STUDENTS ENTERING ADVANCED PRECALCULUS
SUMMER REVIEW WORK 2015

Because of the cumulative nature of math, you have learned that you need to have mastered concepts and procedures before you can learn new ones. The problems you will be completing will help you review material from Advanced Algebra II.

This assignment should take you approximately 3-5 hours, depending on how well you remember this material. Please do not start it until August. However, give yourself plenty of time as you may need to spend time reviewing your notes and/or examples from Advanced Algebra II.

In an organized fashion, show your work neatly on separate sheets of notebook paper. You may work with another rising Advanced Precal student, but the Honor Code applies: your work must be your own. I have included formulas at the beginning. Don't forget www.khanacademy.org which is a wonderful website that explains mathematical topics through video. Just put in the topic and choose the one you need.

Please have this assignment completed on the first day (half day) of classes. You can expect some kind of evaluation on it but the main reason for doing this is to solidify the concepts you need to do well in Advanced Precalculus.

DIRECTIONS RECAP:
1. Show all work neatly, thoroughly and in pencil on separate paper.
2. Careful documentation of your work is extremely important. (ex: start by copying down the original problem; start by writing out the formula that you are using)
3. Follow directions for each section carefully.
4. Do not use your graphing calculator unless told otherwise.
5. You may work together on this assignment. In fact, I encourage you to work with one or two other students. Help each other but don't copy someone's work as that will not benefit you when you take the test.

Good luck! I look forward to working with you!

Mrs. Davis

Formulas That You Need To Know

Linear

Slope between two points: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Point –Slope Form: $y - y_1 = m(x - x_1)$ Slope-Intercept Form: $y = mx + b$

Standard Form: $Ax + By = C$ (A, B, and C must be integers)

Horizontal Line: $y = k$ (zero slope)   Vertical Line: $x = k$ (no slope)

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Midpoint Formula: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$

Quadratic

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant: $b^2 - 4ac$

$b^2 - 4ac > 0$ 2 real unequal roots
$b^2 - 4ac = 0$ 1 double real root
$b^2 - 4ac < 0$ 2 complex conjugate roots

Sum of Roots: $-\dfrac{b}{a}$ Product of Roots: $\dfrac{c}{a}$

Standard Form of a Parabola: $y = ax^2 + bx + c$ or $x = ay^2 + by + c$

Vertex Form of a Parabola: $y = a(x - h)^2 + k$ or $x = a(y - k)^2 + h$

$a = \dfrac{1}{4p}$; p is the distance from vertex to focus or p is the distance from vertex to directrix

Solving different types of equations:

Quadratic equation – factor, complete the square, quadratic formula
Quadratic-like equation – use a dummy variable
Rational equation – remember to check solutions in original equation for any restrictions on the domain
Radical equation – remember to check solutions for possible extraneous roots

Exponential and Logarithmic

Exponential Function: $y = b^x$ with $b > 0, \neq 1$; $Domain : \Re$ $Range : \Re > 0$

Logarithmic Function: $y = \log_b x$ with $b > 0, \neq 1$; $Domain : \Re > 0$ $Range : \Re$

Identities

$\log_b b = 1$ $\ln e = 1$

$\log_b 1 = 0$ $\ln 1 = 0$

$\log_b b^x = x$ $\ln e^x = x$

$b^{\log_b x} = x$ $e^{\ln x} = x$

$\log_b x + \log_b y = \log_b xy$

$\log_b x - \log_b y = \log_b \dfrac{x}{y}$

$k \log_b x = \log_b x^k$

Solving equations

$b^x = b^y ...iff...x = y$

$\log_b x = \log_b y...iff...x = y$

Converting from exponential to logarithmic

$x = b^y \Leftrightarrow y = \log_b x$

Change of Base Formula: $\log_b a = \dfrac{\log_c a}{\log_c b}$ or $\log_b a = \dfrac{\log a}{\log b}$ or $\log_b a = \dfrac{\ln a}{\ln b}$

Corollary: $\log_b a \cdot \log_a c = \log_b c$

Application Formulas: $A(t) = A_0 (1 \pm r)^t$ $A(t) = A_0 e^{kt}$

$A = P\left(1 + \dfrac{r}{n}\right)^{nt}$ $A = Pe^{rt}$

Quadratic Relations:

