Unit 1 Notes

Geometry
Points, Lines, Planes, & Angles

www.njctl.org

www.njctl.org

Table of Contents

click on the topic to go to that section

Table of Contents

Table of Contents for Videos Demonstrating Constructions

click on the topic to go to that video

Table of Contents Video

Basic History of Geometry

Aug 13-1:47 PM

Euclidean Geometry

This course will use the basic thinking developed by Euclid.

We will attempt to make clear and distinguish between:

- What we have assumed to be true, and cannot prove
- What follows from what we have previously assumed or proven

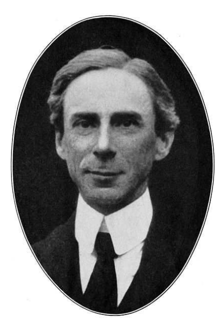
That is the reasoning that makes geometric thinking so valuable. Always *question every idea* that's presented, that's what Euclid and those who invented geometry would have wanted.

Points, Lines, & Planes

Euclidean Geometry

This also represents a path to logical thinking, which British philosopher Bertrand Russell showed is identical to mathematical thinking.

Click on the image to watch a short video of Bertrand Russell's message to the future which was filmed in 1959.



Did you hear anything that sounded familiar?

What was it?

Points, Lines, & Planes

Euclidean Geometry

Euclid's assumptions are **axioms**, **postulates** and **definitions**.

You won't be expected to memorize them, but to use them to develop further understanding.

Major ideas which are proven are called **Theorems**.

Ideas that easily follow from a theorem are called **Corollaries**.

Points, Lines, & Planes

Euclidean Geometry

The five axioms are very general, apply to the entire course, and don't depend on the definitions or postulates, so we'll review them in this unit.

The postulates and definitions are related to specific topics, so we'll introduce them as required.

Also, additional modern terms which you will need to know will be introduced as needed.

Points, Lines, & Planes

Euclid's Axioms (Common Understandings)

Euclid called his axioms was "Common Understandings."

They seem so obvious to us now, and to him then, that the fact that he wrote them down as his assumptions reflects how carefully he wanted to make clear his thinking.

He didn't want to assume even the most obvious understandings without indicating that he was doing just that.

Points, Lines, & Planes

Euclid's Axioms (Common Understandings)

This careful rigor is what led to this approach changing the world.

Great breakthroughs in science, mathematics, engineering, business, etc. are made by people who question what seems obviously true...but turns out to not always be true.

Without recognizing the assumptions you are making, you're not able to question them...and, sometimes, not able to move beyond them.

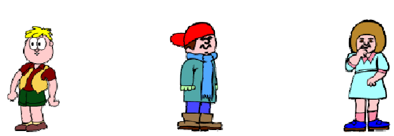
Points, Lines, & Planes

Euclid's First Axiom

Things which are equal to the same thing are also equal to one another.

For example:

If I know that Tom and Bob are the same height, and I know that Bob and Sarah are the same height...what other conclusion can I come to?



Jul 18-10:22 PM

Euclid's Second Axiom

If equals are added to equals, the whole are equal.

For example:

If you and I each have the same amount of money, let's say $20, and we each earn the same additional amount, let's say $2,

then we still each have the same amount of money as each other, in this case $22.

Jul 18-10:22 PM

Euclid's Third Axiom

If equals be subtracted from equals, the remainders are equal.

This is just like the second axiom.

Come up with an example on your own. Look back at the second axiom if you need a hint.

Jul 18-10:22 PM

Euclid's Fourth Axiom

Things which coincide with one another are equal to one another.

For example:

If I lay two pieces of wood side by side and both ends and all the points in between line up, I would say they have equal lengths.

Jul 18-10:22 PM

Euclid's Fifth Axiom

The whole is greater than the part.

For example:

If an object is made up of more than one part,

then the object has to be larger than any of those parts.

Jul 18-10:22 PM

Euclid's Axioms (Common Understandings)

First Axiom: Things which are equal to the same thing are also equal to one another.

Second Axiom: If equals are added to equals, the whole are equal.

Third Axiom: If equals be subtracted from equals, the remainders are equal.

Fourth Axiom: Things which coincide with one another are equal to one another.

Fifth Axiom: The whole is greater than the part.

Jul 18-10:22 PM

Points, Lines, & Planes

Return to Table of Contents

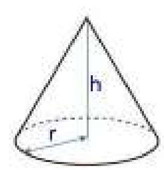
Points, Lines, & Planes

Definitions

Definitions are words or terms that have an agreed upon meaning; they cannot be derived or proven.

The definitions used in geometry are idealizations, they do not physically exist.

Definitions state the meaning of a term or idea.



When we draw objects based on these definitions, that is just to help visualize them. However, imaginary geometric objects can be used to develop ideas that can then be made into real objects.

Points, Lines, & Planes

Definitions

Definitions are ***reversible.***

Example:

If a point is the midpoint of a segment, then the point divides the segment into two congruent segments.

or

If a point divides a segment into two congruent segments, then the point is the midpoint of the segment.

In some problems, the first definition is necessary in a proof, in others, the second definition is necessary.

Points, Lines, & Planes

Points

Definition 1: A point is that which has no part.

A point is infinitely small.

It cannot be divided into smaller parts.

It is a location in space, without dimensions.

It has no length, width or height.

Points, Lines, & Planes

Points

Definition 1: A point is that which has no part.

Look at this dot. Why can it not be considered a point? Discuss your answer with a partner.

Points, Lines, & Planes

Points

A point is represented by a dot.

The dot drawn on a page has dimensions, but the point it represents does not.

A point can be imagined, but not drawn.

Only the position of the point is shown by the dot.

Points are usually labeled with a capital letter (e.g. A, B, C).

C

A B

Points, Lines, & Planes

Lines

Definition 2: A line is breadthless length.

A line is defined to have length, but no width or height.

The line drawn on a page has width, but the idea of a line does not.

Lines can be thought of as an infinite number of points with no space between them.

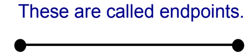
Points, Lines, & Planes

Lines

Definition 3: The ends of a line are points.

A line consists of an infinite number of points laid side by side, so at either end of a line are points.

These are called endpoints.



Even though this is how we correctly depict a line with endpoints, why is is not accurate?

Points, Lines, & Planes

Unit 1 Notes

Lines

Definition 4: A straight line is a line which lies evenly with the points on itself.

In a straight line the points lie next to one another without bending or turning in any direction.

While a line can follow any path, in this course we will use the term "line" to mean a straight line, unless otherwise indicated.

Points, Lines, & Planes

Lines

A postulate is an unproven assumption.

First Postulate: To draw a line from any point to any point.

This postulate indicates that given any two points, it is possible to draw a line between them.

Aside from letting us connect two points with a line, it also allows us to extend any line as far as we choose since points could be located at any point in space.

Jul 19-12:05 PM

Lines

Second Postulate: To produce a finite straight line continuously in a straight line.

This postulate indicates that the line drawn between any two points can be a straight line.

This allows the use of a straight edge to draw lines.

A straight edge is a ruler without markings.

Note: Any object with a straight edge can be used.

Jul 19-12:05 PM

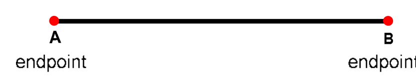
Line Segments

Using these definitions and postulates we can *first draw two points* (the endpoints) and then draw a straight line between them using a straight edge.

A line drawn in this way is called a line segment.

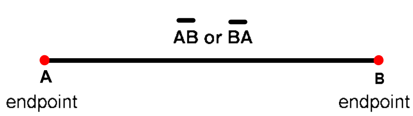
It has finite length, a beginning and an end.

At each end of the segment there is an endpoint, as shown below:



Line Segments

Naming Line Segments



A line segment is named by its two endpoints.

The order of the endpoints doesn't matter.

For instance, $\overline{AB}$ and $\overline{BA}$ are different names for the same segment.

Line Segments

Lines

A straight line, which extends to infinity in both directions, can be created by extending a line segment in both directions.

This is allowed by our definitions and postulates by imagining connecting each endpoint of the segment to other points that lie beyond it, in both directions.

Points, Lines, & Planes

Unit 1 Notes

Lines

In this example, line Segment AB is extended in both directions to create Line AB.

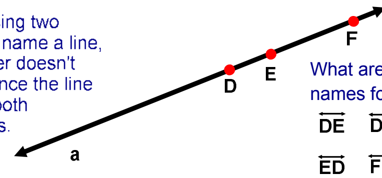
Points, Lines, & Planes

Naming Lines

A line is named by using any two points on it OR by using a single lower-case letter.

Arrowheads in the symbol above the points in the name of the line show that the line continues without end in opposite directions.

When using two points to name a line, their order doesn't matter since the line goes in both directions.



What are 7 valid names for this line?

$\overleftrightarrow{DE}$ $\overleftrightarrow{DF}$ $\overleftrightarrow{EF}$

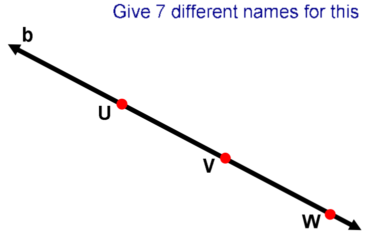
$\overleftrightarrow{ED}$ $\overleftrightarrow{FD}$ $\overleftrightarrow{FE}$

a

Points, Lines, & Planes

Example

Give 7 different names for this line.



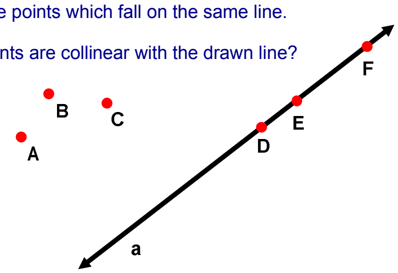
Answer

Points, Lines, & Planes

Collinear Points

Collinear points are points which fall on the same line.

Which of these points are collinear with the drawn line?



Answer

Points, Lines, & Planes

Collinear Points

Is it possible for any two points to not be collinear on at least one line?

Come up with an answer at your table.
Remember, only use facts to make your argument!

Answer

Points, Lines, & Planes

1 How many points are needed to define a line?

Answer

Jul 18-10:49 PM

2 Can there be two points which are not collinear on some line?

Answer

Yes

No

Jul 18-10:53 PM

3 Can there be three points which are not collinear on some line?

Answer

Yes

No

Jul 18-10:53 PM

Intersecting Lines

Is it possible for two different lines to intersect at more than one point?

A good technique to prove whether this is possible is called either

Argumentum ad absurdum

or

Reductio ad absurdum

Points, Lines, & Planes

Intersecting Lines

Argumentum ad absurdum

or

Reductio ad absurdum

These are two Latin terms which refer to the same powerful approach, an ***indirect proof***.

First, your assume something is true. Then you see what logically follows from that assumption. If the conclusion is absurd, the assumption was false, and disproven.

Points, Lines, & Planes

Intersecting Lines

Is it possible for two different lines to intersect at more than one point?

Let's assume that two different lines can share more than one point and see where that leads us.

Let's name the two points which are shared A and B.

We could connect A and B with a line segment, since we can draw a line segment between any two points.

That segment would overlap both our original lines between A and B, since they are all straight lines and all include A and B.

Points, Lines, & Planes

Intersecting Lines

We could then extend our Segment AB infinitely in both directions and our new Line AB would overlap our original two lines to infinity in both directions.

If they share all the same points, they are the same lines, just with different names.

But we assumed that the two original lines were different lines sharing two points.

Points, Lines, & Planes

Intersecting Lines

Is it possible for two different lines to intersect at more than one point?

But we have concluded that they are the same line, not different lines.

It is impossible for them to be both different lines and the same lines.

So, our assumption is proven false and the opposite assumption must be true.

Two different lines cannot share two points.

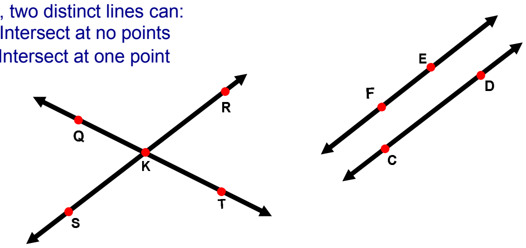
Points, Lines, & Planes

Intersecting Lines

Is it possible for two different lines to intersect at more than one point?

So, two distinct lines can:

- Intersect at no points
- Intersect at one point



Points, Lines, & Planes

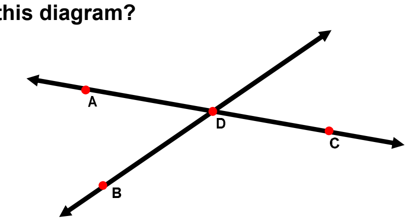
4 What is the maximum number of points at which two distinct lines can intersect?

Answer

Jul 18-10:49 PM

5 Which sets of points are collinear on the lines drawn in this diagram?

A B, D, A
B C, D, B
C D, A, C
D none

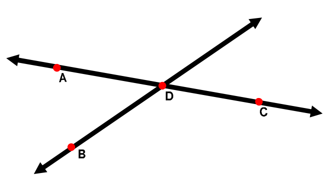


Answer

Jul 18-10:59 PM

6 At which point, or points, do the drawn lines intersect?

A A and D
B A and C
C D
D none



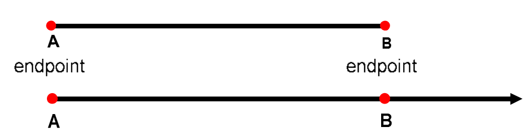
Answer

Jul 18-10:59 PM

Rays

A Ray is created by extending a line segment to infinity in just one direction. It has a point at one end, its endpoint, and extends to infinity at the other.

Below, the segment AB is extended to infinity, beyond Point B, to create Ray AB.



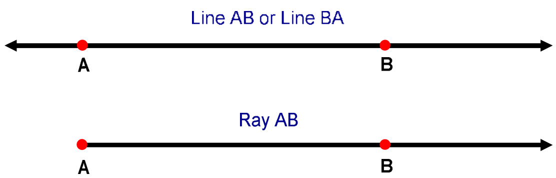
Points, Lines, & Planes

Naming Rays

When naming a ray the first letter is the point where the ray begins and the second is any other point on the ray.

The order of the letters matters for rays, while it doesn't for lines.

Why do you think the order of the letters matter for rays?

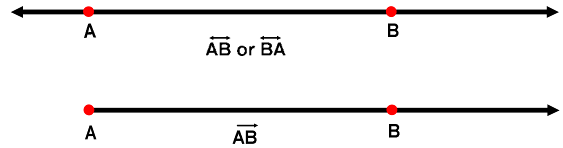


Points, Lines, & Planes

Naming Rays

Also, instead of the double-headed arrows which are used for lines, rays are indicated by a single-headed arrow.

The arrow points from the endpoint of the ray to infinity.

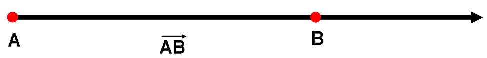


Points, Lines, & Planes

Naming Rays

Segment AB can be extended in either in either direction.

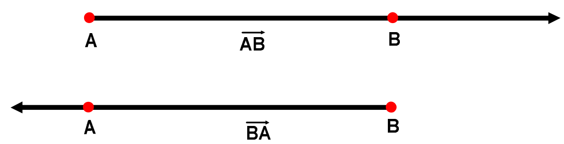
We can extend it at B to get ray AB.



Or, we can extend it at A to get Ray BA.

Points, Lines, & Planes

Naming Rays



Rays AB and BA are NOT the same.
What is the difference between them?

