interpret the following calculator scenarios. Do not use your calculator to solve these problems as it defeats the purpose. Use them to check after you have arrived at a solution that you believe to be correct.

a. If possible, determine the mode setting (degree or radian) of the calculator for each calculation.





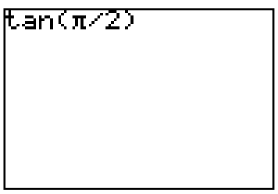


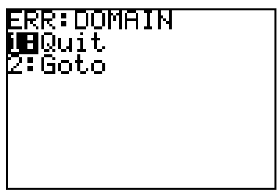


Mode: ___________ Mode: ___________ Mode: ___________ Mode: ___________

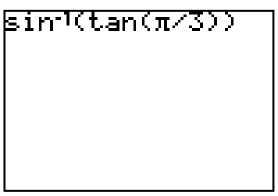
b. Explain why the inputs on the first screen cause the error messages on the second screen.

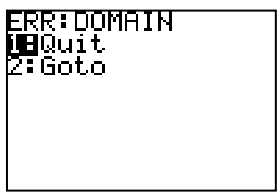
i.





ii.





c. You forget your graphing calculator and are given a loaner. Unfortunately, the SIN button is the only one of the three trig buttons that works. How could you use this calculator to evaluate the following expressions?

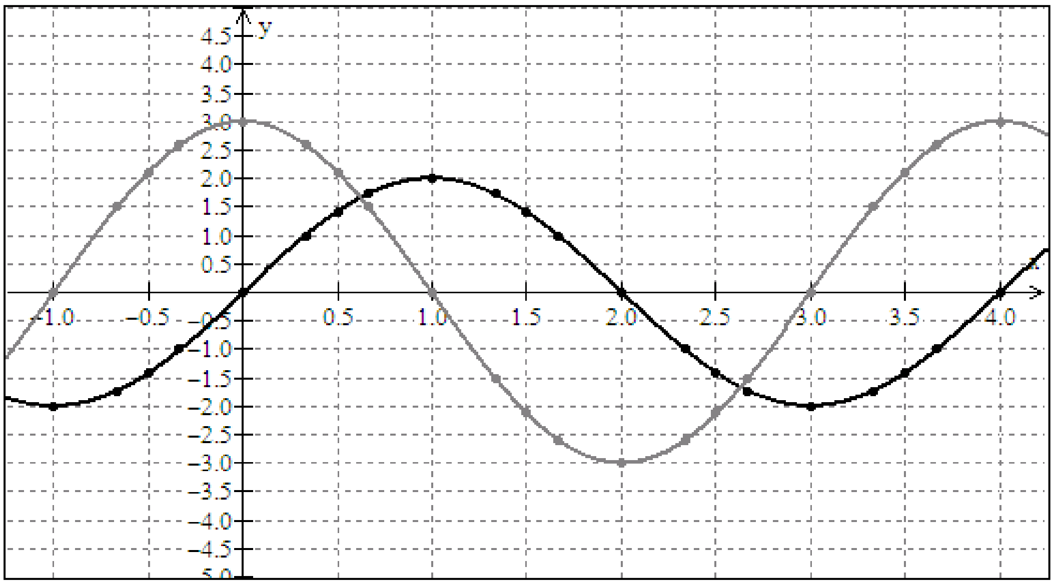
i. $\tan(37^\circ)$

ii. $\tan^{-1}(2.5)$

5. The grid shows graphs of $f(x) = \_\_\_\_ \sin(\_\_\_\_ x)$ and $g(x) = \_\_\_\_ \cos(\_\_\_\_ x)$

a. Fill in the blanks in the equations above.

b. Use the given graphs to sketch a reasonable graph of $h(x) = f(x) + g(x)$.

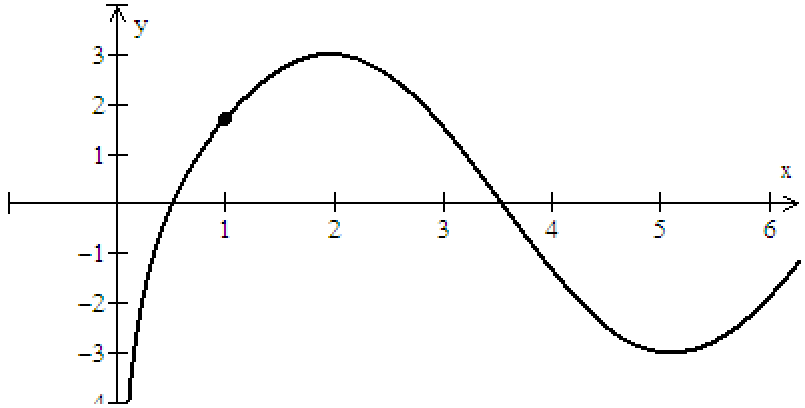


c. Find values of A and ϕ such that $h(x) = A\cos(\phi)\sin(\frac{\pi}{2}x) + A\sin(\phi)\cos(\frac{\pi}{2}x)$

d. Use an identity to rewrite you equation for $h(x)$ in terms of a single *transformed* sine function.

6. The graph below shows the ***continuous***, ***differentiable*** function $f(x)=\begin{cases} A\ln(2x) & x<1 \\ 3\sin(x+\phi) & x\geq 1 \end{cases}$

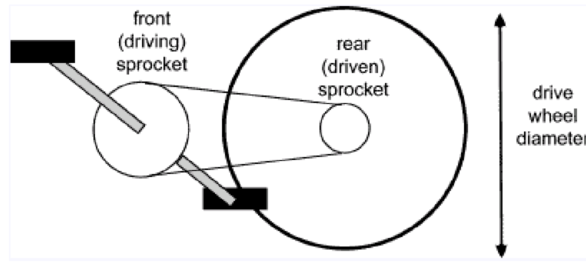
a. Find the appropriate values of A and ϕ



b. Find an additional function $q(x)$ such that the function $f(x)=\begin{cases} A\ln(2x)+q(x) & x<1 \\ 3\sin(x+\phi) & x\geq 1 \end{cases}$ is ***continuous and twice differentiable***. Think about how Taylor polynomials work to get started (and you can't use $q(x)=3\sin(x+\phi)-A\ln(2x)$).

7. Consider this simplified diagram of a bicycle drive train. Assume the drive wheel has a diameter of 30 inches, the rear sprocket has a diameter of 5 inches, the front sprocket has a diameter of 7 inches and the crank arms are 12 inches long.

a. If the rider is pedaling and moving at a constant speed of 16 miles per hour, how long does it take her foot to make one revolution? Try to solve the problem without ever converting to degrees.



b. At $t = 0$ her left crank arm makes an angle of $\frac{\pi}{3}$ with respect to the horizontal direction of travel. Write down an equation for the function $h(t)$ that gives the height h of her left foot off the ground at time t.

c. What are the vertical and horizontal components of the velocity of her foot at $t = 1$?

8. Find an infinite polynomial for $f(x)=\dfrac{1}{\sqrt{1+x}}$ centered at $x=0$. First write your expression as $f(x)=0^{\text{th}}$ term + 1^{st} term + 2^{nd} term + 3^{rd} term + 4^{th} term + 5th term + ... After you get this written out, go ahead and try to get $f(x)$ in terms of a sigma notation expression.

Hint: $9\cdot7\cdot5\cdot3\cdot1=\dfrac{10!}{2^5\cdot5!}$.

9. The figure below shows a graph of the relation $y^2 + 3y = -4\sin(x)$. It also shows a quadratic function that approximates this curve near $x = \frac{\pi}{6}$. Find the equation for this quadratic function.