Equations of a Circle: $x^2 + y^2 = r^2;$ center $= (0,0)$ and radius $= r$

$(x-h)^2 + (y-k)^2 = r^2;$ center $= (h,k)$ and radius $= r$

$x^2 + y^2 + ax + by + c = 0;$ must complete the square in x and y

Equations of an Ellipse:

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1; \quad \text{center}=(h,k),$$

sum of focal radii $=2a$,
foci $=(\pm c,0)$, and
$a^2=b^2+c^2$; $a>b$ and $a>c$

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1; \quad \text{center}=(h,k),$$

sum of focal radii $=2a$,
foci $=(0,\pm c)$, and
$a^2=b^2+c^2$; $a>b$ and $a>c$

Equations of a Hyperbola:

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1; \quad \text{center}=(h,k),$$

difference of focal radii $=2a$,
foci $=(\pm c,0)$, and
$c^2=a^2+b^2$; $c>a$ and $c>b$

$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1; \quad \text{center}=(h,k),$$

difference of focal radii $=2a$,
foci $=(0,\pm c)$, and
$c^2=a^2+b^2$; $c>a$ and $c>b$

$xy=k$; \qquad\qquad simple hyperbola:

Sequences, Series, and Binomial Theorem
Arithmetic Sequence: \qquad\qquad\qquad\qquad **Arithmetic Series:**

$a_n=a_1+(n-1)d$ \qquad $a_n=$ n^{th} term \qquad\qquad $S_n=\dfrac{n(a_1+a_n)}{2}$ or $S_n=\dfrac{n[2a_1+(n-1)d]}{2}$

$a_1=$ first term \qquad\qquad\qquad $S_n=$ sum of n terms

$d=$ common difference \qquad\qquad\qquad $a_1=$ first term

$n=$ # of terms \qquad\qquad\qquad\qquad $a_n=$ n^{th}

\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad $n=$ # of terms

Geometric Sequence:

$$a_n = a_1 r^{n-1}$$

$a_n =$ n^{th} term

$a_1 =$ first term

$n =$ # of terms

$r =$ common ratio

Geometric Series:

$$S_n = \frac{a_1\left(1 - r^n\right)}{1 - r}$$

$S_n =$ sum of n terms

$a_1 =$ first term

$n =$ # of terms

$r =$ common ratio $(r \neq 1)$

$$S_n = \frac{a_1 - r a_n}{1 - r}$$

$S_n =$ sum of n terms

$a_1 =$ first term

$a_n =$ n^{th} term

$r =$ common ratio $(r \neq 1)$

Infinite Geometric Series:

$$S = \frac{a_1}{1 - r}$$

$S =$ sum of infinite series

$a_1 =$ first term

$r =$ common ratio; $|r| < 1$ *"very important"*

Linear and Quadratic Functions

I. **Solve each equation** over complex numbers using the **most appropriate method** (factoring, completing the square, quadratic formula, squaring both sides, dummy variable). Do NOT approximate any irrational roots. Start by copying the original problem on your own college rule paper.

1. $4x^2 + 3 = 4x$

2. $1 = \sqrt{x+4} - \sqrt{x-1}$

3. $x - 11\sqrt{x} + 30 = 0$

4. $\left(x^2 - 3x\right)^2 - 3\left(x^2 - 3x\right) = 10$

5. $x^2 + 2x\sqrt{3} - 3 = 0$

6. $3 - 2|x-2| < -9$

7. $\begin{cases} \dfrac{x}{y} = -2 \\ \dfrac{2x+y}{x-y} = -2 \end{cases}$

8. $\begin{cases} y^2 - 6x^2 = 4 \\ y = 3x^2 - 2 \end{cases}$

9. $3x^3 + 5x^2 - 26x + 8 < 0$ (syn division)

II. **State** the x and y-intercepts, vertex, axis of symmetry, focus, equation of directrix. **Sketch** a graph. Use your calculator to approximate any irrational roots to the nearest tenth.

1. $y = -\dfrac{1}{12}(x+3)^2 + 2$

2. $x = \dfrac{1}{4}y^2 + y - 3$

III. **Determine the equation of the following:**

1. perpendicular bisector of $\overline{AB}$ given $A(2,-3)$ and $B(6,7)$.

   (give your equation in slope-intercept form)

2. linear function $f(x)$ with $f(-1) = -3$ and $f(-4) = 12$.