Points, Lines, & Planes

Opposite Rays

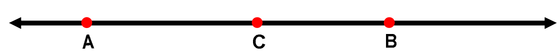
Opposite rays are defined as being two rays with a common endpoint that point in opposite directions and form a straight line.

Below, suppose point C is between points A and B.

Rays CA and CB are *opposite rays*.

Points, Lines, & Planes

Collinear Rays

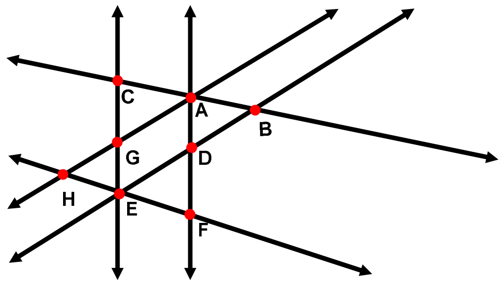


Recall: Since A, B, and C all lie on the same line, we know they are collinear points.

Similarly, rays are also called collinear if they lie on the same line.

Points, Lines, & Planes

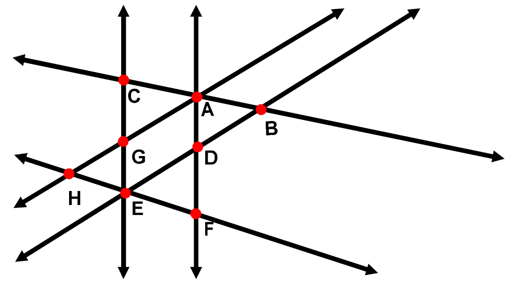
7 Name a point which is collinear with points G & H.



Answer

Aug 27-9:19 AM

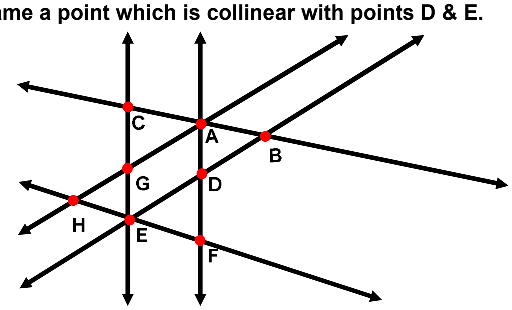
8 Name a point which is collinear with points D & A.



Answer

Aug 27-9:19 AM

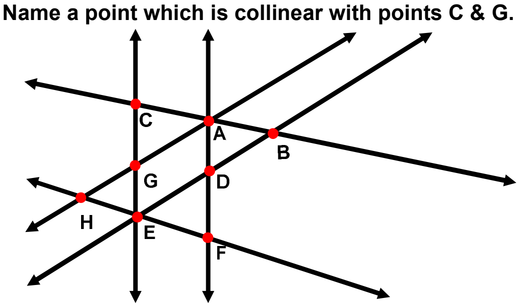
9 Name a point which is collinear with points D & E.



Answer

Aug 27-9:19 AM

10 Name a point which is collinear with points C & G.



Answer

Aug 27-9:19 AM

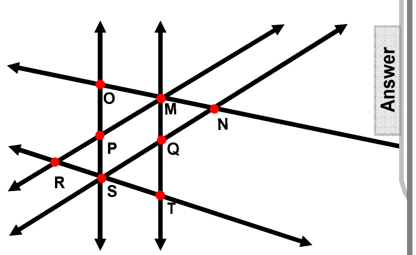
11 Name an opposite ray to Ray MN.

A Ray MQ

B Ray MO

C Ray RO

D Ray PR

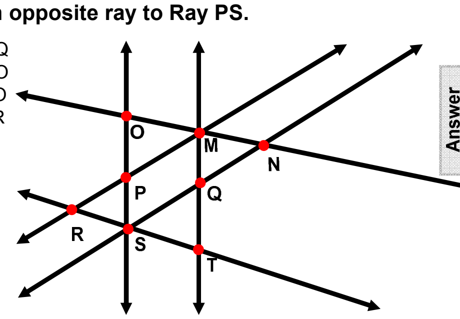


Answer

Aug 25-8:23 PM

12 Name an opposite ray to Ray PS.

A Ray MQ
B Ray MO
C Ray PO
D Ray PR

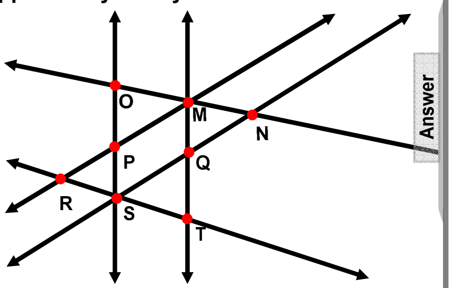


Answer

Jul 18-11:05 PM

13 Name an opposite ray to Ray PM.

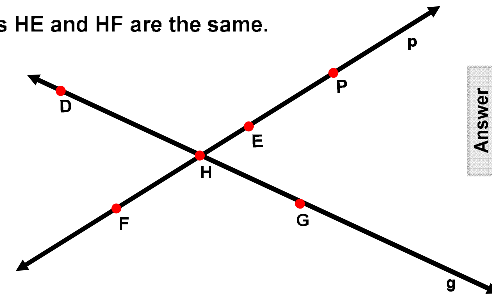
A Ray MQ
B Ray MO
C Ray PO
D Ray PR



Jul 18-11:05 PM

14 Rays HE and HF are the same.

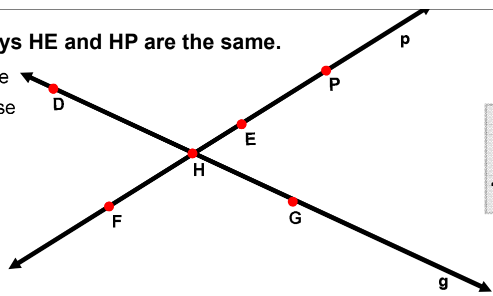
True
False



Points, Lines, & Planes

15 Rays HE and HP are the same.

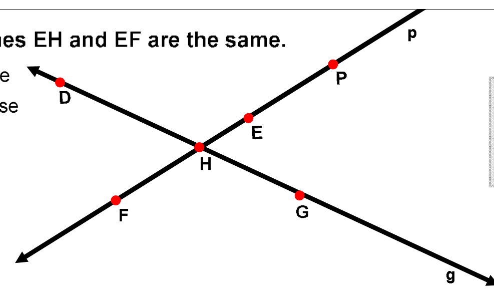
True
False



Points, Lines, & Planes

16 Lines EH and EF are the same.

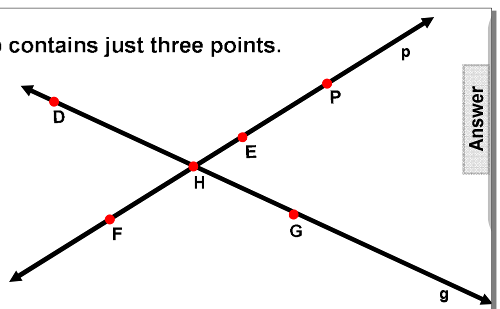
True
False



Points, Lines, & Planes

17 Line p contains just three points.

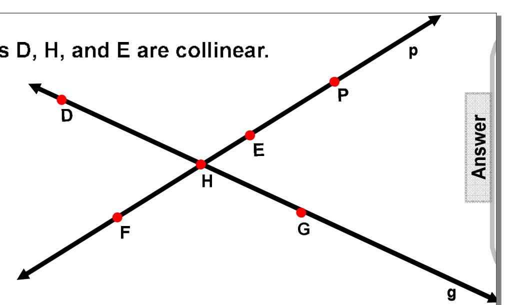
True
False



Points, Lines, & Planes

18 Points D, H, and E are collinear.

True
False

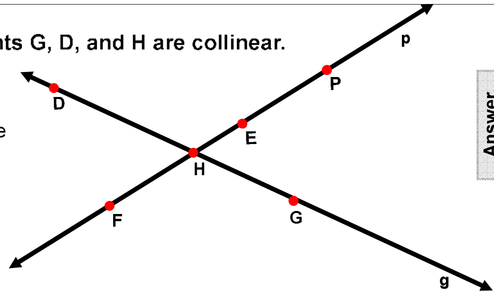


Points, Lines, & Planes

19 Points G, D, and H are collinear.

True

False



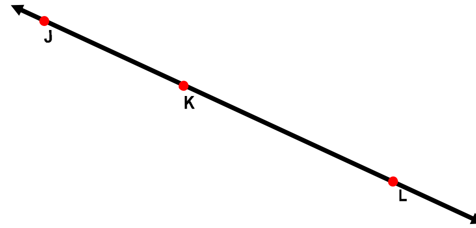
Answer

Points, Lines, & Planes

20 Are ray LJ and ray JL opposite rays?

Yes

No



Answer

Points, Lines, & Planes

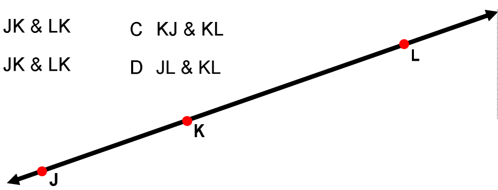
21 Which of the following are opposite rays?

A JK & LK

B JK & LK

C KJ & KL

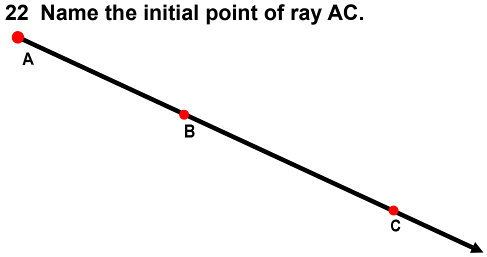
D JL & KL



Answer

Points, Lines, & Planes

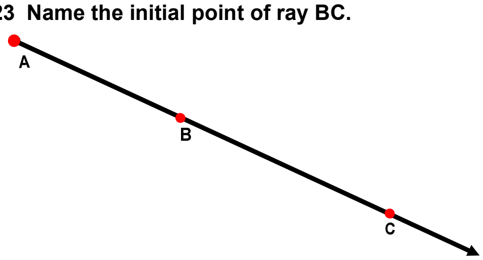
22 Name the initial point of ray AC.



Answer

Aug 27-10:02 AM

23 Name the initial point of ray BC.



Answer

Aug 27-10:02 AM

Planes

Definition 5: A surface such that if any two points on the surface are connected by a line, all points of the line are also on the surface.

A plane is a flat surface that has no thickness or height.

It can extend infinitely in the directions of its length and breadth/ width, just as the lines that lie on it may.

PLANES

Planes

Recall that points which fall on the same line are called collinear points.

With that in mind, what do you think points on the same plane are called?

Points, Lines, & Planes

Planes

Definition 6: The edges of a surface are lines.

Just as the ends of lines are points, the edges of planes are lines.

Points, Lines, & Planes

Planes

Definition 7: A plane surface is a surface which lies evenly with the straight lines on itself.

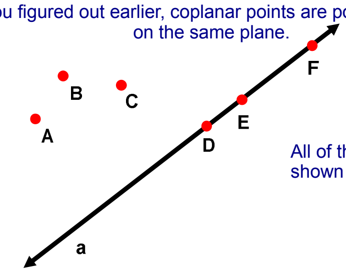
This indicates that the surface of the plane is flat so that lines on the plane will lie flat on it.

Planes can be horizontal, vertical or diagonal.

Points, Lines, & Planes

Coplanar Points and Lines

As you figured out earlier, coplanar points are points which fall on the same plane.



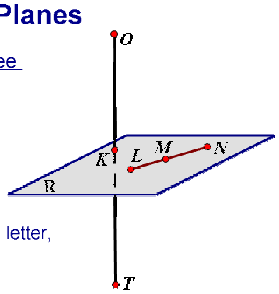
All of the lines and points shown here are coplanar.

Points, Lines, & Planes

Naming Planes

Planes can be named by <u>any three points that are not collinear</u>.

What are 3 examples of how this plane can be named?



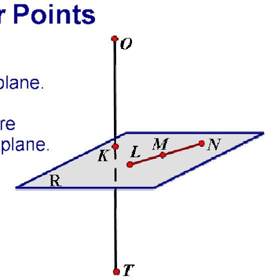
Also, it can be named by the single letter, "Plane R."

Points, Lines, & Planes

Coplanar Points

Coplanar points lie on the same plane.

In this case, Points K, M, and L are coplanar and lie on the indicated plane.



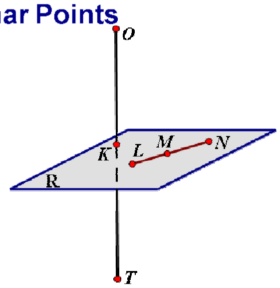
Points, Lines, & Planes

Coplanar Points

While points O, K, and L do not lie on the indicated plane, they are coplanar with one another.

Can you imagine a plane in which they are coplanar?

Can you draw it on the image? What could be a name for that plane?

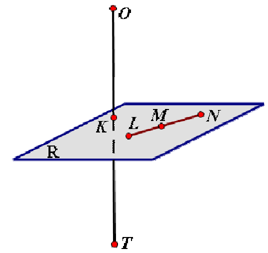


Points, Lines, & Planes

Coplanar Points

Is it possible for any three points to not be coplanar with one another?

Try and find 3 points on this diagram which are not coplanar.



Answer

Points, Lines, & Planes

24 How many points are needed to define a plane?

Answer

Jul 18-10:49 PM

25 Can there be three points which are not coplanar on any plane?

Yes
No

Answer

Jul 18-10:53 PM

26 Can there be four points which are not coplaner on any plane?

Yes
No

Answer

Jul 18-10:53 PM

Intersecting Planes

What would the intersection of two planes look like?

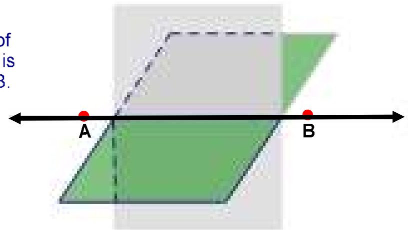
Hint: the walls and ceiling of this room could represent planes.

Answer

Points, Lines, & Planes

Intersecting Planes

The intersection of these two planes is shown by Line AB.

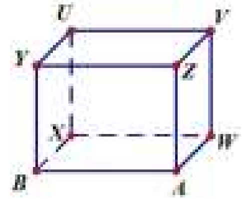


Try to imagine how two planes could intersect at a point, or in any other way than a line.

Points, Lines, & Planes

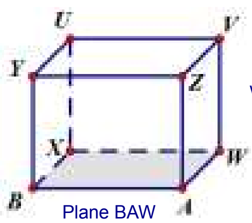
Various Planes Defined by 3 points

Imagine or shade in Plane BAW in the below drawing.



Points, Lines, & Planes

Various Planes Defined by 3 points

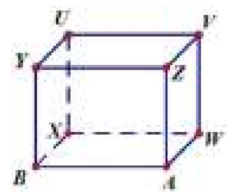


What are the 3 other ways you can name this same plane?

Points, Lines, & Planes

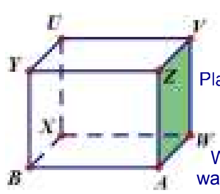
Various Planes Defined by 3 points

Imagine or shade in Plane AZW in the below drawing.



Points, Lines, & Planes

Various Planes Defined by 3 points



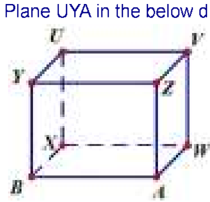
Plane AZW

What are the 3 other ways you can name this same plane?

