



Algebra 2

**Graphing Calculator
and
Spreadsheet Masters**



**Glencoe
McGraw-Hill**

New York, New York
Columbus, Ohio
Chicago, Illinois
Peoria, Illinois
Woodland Hills, California

Glencoe/McGraw-Hill

A Division of The McGraw-Hill Companies



Copyright © by The McGraw-Hill Companies, Inc. All rights reserved.
Printed in the United States of America. Permission is granted to reproduce the material contained herein on the condition that such material be reproduced only for classroom use; be provided to students, teachers, and families without charge; and be used solely in conjunction with *Glencoe Algebra 2*. Any other reproduction, for use or sale, is prohibited without prior written permission of the publisher.

Send all inquiries to:
Glencoe/McGraw-Hill
8787 Orion Place
Columbus, OH 43240

ISBN: 0-07-828020-6

Glencoe Algebra 2
Graphing Calculator and Spreadsheet Masters

2 3 4 5 6 7 8 9 10 024 11 10 09 08 07 06 05 04 03

CONTENTS

RESOURCE MATERIALS

TI-83 Plus Graphing Calculator Template	1
Spreadsheet Template	3

TI-83 Plus Graphing Calculator Handbook

Introduction to the Graphing Calculator	5
Graphing Functions	8
Analyzing Functions.....	11
Graphing Inequalities	16
Matrices	19
Graphing Trigonometric Functions	21
Graphing Special Functions.....	22
Statistics and Statistical Graphs	23

GRAPHING CALCULATOR INVESTIGATIONS

Use with Lesson	Title	Page
1-3	Solving Equations and Checking Solutions	27
2-6	Transformations: Greatest Integer Function.....	29
3-4	Linear Programming.....	31
4-4	Matrices for 30°, 45°, and 60° Rotations	33
5-4	Using Tables to Factor by Grouping.....	35
6-7	Quadratic Inequalities and the Test Menu.....	37
7-6	Rational Root Theorem	39
8-3	Matrices and Equations of Circles.....	41
9-3	Horizontal Asymptotes and Tables.....	43
10-1	Regression Equation Lab	45
11-6	Recursion and Iteration	47
12-5	Probabilities.....	49
13-4	Law of Sines: Ambiguous Case	51
14-2	Sinusoidal Equations.....	53

SPREADSHEET INVESTIGATIONS

Use with Lesson	Title	Page
1-4	Absolute Value Statements	28
2-4	Using Linear Equations	30
3-1	Break-Even Point	32
4-5	Cramer's Rule	34
5-7	Appreciation and Depreciation	36
6-5	Approximating the Real Zeros of Polynomials	38
7-7	Operations on Functions	40
8-2	Parabolas	42
9-4	Variation	44
10-6	Net Present Value	46
11-2	Sequences and Series	48
12-2	Permutations And Combinations.....	50
13-1	Cofunctions	52
14-3	Trigonometric Identities	54
Answers		55

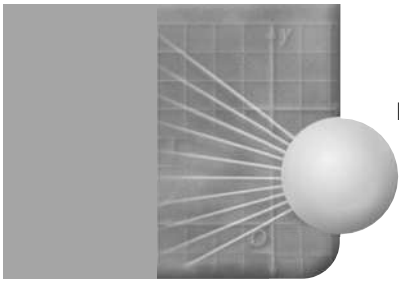
Teacher's Guide to Using the Graphing Calculator and Spreadsheet Masters

This booklet contains a TI-83 Plus Graphing Calculator Handbook, which summarizes many of the graphing calculator skills students might use in pre-algebra. This is a reference tool and does not contain additional exercises for students.

This booklet also includes a Graphing Calculator Investigation and a Spreadsheet Investigation for each chapter in *Glencoe Algebra 2*.

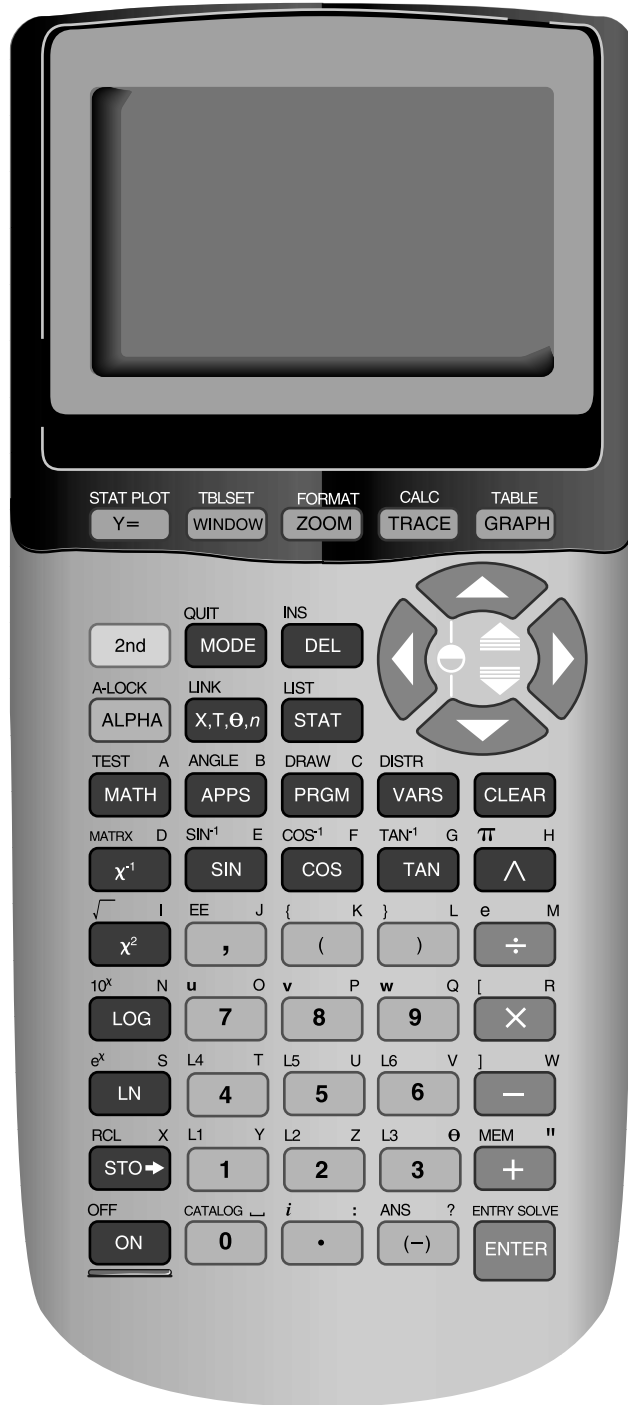
- The graphing calculator activities are written with TI-83 Plus keystrokes provided. If your students use another calculator, they will need to modify their keystrokes to complete the activity.
- The spreadsheet masters were developed for use with Microsoft® Excel. If you use a different spreadsheet application software, you may need to alter the commands given in the activity.

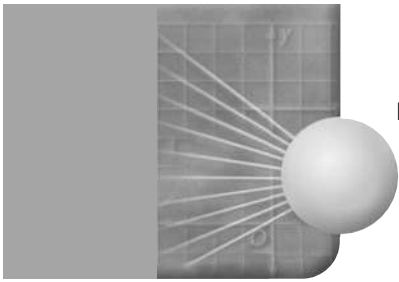
When to Use Each activity should be used as an extension of the lesson to which it is referenced. Because these activities can be done independently of classroom time, they may be used as extra credit or as enrichments for those students who have completed their assignments in a timely manner.



NAME _____ DATE _____ PERIOD _____

TI-83 Plus Graphing Calculator Template





NAME _____ DATE _____ PERIOD _____

Spreadsheet Template

The image shows a spreadsheet window with a grid. The columns are labeled A through F, and the rows are labeled 1 through 21. The window includes a title bar with standard window controls (minimize, maximize, close) and a scroll bar on the right side. The grid is currently empty.

	A	B	C	D	E	F
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						
21						

1

TI-83 Plus Graphing Calculator Handbook

Introduction to the Graphing Calculator

This section introduces you to some commonly-used keys and menus of the calculator.

Setting Preferences

MODE The **MODE** key allows you to select your preferences in many aspects of calculation and graphing. Many of these settings are rarely changed in common usage. This screen shows the default mode settings.

Normal	Sci Eng	← type of numeric notation
Float	0123456789	← number of decimal places in results
Radian	Degree	← unit of angle measure used
Func	Par Pol Seq	← type of graph (function, parametric, polar, sequence)
Connected	Dot	← whether to connect graphed points
Sequential	Simul	
Real	a+bi re ^{θi}	← real, rectangular complex, or polar complex number system
Full	Horiz G-T	← graph occupies full screen, top of screen with HOME screen below, or left side of screen with TABLE on right

To change the preferences, use the arrow keys to highlight your choice and press **ENTER**.

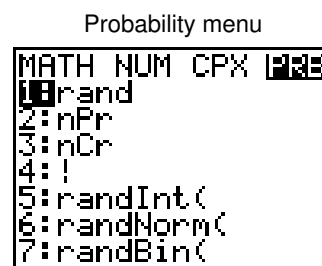
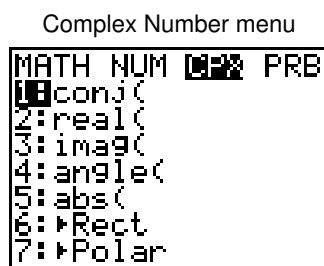
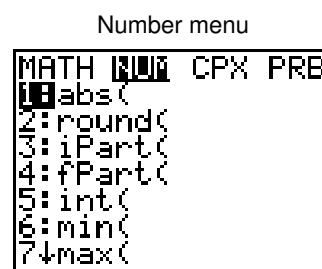
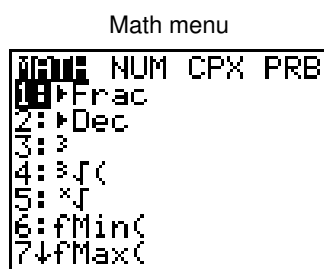
FORMAT The **FORMAT** menu is the second function of **ZOOM** and sets preferences for the appearance of your graphing screen. The default screen is shown below.

RectGC	PolarGC	← rectangular or polar coordinate system
CoordOn	CoordOff	← whether to display the cursor coordinates on screen
GridOff	GridOn	← whether to show a grid pattern on screen
AxesOn	AxesOff	← whether to show the axes
LabelOff	LabelOn	← whether to label the axes
ExprOn	ExprOff	← whether to show the equation being graphed

You can change your preferences in the **FORMAT** menu in the same way you change **MODE** settings.

Using Menus

Many keys on the calculator access menus from which you can select a function, command, or setting. Some keys access multiple menus. You can use the right and left arrow keys to scroll through the different menu names located at the top of the screen. As each menu name is highlighted, the choices on the screen change. The screens on the next page show various menus accessed by using **MATH**.



To select a choice in a menu, either use the arrow keys to highlight your choice and press **ENTER** or simply press the number or letter of your selection. Notice that entry 7 in the first screen has a down arrow instead of a colon after the 7. This signifies there are more entries in the menu.

Alternate Function Keys

Whenever an alternate function is indicated in the keystrokes of this appendix, we will use brackets to show that the function is listed above a key.

Above most keys are one or two additional labels representing commands, menus, letters, lists, or operational symbols. These are accessed by using **2nd** or **ALPHA**.

- **2nd** accesses the commands on the left above the key. Note that these commands and **2nd** are the same color.
- **ALPHA** accesses the commands on the right above each key. These commands and **ALPHA** are also the same color.
- Pressing **2nd** **ALPHA** engages the **[A-LOCK]** or Alpha Lock command. This enables you to select consecutive **ALPHA** commands without pressing **ALPHA** before each command. This is especially useful when entering programs.

Each letter accessed by using **ALPHA** can be used to enter words or labels on the screen, but can also be used as a variable. A value can be stored to each variable.

Computation

A graphing calculator is also a scientific calculator. That is, it follows the order of operations when evaluating entries. Unlike some scientific calculators, the graphing calculator displays every entry in the expression.

Before pressing **ENTER** to evaluate the expression, you can use the arrow keys to scroll through the expression to make corrections. Corrections can be made in three ways.

- Use **DEL** to delete any unwanted entries.
- Use **2nd** **[INS]** to insert omitted entries.
- “Type” over an incorrect entry. This overprints any entries and does not shift the entries to the right as a word processor does.

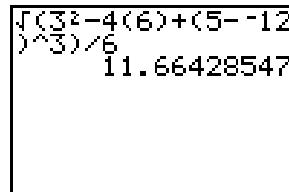
If you have an expression that you wish to evaluate repeatedly with a change in one part of the expression, you can press $\boxed{2\text{nd}} \boxed{\text{ENTRY}}$ after you have pressed $\boxed{\text{ENTER}}$ and the expression will reappear. You can edit it for your next computation. The **ENTRY** command always repeats the last entered expression. You cannot scroll back through previous expressions you have evaluated.

Example 1 Evaluate $\frac{\sqrt{3^2 - 4(6) + [5 - (-12)]^3}}{6}$.

The minus key and the negative key are different keys.

Press: $\boxed{2\text{nd}} \boxed{\sqrt{}} \boxed{3} \boxed{x^2} \boxed{-} \boxed{4} \boxed{(} \boxed{6} \boxed{)} \boxed{+} \boxed{(} \boxed{5} \boxed{-} \boxed{(} \boxed{-} \boxed{12} \boxed{)} \boxed{)} \boxed{\wedge} \boxed{3} \boxed{)} \boxed{\div} \boxed{6} \boxed{\text{ENTER}}$

Note that the square root function automatically includes a left parenthesis. You must enter the right parenthesis to indicate the end of the expression under the radical sign. If you have the decimal in the **Float** mode, as many as 10 digits may appear in the answer.



2 Evaluate each expression if $a = 4$, $b = -5$, $c = 2$, $d = \frac{2}{3}$, and $e = -1.5$.

a. $abc - 3de^4$

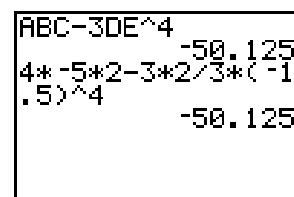
b. $\frac{e + 4a}{c^2 + 8b}$

For a series of expressions that use the same values for the variables, it is often helpful to store the value for each variable into the calculator. You can combine several commands in one line by using the colon after each command. The following commands save the values for variables a , b , c , d and e .

Press: $\boxed{4} \boxed{\text{STO}} \boxed{\text{ALPHA}} \boxed{\text{A}} \boxed{\text{ALPHA}} \boxed{:} \boxed{(-)} \boxed{5} \boxed{\text{STO}} \boxed{\text{ALPHA}} \boxed{\text{B}} \boxed{\text{ALPHA}} \boxed{:} \boxed{2} \boxed{\text{STO}} \boxed{\text{ALPHA}} \boxed{\text{C}} \boxed{\text{ALPHA}} \boxed{:} \boxed{2} \boxed{\div} \boxed{3} \boxed{\text{STO}} \boxed{\text{ALPHA}} \boxed{\text{D}} \boxed{\text{ALPHA}} \boxed{:} \boxed{(-)} \boxed{1.5} \boxed{\text{STO}} \boxed{\text{ALPHA}} \boxed{\text{E}} \boxed{\text{ENTER}}$

a. **Method 1:** Using stored values

$\boxed{\text{ALPHA}} \boxed{\text{A}} \boxed{\text{ALPHA}} \boxed{\text{B}} \boxed{\text{ALPHA}} \boxed{\text{C}} \boxed{-} \boxed{3} \boxed{\text{ALPHA}} \boxed{\text{D}} \boxed{\text{ALPHA}} \boxed{\text{E}} \boxed{\wedge} \boxed{4} \boxed{\text{ENTER}}$

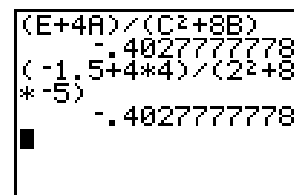


Method 2: Entering computations

$\boxed{4} \boxed{\times} \boxed{(-)} \boxed{5} \boxed{\times} \boxed{2} \boxed{-} \boxed{3} \boxed{\times} \boxed{2} \boxed{\div} \boxed{3} \boxed{\times} \boxed{(} \boxed{(-)} \boxed{1.5} \boxed{)} \boxed{\wedge} \boxed{4} \boxed{\text{ENTER}}$

b. **Method 1:** Using stored values

$\boxed{(} \boxed{\text{ALPHA}} \boxed{\text{E}} \boxed{+} \boxed{4} \boxed{\text{ALPHA}} \boxed{\text{A}} \boxed{)} \boxed{\div} \boxed{(} \boxed{\text{ALPHA}} \boxed{\text{C}} \boxed{x^2} \boxed{+} \boxed{8} \boxed{\text{ALPHA}} \boxed{\text{B}} \boxed{)} \boxed{\text{ENTER}}$



Method 2: Entering computations

$\boxed{(} \boxed{(-)} \boxed{1.5} \boxed{+} \boxed{4} \boxed{\times} \boxed{4} \boxed{)} \boxed{\div} \boxed{(} \boxed{2} \boxed{x^2} \boxed{+} \boxed{8} \boxed{\times} \boxed{(-)} \boxed{5} \boxed{)} \boxed{\text{ENTER}}$

2

TI-83 Plus Graphing Calculator Handbook

Graphing Functions

Most functions can be graphed by using the $\boxed{Y=}$ key. The viewing window most often used for non-trigonometric functions is the standard viewing window $[-10, 10]$ scl:1 by $[-10, 10]$ scl:1, which can be accessed by selecting **6:ZStandard** on the **ZOOM** menu. Then the window can be adjusted so that a **complete graph** can be viewed. A complete graph is one that shows the basic characteristics of the parent graph.

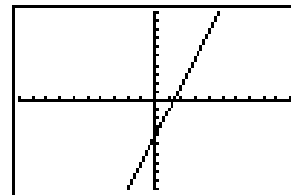
Example 1 Linear Functions A complete linear graph shows the x - and y -intercepts.

a. Graph $y = 3x - 4$ in the standard viewing window.

Press: $\boxed{Y=}$ 3 $\boxed{X,T,\theta,n}$ $\boxed{-}$ 4 \boxed{ZOOM} 6

If your calculator is already set for the standard viewing window, press \boxed{GRAPH} instead of \boxed{ZOOM} 6.

Both the x - and y -intercepts of the linear graph are viewable in this window, so the graph is complete.



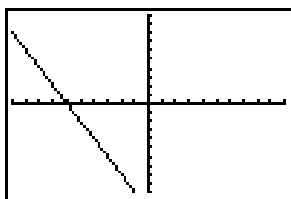
$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1

b. Graph $y = -2(x + 5) - 2$.

