Lab 3: Robots on Racetracks
15-424/15-624 Foundations of Cyber-Physical Systems
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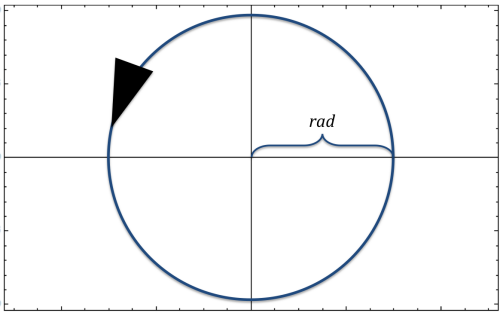
Betabot Due Date: 10/16/14 (**Thursday!**), worth 20 points
Veribot Due Date: 10/22/14 (**Wednesday!**), worth 80 points

1. **Practice with Diff Cuts, Invariants, and Weakening in KeYmaera**

   Prove the following $\mathsf{d}\mathcal{L}$ formula using KeYmaera. You **may not** use the start button or the "apply rules automatically" proof rule until no hybrid programs or quantifiers remain in your property. You also may not use the ODE Solve rule, but instead use the Differential Invariant, Differential Cut, and Differential Weakening rules. The idea here is to recreate the written proof you completed for assignment 4, question 2 of the same property, but now doing so in KeYmaera. Save your resulting proof as `L3Q1_andrewid.proof`.

$$\vdash (x \geq 0 \wedge y \geq 0 \wedge z \geq 0) \rightarrow [x' = y, y' = z, z' = x^2]x \geq 0$$

In questions 2 and 3, you will design controllers to drive your robots around a circular racetrack centered at the origin and with radius *rad*. In both questions, you will design a controller that satisfies the safety specifications given. The properties you prove are safety properties (e.g. stay on the track and don't hit the obstacle), but you should also keep in mind the efficiency of your controller (e.g. can your controlled robot reach the obstacle). So, for example, a robot that just stays still will satisfy all the safety properties, but would be rather useless. To prepare you for lab 4, we **strongly encourage you to use cartesian coordinates** (as opposed to polar coordinates) for this problem.



2. **Lap time (empty racetrack)**

   In this question, your robot should be able to drive *counter-clockwise* on the track without leaving it. In other words, the robot should always be on the circle with radius *rad* centered at the origin. The racetrack is completely empty, so the robot may choose to travel at any speed. This should be *linear* velocity, not angular. Your HP should include a loop so that the controller may execute more than once.

   (a) Design a hybrid program to keep your robot on the track, then create a formula which, if proved, would guarantee that your controller never causes the robot to drift off the circle. You will need to determine appropriate initial conditions to make this property provable; however, try to prove the strongest property you can by making the initial conditions as general as possible.

   (b) Use KeYmaera to prove that your controller keeps the robot on the track. Submit the .key file you proved and the proof file as `L3Q2_andrewid.key` and `L3Q2_andrewid.proof`.

(c) **Question:** Solve the differential equation you used to model the physical dynamics of the system. Could you rewrite an equivalent hybrid program using this solution and the assignment operator instead of a continuous evolution? Why or why not? Submit a *brief* answer in `L3_andrewid.txt`. Also *briefly* explain the intuition behind your differential equations.

Hint: Since the next question is more complicated, try allowing your robot to accelerate, and proving it still remains on the racetrack. This ensures that he does not spiral away from the racetrack. KeYmaera should be able to prove acceleration around the racetrack automatically.

3. **Racetrack Debris (static obstacle avoidance)**

For this question, a frustrated student left their computer on the track, while it was running KeYmaera to prove a previous lab. Your goal is to develop a robot that moves counter-clockwise around the track but stops before running over the student's computer (since you feel his pain). Of course, you can't start your robot at 100 mph just feet behind the computer and expect the system to still be safe, so you may define some initial conditions on the distance between your robot and the relative position of the computer. If these initial conditions are satisfied, your controller should be able to bring your robot to a stop before it hits the computer. The bounds on acceleration and braking for your controlled robot are $A > 0$ and $-B < 0$. *Your controller should be time-triggered.*

The computer is at the leftmost position on the track, since that's closest to the nearest place where the student could get coffee. **We strongly suggest you start with very conservative controllers that you can prove**, and only then try to make them more efficient.

(a) Design a hybrid program to keep your robot on the track and stop before the position of the obstacle. Create a formula which, if proved, would guarantee that your controller satisfies these safety properties.

(b) Use KeYmaera to prove the safety properties. Submit the .key file you proved and the proof file as `L3Q3_andrewid.key` and `L3Q3_andrewid.proof`.

(c) In `L3_andrewid.txt`, briefly explain your control decision **(Betabots)** and any tricks used in your proof **(Veribots)**.

(d) **(Veribots)** Suppose you've proven this controller to be safe. Would it work on a circle in which the obstacle was dropped at an arbitrary spot? How and why? Write your answer in `L3_andrewid.txt`.

4. **Racetrack Debris (Extra credit, Veribots only)**

Because meta-arguments, like in Q3.d), aren't formally KeYmaera proved, for extra credit (*only for Veribots*), you don't get to assume that the student's computer was dropped at the leftmost position of the racetrack. The befuddled student might have dropped it anywhere on the racetrack!

Submit `L3Q4_andrewid.key` and `L3Q4_andrewid.proof` for the Veribots deadline. In `L3_andrewid.txt`, briefly explain how proving this generalisation was different from the previous question.

5. **Submission Checklist**

Submit a zip file on autolab containing the following files. If you are working in a group, **include both andrewIDs**. Both members of the group may submit, which might get you two simulations instead of one.

Submission (some subset of the following):

```
L3Q1_andrewid.key      L3Q2_andrewid.key      L3Q3_andrewid.key
L3Q1_andrewid.proof    L3Q2_andrewid.proof    L3Q3_andrewid.proof
L3_andrewid.txt
```