Points, Lines, & Planes

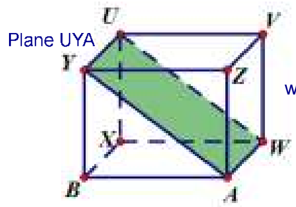
Various Planes Defined by 3 points

Draw Plane UYA in the below drawing.



Points, Lines, & Planes

Various Planes Defined by 3 points

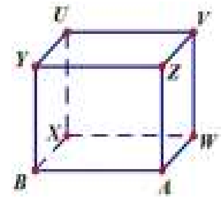


What are the 3 other ways you can name this same plane?

Points, Lines, & Planes

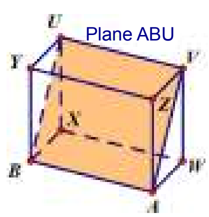
Various Planes Defined by 3 points

Imagine or draw Plane ABU in the below drawing.



Points, Lines, & Planes

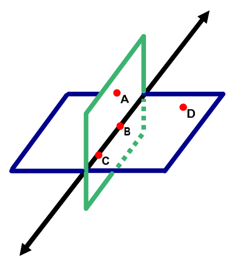
Various Planes Defined by 3 points



What are the 3 other ways you can name this same plane?

Points, Lines, & Planes

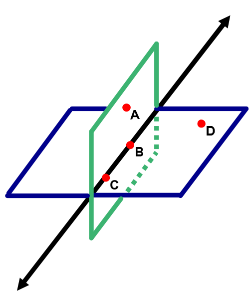
27 Name the point that is not in plane ABC.



Answer

Aug 27-10:23 AM

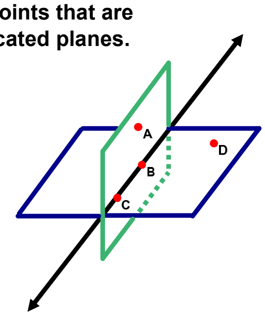
28 Name the point that is not in plane DBC.



Answer

Aug 27-10:23 AM

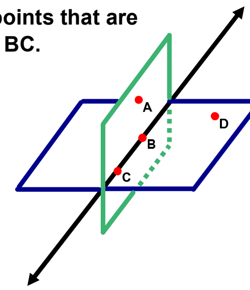
29 Name two points that are in both indicated planes.



Answer

Aug 27-10:25 AM

30 Name two points that are not on Line BC.



Answer

Aug 27-10:25 AM

31 Line BC does not contain point R. Are points R, B, and C collinear? Draw the situation if it helps.

Yes

No

Answer

Points, Lines, & Planes

32 Plane LMN does not contain point P. Are points P, M, and N coplanar?

Yes

No

Answer

Points, Lines, & Planes

33 Plane QRS contains line QV. Are points Q, R, S, and V coplanar? (Draw a picture)

Yes

No

Answer

Points, Lines, & Planes

34 Plane JKL does not contain line JN. Are points J, K, L, and N coplanar?

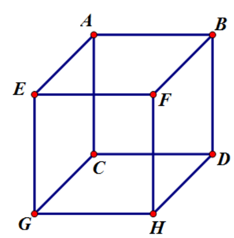
Yes

No

Answer

Points, Lines, & Planes

35 Line BA and line DB intersect at Point ____.



Answer

Aug 27-10:31 AM

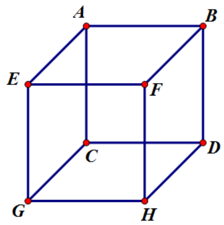
36 **Which group of points are noncoplanar with points A, B, and F on the cube below.**

A E, F, B, A

B A, C, G, E

C D, H, G, C

D F, E, G, H



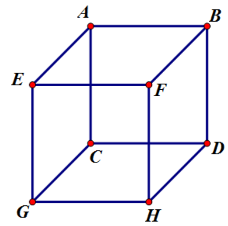
Answer

Points, Lines, & Planes

37 **Are lines EF and CD coplanar on the cube below?**

Yes

No



Answer

Points, Lines, & Planes

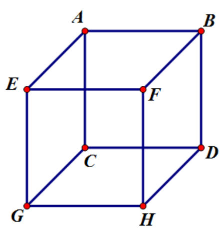
38 **Plane ABC and plane DCG intersect at _____?**

A C

B line DC

C Line CG

D they don't intersect



Answer

Points, Lines, & Planes

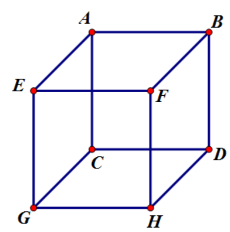
39 **Planes ABC, GCD, and EGC intersect at _____?**

A line GC

B point C

C point A

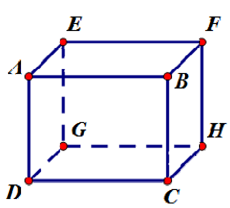
D line AC



Answer

Points, Lines, & Planes

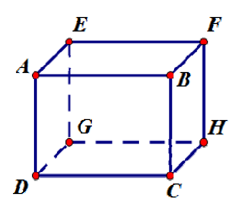
40 **Name another point that is in the same plane as points E, G, and H.**



Answer

Aug 27-10:36 AM

41 **Name a point that is coplanar with points E, F, and C.**



Answer

Aug 27-10:36 AM

42 Intersecting lines are __________ coplanar.

A Always

B Sometimes

C Never

Answer

Points, Lines, & Planes

43 Two planes ____________ intersect at exactly one point.

A Always

B Sometimes

C Never

Answer

Points, Lines, & Planes

44 A plane can __________ be drawn so that any three points are coplaner

A Always

B Sometimes

C Never

Answer

Points, Lines, & Planes

45 A plane containing two points of a line __________ contains the entire line.

A Always

B Sometimes

C Never

Answer

Points, Lines, & Planes

46 Four points are ____________ noncoplanar.

A Always

B Sometimes

C Never

Answer

Points, Lines, & Planes

47 Two lines ________________ meet at more than one point.

A Always

B Sometimes

C Never

Answer

Points, Lines, & Planes

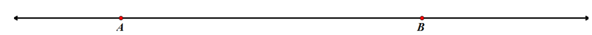
Line Segments

Return to Table of Contents

Line Segments

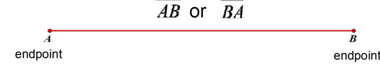
Line Segments

$\overline{AB}$ or $\overline{BA}$



Line segments are portions of a line.

$\overline{AB}$ or $\overline{BA}$



$\overline{AB}$ is read "*segment* AB"

Line Segment $\overline{AB}$ or $\overline{BA}$ are different names for the same segment. It consists of the endpoints A and B and all the points on the line between them.

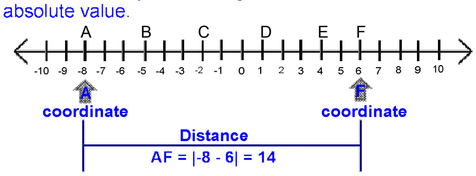
Line Segments

Ruler Postulate

On a number line, every point can be paired with a number and every number can be paired with a point.

Coordinates indicate the point's position on the number line.

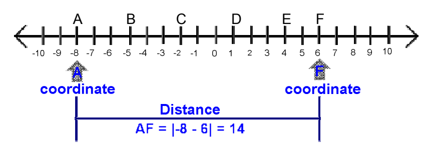
The symbol AF stands for the length of $\overline{AF}$. This distance from A to F can be found by subtracting the two coordinates and taking the absolute value.



Line Segments

Why did we take the Absolute Value when calculating distance?

In our previous slide, we were seeking the distance between two points. Distance is a physical quantity that can be measured - distances cannot be negative.



When you take the absolute value between two numbers, the order in which you subtract the two numbers <u>does not matter.</u>

Line Segments

Definition: Congruence

Equal in size and shape. Two objects are congruent if they have the same dimensions and shape.

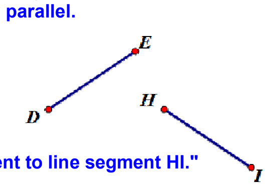
Roughly, 'congruent' means 'equal', but it has a precise meaning that you should understand completely when you consider complex shapes.

Line Segments

Congruent Segments

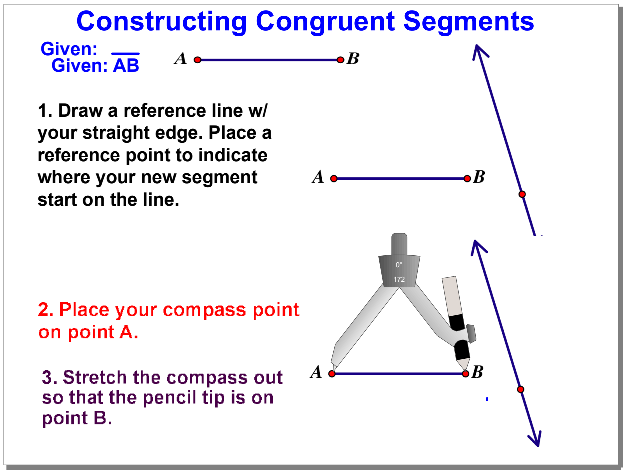
Line Segments are congruent if they have the same length. Congruent lines can be at any angle or orientation on the plane; they do not need to be parallel.

$\overline{DE} \cong \overline{HI}$

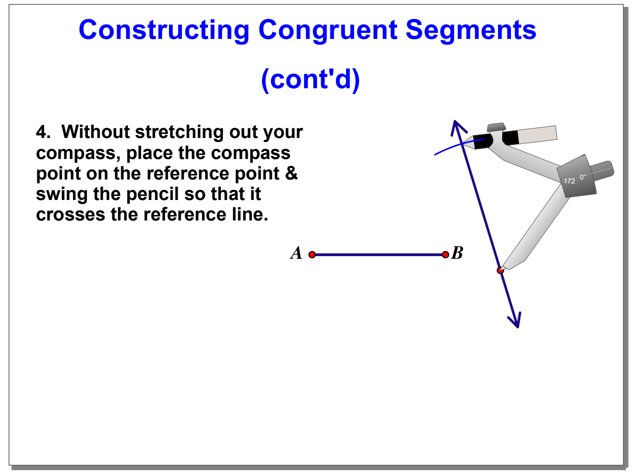


Read as:
"The line segment DE is congruent to line segment HI."

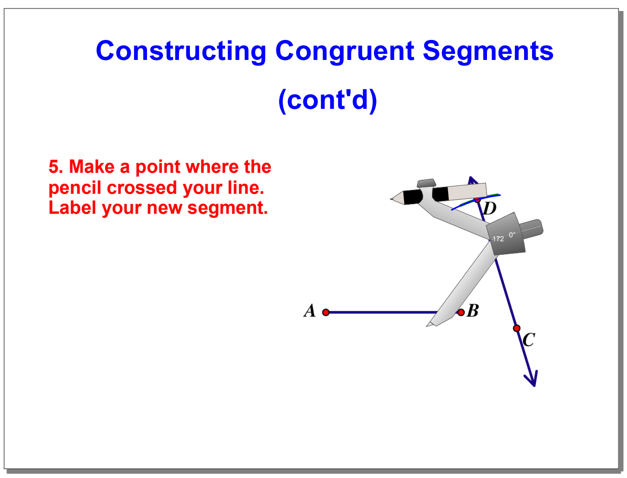
Line Segments



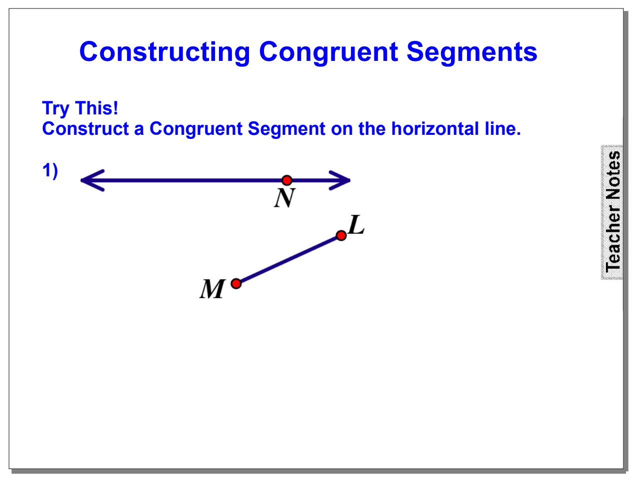
Line Segments



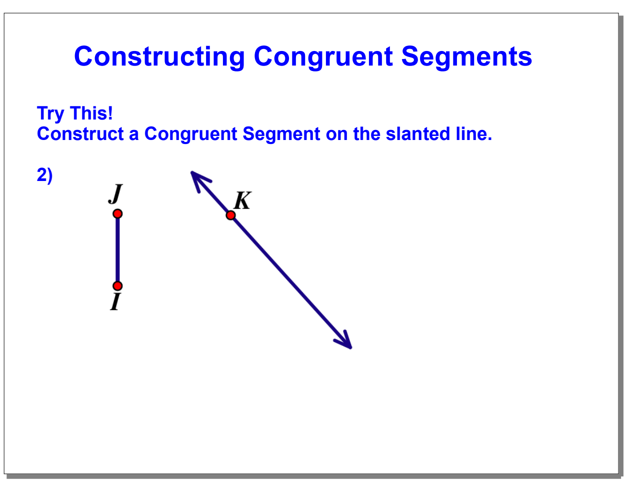
Line Segments



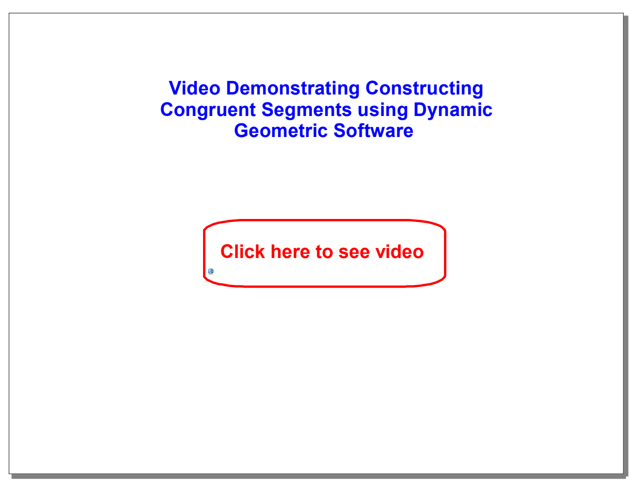
Jul 19-9:30 AM



Line Segments



Line Segments

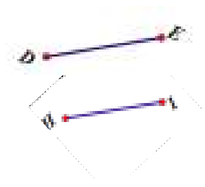


Video

Definition: Parallel Lines

Lines are parallel if they lie in the same plane, and are the same distance apart over their entire length.

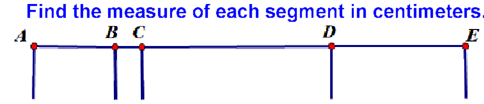
That is, they do not intersect.