Press: $\boxed{Y=}$ $\boxed{(-)}$ 2 $\boxed{(}$ $\boxed{X,T,\theta,n}$ $\boxed{+}$ 5 $\boxed{)}$ $\boxed{-}$ 2 \boxed{GRAPH}

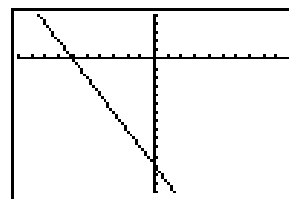
When this equation is graphed in the standard viewing window (Figure 1), a complete graph is not visible. The graph indicates that the y -intercept is less than -10 . You can experiment with the **Ymin** setting or you can rewrite the equation in $y = mx + b$ form, which would be $y = -2x - 12$. The y -intercept is -12 , so **Ymin** should be less than -12 . Remember that **Xmax** and **Ymax** can be less than 10 so that your screen is less compressed. Use the **WINDOW** menu to change the parameters, or settings, and press \boxed{GRAPH} to view the result. There are many windows that will enable you to view the complete graph (Figure 2).

Figure 1



$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1

Figure 2



$[-10, 10]$ scl:1 by $[-15, 5]$ scl:1

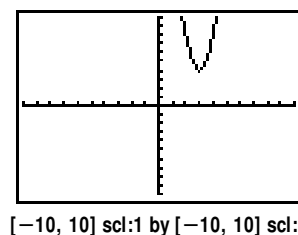
2 Quadratic Functions When graphing quadratic functions, a complete graph includes the vertex of the parabola and enough of the graph to determine if it opens upward or downward.

(continued on the next page)

Graph $y = 4(x - 3)^2 + 4$.

Press: $\boxed{Y=}$ $\boxed{4}$ $\boxed{(}$ $\boxed{X,T,\theta,n}$ $\boxed{-}$ $\boxed{3}$ $\boxed{)}$ $\boxed{x^2}$ $\boxed{+}$ $\boxed{4}$ \boxed{GRAPH}

While the standard viewing window shows a complete graph, you may want to change the viewing window to see more of the graph.

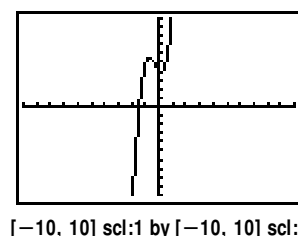


Example 3 Polynomial Functions The graphs of other polynomial functions are complete when their maxima, minima, and x -intercepts are visible.

a. Graph $y = 5x^3 + 4x^2 - 2x + 4$.

Press: $\boxed{Y=}$ $\boxed{5}$ $\boxed{X,T,\theta,n}$ $\boxed{\wedge}$ $\boxed{3}$ $\boxed{+}$ $\boxed{4}$ $\boxed{X,T,\theta,n}$ $\boxed{x^2}$ $\boxed{-}$ $\boxed{2}$ $\boxed{X,T,\theta,n}$ $\boxed{+}$ $\boxed{4}$ \boxed{GRAPH}

A complete graph is shown in the standard viewing window. You may want to redefine your window to observe the intercepts, maximum, and minimum points more closely.



b. Graph $y = x^4 - 13x^2 + 36$.

Press: $\boxed{Y=}$ $\boxed{X,T,\theta,n}$ $\boxed{\wedge}$ $\boxed{4}$ $\boxed{-}$ $\boxed{13}$ $\boxed{X,T,\theta,n}$ $\boxed{x^2}$ $\boxed{+}$ $\boxed{36}$ \boxed{GRAPH}

The standard viewing window (Figure 1) does not show a complete graph. It seems that only the y parameters need to be adjusted. Experiment to find a window that is suitable. Figure 2 shows a sample.

Figure 1

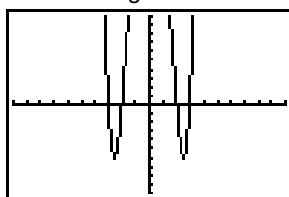
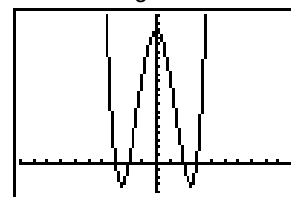


Figure 2



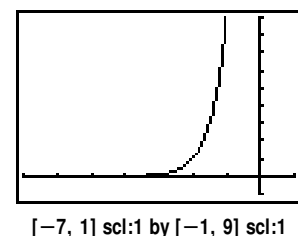
4 Exponential Functions A complete graph of an exponential function shows the curvature of the graph and the y -value that it approaches.

Graph $y = 9^2 + x$.

Press: $\boxed{Y=}$ $\boxed{9}$ $\boxed{\wedge}$ $\boxed{(}$ $\boxed{2}$ $\boxed{+}$ $\boxed{X,T,\theta,n}$ $\boxed{)}$ \boxed{GRAPH}

Note that you must use parentheses to group the terms that make up the exponent.

A complete graph seems to appear in the second quadrant of the standard viewing window. Vary the **WINDOW** settings to view the graph more closely.

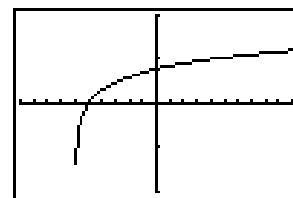


Example 5 Logarithmic Functions A complete graph of a logarithmic function shows the curvature of the graph and the values, or locations, of the asymptotes that the curve approaches.

a. Graph $y = \log(x + 6)$.

Press: $\boxed{Y=}$ $\boxed{\text{LOG}}$ $\boxed{X,T,\theta,n}$ $\boxed{+}$ $\boxed{6}$ $\boxed{)}$
 $\boxed{\text{GRAPH}}$

An entire graph appears in the standard viewing window, but is very small. Redefine the **WINDOW** parameters for y , so that the graph is more visible.



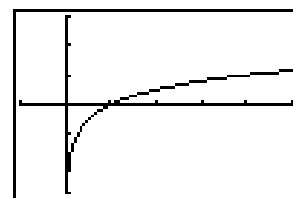
$[-10, 10]$ scl:1 by $[-2, 2]$ scl:1

b. Graph $y = \log_4 x$.

To graph a logarithmic function with a base other than 10, you must first change the function by using the change of base formula,
 $\log_a x = \frac{\log x}{\log a}$.

Press: $\boxed{Y=}$ $\boxed{\text{LOG}}$ $\boxed{X,T,\theta,n}$ $\boxed{)}$ $\boxed{\div}$ $\boxed{\text{LOG}}$
 $\boxed{4}$ $\boxed{)}$ $\boxed{\text{GRAPH}}$

An entire graph appears in the standard viewing window, but is very small. Redefine the **WINDOW** parameters, so that the graph is more visible.



$[-1, 5]$ scl:1 by $[-3, 3]$ scl:1

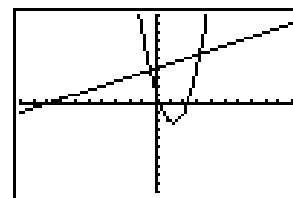
You can graph multiple functions on a single screen. Each function is denoted by $Y_1=$, $Y_2=$, $Y_3=$, and so on, in the $Y=$ menu. To graph more than one function, press $\boxed{\text{ENTER}}$ at the end of each function you are entering and the cursor will move to the next function to be entered.

Example 6 Systems of Equations

Graph $y = 0.5x + 4$ and $y = 2x^2 - 5x + 1$.

Press: $\boxed{Y=}$ $\boxed{0.5}$ $\boxed{X,T,\theta,n}$ $\boxed{+}$ $\boxed{4}$ $\boxed{\text{ENTER}}$ $\boxed{2}$ $\boxed{X,T,\theta,n}$ $\boxed{x^2}$
 $\boxed{-}$ $\boxed{5}$ $\boxed{X,T,\theta,n}$ $\boxed{+}$ $\boxed{1}$ $\boxed{\text{GRAPH}}$

The standard viewing window shows that the line and parabola intersect in two points.



$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1

3

TI-83 Plus Graphing Calculator Handbook

Analyzing Functions

In addition to graphing a function, you can use other tools on a graphing calculator to analyze functions. One of those tools is a function table.

Example 1 How to Use a Table You may complete a table manually or automatically. To create a table for one or more functions, you must first enter each function into the **Y=** list. Then set up and create the table.

The function values are the dependent variable values.

a. Use a table to evaluate the function $y = 4x^2 - 2x + 7$ for $\{-9, -4, 0, 1, 5\}$.

In this case you only need to evaluate the function for selected values, so use the **TBLSET** menu to have the calculator ask for the values of the independent variable (domain) and find the function value (range) automatically.

Press: $\boxed{Y=}$ $\boxed{4}$ $\boxed{X,T,\theta,n}$ $\boxed{x^2}$ $\boxed{-}$ $\boxed{2}$ $\boxed{X,T,\theta,n}$ $\boxed{+}$
 $\boxed{7}$ $\boxed{2nd}$ $\boxed{[TBLSET]}$ $\boxed{\blacktriangledown}$ $\boxed{\blacktriangledown}$ $\boxed{\blacktriangleright}$ \boxed{ENTER}
 $\boxed{2nd}$ $\boxed{[TABLE]}$ $\boxed{(-)}$ $\boxed{9}$ \boxed{ENTER} $\boxed{(-)}$ $\boxed{4}$
 \boxed{ENTER} $\boxed{0}$ \boxed{ENTER} $\boxed{1}$ \boxed{ENTER} $\boxed{5}$ \boxed{ENTER}

X	Y1	
-9	349	
-4	99	
0	7	
1	97	
5	97	
X=		

b. Use a table to evaluate the functions $y = 5x^2 - x + 1$ and $y = 6 - x^3$ for the integers from -3 to 3 , inclusive.

When you want to evaluate a function for a range of values, have the calculator find both the values of the independent variable and the function values automatically. In **Table Setup**, enter the initial number of the domain as the **TblStart** value and the increment between the values of the independent variable as ΔTbl . Entering more than one function in the **Y=** list allows you to evaluate all of the functions in one table.

Press: $\boxed{Y=}$ $\boxed{5}$ $\boxed{X,T,\theta,n}$ $\boxed{x^2}$ $\boxed{-}$ $\boxed{X,T,\theta,n}$ $\boxed{+}$
 $\boxed{1}$ \boxed{ENTER} $\boxed{6}$ $\boxed{-}$ $\boxed{X,T,\theta,n}$ $\boxed{\wedge}$ $\boxed{3}$ $\boxed{2nd}$
 $\boxed{[TBLSET]}$ $\boxed{(-)}$ $\boxed{3}$ \boxed{ENTER} $\boxed{1}$ \boxed{ENTER}
 \boxed{ENTER} $\boxed{2nd}$ $\boxed{[TABLE]}$

X	Y1	Y2
-3	49	33
-2	23	14
-1	7	7
0	1	6
1	4	5
2	19	2
3	43	-21
X = -3		

Once you create a table, you can scroll through the values using the arrow keys.

ZOOM allows you to quickly adjust the viewing window of a graph in different ways. The effect of each choice on the **ZOOM** menu is shown on the next page.

- 1: ZBox** Allows you to draw a box to define the viewing window
- 2: Zoom In** Magnifies the graph around the cursor
- 3: Zoom Out** Views more of a graph around the cursor
- 4: ZDecimal** Sets ΔX and ΔY to 0.1
- 5: ZSquare** Sets equal-sized pixels on the x - and y -axes
- 6: ZStandard** Sets the standard viewing window, $[-10, 10]$ scl:1 by $[-10, 10]$ scl:1
- 7: ZTrig** Sets the built-in trig window, $[-\frac{47}{24}\pi, \frac{47}{24}\pi]$ scl: $\frac{\pi}{2}$ by $[-4, 4]$ scl:1 for radians or $[-352.5, 352.5]$ scl:90 by $[-4, 4]$ scl:1 for degrees
- 8: ZInteger** Sets integer values on both the x - and y -axes
- 9: ZoomStat** Sets values for displaying all of the data in the current stat lists
- 0: ZoomFit** Fits Ymin and Ymax to show all function values for Xmin to Xmax

Example 2 Using **ZOOM** to Graph in the Standard and Square Windows

Graph the circle with equation $x^2 + y^2 = 16$ in the standard viewing window. Then use ZSquare to view the graph in a square screen.

First solve the equation for y in order to enter it into the **Y=** list.

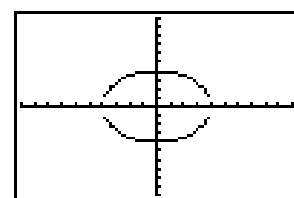
$$x^2 + y^2 = 16 \rightarrow y = \pm \sqrt{16 - x^2}$$

The two pieces of the graph can be entered at one time using $\{-1, 1\}$. This expression tells the calculator to graph -1 and 1 times the function.

Press: **Y=** **2nd** **[]** **(-)** **1** **,** **1** **2nd** **[]** **2nd** **[√]** **]** **16** **-** **X,T,θ,n** **x²** **)**
ZOOM **6**

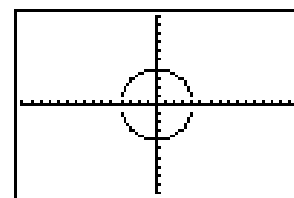
The circle is distorted when viewed in the standard viewing window.

Press: **ZOOM** **5**



$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1

Using **ZSquare** makes the circle appear as a circle.



$[-15.16, 15.16]$ scl:1 by $[-10, 10]$ scl:1

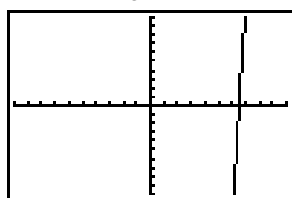
Example 3 Using **ZOOM** to Zoom In and Out Graph $y = 0.5x^3 - 3x^2 - 12$ in the standard viewing. Zoom out to view a complete graph. Then zoom in to approximate the y-intercept of the graph to the nearest whole number.

Press: $Y=$ 0.5 X,T,θ,n \wedge 3 $-$ 3 X,T,θ,n x^2 $-$ 12 **ZOOM** 6

The complete graph is not shown in the standard viewing window. (Figure 1) When you zoom out or in, the calculator allows you to choose the point around which it will zoom. Zooming out around the origin once allows a complete graph to be shown. (Figure 2)

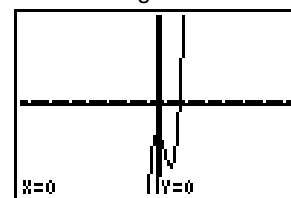
Press: **ZOOM** 3 **ENTER**

Figure 1



$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1

Figure 2

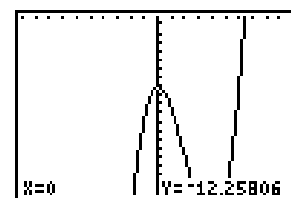


$[-40, 40]$ scl:1 by $[-40, 40]$ scl:1

Now zoom in to approximate the y-intercept. Choose a point close to the intercept by using the arrow keys.

Press: **ZOOM** 2 **ENTER**

The y-intercept appears to be about -12 . Zooming in again may allow you to make a closer approximation.



$[-10, 10]$ scl:1 by $[-24.19, -4.19]$ scl:1

The **TRACE** feature allows you to move the cursor along a graph and display the coordinates of the points on the graph.

Example 4 Using **TRACE** Graph $y = 4x + 2$ and $y = -3x^2 - x + 5$. Use the **TRACE** feature to approximate the coordinates of the intersection of the graphs in the first quadrant. Then evaluate $y = -3x^2 - x + 5$ for $x = 1.7$.

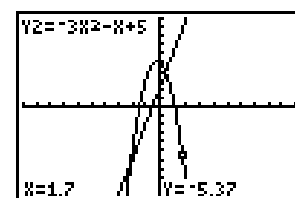
*Make sure that **CoordOn** is highlighted in the **FORMAT** menu to display the cursor coordinates as you trace.*

Press: $Y=$ 4 X,T,θ,n $+$ 2 **ENTER** $(-)$ 3 X,T,θ,n x^2 $-$ X,T,θ,n $+$ 5 **TRACE**

Move the cursor along the graphs using \blacktriangleleft and \blacktriangleright .

Pressing 2^{nd} \blacktriangleleft or 2^{nd} \blacktriangleright moves the cursor more quickly. If your cursor moves off of the screen, the calculator will automatically update the viewing window so that the cursor is visible. Use \blacktriangledown and \blacktriangleup to move from one function to the other. The intersection is at about $(0.4, 4)$.

To evaluate a function for a value and move to that point, place the cursor on the function graph. Then enter the value and press **ENTER**. When $x = 1.7, y = -5.37$ for $y = -3x^2 - x + 5$.



$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1

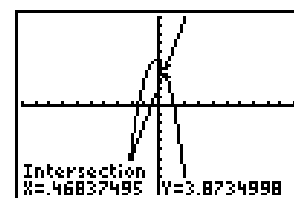
Using **TRACE** to locate the intersection points of the graphs of two functions gives you an approximation of the coordinates. For more accurate coordinates, you can use the **intersect** option on the **CALC** menu.

Example 5 Finding Intersection Points Use **5:intersect** on the **CALC** menu to find the coordinates of the intersection of the graphs of $y = 4x + 2$ and $y = -3x^2 - x + 5$.

If you do not have the functions graphed, enter the functions into the **Y=** list and press **GRAPH**. Then find the coordinates of the intersection.

Press: **2nd** **[CALC]** **5**

Place the cursor on one graph and press **ENTER**. Then move the cursor to the other graph and press **ENTER**. To guess at the intersection or enter an x -value and press **ENTER**. If there is more than one intersection point, the calculator will find the one closest to your guess. The cursor will move to the intersection point and the coordinates will be displayed.



$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1

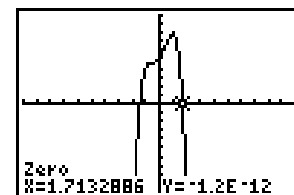
*The intersection of the graphs must appear on the screen to find the coordinates when using **intersect**.*

The **CALC** menu also allows you to find the zeros of a function.

Example 6 Finding Zeros Find the zeros of $f(x) = -2x^4 + 3x^2 + 2x + 5$.

Press: **Y=** **(-)** **2** **[X,T,θ,n]** **^** **4** **+** **3** **[X,T,θ,n]** **x²** **+** **2** **[X,T,θ,n]** **+** **5** **2nd** **[CALC]** **2**

The calculator can find one zero at a time. Use the arrow keys or enter a value to choose the left bound for the interval in which the calculator will search for the zero and press **ENTER**. Choose the right bound and press **ENTER**. Select a point near the zero using the arrow keys or by entering a value and press **ENTER**. Repeat with another interval to find the other zero. The zeros of this function are about -1.42 and 1.71 .



$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1

Real-world application problems often require you to find the relative minimum or maximum of a function. You can use **3:minimum** or **4:maximum** features on the **CALC** menu of a graphing calculator to solve these problems.