   (give your equation in slope-intercept form in terms of f(x))

3. quadratic equation with roots $\dfrac{-4 \pm 2i\sqrt{3}}{3}$.

   (give your equation in $ax^2 + bx + c = 0$ form)

4. parabola with focus $(-1,-5)$ and directrix $x = 9$.

   (give your equation in $y = a(x-h)^2 + k$ form)

5. quadratic function $h(x)$ with zeros -3 and 7 and *maximum value* of 6.

   (give your equation in $y = a(x-h)^2 + k$ form)

IV. **Applications** of linear equations and quadratic functions:

1. Find the equation of the line that contains the point $(4,-6)$ and the point of intersection of the lines $3x+4y=2$ and $2x-y=5$. (leave your equation in standard form)

2. Suppose you have 160 meters of fencing to make five side-by-side rectangular enclosures. If its width is w, express the area of this figure in terms of w. What is the maximum area that you can enclose?

3. From a platform 45 meters above the ground, a ball is thrown upward with an initial speed of 40 m/s. The approximate height of the ball above the ground t seconds later is given by $h(t)=45+40t-5t^2$. You may solve this problem by hand or by using your calculator. If you use your calculator, state what keys you used to arrive at your answers.
   A) What is the maximum height reached by this ball?
   B) After how many seconds does the ball hit the ground?
   C) How high is the ball after 1.25 seconds?
   D) Draw a graph of $h(t)$. What domain and range make sense in the problem situation?

4. The table below shows the number of CDs shipped in the United States over the years 1986 – 1991. (These are net figures; returns from the retailers have been accounted for)

Year	1986	1987	1988	1989	1990	1991
CDs (*million*)	53	102.1	149.7	207.2	286.5	373.3

   A) Find the linear regression model for these data rounding slope and y-intercept to the nearest thousandth.
   B) What is the meaning of slope in context to this problem?
   C) Use your model to estimate the number of CDs that might have been shipped in 2006.
   D) What is the correlation coefficient and what does the correlation coefficient tell you about the accuracy of your model?

Exponential and Logarithmic Functions

I. **Simplify**: (no calculators)

1. $\log_8 128$

2. $\left(3^{-2} + 4^{-2}\right)^{-1/2}$

3. $\log_6 6\sqrt[3]{1/36}$

4. $\ln \sqrt[5]{e^{-3}}$

5. $243^{\log_3 2}$

6. $\sqrt[10]{32} \div \sqrt[6]{4}$

II. **Solve**: If you must use your calculator, find the value of x to the nearest hundredth.

Remember: $y = b^x$ have restrictions on b and y

$y = \log_b x$ has restrictions on b and x

1. $27^{4-x} = \left(\dfrac{1}{81}\right)^{x-1}$

2. $9^{x-4} = 7.13$

3. $e^{2x+1} - 1 = 6$

4. $\log_4(x-4) + \log_4 x = \log_4 5$

5. $e^{2x} - 13e^x - 48 = 0$

6. $2\log_3 x = \log_3(x-2) + 2$

III. **Applications** of exponential and logarithmetic functions: Calculator-active.

1. Will wants to purchase a car in 6 years. He needs a down payment of $6600. Determine the continuously compounded interest rate at which he would have to deposit $3600 so that he will have the money he needs at the end of 6 years?

2. A bacteria colony grows from 2,000 to 1,500,000 in 30 days. Find out how many days (nearest tenth of a day) it takes this particular bacteria to quadruple.

3. The MacDonald family wants to give Daniel $36,000 when he is in college. They now have $25,000 to invest. Determine how many years (nearest tenth of a year) it will take them to achieve their goal given that they invest this amount at 6.25% compounded monthly.

4. Suppose a car is presently worth $10,150 and was bought new five years ago for $25,000. Assuming that the car depreciates the same amount each year, find the rate of depreciation, rounding to the nearest tenth of a percent.

IV. **Sketch** a graph. Use a table if necessary. Check with your calculator.

1. $f(x) = \left(\dfrac{1}{3}\right)^x$

    $g(x) = -\left(\dfrac{1}{3}\right)^x$

    $h(x) = \left(\dfrac{1}{3}\right)^{-x}$

    $j(x) = \left(\dfrac{1}{3}\right)^{x+3}$

2. $f(x) = e^x$

    $g(x) = \ln(x)$

    $h(x) = |\ln x|$

    $j(x) = \ln(x-4)$

3.

