# INTRODUCTION TO STATISTICS: ENHANCEMENT THROUGH THE USE OF THE GRAPHING CALCULATOR 

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The graphing calculator is one example of technology assisting students in the development of deeper understanding and richer conceptualization. The emphasis in class is in learning how to think in statistics.

Today mathematics is too often a barrier that discourages students from making ambitious career choices. This is particularly true for minorities and women students. Major initiatives in mathematics and statistics of this kind will play a crucial role in strengthening mathematics and statistical education at all levels, and hence in assuring that more students will succeed in these two careers.

Statistics 100. Introduction to Statistical Reasoning at the U of Michigan is designed to provide an overview of the field of statistics. Course topics include methods of analyzing and summarizing data, statistical reasoning as a means of learning from observations (experimental or sample), and techniques for dealing with uncertainties in drawing conclusions from collected data. The chief tenant supporting the use of the graphing calculator in the course is the motto: "learn by discovery." Research shows that once the students visualize the problem with a graph, the problem is easier to solve. Students can be lead to many "discoveries" on the spot. The ability to change the setting of the problem, and have immediate results and feedback, allows the students to think through many new ideas in one class period. These new ideas will be timely and relevant, and take advantage of the "moment of readiness."

The use of graphing calculators eliminates much time spent on the manipulation of the formulae, and frees up time for teaching concepts and problem solving. (It also alleviates agony of many of the students who are intimidated by mathematics and calculations.) With this technology every one of the 150 students per lecture can have the calculator in his or her hands during the instructor's lecture. The lecturer uses a big screen simultaneously. The use of the calculator also accommodates working groups. Compared to computers, the use of calculators is cost effective and flexible. Some of the Statistics 100 students have already purchased the calculator, since it is required for Calculus I or II in the Mathematics Department.

The use of the graphing calculator in Statistics was "borrowed" from calculus reform. Its application in the UM Department is fluid; it is a "natural fit" and has impacted the teaching style in the department profoundly.

The tremendous advantage of using the graphing calculator is the immediate feedback to inquiries. This time maximization allows students to spend more classroom time applying and extending the skills than was previously possible when time consuming calculations were required. Instead of spending time calculating, students spend time thinking. This allows students the opportunity to problem solve with immediate results, and then either review or expand the thinking required to solve the problem. It becomes more apparent that there may be more than one way to solve a problem, and there may be more than one answer that is correct.

## PEDAGOGICAL PRINCIPLES

We use the following principles in teaching, some of which are from the literature, and some of which we have developed after years of teaching.

- The most effective time for learning, retention, and ability to utilize material later is that developmental "moment of readiness." Therefore, that is the time to respond to questions, solve problems, straighten out mistakes in learning (Hilliard).
- Students must be tested on material while still at the beginning of the learning curve, so that errors can be noted immediately, and corrected before they become an erroneous part of the statistical thinking and understanding.
- All information about statistics is disseminated in sequences and sizes that mirror students' learning ability. As their incremental knowledge increases, one can move faster, and also, in larger conceptual blocks.
- Problems should include content which is relevant to real life.
- Quizzes test on all previous material, with emphasis on the most current. This approach avoids the feeling that all previous assumptions about statistics are to be erased, that class "starts over" with this instructor. (Remove obsolete and not useful paradigms about statistics.)
- The students work in small group settings. Although there is much advice in the literature about how to form groups, we prefer to do it at random. During the
first day of class numbers are placed on the desks; the students pick a number at random at the door as they enter, and sit at the corresponding seat. The students then introduce themselves to the group through exercises, and share phone numbers. The following day students are invited to change seats if they prefer, although the majority choose to continue with the original group. They are also encouraged to work together on homework. The environment is conducive to learning as well as enjoyment.

The students debate with and question each other to arrive at correct solutions. The group must share, discuss and agree on their combined answers, and then this is presented as the group's proposed response to the problem. This means that the group must review the statistical principles involved over and over, thus learning them more fully, as they try to convince the other group members about their suggested answers. If questions come up from the group or any individual student, the instructor redirects the questions back to the rest of the class. There needs to be a variety of methods to enable students to depend on their own understandings and abilities to move all the way to a solution. Also, it is worth using the class to expand the student's awareness of resources other than the teacher. Should no one have the answer, the instructor would of course provide it or provide a source to find it. The students are encouraged to participate, to talk, to debate, to disagree, to argue.

- Literature indicates that when the teaching objectives of learning for understanding and involves higher order thinking, task arrangements and instruction that constrain and routinize interaction will be less productive than arrangements and instruction that foster maximum interaction, mutual exchange, and elaborated discussions (Cohen).
- Believe that student can achieve, and that you can teach them successfully.


## GETTING STARTED

The reform in the Statistics 100 followed the reform in calculus in the Mathematics Department, in which Aliaga also participated. The support of the University of Michigan Department of Statistics was critical in the acceptance and implementation of the use of technology in teaching. The two large classes in Statistics are 100 (for non-
majors) and 402 (required for majors and research degrees). Each senior faculty is required to teach either course.

## PEDAGOGY

Using the graphing calculator affects pedagogy. Students are able to see the function in a graph. This visualization allows them to more readily comprehend the scope and purpose of the function. They can prove, predict and verify statements. The calculator can also produce the simple statistical plots that help the students learn about analyzing data. While a computer can do all of these things, it is not financially or logistically practical for students to have access to a computer during large lecture classes.