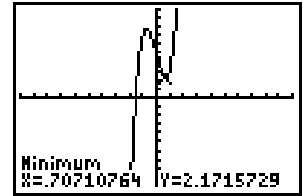
Example 7 Finding Maxima and Minima Determine the relative minimum and the relative maximum for the graph of $f(x) = 4x^3 - 6x + 5$.

First graph the function.

Press: $\boxed{Y=}$ 4 $\boxed{X,T,\theta,n}$ $\boxed{\wedge}$ 3 $\boxed{-}$ 6 $\boxed{X,T,\theta,n}$ $\boxed{+}$ 5 \boxed{ZOOM} 6

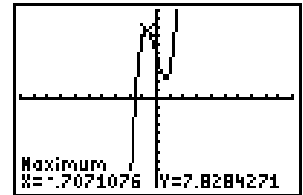
To find the relative minimum press $\boxed{2nd}$ $\boxed{[CALC]}$ 3.

Similar to finding a zero, choose the left and right bound of the interval and guess the minimum or the maximum. The point at about (0.71, 2.17) is a relative minimum.



[-10, 10] scl:1 by [-10, 10] scl:1

Use a similar method to find the relative maximum, by pressing $\boxed{2nd}$ $\boxed{[CALC]}$ 4. The point at about (-0.71, 7.83) is a relative maximum.



[-10, 10] scl:1 by [-10, 10] scl:1



TI-83 Plus Graphing Calculator Handbook

Graphing Inequalities

Most linear and nonlinear inequalities can be graphed using the $\boxed{Y=}$ key and selecting the appropriate graph style in the $Y=$ editor. To select the appropriate graph style, select the graph style icon in the first column of the $Y=$ editor and press $\boxed{\text{ENTER}}$ repeatedly to rotate through the graph styles.

- To shade the area above a graph, select the Above style icon, \blacktriangledown .
- To shade the area below a graph, select the Below style icon, \blacktriangle .

Before graphing an inequality, clear any functions in the $Y=$ list by pressing $\boxed{Y=}$ and then using the arrow keys and the $\boxed{\text{CLEAR}}$ key to select and clear all functions. If you do not wish to clear a function, you can turn that particular graph off by using the arrow keys to position the cursor over that function's = sign and then pressing $\boxed{\text{ENTER}}$ to change the selection status.

Example 1 Linear Inequalities

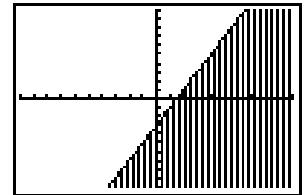
a. Graph $y \leq 2x - 3$ in the standard viewing window.

First enter the boundary equation $y = 2x - 3$ into the $Y=$ list.

Press: $\boxed{Y=}$ $\boxed{2}$ $\boxed{X,T,\theta,n}$ $\boxed{-}$ $\boxed{3}$

Next, press the \blacktriangleleft key until the icon before = flashes. Press $\boxed{\text{ENTER}}$ until the icon changes to the Below style icon, \blacktriangle , for “ $y \leq$ ”.

Finally, if your calculator is not already set for the standard viewing window, press $\boxed{\text{ZOOM}}$ 6. Otherwise, press $\boxed{\text{GRAPH}}$.



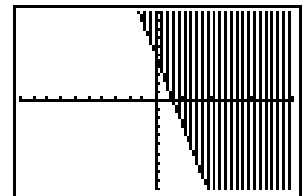
$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1

b. Graph $y \geq -4x + 5$ in the standard viewing window.

Press: $\boxed{Y=}$ $\boxed{(-)}$ $\boxed{4}$ $\boxed{X,T,\theta,n}$ $\boxed{+}$ $\boxed{5}$

Next, press the \blacktriangleleft key until the icon before = flashes.

Then press $\boxed{\text{ENTER}}$ until the icon changes to the Above style icon, \blacktriangledown , since the inequality asks for “ $y \geq$ ”. Finally, press $\boxed{\text{GRAPH}}$.



$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1

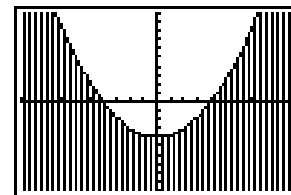
Example 2 Nonlinear Inequalities The procedure for graphing nonlinear inequalities is the same as that of graphing linear inequalities.

a. Graph $y \leq 0.25x^2 - 4$ in the standard viewing window.

Press: $\boxed{Y=}$ $\boxed{0.25}$ $\boxed{X,T,\theta,n}$ $\boxed{x^2}$ $\boxed{-}$ $\boxed{4}$

Next, select the Below style icon, \blacktriangledown , since the inequality asks for “ $y \leq$ ”, and then press

$\boxed{\text{GRAPH}}$.



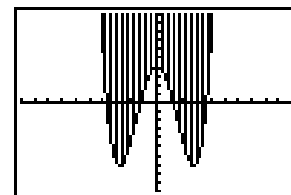
$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1

b. Graph $y \geq 0.2x^4 - 3x^2 + 4$.

Press: $\boxed{Y=}$ $\boxed{0.2}$ $\boxed{X,T,\theta,n}$ $\boxed{\wedge}$ $\boxed{4}$ $\boxed{-}$
 $\boxed{X,T,\theta,n}$ $\boxed{\wedge}$ $\boxed{2}$ $\boxed{+}$ $\boxed{4}$

Next, select the Above style icon, \blacktriangledup , since the inequality asks for “ $y \geq$ ”. Then press

$\boxed{\text{GRAPH}}$.



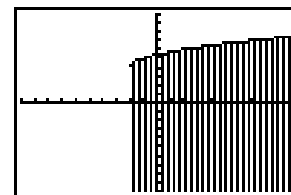
$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1

c. Graph $y \leq \sqrt{x + 2} + 4$.

Press: $\boxed{Y=}$ $\boxed{2nd}$ $\boxed{[\sqrt{\quad}]}$ $\boxed{X,T,\theta,n}$ $\boxed{+}$ $\boxed{2}$ $\boxed{)}$ $\boxed{+}$ $\boxed{4}$

Next, select the Below style icon, \blacktriangledown , since the inequality asks for “ $y \leq$ ”. Then press

$\boxed{\text{GRAPH}}$.



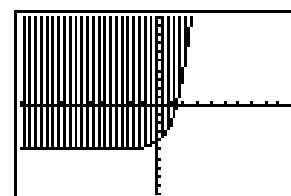
$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1

d. Graph $y \geq 3^x - 5$.

Press: $\boxed{Y=}$ $\boxed{3}$ $\boxed{\wedge}$ $\boxed{X,T,\theta,n}$ $\boxed{-}$ $\boxed{5}$

Next, select the Above style icon, \blacktriangledup , since the inequality asks for “ $y \geq$ ”. Then press

$\boxed{\text{GRAPH}}$.



$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1

Graphing systems of inequalities on a graphing calculator is similar to graphing systems of equations.

Example 3 Graph the system of inequalities.

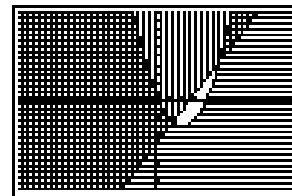
$$y \geq 2x - 5$$

$$y \leq x^2 - 4x + 1$$

Method 1: Shading Options in Y=

Press: $\boxed{Y=}$ $\boxed{2}$ $\boxed{X,T,\theta,n}$ $\boxed{-}$ $\boxed{5}$ $\boxed{\text{ENTER}}$ $\boxed{X,T,\theta,n}$ $\boxed{x^2}$
 $\boxed{-}$ $\boxed{4}$ $\boxed{X,T,\theta,n}$ $\boxed{+}$ $\boxed{1}$

Select the Above style icon, \blacktriangledown , for $y \geq 2x - 5$ and the Below style icon, \blacktriangle , for $y \leq x^2 - 4x + 1$. Then press $\boxed{\text{GRAPH}}$.



$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1

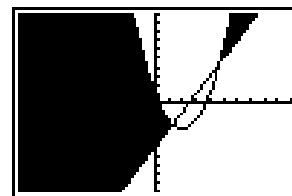
Notice that the first inequality is indicated using vertical lines and the second inequality uses horizontal lines. The solution to the system is shown by the intersection of the shaded areas.

Method 2: Using the Shade Command

Some systems of inequalities can be graphed by using the **Shade(** command and entering a function for a lower boundary and a function for the upper boundary of the inequality. The calculator first graphs both functions and then shades above the first function entered and below the second function entered.

Before graphing an inequality using the **Shade(** command, clear any graphics from the viewing window by pressing $\boxed{2nd}$ $\boxed{\text{DRAW}}$ $\boxed{1}$ $\boxed{\text{ENTER}}$. Also clear any equations in the **Y=** list. If not already there, return to the home screen by pressing $\boxed{2nd}$ $\boxed{\text{QUIT}}$.

Press: $\boxed{2nd}$ $\boxed{\text{DRAW}}$ $\boxed{7}$ $\boxed{2}$ $\boxed{X,T,\theta,n}$ $\boxed{-}$ $\boxed{5}$ $\boxed{,}$ $\boxed{X,T,\theta,n}$
 $\boxed{x^2}$ $\boxed{-}$ $\boxed{4}$ $\boxed{X,T,\theta,n}$ $\boxed{+}$ $\boxed{1}$ $\boxed{)}$ $\boxed{\text{ENTER}}$



$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1

The Shade(command can only be used with two inequalities which can be written with “ $y \leq$ ” in one inequality and “ $y \geq$ ” in the other.

5 TI-83 Plus Graphing Calculator Handbook

Matrices

A graphing calculator can perform operations with matrices. It can also find determinants and inverses of matrices. The **MATRIX** menus are accessed using $\boxed{2\text{nd}} \boxed{[\text{MATRIX}]}$.

There are three menus in the **MATRIX** menu.

- The **NAMES** menu lists the matrix locations available. There are ten matrix variables, [A] through [J].
- The **MATH** menu lists the matrix functions available.
- The **EDIT** menu allows you to define matrices.

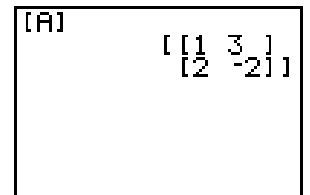
A matrix with dimension 2×3 indicates a matrix with 2 rows and 3 columns. Depending on available memory, a matrix may have up to 99 rows or columns.

Example 1 Entering a Matrix Enter matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$.

To enter a matrix into your calculator, choose the **EDIT** menu and select the matrix name. Then enter the dimensions and elements of the matrix.

Press: $\boxed{2\text{nd}} \boxed{[\text{MATRIX}]} \boxed{\leftarrow} \boxed{\text{ENTER}} \boxed{2} \boxed{\text{ENTER}}$
 $\boxed{2} \boxed{\text{ENTER}} \boxed{1} \boxed{\text{ENTER}} \boxed{3} \boxed{\text{ENTER}} \boxed{2} \boxed{\text{ENTER}} \boxed{(-)} \boxed{2}$

Press $\boxed{2\text{nd}} \boxed{[\text{QUIT}]}$ to return to the **HOME** screen. Then press $\boxed{2\text{nd}} \boxed{[\text{MATRIX}]} \boxed{\text{ENTER}}$
 $\boxed{\text{ENTER}}$ to display the matrix.

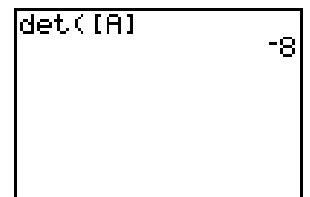


You can find the determinant and inverse of a matrix very quickly with a graphing calculator.

Example 2 Determinant and Inverse of a Matrix

a. Find the determinant of matrix A.

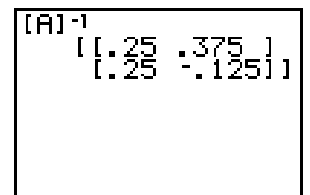
Press: $\boxed{2\text{nd}} \boxed{[\text{MATRIX}]} \boxed{\rightarrow} \boxed{1} \boxed{2\text{nd}} \boxed{[\text{MATRIX}]} \boxed{1} \boxed{\text{ENTER}}$



The determinant of matrix A is -8 .

b. Find the inverse of matrix A.

Press: $\boxed{2\text{nd}} \boxed{[\text{MATRIX}]} \boxed{1} \boxed{X^{-1}} \boxed{\text{ENTER}}$



$$A^{-1} = \begin{bmatrix} 0.25 & 0.375 \\ 0.25 & -0.125 \end{bmatrix}$$

Example 3 Operations with Matrices

Enter matrix $B = \begin{bmatrix} 2 & 3 & 6 \\ 4 & -8 & 5 \end{bmatrix}$. Then perform each operation.

a. $\frac{1}{2}B$

First, enter matrix B .

Press: $\boxed{2\text{nd}} \boxed{[\text{MATRX}]} \boxed{\leftarrow} \boxed{2} \boxed{2} \boxed{\text{ENTER}}$
 $\boxed{3} \boxed{\text{ENTER}} \boxed{2} \boxed{\text{ENTER}} \boxed{3} \boxed{\text{ENTER}} \boxed{6} \boxed{\text{ENTER}}$
 $\boxed{4} \boxed{\text{ENTER}} \boxed{(-)} \boxed{8} \boxed{\text{ENTER}} \boxed{5} \boxed{2\text{nd}} \boxed{[\text{QUIT}]}$

```
MATRIX[B] 2 x3
[[ 2  3  6 ]
 [ 4 -8  5 ]
2,3=5
```

Then find $\frac{1}{2}B$.

Press: $\boxed{.5} \boxed{2\text{nd}} \boxed{[\text{MATRX}]} \boxed{2} \boxed{\text{ENTER}}$

$$\frac{1}{2}B = \begin{bmatrix} 1 & 1.5 & 3 \\ 2 & -4 & 2.5 \end{bmatrix}$$

```
.5[B]
[[ 1  1.5  3 ]
 [ 2 -4  2.5 ]
```

b. AB

Press: $\boxed{2\text{nd}} \boxed{[\text{MATRX}]} \boxed{1} \boxed{2\text{nd}} \boxed{[\text{MATRX}]}$
 $\boxed{2} \boxed{\text{ENTER}}$

$$AB = \begin{bmatrix} 14 & -21 & 21 \\ -4 & 22 & 2 \end{bmatrix}$$

```
[A][B]
[[ 14 -21 21 ]
 [ -4  22  2 ]
```

c. A^2

Press: $\boxed{2\text{nd}} \boxed{[\text{MATRX}]} \boxed{1} \boxed{x^2} \boxed{\text{ENTER}}$

$$A^2 = \begin{bmatrix} 7 & -3 \\ -2 & 10 \end{bmatrix}$$

```
[A]^2
[[ 7  -3 ]
 [ -2 10 ]
```

d. $AB + B$

Press: $\boxed{2\text{nd}} \boxed{[\text{MATRX}]} \boxed{1} \boxed{2\text{nd}} \boxed{[\text{MATRX}]}$
 $\boxed{2} \boxed{+} \boxed{2\text{nd}} \boxed{[\text{MATRX}]} \boxed{2} \boxed{\text{ENTER}}$

$$AB + B = \begin{bmatrix} 16 & -18 & 27 \\ 0 & 14 & 7 \end{bmatrix}$$

```
[A][B]+[B]
[[ 16 -18 27 ]
 [  0  14  7 ]
```

6

TI-83 Plus Graphing Calculator Handbook

Graphing Trigonometric Functions

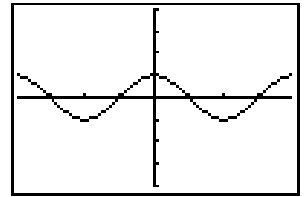
Trigonometric functions and the inverses of trigonometric functions can be graphed using $Y=$. The functions and their inverses can be graphed in degrees or radians. You must set the calculator in **Radian** or **Degree** mode. The standard viewing window for trigonometric functions can be set by pressing $ZOOM$ 7:Trig, which automatically adjusts the x - and y -axes scales for degrees or radians.

Example 1 a. Using Degrees Graph $y = \cos x$.

First, set the calculator in degree mode by pressing $MODE$ \blacktriangledown \blacktriangledown \blacktriangleright $ENTER$.

Now enter and graph the function. Press

$Y=$ COS (X,T,θ,n) $)$ $ZOOM$ 7.



$[-352.5, 352.5]$ scl:90 by $[-4, 4]$ scl:1

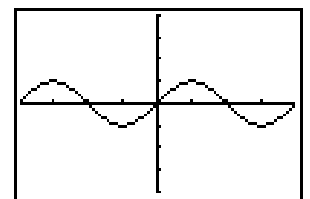
b. Using Radians Graph $y = \sin x$.

Change to radian mode by pressing $MODE$

\blacktriangledown \blacktriangledown $ENTER$. Press $Y=$ $CLEAR$ to delete the function entered in part a. Then press

SIN (X,T,θ,n) $)$ to enter the new function.

Next, press $ZOOM$ 7 to set the viewing window to accommodate radian mode and graph the function.



$[-2, 2]$ scl: $\frac{\pi}{2}$ by $[-4, 4]$ scl:1

c. Amplitude, Period, and Phase Shift Graph $y = 2 \sin\left(\frac{1}{2}x - 60^\circ\right)$ using the viewing window $[-540, 540]$ scl:90 by $[-3, 3]$ scl:1. Then state the amplitude, period, and phase shift.

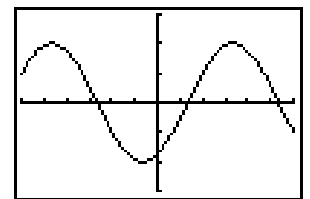
Make sure the calculator is in degree mode.

Press: $Y=$ 2 SIN (X,T,θ,n) \div 2 $-$ 60 $)$ $GRAPH$

The amplitude of the function is $\frac{|-2 - 2|}{2}$

or 2. The period is $\frac{360}{\frac{1}{2}}$ or 720° , and

the phase shift is $\frac{-60}{\frac{1}{2}}$ or -120° .



$[-540, 540]$ scl:90 by $[-3, 3]$ scl:1

7

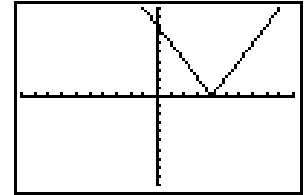
TI-83 Plus Graphing Calculator Handbook

Graphing Special Functions

Most special functions can be graphed using the $Y=$ key. The absolute value function **abs**(and the greatest integer function **int**(can be found in the **MATH NUM** menu.

Example 1 Absolute Value Graph $y = 2|x - 4|$.

Press $Y=$ 2 **MATH** \blacktriangleright 1 **X,T,θ,n** $-$ 4 **)**
ZOOM 6 to graph the function in the standard viewing window.