Graphing and more…

I. Sketch the graph of each of the following: Be sure you label points on your coordinate plane.

1. $\dfrac{(x+1)^2}{4} - \dfrac{(y-2)^2}{25} = 1$ (name vertices, foci, & equation of asymptotes)

2. $9x^2 + 4y^2 - 36x + 8y + 4 = 0$ (name vertices, foci, length of minor axis,& eccentricity)

3. $2y = \sqrt{16 + x^2}$

4. $f(x) = \dfrac{1}{x} + 2$ 5. $r(x) = \sqrt{x - 10}$

6. $h(x) = 4 - |x|$ 7. $j(x) = -\sqrt{-x}$

8. $f(x) = (x+3)^3 - 4$ 9. $g(x) = -(x-4)^2 + 2$

10. $f(x) = -3[x+2]$ 11. $f(x) = \begin{cases} 4, & if\ x \le 0 \\ 4 - x^2, & if\ 0 < x \le 2 \\ 2x - 6, & if\ x > 2 \end{cases}$

II. Find the equation of
1. a hyperbola with foci at $(-2,1)$ and $(8,1)$ and major axis of length 8.
2. an ellipse with vertices $(3,11)$ and $(3,-5)$; foci at $(3,7)$ and $(3,-1)$.
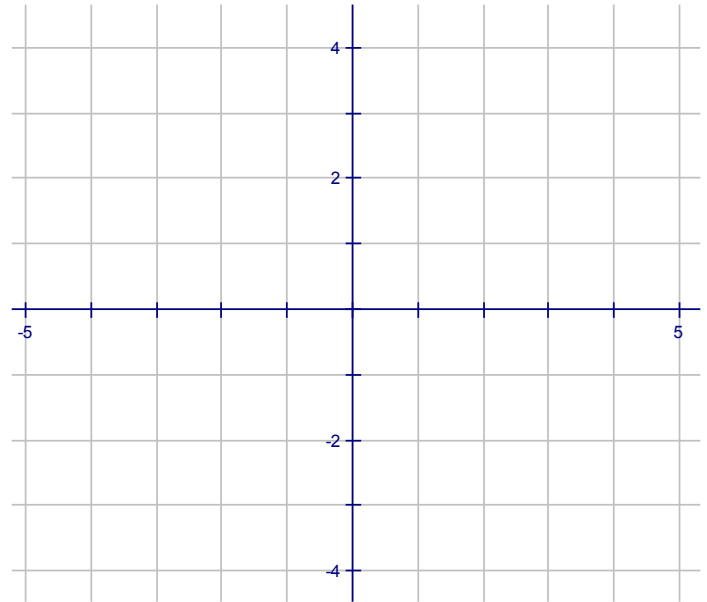
III. Determine the domain of each of the following.

1. $f(x) = \dfrac{-2x^2 - 6}{x^2 - x - 6}$ 2. $h(x) = \log_2(x^2 - 2x - 3)$ 3. $g(x) = \dfrac{\sqrt{x-4}}{|x|-5}$

IV. For the function $h(x) = \sqrt{x} + 1$, find $h^{-1}(x)$, find the domain and range of $h(x)$ and $h^{-1}(x)$, and graph the functions on the given axis.

$h^{-1}(x) = $_____

Domain of $h(x)$: _____   Range of $h(x)$: _____
Domain of $h^{-1}(x)$: _____   Range of $h^{-1}(x)$: _____

Sequences and Series

I. Problems:Be sure you use the arithmetic and geometric sequences and series formulas.
1. Insert 3 arithmetic means between 1 and 49.
2. Find the tenth term of the geometric sequence $11, -22, 44, -88$.
3. Find the sum of the first twenty-four terms of the arithmetic series $46+39+32+25+....$
4. Find the sum of the infinite geometric series $250+150+90+54+....$
5. Evaluate: (a) $\sum_{j=1}^{50}(3j-2)$ (b) $\sum_{n=5}^{11}2\cdot3^{n-1}$
6. Find the sum of the positive three-digit even integers.
7. The cost of an annual subscription to a magazine is $42 this year. The cost is expected to rise by 7.4% each year. What will be the cost 6 years **from now**?   (*be careful*)