We use two overhead projectors when presenting problems. One of these projectors shows the stated problem, while the other shows the solution as it is being solved by either a student or instructor. Pages from the book are projected so students are well oriented, and time is not wasted copying the problem. Students spend their class time on solutions. The pages of the book are formatted to accommodate this projected use.

## Instructors

Statistics faculty rotate through the instructor assignments of teaching the largest classes, Statistics 100 and Statistics 402. Each have approximately 150 students in their class. Many of these faculty are established, tenured professors who are used to teaching in traditional methods, and many are greatly involved in research. The students use "real life" data gathered from current newspaper articles, and data gathered from the student class.

The Graduate Student Instructors (GSIs) meet once per week with the students in labs. They facilitate the hands-on experience of the students, solving projects and performing experiments, using both the TI calculator and Macintosh computers.

While the faculty acceptance of the use of the graphing calculator has been great, there have been a few members who have been reluctant to use it. Some faculty who do much research are more reliant on the computer, and find the transition a bit annoying.

There are no problems supplying nor maintaining the calculators. One set of batteries last approximately one semester. The calculators used in class are portable, userfriendly and affordable. They are cheap enough to expect students to purchase their own. While students can buy previously-used calculators, there are not many available since the
students want to keep them. The highly interactive nature of the class accommodated a great deal of communication between teacher and students, making the students feel more engaged in discussions and more comfortable asking questions.

One other aspect of technology in the course is the use of videotapes series.
Portions of the series are used intermittently to give an overview and to motivate students by showing the application of statistics techniques in real life.

## Sample material

Problem 1. A population consists of 12 people. These 12 people are listed in the table below. The two columns in the table divide up the population into 2 strata (I and II). The population is also divided into 3 rows, each row is a cluster. Below each name in the table is the person's corresponding ID number.

POPULATION

## STRATUM I STRATUM 2

| Row 1 | Ann <br> $(1)$ | Betty <br> $(2)$ | George <br> $(3)$ | John <br> $(4)$ |
| :---: | :---: | :---: | :---: | :---: |
| Row 2 | Carrie | Donna | Bob | Steve |
| $(8)$ | $(9)$ | $(6)$ | $(7)$ |  |
| Row 3 | Ellen | Fran | Paul | Tom |
|  | $(9)$ | $(10)$ | $(11)$ | $(12)$ |


| Thus, | Row 1 consists of: | Ann | Betty George John |  |
| ---: | :--- | :--- | :--- | :--- | :--- |
| Row 2 consists of: | Carrie Donna Bob Steve |  |  |  |
| Row 3 consists of: | Ellen Fran Paul Tom |  |  |  |
|  |  |  |  |  |
| Stratum I consists of: Ann | Betty | Stratum II consists of: George | John |  |
|  | Carrie Donna | Bob | Steve |  |
|  | Ellen | Fran | Paul | Tom |

ID Numbers: (1) (2) (3) (4) (5) (6) (7) (8) (9) (10) (11) (12)
A sample of 4 people was obtained from this population. Listed below are three different samples. Consider the following sampling methods: a) simple random sampling(SRS), b) stratified random sampling, with equal sample sizes from each stratum, c) cluster sampling by rows. For each sample determine which sampling
method(s) could have generated that sample, by circling yes or no for each. Hint: more than one method is possible.

Sample Method

| Sample | a) SRS | b) Stratified | c) Cluster |
| :--- | :---: | :---: | :---: | :---: |
| Carrie, Donna, Bob, Steve | Yes No | Yes No | Yes No |
| Ann, Fran, Carrie, Betty | Yes No | Yes No | Yes No |
| Carrie, Donna, George, Tom | Yes No | Yes No | Yes No |

b) Using seed $=1005$. Select one cluster at random.

Cluster 1=Row1 Cluster 2=Row2 Cluster 3=Row3
Who are the people in the sample?
c) Using the ID numbers of the people as levels, select a 1-6 systematic sample.

Use seed $=64$
Who are the people in the sample?

Problem. 2. Population A is divided in Strata 1 and Strata 2
Strata 1 contains all the women. Strata 2 contains all the men
The variable of interest is income
(a) Strata 2 contains more people than Strata 1

Circle one: True False Can't tell Explain
(b) The distribution of Variable Income in Strata 1 is symmetric

Circle one: True False Can't tell Explain

## POPULATION A

Strata $1 \quad$ Strata 2


Problem 3. How does the Correlation Coefficient between two variables X and Y $r(X, Y)$ change? Using your graphing calculator investigate the following cases. Consider the following distributions for X and Y . Calculate $\mathrm{r}(\mathrm{X}, \mathrm{Y})=$

| X | 2 | 3 | 4 | 5 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y | 1 | 2 | 2 | 3 | 4 | 5 |

Define $\mathrm{M}=-3 \mathrm{X}-1$ and $\mathrm{N}=-\mathrm{Y}-1$ Calculate $\mathrm{r}(\mathrm{M}, \mathrm{N})=$
Define $\mathrm{T}=4 \mathrm{X}+5 \quad$ Calculate $\mathrm{r}(\mathrm{X}, \mathrm{N})=$
Define $\mathrm{L}=7 \mathrm{Y}+2$ Calculate $\mathrm{r}(\mathrm{L}, \mathrm{T})=$ Write up what you have observed.