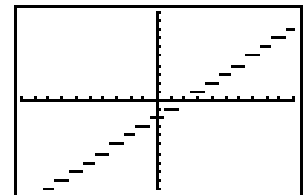


$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1

2 Greatest Integer Function Graph $y = \llbracket x - 1.5 \rrbracket$.

First, make sure the calculator is set for dot plotting rather than the connected plotting used in most other functions. Press **MODE**, highlight **Dot**, and press **ENTER**.

Then, enter the function. Press $Y=$ **MATH**
 \blacktriangleright 5 **X,T,θ,n** $-$ 1.5 **)** **ZOOM** 6. If your calculator is already set for the standard viewing window, press **GRAPH** instead of **ZOOM** 6.



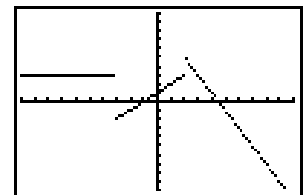
$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1

The **TEST** menu allows you to graph other piecewise functions. Enter the pieces of the function as a sum of the products of each piece of the function and its domain. For example, $y = \begin{cases} 5 & \text{if } x < 2 \\ 4x & \text{if } x > 2 \end{cases}$ is entered as $(5)(X<2) + (4X)(X>2)$ in the **Y=** menu.

Example 3 Piecewise Function Graph $y = \begin{cases} 3 & \text{if } x \leq -3 \\ 1 + x & \text{if } -3 < x \leq 2 \\ 9 - 2x & \text{if } x > 2 \end{cases}$

Place the calculator in **Dot** mode. Then enter the function in the **Y=** list using the **TEST** menu options.

Press: $Y=$ (3) (**X,T,θ,n** 2nd **[TEST]**
 6 ($-$) 3) + (1 + **X,T,θ,n**)
 (($-$) 3 2nd **[TEST]** 5 **X,T,θ,n**)
 (**X,T,θ,n** 2nd **[TEST]** 6 2) +
 (9 $-$ 2 **X,T,θ,n**) (**X,T,θ,n** 2nd
[TEST] 3 2) **ZOOM** 6



$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1

9

TI-83 Plus Graphing Calculator Handbook

Statistics and Statistical Graphs

A graphing calculator allows you to enter a set of data and generate statistics and statistical graphs. Before you enter data values, make sure you clear the **Y=** list, **L1** and **L2**, and the graphics screen. Clear the **Y=** list by pressing **Y=** **CLEAR**. Use the **▼** key to select additional equations and clear them also. To clear **L1** and **L2**, press **STAT** 4 **2nd** **[L1]** **,** **[L2]** **ENTER**. If you need to clear the graphics screen, press **2nd** **[DRAW]** 1 **ENTER**.

Example 1 Enter Data into Lists Enter the following data into a graphing calculator.

49 53 54 54 56 55 57 61 51 58 41 59 54 50 60 44

Press: **STAT** 1 49 **ENTER** 53 **ENTER** 54 **ENTER** 54 **ENTER**
 56 **ENTER** 55 **ENTER** 57 **ENTER** 61 **ENTER**
 51 **ENTER** 58 **ENTER** 41 **ENTER** 59 **ENTER**
 54 **ENTER** 50 **ENTER** 60 **ENTER** 44 **ENTER**

ENTER

L1	L2	L3	1
49			
53			
54			
54			
56			
55			
57			
61			
51			
58			
41			
59			
54			
50			
60			
44			
L1(17) =			

You can use the up and down arrow keys to scroll through the list.

2 Find Mean, Median, and Mode Find the mean, median, and mode of the data in Example 1.

Press **STAT** **▶** 1 **ENTER**. This function displays many statistics about the data. \bar{X} denotes the mean. Scroll down to find the median.

mean →

1-Var Stats
$\bar{x}=53.5$
$\Sigma x=856$
$\Sigma x^2=46252$
$Sx=5.513619501$
$\sigma x=5.338539126$
$n=16$

median →

1-Var Stats
$n=16$
minX=41
Q1=50.5
Med=54
Q3=57.5
maxX=61

The calculator does not have a function to determine the mode. You can find the mode by examining the data. First sort the data to write them in order from least to greatest.

Press: **STAT** 2 **2nd** **[L1]** **)** **ENTER**

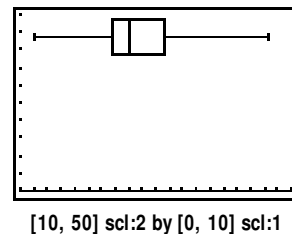
Then scroll through the data by pressing **STAT** 1 and using the **▲** and **▼** keys. You will find that the mode is 54.

Examples 3 Box-and-Whisker Plots

a. Draw a box-and-whisker plot for the data.

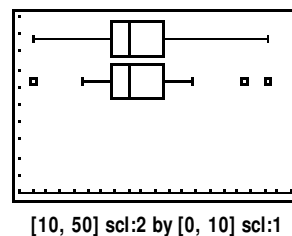
30.2 29.0 26.2 25.8 23.8 43.0 19.8 19.4
26.0 46.6 26.8 22.8 35.4 25.2 12.2 31.4

Set the viewing window. Next, set the plot type. Press $\boxed{2\text{nd}} \boxed{[\text{STAT PLOT}]} \boxed{1} \boxed{\text{ENTER}}$
 $\boxed{\blacktriangledown} \boxed{\blacktriangleright} \boxed{\blacktriangleright} \boxed{\blacktriangleright} \boxed{\blacktriangleright}$ to highlight $\boxed{\text{I-Box}}$ and press $\boxed{\text{ENTER}}$. Make sure **L1** is entered in **Xlist**.
If not, move the cursor to highlight and press $\boxed{2\text{nd}} \boxed{[\text{L1}]} \boxed{\text{ENTER}}$. Then, enter the data into **L1**. Press $\boxed{\text{STAT}} \boxed{1} \boxed{30.2} \boxed{\text{ENTER}}$
 $\boxed{29} \boxed{\text{ENTER}}$... $\boxed{31.4} \boxed{\text{ENTER}}$ $\boxed{\text{GRAPH}}$.



b. Draw a box-and-whisker plot with outliers using the data.

Without clearing the lists or graphic screen, press $\boxed{2\text{nd}} \boxed{[\text{STAT PLOT}]} \boxed{2} \boxed{\text{ENTER}}$ $\boxed{\blacktriangledown} \boxed{\blacktriangleright}$
 $\boxed{\blacktriangleright} \boxed{\blacktriangleright}$, to highlight $\boxed{\text{I-Box+Out}}$, and press $\boxed{\text{ENTER}}$.
Make sure **L1** is entered in **Xlist**.
Press $\boxed{\text{GRAPH}}$.



c. Find the upper and lower quartiles, the median, and the outliers.

Press $\boxed{\text{TRACE}}$ and use $\boxed{\blacktriangleleft}$ and $\boxed{\blacktriangleright}$ to move the cursor along the graph. The values will be displayed. For this data, the upper quartile is 30.8, the lower quartile is 23.3, the median is 26.1, and the outliers are 43 and 46.6.

4 Histograms

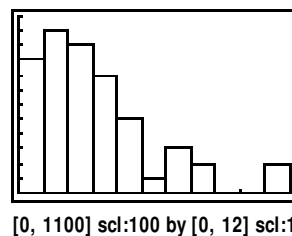
a. Use the data on the number of public libraries in each state and Washington, D.C., to make a histogram.

273 102 159 196 1030 235 244 30 27 428 366 49 141
772 427 554 372 188 322 273 187 491 659 361 243 346
110 283 78 238 455 92 1067 352 86 684 192 201 640
74 180 134 284 753 96 204 308 309 174 451 74

Enter the data in **L1**. Press $\boxed{\text{STAT}} \boxed{1} \boxed{273} \boxed{\text{ENTER}}$ $\boxed{102} \boxed{\text{ENTER}}$... $\boxed{74} \boxed{\text{ENTER}}$.

Set the viewing window. Choose **Xmin**, **Xmax**, and **Xscl** to determine the number of bars in the histogram. For this data, the least value is 27 and the greatest is 1067. If **Xmin** = 0, **Xmax** = 1100, and **Xscl** = 100, the histogram will have 11 bars each representing an interval of 100.

Choose the type of graph. Press $\boxed{2\text{nd}}$
 $\boxed{[\text{STAT PLOT}]} \boxed{1} \boxed{\text{ENTER}}$ $\boxed{\blacktriangledown} \boxed{\blacktriangleright} \boxed{\blacktriangleright} \boxed{\text{ENTER}}$ $\boxed{\blacktriangledown}$
 $\boxed{2\text{nd}} \boxed{[\text{L1}]} \boxed{\text{ENTER}}$ $\boxed{\blacktriangledown} \boxed{1} \boxed{\text{ENTER}}$. Then press $\boxed{\text{GRAPH}}$ to draw the histogram.



Make sure you have cleared the **Y=** list, **L1** and **L2**, and the graphic screen.

b. Use the data in the table to draw a histogram and its frequency polygram.

Class Limits	Frequency
155–170	7
170–185	15
185–200	34
200–215	38
215–230	42
230–245	35
245–260	33
260–275	21
275–290	18
290–305	6
305–320	3

Enter the class marks as **L1**. Press **STAT**

1 162.5 **ENTER** 177.5 **ENTER** ... 312.5 **ENTER**.

Move the cursor to **L2**. Enter the frequencies.

Press 7 **ENTER** 15 **ENTER** ... 3 **ENTER** **2nd** **[QUIT]**.

Set the viewing window. Use the minimum and maximum of the class limits for **Xmin** and **Xmax**. Use the size of the intervals for **Xscl**. Choose the y-axis values to show the complete histogram.



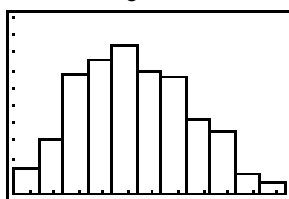
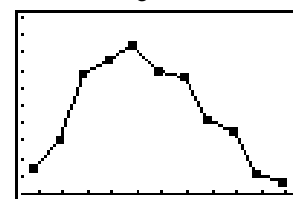
Set the plot type. Press **2nd** **[STAT PLOT]** 1 **ENTER** **▼** **▶** **▶** **ENTER** **▼** **2nd** **[L1]** **ENTER** **▼** **2nd** **[L2]** **ENTER**. Then press **GRAPH**. (Figure 1) Without clearing the lists or graphic screen, press **2nd** **[STAT PLOT]** 1 **ENTER** **▼** **▶** to highlight , and press **ENTER**. Make sure **L1** is entered in **Xlist:** and **L2** is the **Ylist**. Choose  as the mark. Press **GRAPH**.

Figure 1



[155, 320] scl:15 by [0, 50] scl:5


Figure 2

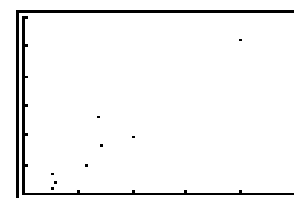


[155, 320] scl:15 by [0, 50] scl:5

Example 5 Scatter Plot, Connected Line Scatter Plot, and Regression Line

a. Use these data to draw a scatter plot: (20.0, 5.2), (10.2, 1.9), (7.3, 1.6), (6.8, 2.6), (5.9, 1.0), (2.6, 0.7), (2.8, 0.35), (2.7, 0.15).

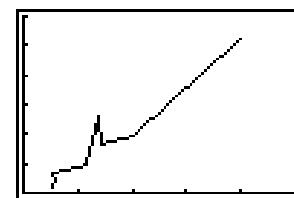
Clear previous data and graphs and set the viewing window. Enter the x-values into **L1** and the y-values into **L2**. Then draw the scatter plot by pressing **2nd** **[STAT PLOT]** 1 **ENTER** **▼** to highlight , and press **ENTER**. Make sure **L1** is the **Xlist:** and **L2** is the **Ylist:**. Then press **GRAPH**.



[0, 25] scl:5 by [0, 6] scl:1

b. Use the data to draw a line graph.

Press **2nd** **[STAT PLOT]** 1 **▼** **▶** to highlight , and press **ENTER** **GRAPH**.



[0, 25] scl:5 by [0, 6] scl:1

c. Draw a regression line for the data in the table.

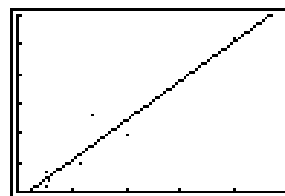
Set the plot to display a scatter plot by pressing 2nd [STAT PLOT] 1 \blacktriangledown [ENTER] 2nd [QUIT].

To calculate the coefficients of regression press [STAT] \blacktriangleright 4 [ENTER].

Then, write the equation of the regression line. You can automatically enter the regression equation in the **Y=** list.

Press Y= [VARS] 5 \blacktriangleright \blacktriangleright 1. Finally, graph the regression line by pressing [GRAPH].

```
LinReg
y=ax+b
a=.2695895968
b=-.2771341868
r2=.9089138336
r=.9533697255
```



[0, 25] scl:5 by [0, 6] scl:1

There are also regression models for analyzing data that are not linear built into the calculator.

Example 6 Nonlinear Regression Find a sine regression equation to model the data in the table. Graph the data and the regression equation.

x	1	2	3	4	5	6	7	8	9	10	11	12
y	39	42	45	48	54	59	63	64	59	52	44	40

Enter the data into lists **L1** and **L2**. Press [STAT] 1 [ENTER] 2 [ENTER] ... 12 [ENTER] \blacktriangleright 39 [ENTER] 42 [ENTER] ... 40 [ENTER].

Find the regression statistics.

Press: [STAT] \blacktriangleright [ALPHA] [C] [ENTER]

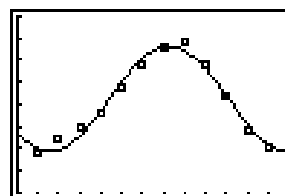
Enter the regression equation into the **Y=** list.

Press: Y= [VARS] 5 \blacktriangleright \blacktriangleright 1

```
SinReg
y=a*sin(bx+c)+d
a=11.83736682
b=.5543464048
c=-2.449519342
d=51.35457494
```

Then format the scatter plot to graph the data by pressing 2nd [STAT PLOT] 1 \blacktriangledown [ENTER].

Make sure that **L1** is chosen as the **Xlist** and **L2** is chosen as the **Ylist**. Set the viewing window. Press [GRAPH] to see the scatter plot and the graph of the regression equation.



[0, 13] scl:1 by [30, 70] scl:5

1

Graphing Calculator Investigation

Solving Equations and Checking Solutions

(Use with Lesson 1-3.)

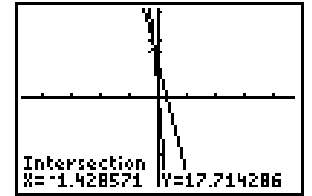
When solving equations, checking the solutions is an important process. A graphing calculator can be used to check the solution of an equation.

Example 1 Solve $-2(5y - 1) - y = -4(y - 3)$.

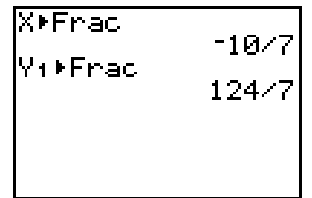
Graph the expression on the left side of the equation in **Y1** and the expression on the right side of the equation in **Y2**. Choose an appropriate view window so that the intersection of the graphs is visible. Then use the intersect command to find the coordinates of the common point.

Keystrokes: $Y=$ $(-)$ 2 $($ 5 X,T,θ,n $-$ 1 $)$ $-$ X,T,θ,n $ENTER$ $(-)$ 4 $($ X,T,θ,n $-$ 3 $)$ $ZOOM$ 6 $ZOOM$ 8 $ENTER$ 2^{nd} $[CALC]$ 5 $ENTER$ $ENTER$ $ENTER$ 2^{nd} $[QUIT]$ X,T,θ,n $MATH$ $ENTER$ $ENTER$.

The x -coordinate, $-\frac{10}{7}$, is the solution to the equation. The y -coordinate is the value of both sides of the equation when $x = -\frac{10}{7}$.



$[-47, 47]$ scl:10 by $[-31, 31]$ scl:10

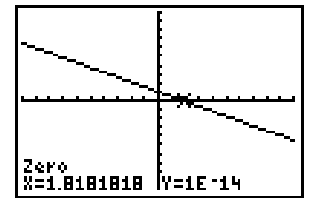


Example 2 Solve $\frac{x}{5} - \frac{x}{4} = \frac{1}{2}(x - 2)$.

Graph the expression on the left side of the equation in **Y1** and the expression on the right side of the equation in **Y2**. Enter **Y1 - Y2** in **Y3**. Then graph the function in **Y3**. Use the zero function under the **CALC** menu to determine where the graph of **Y3** equals zero. This point will be the solution.

Keystrokes: $Y=$ $($ X,T,θ,n \div 5 $)$ $-$ $($ X,T,θ,n \div 4 $)$ $ENTER$ $($ 1 \div 2 $)$ $($ X,T,θ,n $-$ 2 $)$ $ENTER$ $VARS$ \blacktriangleright $ENTER$ $ENTER$ $-$ $VARS$ \blacktriangleright $ENTER$ 2 \blacktriangle \blacktriangle \blacktriangleleft $ENTER$ \blacktriangle $ENTER$ $ZOOM$ 6 2^{nd} $[CALC]$ 2.

Use arrow keys and enter to set the bound prompts. The solution is $x = \frac{20}{11}$.



$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1

Exercises

Solve each equation.

1. $-3(2w - 7) = 9 - 2(5w + 4)$ 2. $1.5(4 - x) = 1.3(2 - x)$ 3. $\frac{1}{4}(a + 2) = \frac{1}{6}(5 - a)$

4. $3(2z + 25) - 2(z - 1) = 78$ 5. $\frac{m - 4}{3} - \frac{3m - 1}{5} = 1$ 6. $\frac{x + 5}{2} + \frac{1}{2} = 2x - \frac{x - 3}{8}$

1

Spreadsheet Investigation

Absolute Value Statements (Use with Lesson 1-4.)

You can use a spreadsheet to try several different values in an equation to help you determine whether the statement is *sometimes*, *always*, or *never* true. Remember that showing that a statement is true for some values does not prove that it is true for all values. However, finding one value for which a statement is false proves that it is not true for all values.

Example

Determine whether $c|a + b| = |ca + cb|$ is *sometimes*, *always*, or *never* true.

Try a number of values for a , b , and c to determine whether the statement is true or false for each set of values.

Step 1 Use Columns A, B, and C for the values of a , b , and c . Choose several sets of values including positive and negative numbers, and zero.

Step 2 Use Column D to test the equation. A formula such as $C2*ABS(A2+B2) = ABS(C2*A2+C2*B2)$ in cell D2 returns TRUE if the equation is true.