Line Segments

Example

Find the measure of each segment in centimeters.



cm

a. $\overline{CE}$ = click

b. $\overline{AB}$ = click

Line Segments

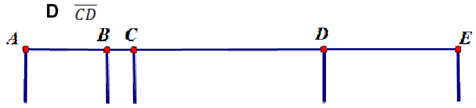
48 Find a segment that is 4 cm long

A $\overline{DA}$

B $\overline{BD}$

C $\overline{DE}$

D $\overline{CD}$



cm

Answer

Line Segments

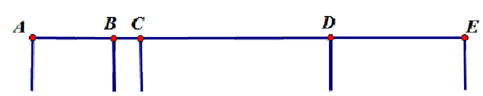
49 Find a segment that is 6.5 cm long

A $\overline{DA}$

B $\overline{BE}$

C $\overline{DE}$

D $\overline{CD}$



cm

Answer

Line Segments

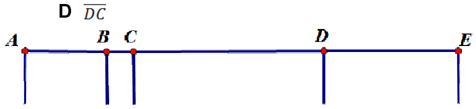
50 Find a segment that is 3.5 cm long

A $\overline{AC}$

B $\overline{BE}$

C $\overline{BD}$

D $\overline{DC}$



cm

Answer

Line Segments

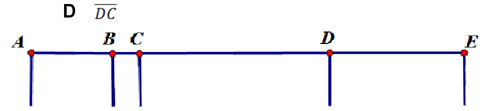
51 Find a segment that is 2 cm long

A $\overline{DE}$

B $\overline{CA}$

C $\overline{BD}$

D $\overline{DC}$



cm

Answer

Line Segments

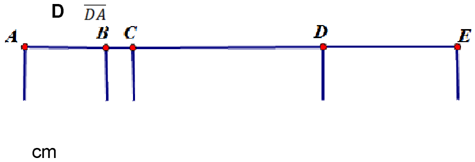
52 Find a segment that is 5.5 cm long

A $\overline{CE}$

B $\overline{EB}$

C $\overline{DB}$

D $\overline{DA}$



Answer

Line Segments

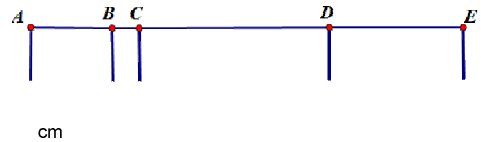
53 If point F was placed at 3.5 cm on the ruler, how far from point E would it be?

A 5 cm

B 4 cm

C 3.5 cm

D 4.5 cm



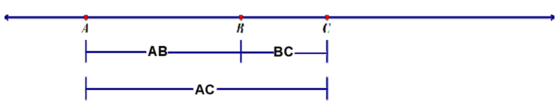
Answer

Line Segments

Segment Addition Postulate

If B is between A and C, Then $\overline{AB} + \overline{BC} = \overline{AC}$.

Or, If $\overline{AB} + \overline{BC} = \overline{AC}$, then B is between A and C.



Simply said, if you take one part of a segment (AB), and add it to another part of the segment (BC), you get the entire segment.

The whole is equal to the sum of its parts.

Line Segments

Example

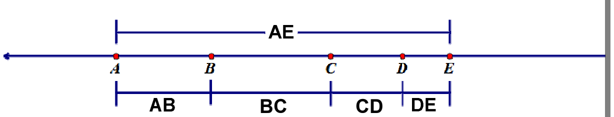
The segment addition postulate works for three or more segments if all the segments lie on the same line (i.e. all the points are collinear).

In the diagram, $\overline{AE} = 27$, $\overline{AB} = \overline{CD}$, $\overline{DE} = 5$, and $\overline{BC} = 6$

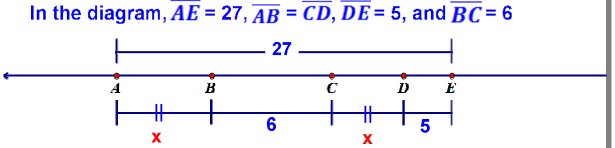
Find $\overline{CD}$ and $\overline{BE}$

Line Segments

Start by filling in the information you are given



In the diagram, $\overline{AE} = 27$, $\overline{AB} = \overline{CD}$, $\overline{DE} = 5$, and $\overline{BC} = 6$



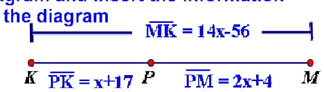
Can you finish the rest? $\overline{CD}$ click click click $\overline{BE}$ click

Line Segments

Example

K, M, and P are collinear with P between K and M. $\overline{PM} = 2x + 4$, $\overline{MK} = 14x - 56$, and $\overline{PK} = x + 17$. Solve for x

1) Draw a diagram and insert the information given into the diagram



2) From the segment addition postulate, we know that $\overline{KP} + \overline{PM} = \overline{MK}$ (the parts equal the whole)

3) Solve for x

$(x + 17) + (2x + 4) = 14x - 56$

Answer

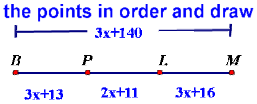
Line Segments

Example

P, B, L, and M are collinear and are in the following order:
a) P is between B and M
b) L is between M and P

Draw a diagram and solve for x, given:
$\overline{ML}$ = 3x +16, $\overline{PL}$ = 2x +11, $\overline{BM}$ = 3x +140, and $\overline{PB}$ = 3x + 13

1) First, arrange the points in order and draw a diagram
a) BPM
b) BPLM



2) Segment addition postulate gives
3x+13 + 2x+11 + 3x+16 = 3x+140

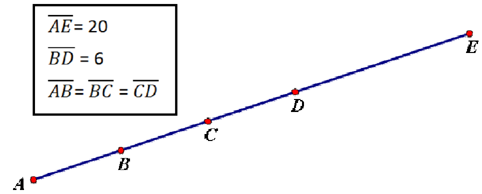
3) Combine like terms and isolate/solve for the variable x

Answer

Line Segments

For next six questions we are given the following information about the collinear points:

$\overline{AE}$ = 20
$\overline{BD}$ = 6
$\overline{AB}$ = $\overline{BC}$ = $\overline{CD}$



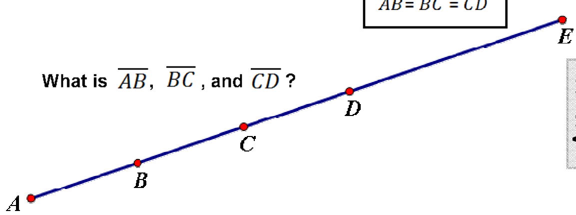
Hint: always start these problems by placing the information you have into the diagram.

Line Segments

54 We are given the following information about the collinear points:

$\overline{AE}$ = 20
$\overline{BD}$ = 6
$\overline{AB}$ = $\overline{BC}$ = $\overline{CD}$

What is $\overline{AB}$, $\overline{BC}$, and $\overline{CD}$?



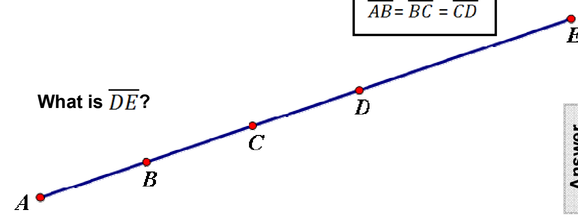
Answer

Line Segments

55 We are given the following information about the collinear points:

$\overline{AE}$ = 20
$\overline{BD}$ = 6
$\overline{AB}$ = $\overline{BC}$ = $\overline{CD}$

What is $\overline{DE}$?



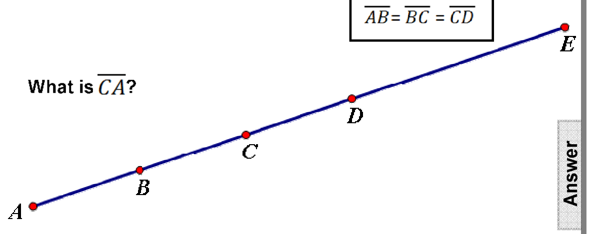
Answer

Line Segments

56 We are given the following information about the collinear points:

$\overline{AE}$ = 20
$\overline{BD}$ = 6
$\overline{AB}$ = $\overline{BC}$ = $\overline{CD}$

What is $\overline{CA}$?



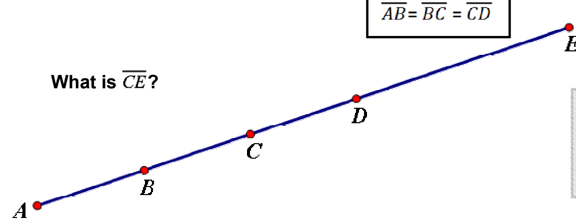
Answer

Line Segments

57 We are given the following information about the collinear points:

$\overline{AE}$ = 20
$\overline{BD}$ = 6
$\overline{AB}$ = $\overline{BC}$ = $\overline{CD}$

What is $\overline{CE}$?



Answer

Line Segments

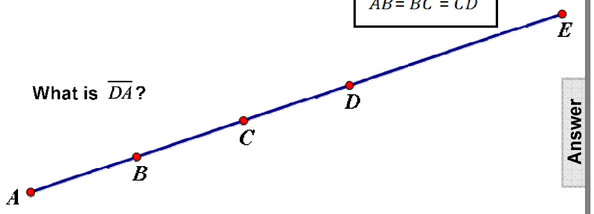
58 We are given the following information about the collinear points:

$\overline{AE} = 20$

$\overline{BD} = 6$

$\overline{AB} = \overline{BC} = \overline{CD}$

What is $\overline{DA}$?



Answer

Line Segments

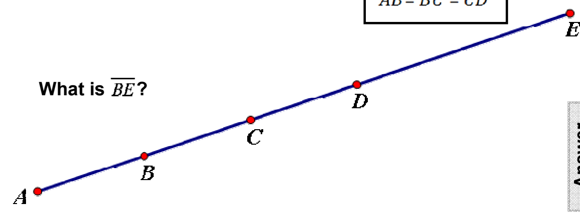
59 We are given the following information about the collinear points:

$\overline{AE} = 20$

$\overline{BD} = 6$

$\overline{AB} = \overline{BC} = \overline{CD}$

What is $\overline{BE}$?



Answer

Line Segments

60 X, B, and Y are collinear points, with Y between B and X. Draw a diagram and solve for x, given:

BX = 6x + 151

XY = 15x - 7

BY = x - 12

Answer

Line Segments

61 Q, X, and R are collinear points, with X between R and Q. Draw a diagram and solve for x, given:

XQ = 15x + 10

RQ = 2x + 131

XR = 7x +1

Answer

Line Segments

62 B, K, and V are collinear points, with K between V and B. Draw a diagram and solve for x, given:

KB = 5x

BV = 15x + 125

KV = 4x +149

Answer

Line Segments

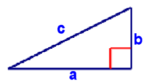
The Pythagorean Theorem

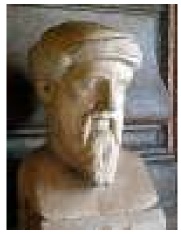
Return to Table of Contents

The Pythagorean Theorem

Pythagorean Theorem

$c^2 = a^2 + b^2$



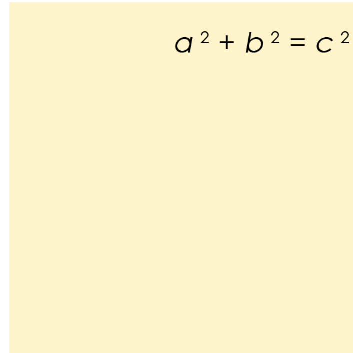


Pythagoras was a philosopher, theologian, scientist and mathematician born on the island of Samos in ancient Greece and lived from c. 570–c. 495 BC.

The theorem that in a right triangle the area of the square on the hypotenuse (the side opposite the right angle) is equal to the sum of the areas of the squares of the other two sides.

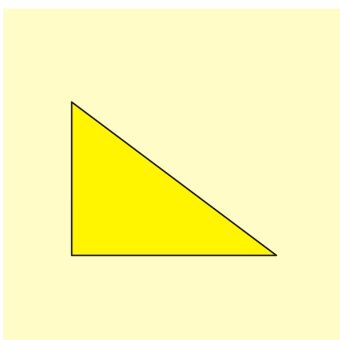
The Pythagorean Theorem

Pythagorean Theory Visual Proof 1



The Pythagorean Theorem

Pythagorean Theory Visual Proof 2

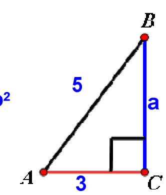


The Pythagorean Theorem

Using the Pythagorean Theorem

In the Pythagorean Theorem, c always stands for the longest side. In a right triangle, the longest side is called the hypotenuse. The hypotenuse is the side opposite the right angle.

$c^2 = a^2 + b^2$



$(5)^2 = a^2 + (3)^2$

$25 = a^2 + 9$

$-9 \qquad -9$

$16 = a^2$

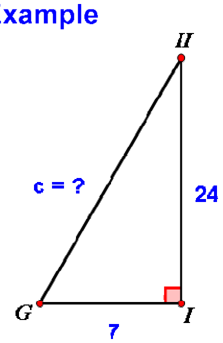
$\sqrt{16} = a$

$4 = a$

You will use the Pythagorean Theorem often.

The Pythagorean Theorem

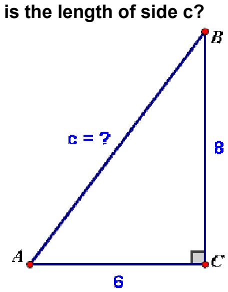
Example



Answer

The Pythagorean Theorem

63 What is the length of side c?

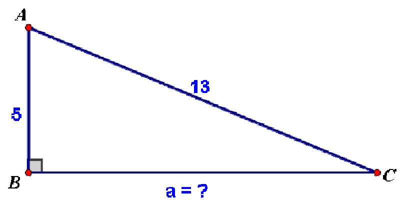


Answer

The longest side of a triangle is called the ___________

The Pythagorean Theorem

64 What is the length of side a?



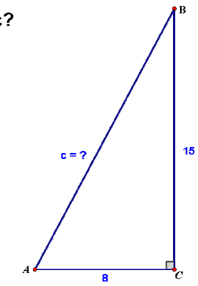
Hint:

click to reveal

Answer

The Pythagorean Theorem

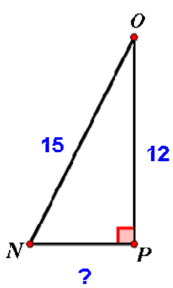
65 What is the length of c?



Answer

The Pythagorean Theorem

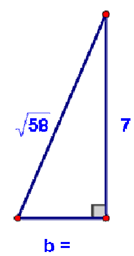
66 What is the length of the missing side?



Answer

The Pythagorean Theorem

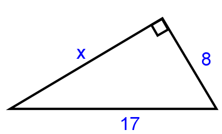
67 What is the length of side b?



Answer

The Pythagorean Theorem

68 What is the measure of x?



Answer

The Pythagorean Theorem

Pythagorean Triples

are three positive **integers** for side lengths that satisfy $a^2 + b^2 = c^2$

(3 , 4 , 5) (5, 12, 13) (6, 8, 10)(7, 24, 25)
(8, 15, 17) (9, 40, 41) (10, 24, 26) (11, 60, 61)
(12, 35, 37) (13, 84, 85) etc.

There are many more.

Remembering some of these combinations may save you some time

The Pythagorean Theorem

Unit 1 Notes

69 A triangle has sides 30, 40 , and 50, is it a right triangle?

Yes

No

Answer

The Pythagorean Theorem

70 A triangle has sides 9, 12 , and 15, is it a right triangle?

Yes

No

Answer

The Pythagorean Theorem

71 A triangle has sides $\sqrt{3}$, 2 , and $\sqrt{5}$, is it a right triangle?

Yes

No

Answer

The Pythagorean Theorem

Distance Between Points

Return to Table of Contents

Distance

Computing distance

Computing the distance between two points in the plane is an application of the Pythagorean Theorem for right triangles.

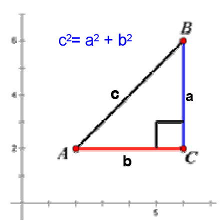
Computing distances between points in the plane is equivalent to finding the length of the hypotenuse of a right triangle.