	A	B	C	D
1	a	b	c	$c a + b = ca + cb $
2	1	1	0	TRUE
3	-1	-1	-1	FALSE
4	2	2	2	TRUE
5	1	1	1	TRUE
6	1	1	-2	FALSE
7	2	1	0	TRUE
8	1	1	2	TRUE
9	0	0	3	TRUE
10				

Through observation of Column D, when c is negative the statement is not true. The absolute value statement, $c|a + b| = |ca + cb|$ is sometimes true; it is true only if $c \geq 0$.

Exercises

Use a spreadsheet to determine whether each absolute value statement is *sometimes*, *always*, or *never* true.

- For all real numbers a and b , $a \neq 0$, $|ax + b| = 0$.
- If a and b are real numbers, then $|a + b| = |a| = |b|$.
- If a and b are real numbers, then $|a + b| = -x$.
- If a and b are real numbers, then $|a| - |b| = a - b$.
- If a and b are real numbers, then $c|a + b| = c|a| + |b|$.

2 Graphing Calculator Investigation

Transformations: Greatest Integer Function

(Use with Lesson 2-6.)

A graphing calculator can be used to display transformations to the greatest integer function. This is done by using the **int(** command under the **MATH: NUM** menu. When graphing the greatest integer function, it is important to set the calculator to **Dot** mode.

Example

Graph $f(x) = \llbracket x \rrbracket$ and $g(x) = \llbracket x \rrbracket + 2$ on the same axes. Evaluate each function for $x = 0.6, x = 1, x = 1.3, x = 2, x = -0.5$, and $x = -1.1$. Compare the graphs of the functions.

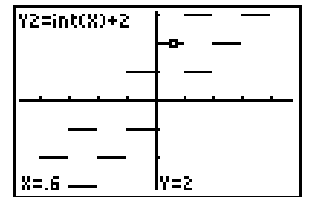
Enter $f(x)$ in **Y1** and $g(x)$ in **Y2**. Graph the functions.

Keystrokes: **Y=** **MATH** **►** 5 **X,T,θ,n** **)** **ENTER** **MATH** **►** 5 **)** **+** 2 **ENTER**
ZOOM 6 **ZOOM** 4.

Use **TRACE** to evaluate each function for the given values.

Keystrokes: **TRACE** 0.6 **ENTER** 1 **ENTER** 1.3 **ENTER** 2 **ENTER** **(-)** 0.5
ENTER **(-)** 1.1 **ENTER** **▲** 0.6 **ENTER** 1 **ENTER** 1.3 **ENTER** 2 **ENTER** **(-)**
 0.5 **ENTER** **(-)** 1.1 **ENTER**.

$f(0.6) = 0, f(1) = 1, f(1.3) = 1, f(2) = 2, f(-0.5) = -1$, and $f(-1.1) = -2$
 $g(0.6) = 2, g(1) = 3, g(1.3) = 3, g(2) = 4, g(-0.5) = -1$, and $g(-1.1) = 0$



The graph of $g(x) = \llbracket x + 2 \rrbracket$ is the same as the graph of $f(x) = \llbracket x \rrbracket$ shifted 2 units up.

Exercises

Graph each function. Evaluate it for $x = 1, x = 1.3$, and $x = 2$. Compare the graph of the function to the graph of $f(x) = \llbracket x \rrbracket$.

- 1. $g(x) = \llbracket x \rrbracket - 3$
- 2. $g(x) = \llbracket x - 2 \rrbracket$
- 3. $g(x) = \llbracket x + 4 \rrbracket$

- 4. $g(x) = \llbracket -x \rrbracket$
- 5. $g(x) = \llbracket 2x \rrbracket$
- 6. $g(x) = 3\llbracket x \rrbracket$

2 Spreadsheet Investigation

Using Linear Equations (Use with Lesson 2-4.)

The slope intercept form of a linear equation is $y = mx + b$, where m is the slope and b is the y -intercept. Recall that the formula for the slope of a line through (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$. You can use the formula for slope and the slope-intercept form to find the value of b .

Example State the slope and y -intercept of the graph of the line through (5, 2) and (4, 1). Then write an equation of the line in slope-intercept form.

Step 1 Use Columns A and B to represent the first point, and Columns C and D to represent the second point on the line. Enter the formula for slope in Column E.

Step 2 Substitute one of the given points into the slope-intercept form and solve for b . Since we know the slope of the line, we can solve for b .

$$\begin{array}{ll} y = mx + b & \text{Slope-intercept form} \\ y_1 = mx_1 + b & \text{Substitute } (x_1, y_1). \\ y_1 - mx_1 = b & \text{Solve for } b. \end{array}$$

Enter this formula into Column F using the names of the spreadsheet cells.

The slope of the line through (5, 2) and (4, 1) is 1 and the y -intercept is -3 . Thus, the equation of the line slope-intercept form is $y = 1x + (-3)$ or $y = x - 3$.

	A	B	C	D	E	F
1	x_1	y_1	x_2	y_2	Slope	y -intercept
2	5	2	4	1	1	-3
3						

Exercises

Use a spreadsheet to find the slope and y -intercept of the line through each pair of points. Then write an equation of the line in slope-intercept form.

1. (0, -5), (2, 5)

2. (4, 2), (-3, -5)

3. (-1, -4), (1, 3)

4. (-4, -9), (8, 3)

5. (12, 9), (10, 10)

6. (-1.5, 3.1), (0.9, 1.9)

7. Does the spreadsheet work when two points have the same x -coordinates? Explain.

3 Graphing Calculator Investigation

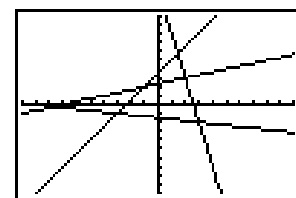
Linear Programming (Use with Lesson 3-4.)

A graphing calculator can store the x - and y -coordinates when using the **intersect** command in the [CALC] menu. This can be displayed on the home screen and used to evaluate an expression with x and y variables. This process is useful in finding the vertices of the feasible region and determining the maximum or minimum value for $f(x, y)$.

Example Graph the system $x - 3y \geq -7$, $5x + y \leq 13$, $x + 6y \geq -9$, $3x - 2y \geq -7$, and $f(x, y) = 4x - 3y$. Find the coordinates of the feasible region. Then find the maximum and minimum values for the system.

Solve each inequality for y . Enter each boundary equation in the **Y=** screen. Find the vertices of the feasible region. Then find the values of $f(x, y)$ to determine the maximum and minimum values.

Keystrokes: $Y=$ () 1 \div 3) X,T, θ ,n + () 7 \div 3) ENTER
 (-) 5 X,T, θ ,n + 13 ENTER (-) () 1 \div 6) X,T, θ ,n - () 3 \div
 2) ENTER () 3 \div 2) X,T, θ ,n + () 7 \div 2) ZOOM 6
 2nd [CALC] 5 ENTER ENTER ENTER 2nd [QUIT] 2nd [{ }
 X,T, θ ,n , ALPHA [Y] , 4 X,T, θ ,n - 3 ALPHA [Y] 2nd [}] ENTER
 GRAPH 2nd [CALC] 5 ∇ ENTER ENTER ENTER 2nd [QUIT]
 2nd [ENTRY] ENTER GRAPH 2nd [CALC] 5 ∇ ∇ ENTER
 ENTER ENTER 2nd [QUIT] 2nd [ENTRY] ENTER GRAPH 2nd
 [CALC] 5 ENTER \blacktriangle \blacktriangle ENTER ENTER 2nd [QUIT] 2nd
 [ENTRY] ENTER.



[-10, 10] scl:1 by [-10, 10] scl:1

	(2 3 -1)
(X, Y, 4X-3Y)	(3 -2 18)
(X, Y, 4X-3Y)	(-3 -1 -9)
(X, Y, 4X-3Y)	(-1 2 -10)

The maximum value of the system is 18 and the minimum value is -10.

Exercises

Graph each system. Find the coordinates of the vertices of the feasible region. Then find the maximum and minimum values for the system.

- | | | |
|---|--|---|
| <p>1. $2x + 3y \geq 6$
 $3x - 2y \geq -4$
 $5x + y \geq 15$
 $f(x, y) = x + 3y$</p> | <p>2. $y \leq 4x + 6$
 $x + 4y \leq 7$
 $2x + y \leq 7$
 $x - 6y \leq 10$
 $f(x, y) = 2x - y$</p> | <p>3. $y \leq 16 - x$
 $0 \leq 2y \leq 17$
 $2x + 3y \geq 11$
 $y \leq 3x + 1$
 $y \geq 2x - 13$
 $y \geq 7 - 2x$
 $f(x, y) = 5x + 6y$</p> |
|---|--|---|

3 Spreadsheet Investigation

Break-Even Point *(Use with Lesson 3-1.)*

You have learned that the break-even point is the point at which the income equals the cost. You can use the formulas and charts in a spreadsheet to find a break-even point.

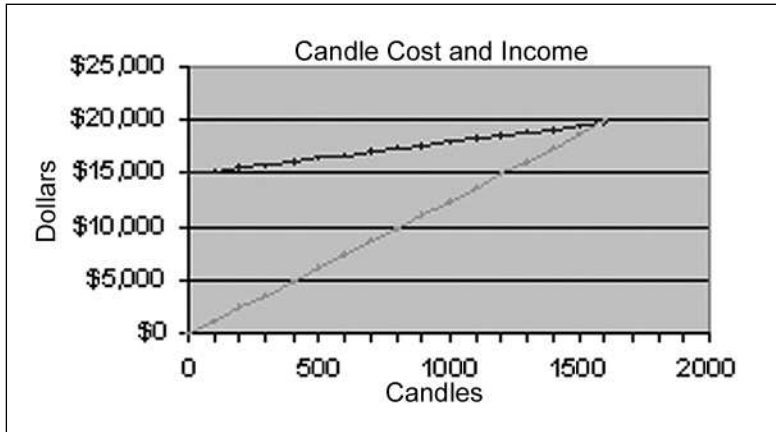
Example Carly Ericson is considering opening a candle business. She estimates that she will have an annual overhead of \$15,000. It costs Carly \$3.00 to make a jar candle, which she sells for \$12.50. What is Carly's break-even point?

Use Column A for the number of candles. Columns B and C are the cost and the income, respectively.

Extend the rows of the spreadsheet to find the point at which the income first exceeds the cost. The break-even point occurs between this point and the previous point. In this case, the break even point occurs between 1500 and 1600 candles.

The chart tool of the spreadsheet allows you to graph the data. The graph verifies the solution.

	A	B	C
1	Candles	Cost	Income
2	0	\$15,000	\$0
3	100	\$15,300	\$1,250
4	200	\$15,600	\$2,500
5	300	\$15,900	\$3,750
6	400	\$16,200	\$5,000
7	500	\$16,500	\$6,250
8	600	\$16,800	\$7,500
9	700	\$17,100	\$8,750
10	800	\$17,400	\$10,000
11	900	\$17,700	\$11,250
12	1000	\$18,000	\$12,500
13	1100	\$18,300	\$13,750
14	1200	\$18,600	\$15,000
15	1300	\$18,900	\$16,250
16	1400	\$19,200	\$17,500
17	1500	\$19,500	\$18,750
18	1600	\$19,800	\$20,000
19			



Exercises

1. If Carly could decrease her annual overhead to \$14,000, what would the break-even point be?
2. Suppose Carly decreases her annual overhead to \$14,000 and increases the price of a candle to \$14.00. What is the new break-even point?

4

Graphing Calculator Investigation

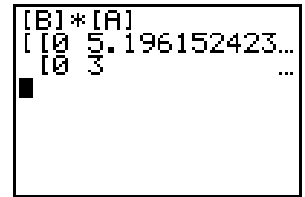
Matrices for 30°, 45°, 60° Rotations (Use with Lesson 4-4.)

The rotation matrix for 90° counterclockwise about the origin is $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

The general rotation matrix for any angle θ counterclockwise about the origin is $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

Example 1 Find the coordinates of the image of $\triangle ABC$ with vertices $A(0, 0)$, $B(6, 0)$ and $C(3, 4)$ after a counterclockwise rotation of 30° about the origin.

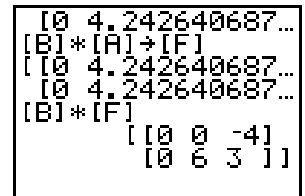
Enter the coordinates of the vertices of the in vertex matrix [A] and the rotation matrix in matrix [B]. Be sure the calculator is set in Degree mode. Keystrokes: `2nd [MATRX] ►► [ENTER] 2 [ENTER] 3 [ENTER] 0 [ENTER] 6 [ENTER] 3 [ENTER] 0 [ENTER] 0 [ENTER] 4 [ENTER] 2nd [MATRX] ►► 2 2 [ENTER] 2 [ENTER] COS 30) [ENTER] (-) SIN 30) [ENTER] SIN 30) [ENTER] COS 30) [ENTER] 2nd [QUIT] 2nd [MATRX] 2 2nd [MATRX] 1 [ENTER]`. Hold `►` to scroll across to see the other coordinates.



The coordinates of the image of $\triangle ABC$ are $A'(0, 0)$, $B'(5.2, 3)$, and $C'(0.6, 5.0)$.

Example 2 Find the coordinates of the image of $\triangle ABC$ with vertices $A(0, 0)$, $B(6, 0)$ and $C(3, 4)$ after a two rotations of 45° counterclockwise about the origin.

In order to rotate the image twice, store the vertex matrix of the first image. Keystrokes: `2nd [MATRX] ►► 2 [ENTER] [ENTER] COS 45) [ENTER] (-) SIN 45) [ENTER] SIN 45) [ENTER] COS 45) [ENTER] 2nd [QUIT] 2nd [MATRX] 2 2nd [MATRX] 1 [ENTER] 2nd [ENTRY] STO► 2nd [MATRX] 6 [ENTER] 2nd [ENTRY] 2nd [ENTRY] ◀ 2nd [MATRX] 6 [ENTER]`.



The vertices of $\triangle A'B'C'$ are $A'(0, 0)$, $B'(0, 6)$, and $C'(-4, 3)$.

Exercises

Quadrilateral $ABCD$ has vertices $A(0, 0)$, $B(4, 0)$, $C(4, 6)$, and $D(0, 6)$. Find the coordinates of the vertices of the image after each counterclockwise rotation. Round to the nearest tenth.

- 1. 45°
- 2. 30°
- 3. 60°
- 4. 120°
- 5. 75°
- 6. 225°

4

Spreadsheet Investigation**Cramer's Rule** (Use with Lesson 4-5.)

You have learned to solve systems of linear equations by using matrix equations and the inverse matrix. Another way to solve systems is to use *Cramer's Rule*. Study the spreadsheet below to discover Cramer's Rule.

	A	B	C
1	6	3	-12
2	5	1	8
3			
4	=A1*B2-B1*A2		
5			
6	=C1*B2-B1*C2		
7			
8	=A1*C2-C1*A2		
9			
10	x =	=(A6/A4)	
11	y =	=(A8/A4)	
12			

To use the spreadsheet to solve a system of equations, write each equation in the form below.

$$ax + by = c$$

The values for the system $6x + 3y = -12$ and $5x + y = 8$ are shown. In the spreadsheet, the values of a , b , and c for the first equation are entered in cells A1, B1, and C1, respectively. The values of a , b , and c for the second equation are entered in cells A2, B2, and C2, respectively.

The values in cells B10 and B11 represent the solution for the system.

Exercises

- Study the formula in cell A4. Write a matrix whose determinant is found using this formula.
- Write matrices whose determinants are found using the formulas in cells A6 and A8.
- Explain how the values of x and y are found using Cramer's rule.

Use the spreadsheet to solve each system of equations.

4. $6x + 3y = -12$
 $5x + y = 8$

5. $5x - 3y = 19$
 $7x + 2y = 8$

6. $8x - 3y = 11$
 $6x + 9y = 15$

7. $0.3x + 1.6y = 0.44$
 $0.4x + 2.5y = 0.66$

8. $3y = 4x + 28$
 $5x + 7y = 8$

9. $y = -0.5x + 4$
 $y = 4x - 5$

5 Graphing Calculator Investigation

Using Tables to Factor by Grouping *(Use with Lesson 5-4.)*

The **TABLE** feature of a graphing calculator can be used to help factor a polynomial of the form $ax^2 + bx + c$.

Example 1 Factor $10x^2 - 43x + 28$ by grouping.

Make a table of the negative factors of $10 \cdot 28$ or 280. Look for a pair of factors whose sum is -43 .

Enter the equation $y = \frac{280}{x}$ in **Y1** to find the factors of 280. Then, find the sum of the factors using $y = \frac{280}{x} + x$ in **Y2**. Set up the table to display the negative factors of 280 by setting $\Delta Tbl =$ to -1 . Examine the results.

Keystrokes: $Y=$ 280 \div X,T,θ,n ENTER VAR \blacktriangleright ENTER ENTER $+$ X,T,θ,n ENTER $2nd$ $[TBLSET]$ $(-)$ 1 ENTER $(-)$ 1 ENTER $2nd$ $[TABLE]$.

X	Y1	Y2
-2	-140	-142
-3	-93.33	-96.33
-4	-70	-74
-5	-56	-61
-6	-46.67	-52.67
-7	-40	-47
-8	-35	-43

Y2 = -43

The last line of the table shows that $-43x$ may be replaced with $-8x + (-35x)$.

$$\begin{aligned} 10x^2 - 43x + 28 &= 10x^2 - 8x + (-35x) + 28 \\ &= 2x(5x - 4) + (-7)(5x - 4) \\ &= (5x - 4)(2x - 7) \end{aligned}$$

Thus, $10x^2 - 43x + 28 = (5x - 4)(2x - 7)$.

Example 2 Factor $12x^2 - 7x - 12$.

Look at the factors of $12 \cdot -12$ or -144 for a pair whose sum is -7 . Enter an equation to determine the factors in **Y1** and an equation to find the sum of factors in **Y2**. Examine the table to find a sum of -7 .

Keystrokes: $Y=$ $(-)$ 144 \div X,T,θ,n ENTER VAR \blacktriangleright ENTER ENTER $+$ X,T,θ,n ENTER $2nd$ $[TBLSET]$ 1 ENTER 1 ENTER $2nd$ $[TABLE]$.

X	Y1	Y2
3	-48	-45
4	-36	-33
5	-28.8	-23.8
6	-24	-18
7	-20.57	-13.57
8	-18	-10
9	-16	-7

Y2 = -7

$$\begin{aligned} 12x^2 - 7x - 12 &= 12x^2 + 9x + (-16x) - 12 \\ &= 3x(4x + 3) - 4(4x + 3) \\ &= (4x + 3)(3x - 4) \end{aligned}$$

Thus, $12x^2 - 7x - 12 = (4x + 3)(3x - 4)$.

Exercises

Factor each polynomial.

1. $y^2 - 20y - 96$
2. $4z^2 - 33z + 35$
3. $4y^2 + y - 18$
4. $6a^2 + 2a - 15$
5. $6m^2 + 17m + 12$
6. $24z^2 - 46z + 15$
7. $36y^2 + 84y + 49$
8. $4b^2 + 36b - 403$

5

Spreadsheet Investigation**Appreciation and Depreciation** (Use with Lesson 5-7.)

When an asset such as a house increases in value over time, it is said to *appreciate*. If the value increases by a fixed percent each year, or other period of time, the amount y of that quantity after t years is given by

$$y = a(1 + r)^t,$$

where a is the initial amount and r is the percent of increase expressed as a decimal. You can use a spreadsheet to investigate future values of an asset.

Example

Michael Blackstock is considering buying a piece of investment property for \$95,000. The homes in the area are appreciating at an average rate of 4% per year. Find the expected value of the home in 1 year, 1 year and 6 months, 4 years, and 6 years and 9 months.

Use rows 1 and 2 to enter the initial amount and the rate of increase. Then use Column A to enter the amounts of time. Enter the numbers of months as a fraction of a year since t is measured in years. Column B contains the formulas for the value of the home.

Format the cells containing the values as currency so that they are displayed as dollars and cents. The expected value of the home after each amount of time is shown in the spreadsheet.

	A	B
1	Initial value =	\$95,000.00
2	Rate =	0.04
3		
4	Years	Value
5	1	\$98,800.00
6	1.5	\$100,756.63
7	4	\$111,136.56
8	6.75	\$123,793.73

Exercises

1. If Mr. Blackstock chooses another property in the neighborhood that costs \$99,900, what are the expected values of that home in the same periods of time?
2. What would Mr. Blackstock's profit be on the \$99,900 home if he sold it after 9 years and 3 months?
3. If an antique chair worth \$165.00 increases in value an average of $3\frac{1}{2}\%$ every year, how much will it be worth next year?
4. Often assets like cars decrease in value over time. This asset is said to *depreciate*. If the value decreases by a fixed percent each year, or other period of time, the amount y of that quantity after t years is given by $y = a(1 - r)^t$, where a is the initial amount and r is the percent of decrease expressed as a decimal. Use a spreadsheet to find the value of a car purchased for \$18,500 after 2 years, 2 years and 6 months, and 4 years and 3 months if the car depreciates at a rate of 12% per year.

6 Graphing Calculator Investigation

Quadratic Inequalities and the Test Menu

(Use with Lesson 6-7.)

The inequality symbols, called *relational operators*, in the **TEST** menu can be used to display the solution of a quadratic inequality. Another method that can be used to find the solution set of a quadratic inequality is to graph each side of an inequality separately. Examine the graphs and use the **intersect** function to determine the range of values for which the inequality is true.

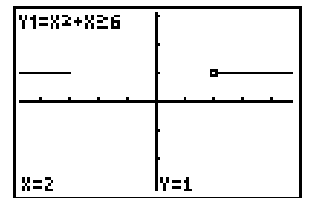
Example 1 Solve $x^2 + x \geq 6$.

Place the calculator in Dot mode. Enter the inequality into **Y1**. Then trace the graph and describe the solution as an inequality.

Keystrokes: $\boxed{Y=}$ $\boxed{2}$ $\boxed{X,T,\theta,n}$ $\boxed{x^2}$ $\boxed{+}$ $\boxed{X,T,\theta,n}$ $\boxed{-}$ $\boxed{6}$ $\boxed{2nd}$ $\boxed{[TEST]}$ $\boxed{4}$ $\boxed{6}$ \boxed{ZOOM} 4.

Use **TRACE** to determine the endpoints of the segments.

These values are used to express the solution of the inequality, $\{x \mid x \geq -3 \text{ or } x \geq 2\}$.



$[-4.7, 4.7]$ scl:1 by $[-3.1, 3.1]$ scl:1

Example 2 Solve $2x^2 + 4x - 5 \leq 3$.

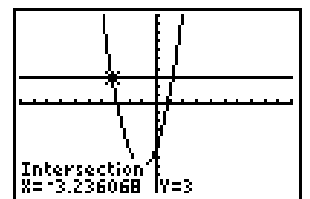
Place the left side of the inequality in **Y1** and the right side in **Y2**. Determine the points of intersection. Use the intersection points to express the solution set of the inequality. Be sure to set the calculator to **Connected** mode.

Keystrokes: $\boxed{Y=}$ $\boxed{2}$ $\boxed{X,T,\theta,n}$ $\boxed{x^2}$ $\boxed{+}$ $\boxed{4}$ $\boxed{X,T,\theta,n}$ $\boxed{-}$ $\boxed{5}$ \boxed{ENTER} $\boxed{3}$ \boxed{ENTER} \boxed{ZOOM} 6.

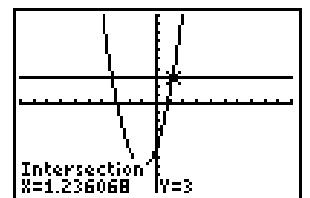
Press $\boxed{2nd}$ $\boxed{[CALC]}$ $\boxed{5}$ and use the $\boxed{\leftarrow}$ key to move the cursor to the left of the first intersection point. Press \boxed{ENTER} . Then move the cursor to the right of the intersection point and press \boxed{ENTER} . One of the values used in the solution set is displayed.

Repeat the procedure on the other intersection point.

The solution is $\{x \mid -3.24 \leq x \leq 1.24\}$.



$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1



$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1

Exercises

Solve each inequality.

1. $-x^2 - 10x - 21 < 0$
2. $x^2 - 9 < 0$
3. $x^2 + 10x + 25 \leq 0$
4. $x^2 + 3x \leq 28$
5. $2x^2 + x \geq 3$
6. $4x^2 + 12x + 9 > 0$
7. $23 > -x^2 + 10x$
8. $x^2 - 4x - 13 \leq 0$
9. $(x + 1)(x - 3) > 0$

6

Spreadsheet Investigation***Approximating the Real Zeros of Polynomials****(Use with Lesson 6-5.)*

You have learned the Location Principle, which can be used to approximate the real zeros of a polynomial.

	A	B	C	D	E	F	G	H	I	J
1										
2	-5	5	x	-5	-3.3333333	-1.6666667	0	1.6666667	3.3333333	5
3			$f(x)$	23	9.1111111	0.7777778	-2	9.1111111	0.7777778	23
4										
5										

In the spreadsheet above, the positive real zero of $f(x) = x^2 - 2$ can be approximated in the following way. Set the spreadsheet preference to manual calculation. The values in A2 and B2 are the endpoints of a range of values. The values in D2 through J2 are values equally in the interval from A2 to B2. The formulas for these values are A2, $A2 + (B2 - A2)/6$, $A2 + 2*(B2 - A2)/6$, $A2 + 3*(B2 - A2)/6$, $A2 + 4*(B2 - A2)/6$, $A2 + 5*(B2 - A2)/6$, and B2, respectively.

Row 3 gives the function values at these points. The function $f(x) = x^2 - 2$ is entered into the spreadsheet in Cell D3 as $D2^2 - 2$. This function is then copied to the remaining cells in the row.

You can use this spreadsheet to study the function values at the points in cells D2 through J2. The value in cell F3 is positive and the value in cell G3 is negative, so there must be a zero between -1.6667 and 0 . Enter these values in cells A2 and B2, respectively, and recalculate the spreadsheet. (You will have to recalculate a number of times.) The result is a new table from which you can see that there is a zero between 1.41414 and 1.414306 . Because these values agree to three decimal places, the zero is about 1.414 . This can be verified by using algebra.

By solving $x^2 - 2 = 0$, we obtain $x = \pm\sqrt{2}$. The positive root is $x = \sqrt{2} = 1.414213\dots$, which verifies the result.

Exercises

- Use a spreadsheet like the one above to approximate the zero of $f(x) = 3x - 2$ to three decimal places. Then verify your answer by using algebra to find the exact value of the root.
- Use a spreadsheet like the one above to approximate the real zeros of $f(x) = x^2 + 2x + 0.5$. Round your answer to four decimal places. Then, verify your answer by using the quadratic formula.
- Use a spreadsheet like the one above to approximate the real zero of $f(x) = x^3 - \frac{3}{2}x^2 - 6x - 2$ between -0.4 and -0.3 .

7

Graphing Calculator Investigation

Rational Root Theorem *(Use with Lesson 7-6.)*

The following program performs synthetic division and displays the depressed polynomial coefficients in rational form. The program will allow the testing of possible rational zeros of a polynomial function.

PROGRAM:SYNTHDIV

```

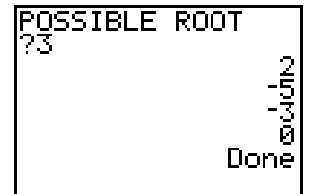
Disp "DEGREE OF DIVIDEND" P+1→P           Q→L2(P)
Input M           Disp "COEFFICIENT" P+1→P
Disp "COEFFICIENTS?" Input A           If P≤M+1
Disp "0=SAME"     A→L1(P)           Goto 3
Disp "1=QUOTIENT" If P<M+1           Stop
Disp "2=NEW"     Goto 1           Lbl 4
Input U           Lbl 2           0→P
Disp "POSSIBLE ROOT" 1→P           Lbl 5
Input R           0→S           1+P→P
If U=0           Lbl 3           L2(P)→L1(P)
Goto 2           L1(P)→F           If P<M+1
If U=1           F+S→Q           Goto 5
Goto 4           Disp Q ► Frac           Goto 2
0→P
Lbl 1           Pause
RQ→S
    
```

Example Find all of the rational zeros of $f(x) = 2x^3 - 11x^2 + 12x + 9$.

Use the program to test possible zeros.

Keystrokes: **PRGM** **▼** **[SYNTHDIV]** **ENTER** **ENTER** **3** **ENTER** **2**
ENTER **1** **ENTER** **2** **ENTER** **(-)** **11** **ENTER** **12** **ENTER** **9** **ENTER**.

Press **ENTER** until the screen displays **Done**.



The column of numbers are the coefficients of the depressed polynomial. Since the last number is not zero, press **ENTER** **3** **ENTER**. Choose 0 for the same coefficients. Press **ENTER** **(-)** **1** then **ENTER** until finished. Repeat this until a zero is found. Then press **ENTER** **2** for the degree of the depressed polynomial and **ENTER** **1** for the quotient.

The zeros are 3, 3, and $\frac{1}{2}$.

Exercises

Find all the zeros of each function.

- | | |
|--|---|
| 1. $f(x) = x^3 - 8x^2 - 23x + 30$ | 2. $f(x) = x^3 - 7x^2 + 2x + 40$ |
| 3. $f(x) = 2x^3 - x^2 - 32x + 16$ | 4. $f(x) = x^4 + x^3 - 11x^2 - 9x + 18$ |
| 5. $p(x) = 3x^4 + 11x^3 + 11x^2 + x - 2$ | 6. $p(x) = x^4 - 2x^3 + x^2 - 8x - 12$ |
| 7. $p(x) = 3x^5 + x^4 - 243x - 81$ | 8. $p(x) = 3x^4 + 13x^3 + 15x^2 - 4$ |

7

Spreadsheet Investigation

Operations on Functions (Use with Lesson 7-7.)

It is possible to perform operations on functions such as addition, subtraction, multiplication and division. You can use a spreadsheet to investigate the relationships among functions.

Consider the functions $f(x) = 3x + 2$, $g(x) = x^2 - 2x$, and $h(x) = x^2 + x + 2$.

Find the function values of each function for several values of x .

Does it appear that $f(x) + g(x) = h(x)$?

Use Column A for the chosen values of x . Columns B, C, and E are $f(x)$, $g(x)$, and $h(x)$ respectively. Use Column D for $f(x) + g(x)$.

For every value of x , $f(x) + g(x) = h(x)$.

	A	B	C	D	E
1	x	f(x)	g(x)	f(x) + g(x)	h(x)
2	-4	-10	24	14	14
3	-2.5	-5.5	11.25	5.75	5.75
4	-1	-1	3	2	2
5	0	2	0	2	2
6	1	5	-1	4	4
7	4	14	8	22	22
8	12	38	120	158	158

Exercises

Study and use the spreadsheet above.

- Find $k(x) = (3x + 2) + (x^2 - 2x)$. How does it compare to $h(x)$?
- Change the functions in the spreadsheet to $f(x) = \frac{x}{2}$, $g(x) = 1 - x^2$, and $h(x) = 1 + \frac{x}{2} - x^2$. How are these functions related? Is it true that $f(x) + g(x) = h(x)$?
- Make a conjecture about $(f + g)(x)$ for any functions $f(x)$ and $g(x)$.
- Make a conjecture about $(f - g)(x)$ for any functions $f(x)$ and $g(x)$. Use the spreadsheet to test your conjecture. Does it appear to be true? Explain your answer.

Find $(f + g)(x)$, $(f - g)(x)$, for each $f(x)$ and $g(x)$. Use the spreadsheet to find function values to verify your solutions. 5-7. See students' spreadsheets.

5. $f(x) = 6x + 8$
 $g(x) = 9 + x$

6. $f(x) = x^2 + 1$
 $g(x) = 3x - 4$

7. $f(x) = 10x^2$
 $g(x) = 6 - x^2$

8 Graphing Calculator Investigation

Matrices and Equations of Circles (Use with Lesson 8-3.)

A graphing calculator can be used to write the equation of a circle in the form $x^2 + y^2 + Dx + Ey + F = 0$ given any three points on the circle.

Example Write the equation of the circle that passes through the given points. Identify the center and radius of each circle.

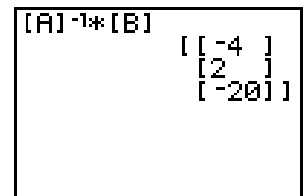
a. $A(5, 3)$, $B(-2, 2)$, and $C(-1, -5)$

Substitute each ordered pair for (x, y) in $x^2 + y^2 + Dx + Ey + F = 0$ to form the a system of equations.

$$5D + 3E + F = -34 \quad -2D + 2E + F = -8 \quad -D - 5E + F = -26$$

Solve the system using a matrix equation to find D , E , and F . Replace the coefficients in the expanded form. Then, complete the square to write the equation in standard form to identify the center and radius.

Keystrokes: `2nd [MATRX] ► ► [ENTER] 3 [ENTER] 3 [ENTER] 5 [ENTER] 3 [ENTER] 1 [ENTER] (-) 2 [ENTER] 2 [ENTER] 1 [ENTER] (-) 1 [ENTER] (-) 5 [ENTER] 1 [ENTER] 2nd [MATRX] ► ► [EDIT] 2 [ENTER] 3 [ENTER] 1 [ENTER] (-) 34 [ENTER] (-) 8 [ENTER] (-) 26 [ENTER] 2nd [QUIT] 2nd [MATRX] [ENTER] x^{-1} × 2nd [MATRX] 2 [ENTER]`



Thus, $D = -4$, $E = 2$, and $F = -20$.

The expanded form is $x^2 + y^2 - 4x + 2y - 20 = 0$.

After completing the square, the standard form is $(x - 2)^2 + (y + 1)^2 = 25$.

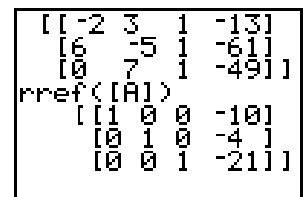
The center is $(2, -1)$, and the radius is 5.

b. $A(-2, 3)$, $B(6, -5)$, and $C(0, 7)$

Find a system of equations. Then enter the equations into an *augmented matrix*. Reduce the matrix to *row reduced echelon form* using the **rref** command. The row reduced echelon form of an augmented matrix will display the solution to the system.

$$-2D + 3E + F = -13 \quad 6D - 5E + F = -61 \quad 7E + F = -49$$

Keystrokes: Enter the system of equations as $[A]$, a 3×4 augmented matrix. Then use the reduced row echelon form by pressing `2nd [MATRX] ► [ALPHA] [B] 2nd [MATRX] [ENTER]) [ENTER]`.



The solution is $D = -10$, $E = -4$, and $F = -21$. The expanded form is $x^2 + y^2 - 10x - 4y - 21 = 0$, standard form is $(x - 5)^2 + (y - 2)^2 = 50$. The center is $(5, 2)$ and the radius is $5\sqrt{2}$.

Exercises

Write the equation of the circle that passes through the given points. Identify the center and radius of each circle.

- $(0, -1)$, $(-3, -2)$, and $(-6, -1)$
- $(7, -1)$, $(11, -5)$, and $(3, -5)$
- $(-2, 7)$, $(-9, 0)$, and $(-10, -5)$

8 Spreadsheet Investigation

Parabolas (Use with Lesson 8-2.)

You have learned many of the characteristics of parabolas with vertical and horizontal axes of symmetry. The information is summarized in the table at the right. You can use what you know to create a spreadsheet to analyze given equations of parabolas.

form of equation	$y = a(x - h)^2 + k$	$x = a(y - k)^2 + h$
vertex	(h, k)	(h, k)
axis of symmetry	$x = h$	$y = k$
focus	$(h, k + \frac{1}{4a})$	$(h + \frac{1}{4a}, k)$
directrix	$y = k - \frac{1}{4a}$	$x = h - \frac{1}{4a}$
direction of opening	upward if $a > 0$, downward if $a < 0$	right if $a > 0$, left if $a < 0$
length of latus rectum	$ \frac{1}{a} $ units	$ \frac{1}{a} $ units

Exercises

The spreadsheet below uses the equation of a parabola in the form $y = a(x - h)^2 + k$ or $x = a(y - k)^2 + h$ to find information about the parabola. x or y is entered in Column D and the values of a , h , and k are entered into Columns A, B, and C respectively.

	A	B	C	D	E	F	G	H	I	J
	a	h	k	y = or x =	Vertex	Length of Latus Rectum (units)	Axis of Symmetry	Focus	Directrix	Direction of Opening
1										
2	0.25	3.00	-1	x	3, -1	4.00	y = -1	4, -1	x = 2	Right
3	3.00	-4.00	2.00	y	-4, 2	0.33	x = -4	-4, 2.08	y = 1.92	Upward
4	-1.00	3.00	2.00	y	3, 2	1.00	x = 3	3, 1.75	y = 2.25	Downward

- Which row represents the equation $y = 3x^2 + 24x + 50$?
- Write the standard form of the equation represented by row 2.
- What formula should be used in cell F2?
- Find the vertex, length of latus rectum, axis of symmetry, focus, directrix, and direction of opening of a parabola with equation $(y - 8)^2 = -4(x - 4)$.

9 Graphing Calculator Investigation

Horizontal Asymptotes and Tables

(Use with Lesson 9-3.)

The line $y = b$ is a horizontal asymptote for the rational function $f(x)$ if $f(x) \rightarrow b$ as $x \rightarrow \infty$ or as $x \rightarrow -\infty$. The horizontal asymptote can be found by using the **TABLE** feature of the graphing calculator.