The Pythagorean Theorem is true for all right triangles. If we know the lengths of two sides of a right triangle then we know the length of the third side.

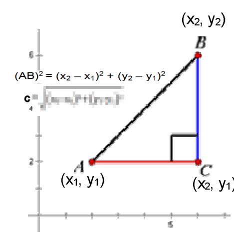
Distance

Relationship between the Pythagorean Theorem & Distance Formula

The Pythagorean Theorem states a relationship among the sides of a right triangle.



The distance formula calculates the distance using the points' coordinates.



Distance

The Distance Formula

The distance 'd' between any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the formula:

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

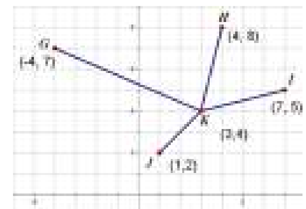
The distance between two points, whether on a line or in a coordinate plane, is computed using the distance formula.

Note: recall that all coordinates are (x-coordinate, y-coordinate).

Distance

Example

Calculate the distance from Point K to Point I
(x_1, y_1) (x_2, y_2)



Label the points - it does not matter which one you label point 1 and point 2.
Your answer will be the same.

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Plug the coordinates into the distance formula

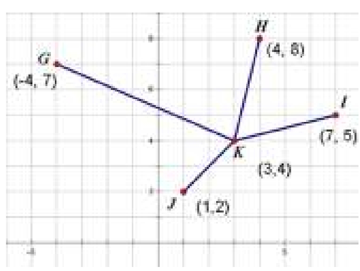
$$KI = \sqrt{(7-3)^2+(5-4)^2}$$

Answer

Distance

72 Calculate the distance from Point J to Point K

A $\sqrt{10}$
B $8\sqrt{2}$
C $2\sqrt{5}$
D $2\sqrt{2}$

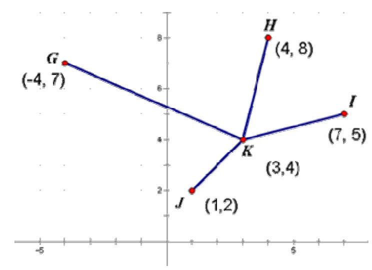


Answer

Distance

73 Calculate the distance from H to K

A $\sqrt{15}$
B $\sqrt{25}$
C 5
D $\sqrt{17}$

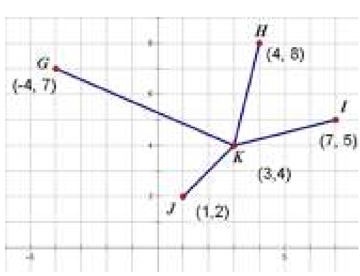


Answer

Distance

74 Calculate the distance from Point G to Point K

A $2\sqrt{10}$
B $2\sqrt{30}$
C $\sqrt{53}$
D $\sqrt{58}$

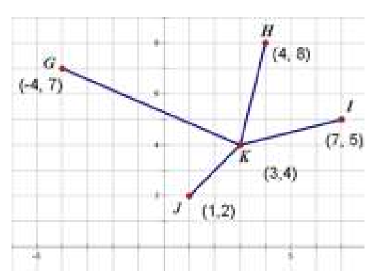


Answer

Distance

75 Calculate the distance from Point I to Point H

A $\sqrt{18}$
B 9
C $2\sqrt{3}$
D $3\sqrt{2}$



Answer

Distance

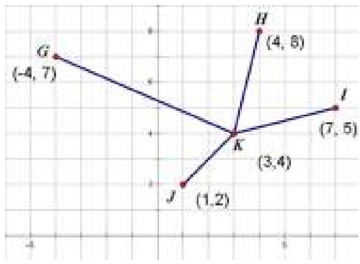
76 Calculate the distance from Point G to Point H

A $\sqrt{17}$

B $\sqrt{65}$

C $\sqrt{256}$

D $\sqrt{257}$



Answer

Distance

Area of Figures in the Coordinate Plane

Return to Table of Contents

Area Coordinate Plane

Calculating Area of Figures in the Coordinate Plane

Area Formulas:

Area of a Triangle: A = $\frac{1}{2}$bh

Area of a Rectangle: A = lw

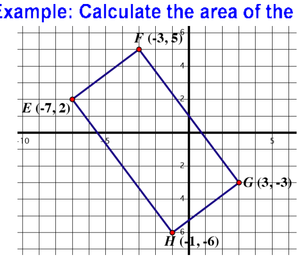
Steps to calculate the area:

1) Calculate the desired distances using the distance formula
   > Ex: base & height in a triangle
   > Ex: length & width in a rectangle

2) Calculate the area of the shape

Area Coordinate Plane

Example: Calculate the area of the rectangle.



Steps to calculate the area:

1) Calculate the desired distances using the distance formula
   > ℓ = $\overline{FG}$ & w = $\overline{EF}$

$d_{FG} = \sqrt{(3--3)^2+(-3-5)^2}$

$= \sqrt{(3+3)^2+(-3-5)^2}$

$= \sqrt{6^2+(-8)^2} = \sqrt{36+64}$

$= \sqrt{100} = 10 = \ell$

$d_{EF} = \sqrt{(-3--7)^2+(5-2)^2}$

$= \sqrt{(-3+7)^2+(5-2)^2}$

$= \sqrt{4^2+3^2} = \sqrt{16+9}$

$= \sqrt{25} = 5$ = w

2) Calculate the area of the shape

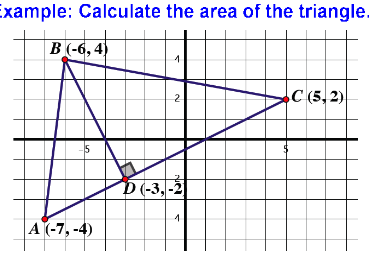
$\ell = 10$ & $w = 5$

$A = 10(5)$

$A = 50 units^2$

Area Coordinate Plane

Example: Calculate the area of the triangle.



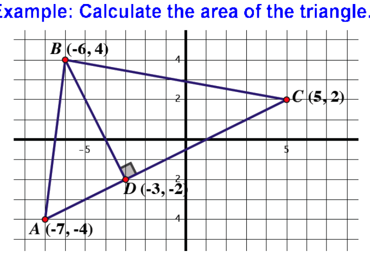
Steps to calculate the area:

1) Calculate the desired distances using the distance formula
   > b = $\overline{AC}$ & h = $\overline{BD}$

Answer

Area Coordinate Plane

Example: Calculate the area of the triangle.



2) Calculate the area of the shape

Answer

Area Coordinate Plane

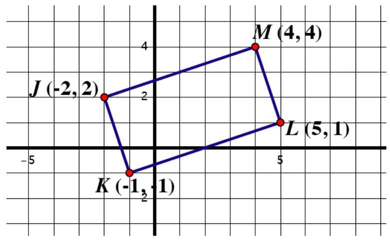
77 **Calculate the area of the rectangle.**

A $2\sqrt{10}$

B $10\sqrt{2}$

C 10

D 20



Answer

Area Coordinate Plane

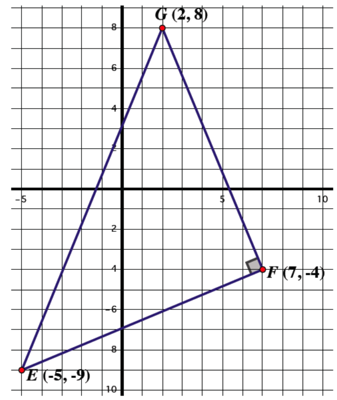
78 **Calculate the area of the triangle.**

A 75

B 144

C 84.5

D 169



Answer

Area Coordinate Plane

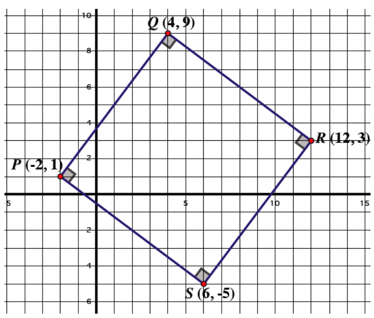
79 **Calculate the area of the rectangle.**

A 10,000

B 100

C 50

D $8\sqrt{5}$



Answer

Area Coordinate Plane

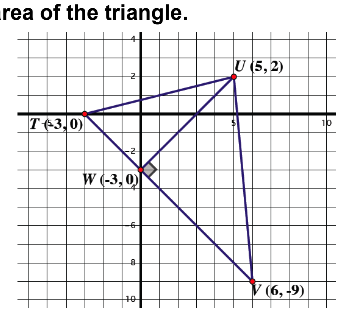
80 **Calculate the area of the triangle.**

A $6\sqrt{5}$

B $15\sqrt{5}$

C 22.5

D 45



Answer

Area Coordinate Plane

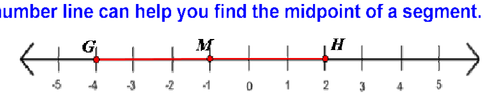
Midpoint Formula

Return to Table of Contents

Midpoints

Midpoint of a line segment

A number line can help you find the midpoint of a segment.



The midpoint of GH, marked by point M, is -1.

Here's how you calculate it using the endpoint coordinates.

Take the coordinates of the endpoint G and H, add them together, and divide by two.

$$\frac{-4+2}{2} = \frac{-2}{2} = -1$$

Midpoints

Midpoint Formula Theorem

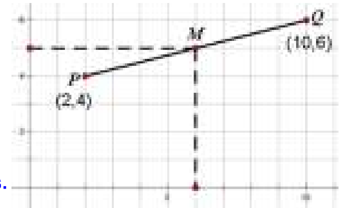
The midpoint of a segment joining points with coordinates (x_1, y_1) and (x_2, y_2) is the point with coordinates

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Midpoints

Calculating Midpoints in a Cartesian Plane

Segment PQ contain the points (2, 4) and (10, 6). The midpoint M of $\overline{PQ}$ is the point halfway between P and Q.



Just as before, we find the average of the coordinates.

$$x = \frac{2+10}{2} = \frac{12}{2} = 6$$

$$y = \frac{4+6}{2} = \frac{10}{2} = 5$$

$$\left(\frac{12}{2}, \frac{10}{2}\right)$$

Remember that points are written with the x-coordinate first. (x, y)

The coordinates of M, the midpoint of PQ, are (6, 5)

Midpoints

81 Find the midpoint coordinates (x,y) of the segment connecting points A(1,2) and B(5,6)

A (4, 3)
B (3, 4)
C (6, 8)
D (2.5, 3)

Answer

Hint:

click

Midpoints

82 Find the midpoint coordinates (x,y) of the segment connecting the points A(-2,5) and B(4, -3)

A (-1, -1)
B (-3, -8)
C (-8, -3)
D (1, 1)

Answer

Midpoints

83 Find the coordinates of the midpoint (x, y) of the segment with endpoints R(-4, 6) and Q(2, -8)

A (-1, 1)
B (1, 1)
C (-1, -1)
D (1, -1)

Answer

Midpoints

84 Find the coordinates (x, y) of the midpoint of the segment with endpoints B(-1, 3) and C(-7, 9)

A (-3, 3)
B (6, -4)
C (-4, 6)
D (4, 6)

Answer

Midpoints

85 Find the midpoint (x, y) of the line segment between A(-1, 3) and B(2,2)

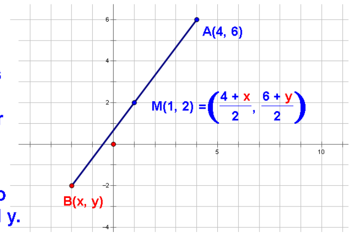
A (3/2, 5/2)
B (1/2, 5/2)
C (1/2, 3)
D (3, 1/2)

Answer

Midpoints

Example: Finding the coordinates of an endpoint of an segment

The midpoint of $\overline{AB}$ is M(1, 2). One endpoint is A(4, 6). Find the coordinates of the other endpoint B(x, y).



Use the midpoint formula to write equations using x and y.

$\frac{4+x}{2} = 1$ <--- x-coordinate of M

$2 = 4 + x$

$-2 = x$

$\frac{6+y}{2} = 2$ <--- y-coordinate of M

$4 = 6 + y$

$-2 = y$

Coordinates of endpoint B is (-2,-2)

Midpoints

86 Find the other endpoint of the segment with the endpoint (7,2) and midpoint (3,0)

A (-1, -2)
B (-2, -1)
C (4, 2)
D (2, 4)

Answer

Midpoints

87 Find the other endpoint of the segment with the endpoint (1, 4) and midpoint (5, -2)

A (11, -8)
B (9, 0)
C (9, -8)
D (3, 1)

Answer

Midpoints

Locus & Constructions

Return to Table of Contents

Locus
&
Constructions

Introduction to Locus

In mathematics, a locus is defined to be the set of points which satisfy a given condition.

Very often, we will set up a condition and solve for the locus of points which meet that condition.

That can be done algebraically, but it can also be done with the use of drawing equipment such as a straight edge and compass.

Locus
&
Constructions

Introduction to Locus

Definition: Locus is a set of points that all satisfy a certain condition, in this course, it is the set of points that are the same distance from something else.

Theorem: locus between two points
All points on the perpendicular bisector of a line segment connecting two points are equidistant from the two points.

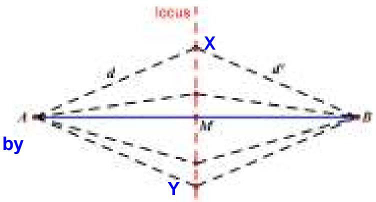
Theorem: locus from a given line
All points equidistant from a given line is a parallel line.

Theorem: locus between two lines
The locus of points equidistant from two given parallel lines is a parallel line midway between them.

Locus
&
Constructions

locus: equidistant from two points

The locus of points equidistant from two points, A and B, is the perpendicular bisector of the line segment determined by the two points.



The distance (d) from point A to the locus is equal to the distance (d') from Point B to the locus. The set of all these points forms the red line and is named the locus.

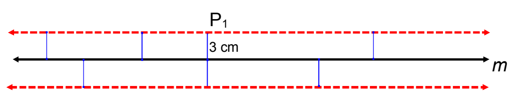
Point M is the midpoint of $\overline{AB}$. X is equidistant (d=d') from A and B. Y lies on the locus; it is also equidistant from A and B.

Locus
&
Constructions

Example:

What is the locus of points 3 cm from a given line?

Step 1: Find a point P_1 that is 3 cm from a given line.



Step 2: Find a second point P_2,third P_3, and so on that are 3 cm from a given line to establish a recognizable pattern.

What is the locus of points? Is there just one?

Sep 16-8:04 AM

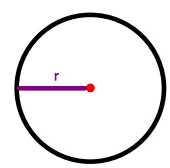
The Circle as a Locus

One important example of a locus is that the set of points which is equidistant from any one point is a circle.

The point from which they are equidistant is the center of the circle.

The distance from the center, is the radius, r, of the circle.

We will learn much more about circles later, but we need to learn a bit now so we can proceed with constructions.



Locus
&
Constructions

Euclid and Circles

Third Postulate: To describe a circle with any center and distance.

This postulate says that we can draw a circle of any radius, placing its center where we choose.

Locus
&
Constructions

Euclid and Circles

Definition 15: A circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure equal one another.

The straight lines referenced here are the radii which are of equal length from the center to the points on the circle

Locus
&
Constructions

Euclid and Circles

Definition 16: And the point is called the center of the circle.

This says that the point that is equidistant from all of the points on a circle is the center of the circle.