Example Find the horizontal asymptote for each function.

a. $f(x) = \frac{1}{x^2 + 4x - 5}$

Enter the function into **Y1**. Place [**TblSet**] in the Ask mode. Enter the numbers 10,000, 100,000, 1,000,000, and 5,000,000 and their opposites in the x -list.

Keystrokes: $\boxed{Y=}$ $\boxed{1}$ $\boxed{\div}$ $\boxed{(}$ $\boxed{X,T,\theta,n}$ $\boxed{x^2}$ $\boxed{+}$ $\boxed{4}$ $\boxed{X,T,\theta,n}$ $\boxed{-}$ $\boxed{5}$ $\boxed{)}$ $\boxed{2nd}$ [**TBLSET**] \blacktriangledown \blacktriangledown \blacktriangleright \boxed{ENTER} $\boxed{2nd}$ [**TABLE**]. Then enter the values for x .

X	Y1
10000	1E-8
100000	1E-10
1E6	1E-12
5E6	1E-14
-10000	1E-8
-1E5	1E-10
-5E6	4E-14

Y1=3.9999968E-14

Notice that as x increases, y approaches 0. Thus, when $y = 0$ is the horizontal asymptote.

b. $f(x) = \frac{3x^2}{2x^2 + 5x - 6}$

Enter the equation into **Y1**. Enter the numbers 10,000, 100,000, 1,000,000, and 5,000,000 and their opposites in the x -list. Note the pattern. As x increases, y approaches 1.5. Thus, $y = 1.5$ is the horizontal asymptote.

X	Y1
10000	1.4996
100000	1.5
1E6	1.5
5E6	1.5
-10000	1.5004
-1E5	1.5
1E6	1.5

Y1=1.49999925

Exercises

Find the horizontal asymptote for each function.

1. $f(x) = \frac{2x}{x + 1}$

2. $f(x) = \frac{x^2 - 1}{2x^2 - 7x + 12}$

3. $f(x) = \frac{6x^3}{2x^3 - 2x^2 - 2}$

4. $f(x) = \frac{2x}{3x^2 - 5x - 1}$

5. $f(x) = \frac{15x^2 - 3x + 7}{x^3}$

6. $f(x) = \frac{x^3 - 8x^2 - 4x + 11}{x^4 - 3x^3 - 4x - 6}$

7. $f(x) = \frac{5x^2 - 3}{x - 2}$

8. $f(x) = \frac{6x^3}{2x^2 - 3x + 6}$

9. $f(x) = \frac{2x - 4}{2}$

9

Spreadsheet Investigation**Variation** (Use with Lesson 9-4.)

You have learned to solve problems involving direct, inverse, and joint variation. Many physical situations involve at least one of these types of variation. For example, according to Newton's law of universal gravitation, the weight of a mass near Earth depends on the distance between the mass and the center of Earth. Study the spreadsheet below to determine the type of variation that exists between the quantity of an astronaut's weight and the distance of the astronaut from the center of Earth.

In the spreadsheet, the values for the astronaut's weight in newtons are entered in the cells in column A, and the values for the astronaut's distance in meters from the center of Earth are entered in cells in column B. Column C contains the astronaut's distance from Earth's surface.

	A	B	C
1	Astronaut's Weight (N)	Distance from Earth's Center (m)	Distance from Earth's Surface (km)
2	734.5643	6,380,000	0
3	712.0675	6,480,000	100
4	548.9825	7,380,000	1000
5	111.4406	16,380,000	10,000
6	2.642112	106,380,000	100,000
7			

Exercises

- Use the values in the spreadsheet to make a graph of the astronaut's weight plotted against the astronaut's distance from Earth's center.
- Based on your graph, is this an inverse or direct variation?
- Write an equation that represents this situation. Let W represent the astronaut's weight, k the constant of variation, and R the distance from Earth's center.
- Use the equation to find the weight of the astronaut at these distances from Earth's surface. (*Hint*: Remember to add these values to the value in cell B2 to find the distance from Earth's center.)
 - 145,300,000 m
 - 65 m
 - 25,600 m
 - 300,800,700 m
 - 6580 m
 - 180,560 m

10

Graphing Calculator Investigation**Regression Equation Lab** (Use with Lesson 10-1.)

A graphing calculator can be used to determine a regression equation that best fits a set of data. This activity requires tiles labeled on one side, and a container.

Collect the Data

Step 1 Place the tiles on the desktop and count the total number. Record the total number. Then place the tiles in the container and gently shake.

Step 2 Pour the tiles onto the desktop, remove all the tiles with a label showing, and set these aside. Count the remaining tiles without the labels showing and return them to the container.

Step 3 Record the data in a table like this one.

Trials x	Number of tiles without label showing y
1	
2	

Step 4 Repeat step 2 and 3 until the number of tiles without labels is zero or the number remains constant.

Step 5 Take the tiles that were set aside in Step 2 and pour them out of the container onto the desktop. Remove the tiles without the label showing and count the tiles with the label showing. Repeat this process until all the tiles have been removed.

Step 6 Record the data in a table like this one.

Trials x	Number of tiles with the label showing y
1	
2	

Analyze the Data

1. Enter trials in **L1** and number of tiles without label showing in **L2**. Enter trials in **L3** and number of tiles with the label showing in **L4**.
2. Use **[STATPLOT]** to make a scatter plot. Make a graph on paper for each plot. Record the window used. Describe the pattern of the points.
3. From the **[STAT]** **[CALC]** menu find the regression equation that best fits the data. Record the two closest equations, rounding values to the nearest hundredths. List and discuss the r and/or r^2 values. Also include the graphs in determining the best-fitting regression equation.
4. Sketch your best-fit regression equation choice for each scatter-plot on paper.
5. Describe any problems with the data or the regression equations.
6. Insert (0, total number of tiles) in the tables and the lists. Describe the effect on the graphs. What happens with **[PwrReg]** and **[ExpReg]** when this ordered pair is inserted? Explain why this occurs?

10

Spreadsheet Investigation

Net Present Value *(Use after Lesson 10-6.)*

You have learned how to use exponential and logarithmic functions to perform a number of financial analyses. Spreadsheets can be used to perform many types of analyses, such as calculating the Net Present Value of expenditures or investments. For example, when a business owner is considering a major purchase, it is a good idea to find out whether the investment will be profitable in the future. Consider the example of a local restaurant-delivery service that is debating whether to buy a van for \$8000. The owners of the company estimate that the van will bring in \$2500 per year over four years. They can use the following formula to find the present value of the future cash flow to find the Net Present Value (NPV), that is, how much the profits would be worth in today's dollars. $NPV = \frac{CF_n}{(1+r)^n}$, where CF_n = the cash flow in period n and r = the cost of capital, which is either the interest that will be paid on a loan or the interest that the money would earn if it were invested.

	A	B	C
1	Cost of Asset	\$8,000.00	
2	Cost of Capital (r)	0.1	
3			
4	Period (n)	CF	CF/(1+r)^n
5	1	\$2,500.00	\$2,272.73
6	2	\$2,500.00	\$2,066.12
7	3	\$2,500.00	\$1,878.29
8	4	\$2,500.00	\$1,707.53
9	Total		\$7,924.66
10	NPV - Cost	-\$75.34	
11			

Exercises

- If the NPV is greater than the cost, the investment will pay for itself. Based on the spreadsheet shown above, would it be cost-effective for the company to buy the van? Explain.
- Four times a year, Josey and Drew publish a magazine. They want to buy a color printer that costs \$1750. The cost of capital for this purchase would be 6%. They are planning to raise the price of their magazine from \$1 to \$2. Create a spreadsheet to determine the NPV for this purchase.
 - The last issue of the magazine sold 500 copies. If each issue of the magazine printed in color sells 100 copies more than the previous issue, is the printer a good investment after one year? Explain.
 - If the sales of the magazine continue to rise at the same rate, is the printer a good investment after two years?
- Calculate the NPV for an investment over a period of six years if the cost of capital is 4.5% and the investment will bring a cash flow of \$750 every year.
 - Would this be a good investment of \$3000? Explain?

11

Graphing Calculator Investigation

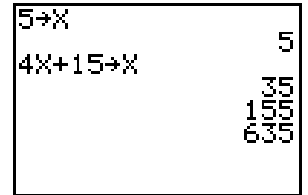
Recursion and Iteration *(Use with Lesson 11-6.)*

A graphing calculator can be used to perform iterations and recursions.

Example 1 Find the first 3 iterates of $f(x) = 4x + 15$ if $x_0 = 5$.

Store x_0 in X. Then enter the expression on the home screen. Store the result to X. Repeat the calculation for each iterate.

Keystrokes: 5 $\boxed{\text{STO}\rightarrow}$ $\boxed{\text{X,T},\theta,n}$ $\boxed{\text{ENTER}}$ 4 $\boxed{\text{X,T},\theta,n}$ $\boxed{+}$ 15 $\boxed{\text{STO}\rightarrow}$ $\boxed{\text{X,T},\theta,n}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$.

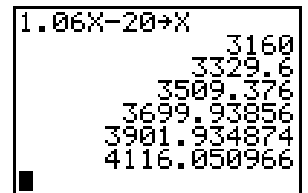


$x_1 = 35$, $x_2 = 155$, and $x_3 = 635$

Example 2 A savings account has an initial balance of \$3000.00. At the end of each year, the bank pays 6% interest and charges a \$20 annual fee. Find the account balance after 6 years.

Store the initial value and enter an expression to calculate the balance at the end of a year.

Keystrokes: 3000 $\boxed{\text{STO}\rightarrow}$ $\boxed{\text{X,T},\theta,n}$ $\boxed{\text{ENTER}}$ 1.06 $\boxed{\text{X,T},\theta,n}$ $\boxed{-}$ 20 $\boxed{\text{STO}\rightarrow}$ $\boxed{\text{X,T},\theta,n}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$.



At the end of six years, the account has a balance of \$4116.05.

Exercises

Find the first three iterates of each function.

1. $f(x) = 6x + 12$ if $x_0 = 5$

2. $f(x) = 2x^2 - 3$ if $x_0 = -1$

3. $f(x) = x^2 - 4x + 5$ if $x_0 = 1$

4. $f(x) = 2x^2 + 2x + 1$ if $x_0 = \frac{1}{2}$

A bank account has an initial balance of \$11,250.00. Interest is paid at the end of each year. Find the account balance under the given interest rate after the stated time period.

5. 3.8%, 2 years

6. 4.75%, 5 years

7. 6.05%, 10 years

8. 7.44%, 15 years

11

Spreadsheet Investigation**Sequences and Series** (Use after Lesson 11-2.)

You have learned about the characteristics of numbers in a sequence. A spreadsheet can calculate a sequence and enable you to find the sum of terms in the series.

Example Create a spreadsheet like the one below and enter the first three terms of a sequence. Find the first ten terms of the sequence. Then find the sum of the first ten terms of the series.

	A	B	C	D	E	F	G	H	I	J	K
1	Symbol	a1	a2	a3							
2	Term	3	2.5	2							
3											

Highlight cells B2 through D2 and move your cursor to any corner of the highlighted cells until a black cross appears. Drag across the row and release it at cell K2. The next values in the sequence will appear in the cells.

To find the sum of the first 10 terms in the series, highlight the cells containing the terms, then click the \sum symbol on the toolbar. The sum will appear in the next cell. Note that this will work for arithmetic series only. The sum of the first ten terms of this series is 7.5

Exercises

1. Create a spreadsheet like the one in the example above. Record the initial sequence as -4 , -1 , and 2 . Repeat the process you followed in the example. What are the next six numbers in the sequence?
2. Describe the steps the spreadsheet program completes to find the next term in the sequence.
3. Use the spreadsheet to find the value for the 16th term in the sequence.
4. Find the sum of the 3rd through 13th terms in the sequence.

12

Graphing Calculator Investigation

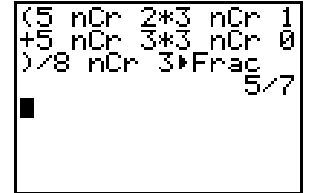
Probabilities *(Use with Lesson 12-5.)*

A graphing calculator can be used to perform calculations involving permutations, combinations, and probability.

Example 1 There are 5 girls and 3 boys on a class committee. A subcommittee of 3 people is being chosen at random. What is the probability that the subcommittee will have at least 2 girls?

$P(\text{at least 2 girls}) = P(2 \text{ girls}) + P(3 \text{ girls})$. Each probability is the product of the combinations of girls and boys divided by the combinations of all the students taken 3 at a time.

Keystrokes: (5 MATH ◀ 3 2 × 3 MATH ◀ 3 1 + 5 MATH ◀ 3 3 × 3 MATH ◀ 3 0) ÷ 8 MATH ◀ 3 3 MATH ENTER ENTER.

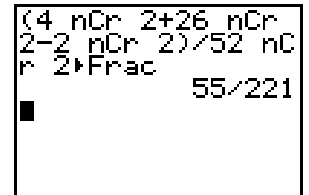


The probability that the subcommittee has at least 2 girls is $\frac{5}{7}$.

Example 2 Two cards are randomly selected from a standard deck of cards. Find the probability that both cards are kings or that both cards are red.

Since these events are mutually inclusive find the combinations of 4 kings taken 2 at a time plus 26 red cards taken 2 at a time minus 2 red kings taken 2 at a time divided by the combinations of 52 cards taken 2 at a time.

Keystrokes: (4 MATH ◀ 3 2 + 26 MATH ◀ 3 2 - 2 MATH ◀ 3 2) ÷ 52 MATH ◀ 3 2 MATH ENTER ENTER.



The probability of choosing 2 kings or two red cards is $\frac{55}{221}$.

Exercises

Find each probability.

- There are 5 girls and 4 boys on the school publications committee. A group of 5 members is being chosen at random to attend a workshop on school newspapers. Find each probability.
 - at least 3 girls
 - 4 girls or 4 boys
 - at least 2 boys
- Two cards are drawn from a standard deck of cards. Find each probability.
 - both queens or both black
 - both kings or both aces
 - both face cards or both black
- Find the probability that a committee of 6 U.S. Representatives selected at random from 7 Democrats and 7 Republicans will have at least 3 Republicans on the committee.
- Three CDs are randomly selected from a collection of 6 rock and 5 rap CDs. Find the probability that at least 2 are rock.

12

Spreadsheet Investigation

Permutations and Combinations *(Use after Lesson 12-2.)*

You have learned the formulas for the number of permutations of n objects taken r at a time, $P(n, r)$, and the number of combinations of n objects taken r at a time, $C(n, r)$. You are going to set up a spreadsheet like the one shown below to perform analyses of these functions.

	A	B	C	D	E	F	G
1	n	5	5	5	5	5	5
2	r	0	1	2	3	4	5
3	$P(n, r)$	1	5	20	60	120	120
4	$C(n, r)$	1	5	10	10	5	1
5							

In the spreadsheet, the values in row 1 represent n , the values in row 2 represent r , and the formulas for $P(n, r)$ and $C(n, r)$ are in rows 3 and 4, respectively.

The formula to calculate $P(n, r)$ is =FACT(B1)/FACT(B1-B2).

FACT is a special function from the function list and should not be entered from the letters on the keyboard. Enter the formula in B3. Then drag the cursor across the row to copy the formula into cells C3 through G3.

The formula for $C(n, r)$ is =FACT(B1)/(FACT(B1-B2)*FACT(B2)) and should be entered in cell B4. Copy the formula into cells C4 through G4.

Exercises

- Compare the values of $P(n, r)$ and $C(n, r)$ for $n = 5$ and $r = 0$ through 5, as well as for two other choices of n and r .
- Several identities hold for $P(n, r)$ and $C(n, r)$. Use the spreadsheet to verify the following identities by finding three examples of each.
 - $P(n, n) = P(n, n - 1)$
 - $C(n + 1, r) = C(n, r - 1) + C(n, r)$
 - $C(n, 0) + C(n, 1) + C(n, 2) + \dots + C(n, n) = 2^n$

13

Graphing Calculator Investigation

Law of Sines: Ambiguous Case *(Use with Lesson 13-4.)*

A graphing calculator can be used to illustrate the Ambiguous Case for the Law of Sines. This program constructs a visual representation of given information. From the drawing, the number of solutions can be determined.

LAWSINES:

```

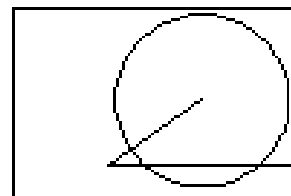
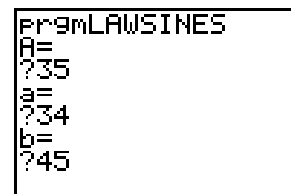
Lbl 1
Disp "A="
Input A
Disp "a="
Input M
Disp "b="
Input B
{0,1,2}→L1:ML1+B→L2
LinReg(ax+b) L1,L2
AxesOff:ClrDraw

bcos(A→D:bsin(A→E
max(1,int((D+a+.999))→Xmax
min(-1,D-int((D+a+.999))→Xmin
int((E+a+.999)→Ymax
min(-1,Ymax-2(Xmax-Xmin)/3) →Ymin
Zsquare
Line(0,0,D,E)
Line(0,0,Xmax,0)
Circle(D,E,a)
    
```

Example In $\triangle ABC$, $A = 35^\circ$, $a = 34$, and $b = 45$. Determine whether $\triangle ABC$ has one, two, or no solutions.

Run the program and enter the given information. Examine the resulting figure for intersection points.

Keystrokes: **PRGM** **▼** to highlight the **LAWSINES** then press **ENTER** **ENTER**. Follow the prompts. $A = 35$ **ENTER** $a = 34$ **ENTER** $b = 45$ **ENTER**. Notice that the circle whose radius is a units intersects the horizontal segment twice. This indicates there are two solutions or two triangles are possible.



Exercises

Determine whether each triangle has one, two, or no possible solutions.

1. $A = 44.3^\circ$, $a = 22$, and $b = 20.1$
2. $A = 126^\circ$, $a = 12$, and $b = 7$
3. $A = 21^\circ$, $a = 2$, and $b = 3$
4. $A = 55^\circ$, $a = 11$, and $b = 15$
5. $A = 112^\circ$, $a = 5$, and $b = 7$
6. $B = 38.6^\circ$, $b = 22.9$, and $c = 33.7$
7. $C = 30^\circ$, $c = 20.2$, and $b = 40.4$
8. $B = 50^\circ$, $b = 13$, and $c = 15$

13

Spreadsheet Investigation

Cofunctions *(Use with Lesson 13-1.)*

The functions of sine and cosine are *cofunctions*. Set up a spreadsheet like the one shown below to investigate the relationships between cofunctions.