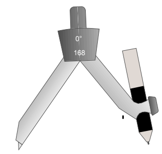
Locus
&
Constructions

Introduction to Constructions

In addition to a pencil, we will be using two tools to construct geometric figures a straight edge and a compass.

A straight edge allows us to draw a straight line, which we are allowed to do between any two points.

A compass allows us to draw a circle.
Try the compass to the right.
You can use the pencil to rotate the compass

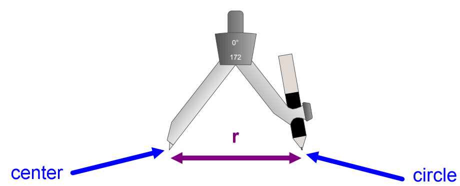


Locus
&
Constructions

Introduction to Constructions

The sharp point of a compass is placed at the center of the circle. The pencil then draws the circle.

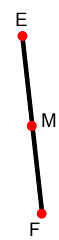
For constructions, we will just draw a small part of a circle, an arc. We do this to take advantage of the fact that every point on that arc is equidistant from the center.We can draw multiple arcs, if needed.



Locus
&
Constructions

Try this!

1) Create a circle using the segment below.

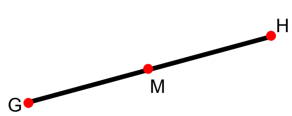


Teacher Notes

Aug 18-8:55 PM

Try this!

2) Create a circle using the segment below.



Aug 18-8:55 PM

Constructing Congruent Segments

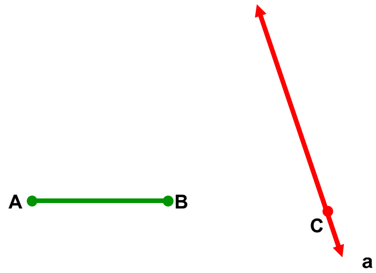
Let's use these tools to create a line segment CD which is congruent with the given line segment AB.

We will first do this with a straight edge and compass.

Line Segments

Constructing Congruent Segments

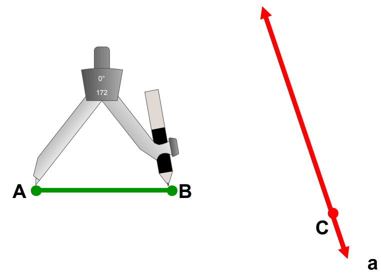
First, use your straight edge to draw a line which is longer than AB and includes Point C, such as Line a below.



Line Segments

Constructing Congruent Segments

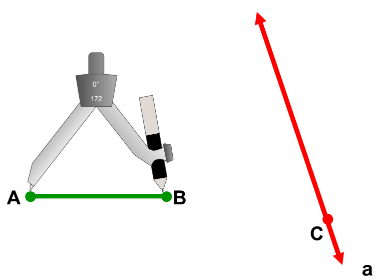
Then, stretch your compass between points A and B.



Line Segments

Constructing Congruent Segments

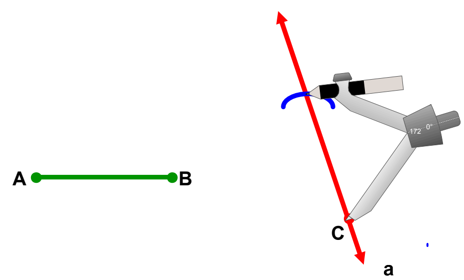
The compass can now be used to draw an arc with any center with the radius of AB, how do you think we could use that to create a congruent segment on Line a with C as an endpoint?



Line Segments

Constructing Congruent Segments

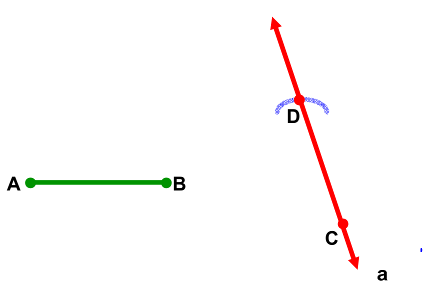
Then, keeping the compass unchanged, place its point at C and make an arc through line a. All the points on that arc are a distance AB from C. The point where the arc intersects the line, is that distance from C and on the line.



Line Segments

Constructing Congruent Segments

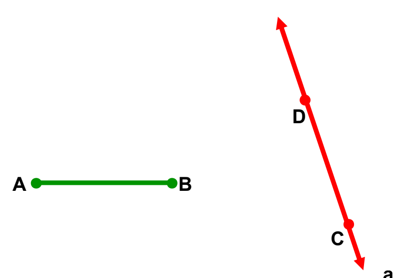
Then, draw Point D at the intersection of the arc and line a. Point D is on the line at a distance of AB from C.



Line Segments

Constructing Congruent Segments

Segment CD is congruent with segment AB, which was our objective.

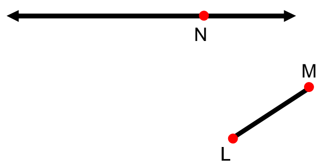


Line Segments

Unit 1 Notes

Try this!

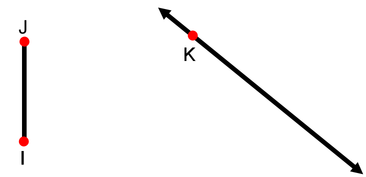
3)Construct a congruent segment on the given line.



Aug 18-8:50 PM

Try this!

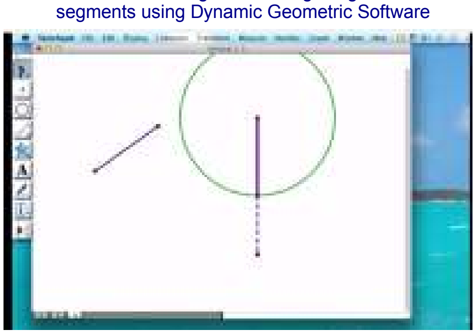
4) Construct a congruent segment on the given line.



Aug 18-8:53 PM

Dynamic Geometric Software

Click on the image below to watch a video demonstrating constructing congruent segments using Dynamic Geometric Software



Video

Bisect a Line Segment

We can find the midpoint of a line segment by constructing the locus between two points.

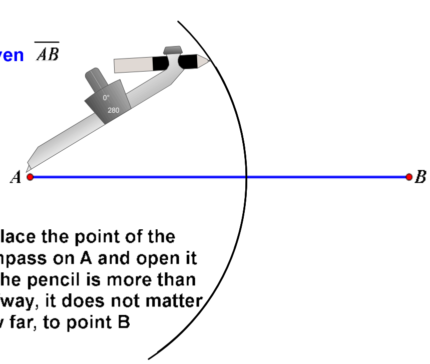
Given $\overline{AB}$

Locus
&
Constructions

Bisect a Line Segment

We can find the midpoint of a line segment by constructing the locus between two points.

Given $\overline{AB}$



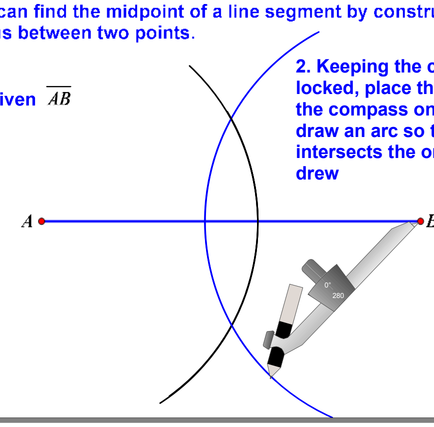
1. Place the point of the compass on A and open it so the pencil is more than halfway, it does not matter how far, to point B

Locus
&
Constructions

Bisect a Line Segment

We can find the midpoint of a line segment by constructing the locus between two points.

Given $\overline{AB}$



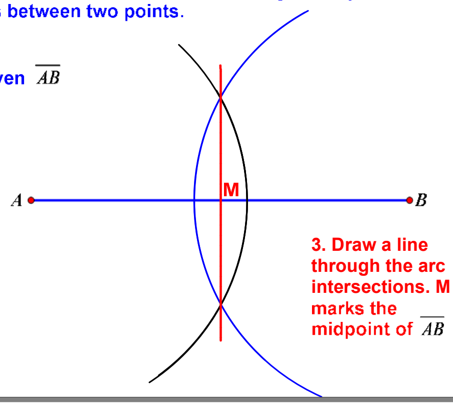
2. Keeping the compass locked, place the point of the compass on B and draw an arc so that it intersects the one you just drew

Locus
&
Constructions

Bisect a Line Segment

We can find the midpoint of a line segment by constructing the locus between two points.

Given $\overline{AB}$



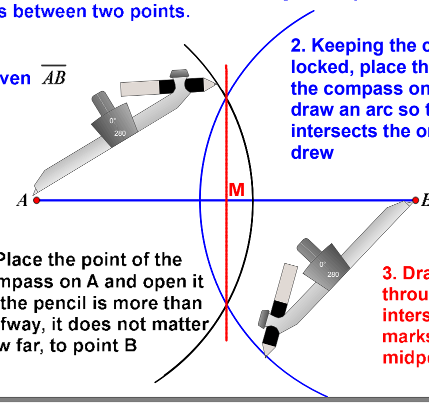
3. Draw a line through the arc intersections. M marks the midpoint of $\overline{AB}$

Locus
&
Constructions

Bisect a Line Segment

We can find the midpoint of a line segment by constructing the locus between two points.

Given $\overline{AB}$



2. Keeping the compass locked, place the point of the compass on B and draw an arc so that it intersects the one you just drew

1. Place the point of the compass on A and open it so the pencil is more than halfway, it does not matter how far, to point B

3. Draw a line through the arc intersections. M marks the midpoint of $\overline{AB}$

Locus
&
Constructions

Bisect a Line Segment

Try this!
Construct the midpoint of the segment below.

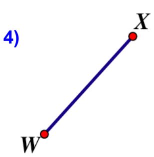
3)

Locus
&
Constructions

Bisect a Line Segment

Try this!
Construct the midpoint of the segment below.

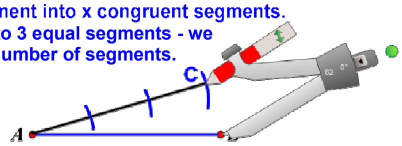
4)



Locus
&
Constructions

Constructions

Dividing a line segment into x congruent segments.
Let us divide AB into 3 equal segments - we could choose any number of segments.



1. From point A, draw a line segment at an angle to the given line, and about the same length. The exact length is not important.

2. Set the compass on A, and set its width to a bit less than 1/3 of the length of the new line.

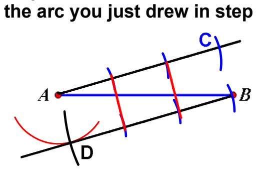
Step the compass along the line, marking off 3 arcs. Label the last C.

3. Set the compass width to CB.

Locus
&
Constructions

4. Using the compass set to CB, draw an arc below A

5. With the compass width set to AC, draw an arc from B intersecting the arc you just drew in step 4. Label this D.



6. Draw a line connecting B with D

7. Set the compass width back to AC and step along DB making 3 new arcs across the line

8. Draw lines connecting the arc along AC and BD. These lines intersect AB and divide it into 3 congruent segments.

Locus
&
Constructions

Example: Construction

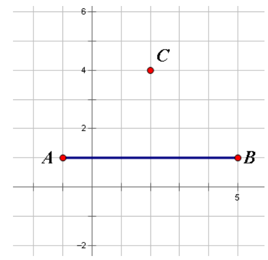
Divide the line segment into 3 congruent segments.

Locus
&
Constructions

88 Point C is on the locus between point A and point B

True

False



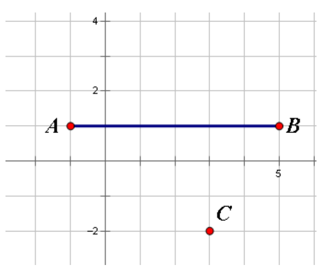
Answer

Locus
&
Constructions

89 Point C is on the locus between point A and point B

True

False



Answer

Locus
&
Constructions

90 How many points are equidistant from the endpoints of $\overline{DE}$?

A 2

B 1

C 0

D infinite

Answer

Locus
&
Constructions

91 You can find the midpoint of a line segment by

A measuring with a ruler

B constructing the midpoint

C finding the intersection of the locus and line segment

D all of the above

Answer

Locus
&
Constructions

92 The definition of locus

A a straight line between two points

B the midpoint of a segment

C the set of all points equidistant from two other points

D a set of points

Answer

Locus
&
Constructions

Videos Demonstrating Constructing Midpoints using Dynamic Geometric Software

Click here to see video using compass and segment tool

Click here to see video using menu options

Video

Construction of a Circle

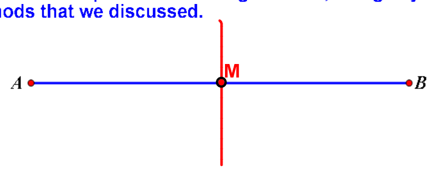
What is a circle?

Answer

Locus & Constructions

Extension: Construction of a Circle

1. Find the midpoint of a line segment 1st, using any of the methods that we discussed.

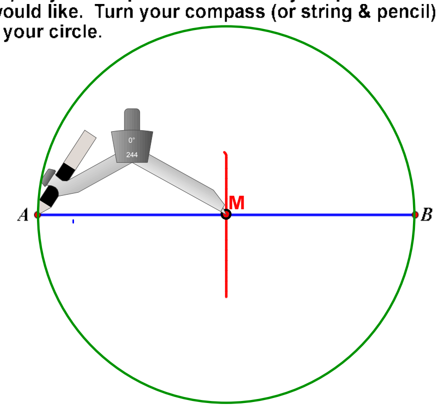


Teacher Notes

Locus & Constructions

Extension: Construction of a Circle

2. Take your compass (or rod, string & pencil) and place the tip (or rod) on your Midpoint. Extend out your pencil as much as you would like. Turn your compass (or string & pencil) 360° to make your circle.

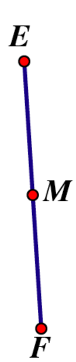


Locus & Constructions

Construction of a Circle

Try this!
Create a circle using the segment below.

9)

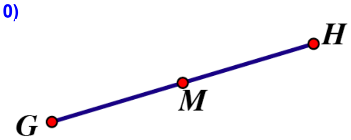


Locus & Constructions

Construction of a Circle

Try this!
Create a circle using the segment below.

10)



Locus & Constructions

Video Demonstrating Constructing a Circle using Dynamic Geometric Software

Click here to see video

Video

Construction of an Equilateral Triangle

What is an equilateral triangle?

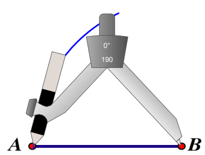
Answer

Locus
&
Constructions

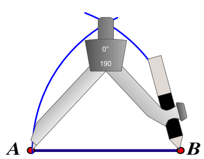
Extension: Construction of an Equilateral Triangle

1. Construct a segment of any length.

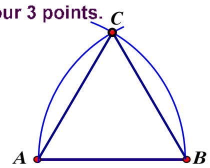
2. Take your compass and place the tip on one endpoint & make an arc.



3. Take your compass and place the tip on the other endpoint & make an arc that intersects your 1st arc.



4. Make a point where your two arcs intersect. Connect your 3 points.

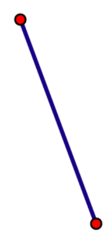


Locus
&
Constructions

Construction of an Equilateral Triangle

Try this!
Create an equilateral triangle using the segment below.

11)

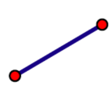


Locus
&
Constructions

Construction of an Equilateral Triangle

Try this!
Create an equilateral triangle using the segment below.

12)

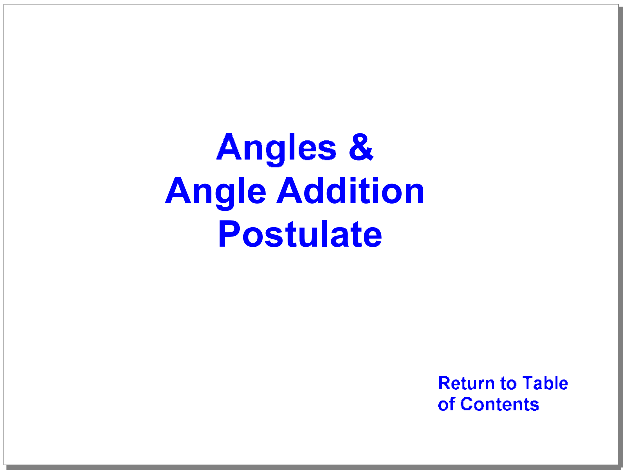


Locus
&
Constructions

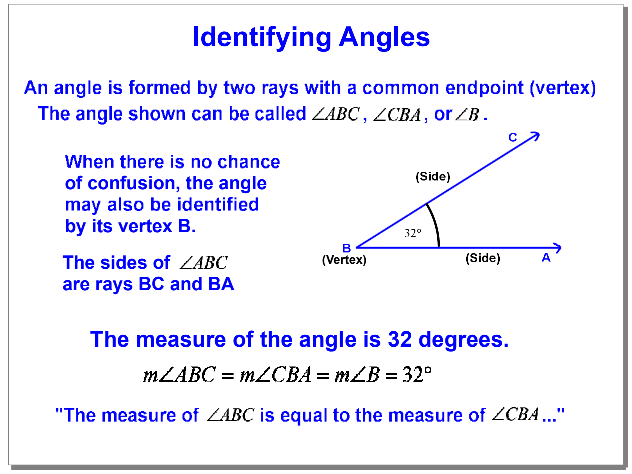
Video Demonstrating Constructing Equilateral Triangles using Dynamic Geometric Software

Click here to see video

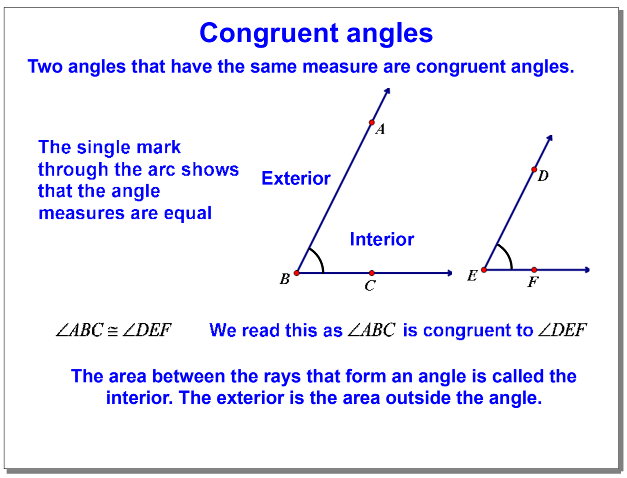
Video



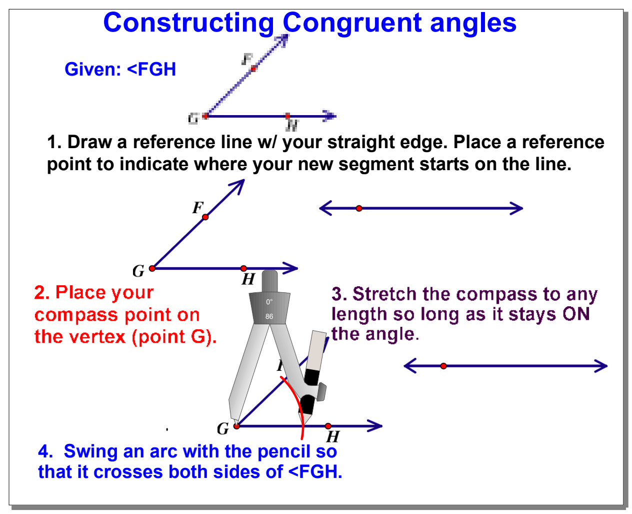
Angles
&
Angle Addition Postulate



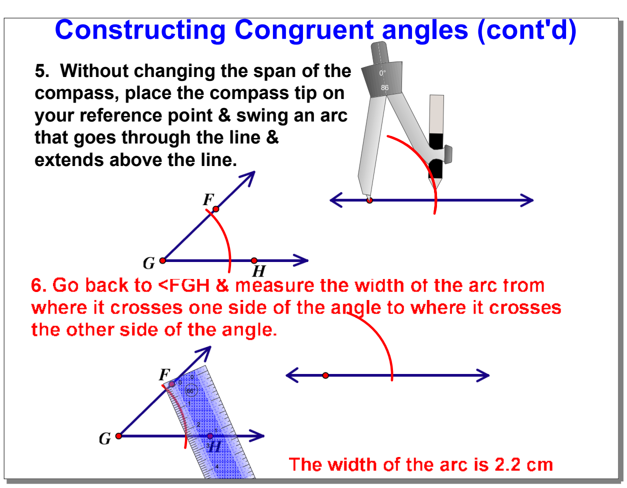
Angles
&
Angle Addition Postulate



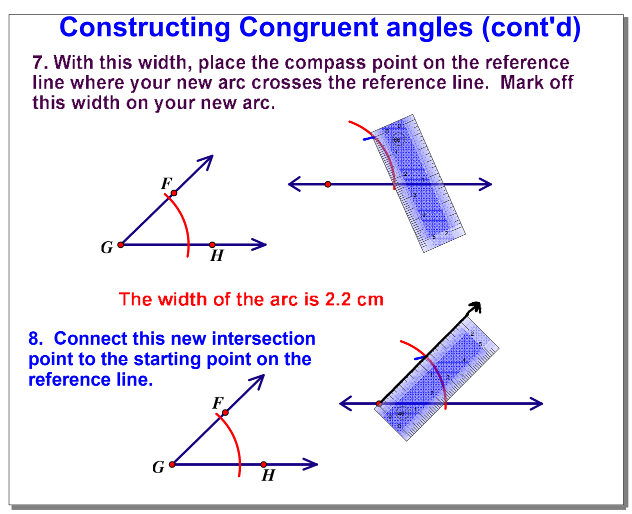
Angles
&
Angle Addition Postulate



Angles
&
Angle Addition Postulate



Angles
&
Angle Addition Postulate

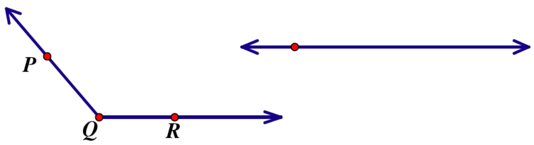


Angles
&
Angle Addition Postulate

Constructing Congruent angles

Try this!
Construct a congruent angle on the given line.

15)

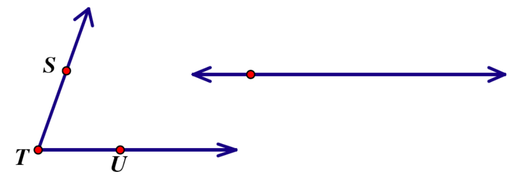


Angles
&
Angle Addition Postulate

Constructing Congruent angles

Try this!
Construct a congruent angle on the given line.

16)



Angles
&
Angle Addition Postulate

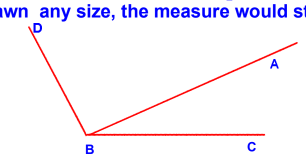
Video Demonstrating Constructing Congruent Angles using Dynamic Geometric Software

Click here to see video

Video

Angle Measures

Angles are measured in degrees, using a protractor.
Every angle has a measure from 0 to 180 degrees.
Angles can be drawn any size, the measure would still be the same.



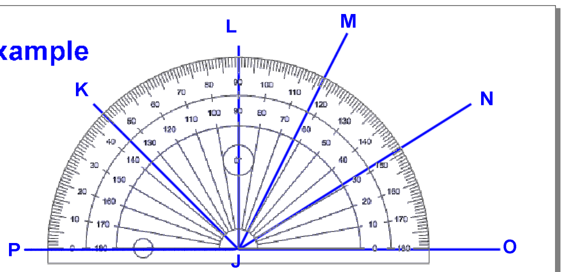
$\angle ABC$ is a 23° degree angle
The measure of $\angle ABC$ is 23° degrees
$m\angle ABC = 60°$

$\angle CBD$ is a 119° degree angle
The measure of $\angle CBD$ is 119° degrees
$m\angle CBD = 119°$

In $\angle ABC$ and $\angle CBD$, notice that the vertex is written in between the sides

Angles
&
Angle Addition Postulate

Example



$m\angle OJN =$ click

$m\angle OJL =$ click

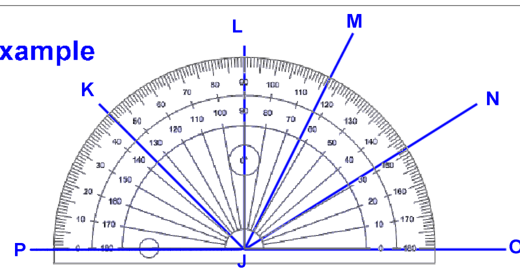
$m\angle OJP =$ click

$m\angle OJM =$ click

$m\angle OJK =$ click

Angles
&
Angle Addition Postulate

Example



Challenge Questions

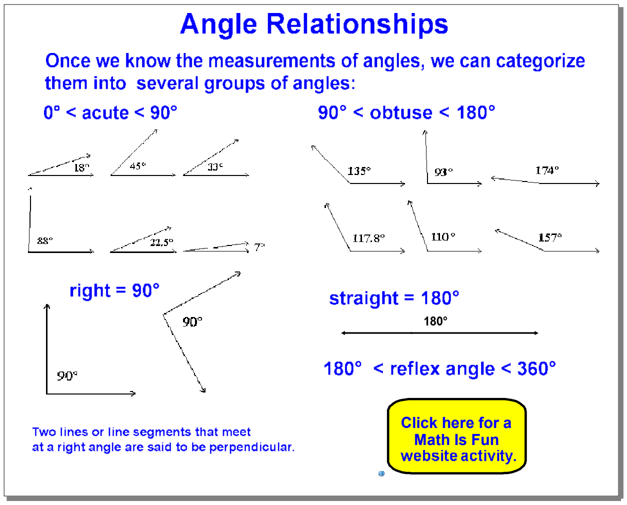
$m\angle NJM =$ click

$m\angle MJL =$ click

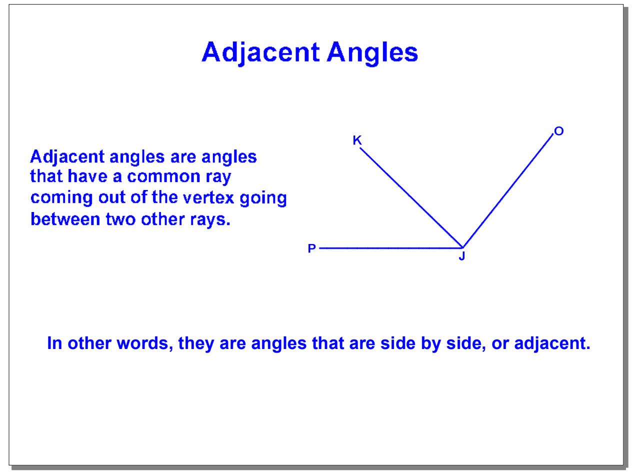
$m\angle LJK =$ click

$m\angle KJP =$ click

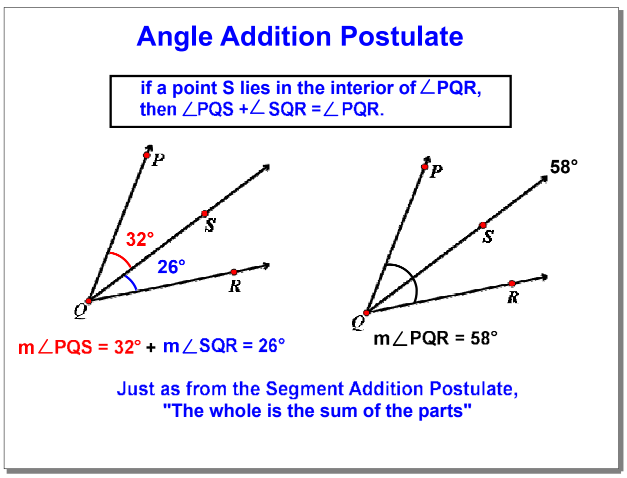
Angles
&
Angle Addition Postulate



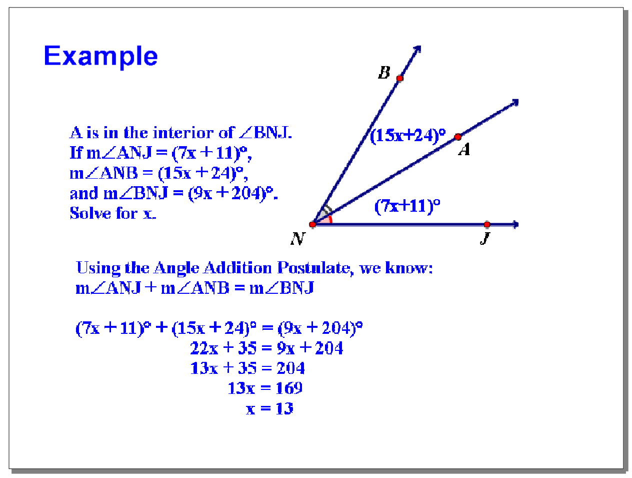
Angles
&
Angle Addition Postulate



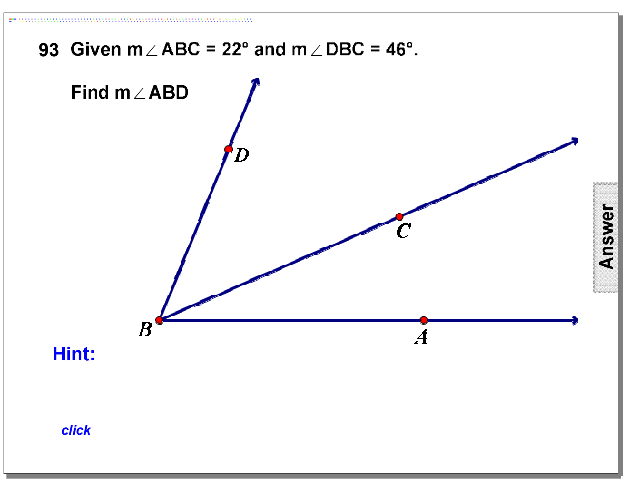
Angles
&
Angle Addition Postulate



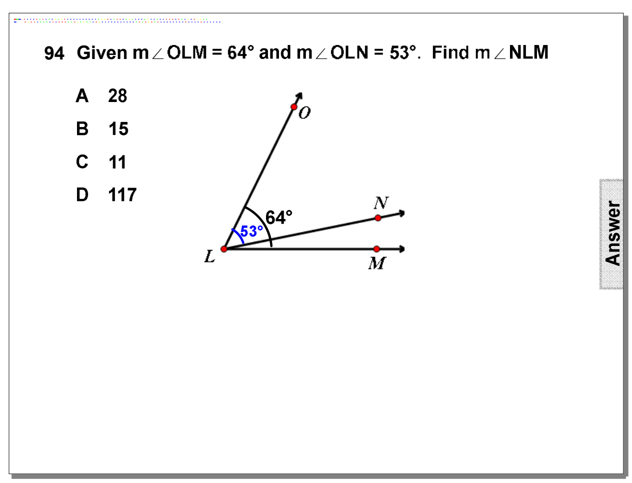
Angles
&
Angle Addition Postulate



Angles
&
Angle Addition Postulate



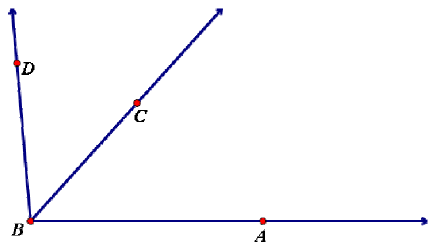
Angles
&
Angle Addition Postulate



Angles
&
Angle Addition Postulate

95 Given m∠ABD = 95° and m∠CBA = 48°.

Find m∠DBC

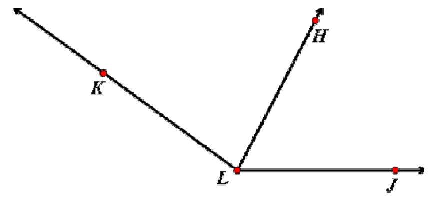


Answer

Angles
&
Angle Addition Postulate

96 Given m∠KLJ = 145° and m∠KLH = 61°.

Find m∠HLJ

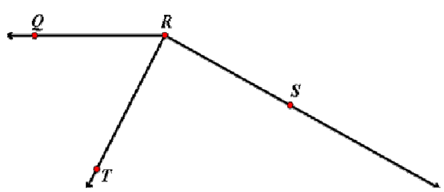


Answer

Angles
&
Angle Addition Postulate

97 Given m∠TRQ = 61° and m∠SRQ = 153°.

Find m∠SRT



Answer

Angles
&
Angle Addition Postulate

98 C is in the interior of ∠TUV.

If m∠TUV = $(10x + 72)^o$,

m∠TUC = $(14x + 18)^o$ and

m∠CUV = $(9x + 2)^o$

solve for *x*.

Answer

Hint:

click

Angles
&
Angle Addition Postulate

99 D is in the interior of ∠ABC.

If m∠CBA = $(11x + 66)^o$,

m∠DBA = $(5x + 3)^o$ and

m∠CBD= $(13x + 7)^o$

solve for *x*.

Answer

Hint:

click

Angles
&
Angle Addition Postulate

100 F is in the interior of ∠DQP.

If m∠DQP = $(3x + 44)^o$,

m∠FQP = $(8x + 3)^o$ and

m∠DQF= $(5x + 1)^o$

solve for *x*

Answer

Angles
&
Angle Addition Postulate

Angle Pair Relationships

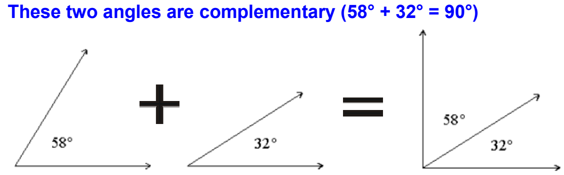
Return to Table of Contents

Angle Pair Relationships

Complementary Angles

A pair of angles are called complementary angles if the sum of their degree measurements equals 90 degrees. One of the angles is said to be the complement of the other.

These two angles are complementary (58° + 32° = 90°)



We can rearrange the angles so they are adjacent, i.e. share a common side and a vertex. Complementary angles do not have to be adjacent.

If two adjacent angles are complementary, they form a right angle

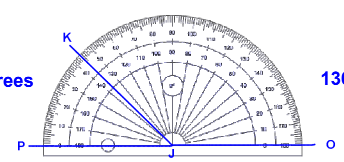
Angle Relationships - Complementary

Supplementary Angles

Supplementary angles are pairs of angles whose measurements sum to 180 degrees.

Supplementary angles do not have to be adjacent or on the same line; they can be separated in space. One angle is said to be the supplement of the other.

44 degrees



136 degrees

Angle Relationships - Supplementary

Example

An angle is 68° more than its complement.
What the angle's measure?

Solution:
Choose a variable for the angle - I'll choose "x"
The complement of this angle x is (90-x) since together they must sum to 90°
(definition of complementary angles)

(90°-x) + x = > 90°-x+x = 90°

So,

x = (90-x) + 68
x = 90 - x + 68
2x = 90 + 68
2x = 158
x = 79

Angle Relationships

Example

Two angles are complementary. The larger angle is twice the size of the smaller angle. What is the measure of both angles?

Let x = the angle

Since the angles are complementary we know their sum must equal 90 degrees.

90 = 2x + x
90 = 3x
30 = x

Angle Relationships

101 An angle is 34° more than its complement.

What is its measure?

Answer

Hint:

click

Angle Relationships

102 An angle is 14° less than its complement.

What is the angle's measure?

Answer

Hint:

click

Angle Relationships

103 An angle is 98 more than its supplement.

What is the measure of the angle?

Answer

Hint:

click

Angle Relationships

104 An angle is 74° less than its supplement.

What is the angle?

Answer

Angle Relationships

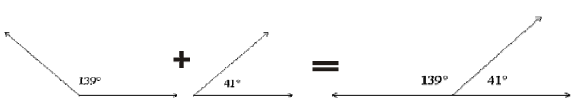
105 An angle is 26° more than its supplement.

What is the angle?

Answer

Angle Relationships

Linear Pair of Angles



If the two supplementary angles are adjacent, having a common vertex and sharing one side, their non-shared sides form a line.

A linear pair of angles are two adjacent angles whose non-common sides on the same line. A line could also be called a straight angle with 180°

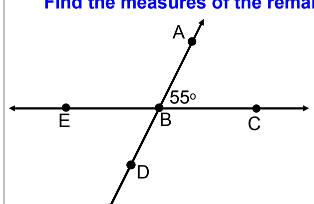
Angle Relationships

Vertical Angles

Vertical Angles: Two angles whose sides form two pairs of opposite rays
- In diagram below, ∠ABC & ∠DBE are vertical angles, and ∠ABE & ∠CBD are vertical angles.

Given: m∠ABC = 55°
Find the measures of the remaining angles.

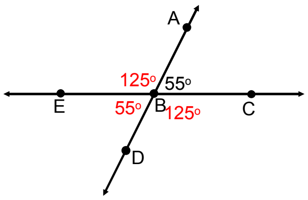
Answer



Angle Relationships

Vertical Angles

Given: m∠ABC = 55°
What do you notice about your vertical angles?

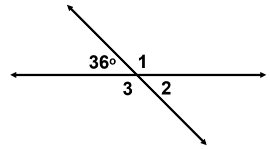


click to reveal theorem

Angle Relationships

Example

Find the m ∠ 1, m ∠ 2 & m ∠ 3. Explain your answer.



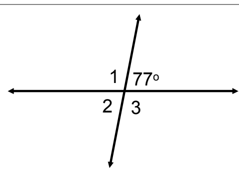
36 + m ∠ 1 = 180
-36 -36
m ∠ 1 = 144°
Linear pair angles are supplementary

m < 2 = 36°; Vertical angles are congruent (original angle & < 2)
m < 3 = 144°; Vertical angles are congruent (< 1 & < 3)

Angle Relationships

106 **What is the m ∠ 1?**

A **77°**
B **103°**
C **113°**
D **none of the above**

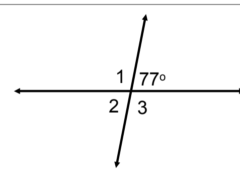


Answer

Angle Relationships

107 **What is the m ∠ 2?**

A **77°**
B **103°**
C **113°**
D **none of the above**

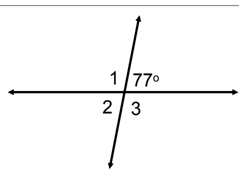


Answer

Angle Relationships

108 **What is the m ∠ 3?**

A **77°**
B **103°**
C **113°**
D **none of the above**

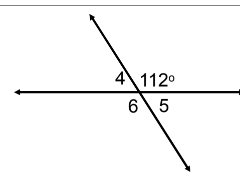


Answer

Angle Relationships

109 **What is the m ∠ 4?**

A **112°**
B **78°**
C **102°**
D **none of the above**



Answer

Angle Relationships

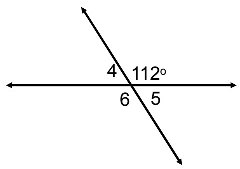
110 What is the m ∠ 5?

A 112°

B 68°

C 102°

D none of the above



Answer

Angle Relationships

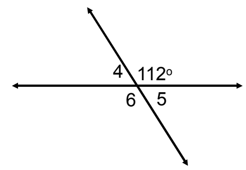
111 What is the m ∠ 6?

A 102°

B 78°

C 112°

D none of the above

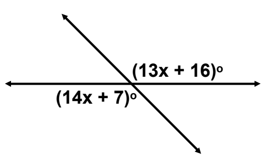


Answer

Angle Relationships

Example

Find the value of x.



The angles shown are vertical angles, so they are congruent

$13x + 16 = 14x + 7$

$-13x \quad -13x$

$16 = x + 7$

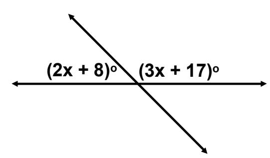
$-7 \quad -7$

$9 = x$

Angle Relationships

Example

Find the value of x.



The angles shown are a linear pair, so they are supplementary

$2x + 8 + 3x + 17 = 180$

$5x + 25 = 180$

$-25 \quad -25$

$\frac{5x}{5} = \frac{155}{5}$

$x = 31$

Angle Relationships

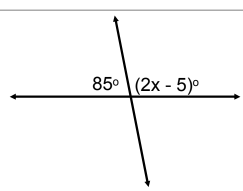
112 Find the value of x.

A 95

B 50

C 45

D 40



Answer

Angle Relationships

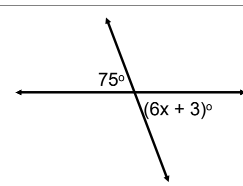
113 Find the value of x.

A 75

B 17

C 13

D 12

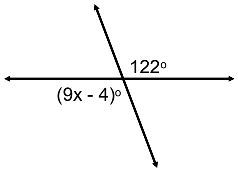


Answer

Angle Relationships

114 Find the value of x.

A 13.1
B 14
C 15
D 122

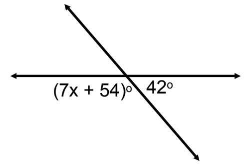


Answer

Angle Relationships

115 Find the value of x.

A 12
B 13
C 42
D 138



Answer

Angle Relationships

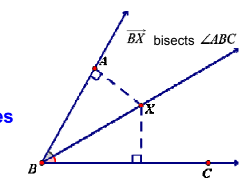
Angle Bisectors & Constructions

Return to Table of Contents

Angle Bisectors

Angle Bisector

An angle bisector is a ray or line which starts at the vertex and cuts an angle into two equal halves



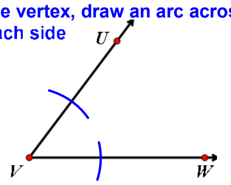
Bisect means to cut it into two equal parts. The 'bisector' is the thing doing the cutting.

The angle bisector is equidistant from the sides of the angle when measured along a segment perpendicular to the sides of the angle.

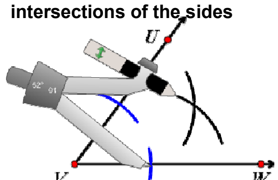
Angle Bisectors

Constructing Angle Bisectors

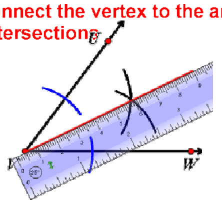
1. With the compass point on the vertex, draw an arc across each side



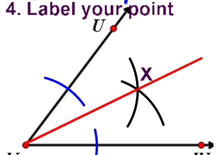
2. Without changing the compass setting, place the compass point on the arc intersections of the sides



3. With a straightedge, connect the vertex to the arc intersections



4. Label your point



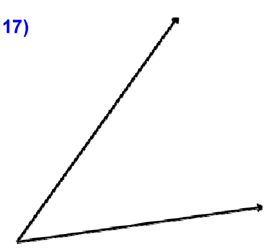
m∠UVX = m∠WVX
∠UVX ≅ ∠WVX

Angle Bisectors

Try this!

Bisect the angle

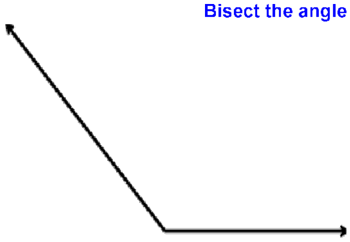
17)



Angle Bisectors

18) Try this!

Bisect the angle



Angle Bisectors

Videos Demonstrating Constructing Angle Bisectors using Dynamic Geometric Software

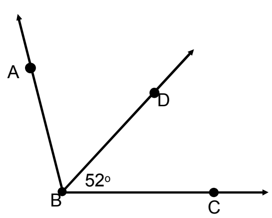
Click here to see video using a *compass and segment tool*

Click here to see video using the *menu options*

Video

Finding the missing measurement.

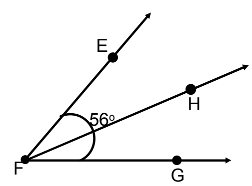
Example: ∠ABC is bisected by $\overrightarrow{BD}$. Find the measures of the missing angles.



Answer

Angle Bisectors

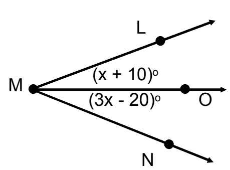
Example: ∠EFG is bisected by $\overrightarrow{FH}$. The m∠EFG = 56°. Find the measures of the missing angles.



Answer

Angle Bisectors

Example: $\overrightarrow{MO}$ bisects ∠LMN. Find the value of x.



Answer

Angle Bisectors

116 $\overrightarrow{JK}$ bisects ∠HJL. Given that m∠HJL = 46°, what is m∠HJK?

Hint:

click to reveal

Answer

Angle Bisectors

117 $\overrightarrow{NP}$ bisects $\angle MNO$ Given that $m\angle MNP = 57°$, what is $m\angle MNO$?

Hint:

click to reveal

Answer

Angle Bisectors

118 $\overrightarrow{RT}$ bisects $\angle QRS$ Given that $m\angle QRT = 78°$, what is $m\angle QRS$?

Answer

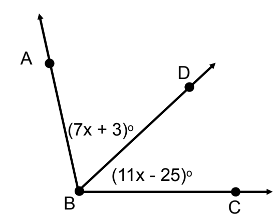
Angle Bisectors

119 $\overrightarrow{VY}$ bisects $\angle UVW$. Given that $m\angle UVW = 165°$, what is $m\angle UVY$?

Answer

Angle Bisectors

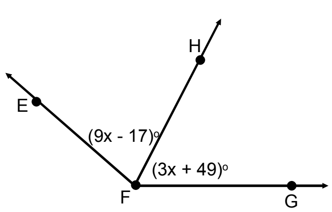
120 $\overrightarrow{BD}$ bisects $\angle ABC$. Find the value of x.



Answer

Angle Bisectors

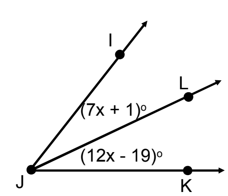
121 $\overrightarrow{FH}$ bisects $\angle EFG$. Find the value of x.



Answer

Angle Bisectors

122 $\overrightarrow{JL}$ bisects $\angle IJK$. Find the value of x.



Answer

Angle Bisectors

Unit 1 Notes

Aug 25-6:29 AM