	A	B	C	D	E	F	G	H
1	A°	0	30	45	60	75	90	120
2	sin A	0	0.5	0.7071068	0.8660254	0.9659258	1	0.8660254
3	cos A	1	0.8660254	0.7071068	0.5	0.259819	6.123E-17	-0.5

In the spreadsheet, the values in row 1 are the angle values in degrees. The values in rows 2 and 3 are the calculated values for the sine and cosine for each angle, respectively. To use the spreadsheet to find the functions for any angle, first enter each function into the spreadsheet in the form shown below.

$$=\text{SIN}(B1*\text{PI}()/180) \quad =\text{COS}(B1*\text{PI}()/180)$$

This form of the formula contains additional information that is necessary for the spreadsheet to use degrees to calculate the answer. Without it, the spreadsheet cannot recognize that the angle is measured in degrees and will return the wrong answer.

Then, complete the spreadsheet by entering the angles up to 360° whose measures are multiples of 30° and 45° .

Exercises

- Use the spreadsheet to make a graph of the sine values for the angles from 0° to 360° . Then make a graph of the cosine values.
- If $f(x)$ and $g(x)$ are cofunctions, then $f(x) = g(90^\circ - x)$. Compare the shapes of the graphs. How can you tell that sine and cosine are cofunctions by their shapes?

14

Graphing Calculator Investigation

Sinusoidal Equations *(Use with Lesson 14-2.)*

A graphing calculator can be used to verify a sinusoidal regression equation in the form $y = a \sin (bx + c) + d$ given four data points. The sinusoidal regression is found under **[STAT]** **[CALC]** **[C]**.

Example

As a person rides a Ferris wheel, the person’s distance from the ground varies sinusoidally with time. Let t be the number of seconds that have elapsed since the Ferris wheel started. The rider’s position when the last seat is filled and the Ferris wheel starts is when $t = 0$. Suppose it takes 3 seconds to reach the top of the Ferris wheel, 43 feet above the ground. The diameter of the wheel is 40 feet, and it makes a revolution every 8 seconds. Create a table of values and write the sinusoidal equation.

t sec.	1	3	5	7	9	11
$h(t)$ ft.	23	43	23	3	23	43

Keystrokes: Enter the data in **L1** and **L2**. Choose an appropriate window. Use **[STATPLOT]** to graph the points. **[STAT]** **[▶]** **[ALPHA]** **[C]**

[2nd] **[L1]** **[,]** **[2nd]** **[L2]** **[,]** **[VARS]** **[▶]** **[ENTER]** **[ENTER]** **[ENTER]**.

$a = 20, b = \frac{\pi}{4}, c = 1, \text{ and } d = 23$

$h(t) = 20 \sin \frac{\pi}{4}(t - 1) + 23$

```
SinReg
y=a*sin(bx+c)+d
a=20
b=.7853981634
c=-.7853981634
d=23
```

Exercises

As the paddlewheel of a steamboat turns, a point on the paddle blade moves so that its distance, h , from the water’s surface is a sinusoidal function of time. The wheel’s diameter is 18 feet, and it completes a revolution every 10 seconds. The height of the point at various times is shown in the table.

t (seconds)	1.5	4	6.5	9	11.5	14
$h(t)$ (feet)	7	16	7	-2	7	16

1. Why is the height the same after 14 seconds as it is after 4 seconds?
2. What are the values of $a, b, c,$ and d ?
3. Write a regression equation.

14

Spreadsheet Investigation**Trigonometric Identities** (Use after Lesson 14-3.)

A trigonometric identity holds for all values of θ where each expression is defined. For example, $\sin \theta \div \cos \theta = \tan \theta$. You have learned to prove algebraically that an equation is an identity. You can use a spreadsheet to test equations for specific values to see if an equation might be an identity.

To use the spreadsheet to test the values of expressions for different angles, enter the angle measures in the cells in Column A, and enter the expressions from the equations you want to test in the columns to the right. First, enter the formula `=RADIANS(A)` in Column B to convert degrees to radians. (Recall that you can do this by entering the formula `RADIANS(A2)` in cell B2, copying cell B2, and pasting to fill the rest of Column B.) In the spreadsheet shown, the formula `SIN(B)/COS(B)` is in the cells in Column C. The cells in Column D contain the formula `TAN(B)`. Column E contains the formula `TAN(-B)`. Notice that the values in Columns C and D agree with the identity stated above.

	A	B	C	D	E
1	Angle in degrees	Angle in radians	$\sin B / \cos B$	$\tan B$	$\tan (-B)$
2	30	0.523598776	0.577350269	0.57735027	-0.57735027
3	45	0.785398163	1	1	-1
4	60	1.047197551	1.732050808	1.73205081	-1.73205081
5	90	1.570796327	1.63246E+16	1.63246E+16	-1.63246E+16
6	120	2.094395102	-1.732050808	-1.73205081	1.73205081
7	135	2.35619449	-1	-1	1
8	150	2.617993878	-0.577350269	-0.57735027	0.57735027
9	180	3.141592654	-1.22515E-16	-1.2251E-16	1.22515E-16
10					

Exercises

1. Study the values in Columns D and E. What identity seems to be possible from this pattern?
2. Enter the formula `SIN(B)` in Column F. Then enter the formula `COS(B)*TAN(B)` in Column G. What identity do these two new columns suggest?
3. Make a column with the formula `SIN(PI()-B)`. What identity do you discover?

NAME _____ DATE _____ PERIOD _____

1 Graphing Calculator Investigation

Solving Equations and Checking Solutions

(Use with Lesson 1-3.)

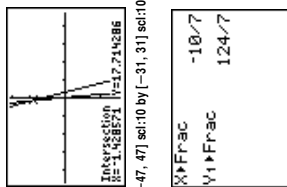
When solving equations, checking the solutions is an important process. A graphing calculator can be used to check the solution of an equation.

Example 1 Solve $-2(5y - 1) - y = -4(y - 3)$.

Graph the expression on the left side of the equation in **Y1** and the expression on the right side of the equation in **Y2**. Choose an appropriate view window so that the intersection of the graphs is visible. Then use the intersect command to find the coordinates of the common point.

Keystrokes: $\sqrt{\square}$ \leftarrow 2 \leftarrow 5 \leftarrow $\frac{\square}{\square}$ \leftarrow 1 \leftarrow) \leftarrow - \leftarrow $\frac{\square}{\square}$ \leftarrow $\frac{\square}{\square}$ \leftarrow ENTER \leftarrow 4 \leftarrow $\frac{\square}{\square}$ \leftarrow 3 \leftarrow) \leftarrow ZOOM \leftarrow 6 \leftarrow ZOOM \leftarrow 8 \leftarrow ENTER \leftarrow 2nd \leftarrow [CALC] \leftarrow 5 \leftarrow ENTER \leftarrow ENTER \leftarrow 2nd \leftarrow [QUIT] \leftarrow $\frac{\square}{\square}$ \leftarrow MATH \leftarrow ENTER \leftarrow ENTER.

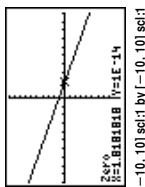
The x-coordinate, $-\frac{10}{7}$, is the solution to the equation. The y-coordinate is the value of both sides of the equation when $x = -\frac{10}{7}$.



Example 2 Solve $\frac{x}{5} - \frac{x}{4} = \frac{1}{2}(x - 2)$.

Graph the expression on the left side of the equation in **Y1** and the expression on the right side of the equation in **Y2**. Enter **Y1 - Y2** in **Y3**. Then graph the function in **Y3**. Use the zero function under the CALC menu to determine where the graph of **Y3** equals zero. This point will be the solution.

Keystrokes: $\sqrt{\square}$ \leftarrow $\frac{\square}{\square}$ \leftarrow 1 \leftarrow) \leftarrow - \leftarrow $\frac{\square}{\square}$ \leftarrow 2 \leftarrow) \leftarrow ENTER \leftarrow VARS \leftarrow ENTER \leftarrow 1 \leftarrow) \leftarrow 2 \leftarrow) \leftarrow $\frac{\square}{\square}$ \leftarrow 2 \leftarrow) \leftarrow ENTER \leftarrow VARS \leftarrow ENTER \leftarrow ENTER \leftarrow 2nd \leftarrow [CALC] \leftarrow 2.



Use arrow keys and enter to set the bound prompts. The solution is $x = \frac{20}{11}$.

Example 3

Solve each equation.

- $-3(2w - 7) = 9 - 2(5w + 4)$ 1. $5(4 - x) = 1.3(2 - x)$ 3. $\frac{1}{2}(a + 2) = \frac{1}{6}(5 - a)$
 $w = -5$ $x = 17$ $a = 5$
- $3(2z + 25) - 2(z - 1) = 78$ 5. $\frac{m - 4}{3} - \frac{3m - 1}{5} = 1$ 6. $\frac{x + 5}{2} + \frac{1}{2} = 2x - \frac{x - 3}{8}$
 $z = \frac{1}{4}$ $m = -8$ $x = \frac{21}{11}$

NAME _____ DATE _____ PERIOD _____

1 Spreadsheet Investigation

Absolute Value Statements

(Use with Lesson 1-4.)

You can use a spreadsheet to try several different values in an equation to help you determine whether the statement is *sometimes, always, or never* true. Remember that showing that a statement is true for some values does not prove that it is true for all values. However, finding one value for which a statement is false proves that it is not true for all values.

Example Determine whether $c|a + b| = |ca + cb|$ is *sometimes, always, or never* true.

Try a number of values for a , b , and c to determine whether the statement is true or false for each set of values.

Step 1 Use Columns A, B, and C for the values of a , b , and c . Choose several sets of values including positive and negative numbers, and zero.

	A	B	C	D
1	a	b	c	$c a + b = ca + cb $
2	1	1	0	TRUE
3	-1	-1	-1	FALSE
4	2	-2	2	TRUE
5	1	1	1	TRUE
6	1	1	-2	FALSE
7	2	1	0	TRUE
8	1	1	2	TRUE
9	0	0	3	TRUE
10				

Step 2 Use Column D to test the equation. A formula such as $C2*ABS(A2+B2) = ABS(C2*A2+C2*B2)$ in cell D2 returns TRUE if the equation is true.

Through observation of Column D, when c is negative the statement is not true. The absolute value statement, $c|a + b| = |ca + cb|$ is sometimes true; it is true only if $c \geq 0$.

Now Discuss

Use a spreadsheet to determine whether each absolute value statement is *sometimes, always, or never* true.

- For all real numbers a and b , $a \neq 0$, $|ax + b| = 0$. **sometimes**
- If a and b are real numbers, then $|a + b| = |a| + |b|$. **sometimes**
- If a and b are real numbers, then $|a + b| = -x$. **never**
- If a and b are real numbers, then $|a| - |b| = a - b$. **sometimes**
- If a and b are real numbers, then $c|a + b| = c|a| + |b|$. **sometimes**

NAME _____ DATE _____ PERIOD _____

1 Graphing Calculator Investigation

Solving Equations and Checking Solutions

(Use with Lesson 1-3.)

When solving equations, checking the solutions is an important process. A graphing calculator can be used to check the solution of an equation.

Example 1 Solve $-2(5y - 1) - y = -4(y - 3)$.

Graph the expression on the left side of the equation in **Y1** and the expression on the right side of the equation in **Y2**. Choose an appropriate view window so that the intersection of the graphs is visible. Then use the intersect command to find the coordinates of the common point.

The x-coordinate, $-\frac{10}{7}$, is the solution to the equation. The y-coordinate is the value of both sides of the equation when $x = -\frac{10}{7}$.

Example 2 Solve $\frac{x}{5} - \frac{x}{4} = \frac{1}{2}(x - 2)$.

Graph the expression on the left side of the equation in **Y1** and the expression on the right side of the equation in **Y2**. Enter **Y1 - Y2** in **Y3**. Then graph the function in **Y3**. Use the zero function under the CALC menu to determine where the graph of **Y3** equals zero. This point will be the solution.

Keystrokes: $\sqrt{\square}$ \leftarrow $\frac{\square}{\square}$ \leftarrow 1 \leftarrow) \leftarrow - \leftarrow $\frac{\square}{\square}$ \leftarrow 2 \leftarrow) \leftarrow ENTER \leftarrow VARS \leftarrow ENTER \leftarrow 1 \leftarrow) \leftarrow 2 \leftarrow) \leftarrow $\frac{\square}{\square}$ \leftarrow 2 \leftarrow) \leftarrow ENTER \leftarrow VARS \leftarrow ENTER \leftarrow ENTER \leftarrow 2nd \leftarrow [CALC] \leftarrow 2.

Use arrow keys and enter to set the bound prompts. The solution is $x = \frac{20}{11}$.

Example 3

Solve each equation.

- $-3(2w - 7) = 9 - 2(5w + 4)$ 2. $1.5(4 - x) = 1.3(2 - x)$ 3. $\frac{1}{2}(a + 2) = \frac{1}{6}(5 - a)$
 $w = -5$ $x = 17$ $a = 5$
- $3(2z + 25) - 2(z - 1) = 78$ 5. $\frac{m - 4}{3} - \frac{3m - 1}{5} = 1$ 6. $\frac{x + 5}{2} + \frac{1}{2} = 2x - \frac{x - 3}{8}$
 $z = \frac{1}{4}$ $m = -8$ $x = \frac{21}{11}$

NAME _____ DATE _____ PERIOD _____

1 Spreadsheet Investigation

Absolute Value Statements

(Use with Lesson 1-4.)

You can use a spreadsheet to try several different values in an equation to help you determine whether the statement is *sometimes, always, or never* true. Remember that showing that a statement is true for some values does not prove that it is true for all values. However, finding one value for which a statement is false proves that it is not true for all values.

Example Determine whether $c|a + b| = |ca + cb|$ is *sometimes, always, or never* true.

Try a number of values for a , b , and c to determine whether the statement is true or false for each set of values.

Step 1 Use Columns A, B, and C for the values of a , b , and c . Choose several sets of values including positive and negative numbers, and zero.

	A	B	C	D
1	a	b	c	$c a + b = ca + cb $
2	1	1	0	TRUE
3	-1	-1	-1	FALSE
4	2	-2	2	TRUE
5	1	1	1	TRUE
6	1	1	-2	FALSE
7	2	1	0	TRUE
8	1	1	2	TRUE
9	0	0	3	TRUE
10				

Step 2 Use Column D to test the equation. A formula such as $C2*ABS(A2+B2) = ABS(C2*A2+C2*B2)$ in cell D2 returns TRUE if the equation is true.

Through observation of Column D, when c is negative the statement is not true. The absolute value statement, $c|a + b| = |ca + cb|$ is sometimes true; it is true only if $c \geq 0$.

Now Discuss

Use a spreadsheet to determine whether each absolute value statement is *sometimes, always, or never* true.

- For all real numbers a and b , $a \neq 0$, $|ax + b| = 0$. **sometimes**
- If a and b are real numbers, then $|a + b| = |a| + |b|$. **sometimes**
- If a and b are real numbers, then $|a + b| = -x$. **never**
- If a and b are real numbers, then $|a| - |b| = a - b$. **sometimes**
- If a and b are real numbers, then $c|a + b| = c|a| + |b|$. **sometimes**

2 Graphing Calculator Investigation

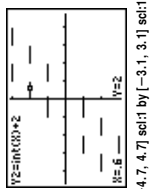
Transformations: Greatest Integer Function

(Use with Lesson 2-6.)

A graphing calculator can be used to display transformations to the greatest integer function. This is done by using the **int()** command under the **MATH: NUM** menu. When graphing the greatest integer function, it is important to set the calculator to **Dot** mode.

Example Graph $f(x) = \lfloor x \rfloor$ and $g(x) = \lfloor x \rfloor + 2$ on the same axes. Evaluate each function for $x = 0.6, x = 1, x = 1.3, x = 2, x = -0.5$, and $x = -1.1$. Compare the graphs of the functions.

Enter $f(x)$ in Y1 and $g(x)$ in Y2. Graph the functions.
 Keystrokes: $\boxed{Y=}$ $\boxed{\text{MATH}}$ $\boxed{\blacktriangleright}$ $\boxed{5}$ $\boxed{\text{X,T,}\theta,\rho}$ $\boxed{\text{ENTER}}$ $\boxed{\text{MATH}}$ $\boxed{\blacktriangleright}$ $\boxed{5}$ $\boxed{+}$ $\boxed{2}$ $\boxed{\text{ENTER}}$
 $\boxed{\text{ZOOM}}$ $\boxed{6}$ $\boxed{\text{ZOOM}}$ $\boxed{4}$.



Use **TRACE** to evaluate each function for the given values.
 Keystrokes: $\boxed{\text{TRACE}}$ $\boxed{0.6}$ $\boxed{\text{ENTER}}$ $\boxed{1}$ $\boxed{\text{ENTER}}$ $\boxed{1.3}$ $\boxed{\text{ENTER}}$ $\boxed{2}$ $\boxed{\text{ENTER}}$ $\boxed{0.5}$
 $\boxed{\text{ENTER}}$ $\boxed{-1.1}$ $\boxed{\text{ENTER}}$ $\boxed{\blacktriangleleft}$ $\boxed{0.6}$ $\boxed{\text{ENTER}}$ $\boxed{1}$ $\boxed{\text{ENTER}}$ $\boxed{1.3}$ $\boxed{\text{ENTER}}$ $\boxed{2}$ $\boxed{\text{ENTER}}$ $\boxed{-}$
 $\boxed{0.5}$ $\boxed{\text{ENTER}}$ $\boxed{-}$ $\boxed{1.1}$ $\boxed{\text{ENTER}}$.
 $f(0.6) = 0, f(1) = 1, f(1.3) = 1, f(2) = 2, f(-0.5) = -1$, and $f(-1.1) = -2$
 $g(0.6) = 2, g(1) = 3, g(1.3) = 3, g(2) = 4, g(-0.5) = -1$, and $g(-1.1) = 0$
 [-4.7, 4.7] sc1: by [-3.1, 3.1] sc1

The graph of $g(x) = \lfloor x + 2 \rfloor$ is the same as the graph of $f(x) = \lfloor x \rfloor$ shifted 2 units up.

EXERCISES

Graph each function. Evaluate it for $x = 1, x = 1.3$, and $x = 2$. Compare the graph of the function to the graph of $f(x) = \lfloor x \rfloor$.

- $g(x) = \lfloor x \rfloor - 3$

 [-4.7, 4.7] sc1: by [-3.1, 3.1] sc1: 1
 $g(1) = -2, g(1.3) = -2, g(2) = -1$
 shifted down 3 units
- $g(x) = \lfloor x - 2 \rfloor$

 [-4.7, 4.7] sc1: by [-3.1, 3.1] sc1: 1
 $g(1) = -1, g(1.3) = -1, g(2) = 0$
 shifted right 2 units
- $g(x) = \lfloor 2x \rfloor$

 [-4.7, 4.7] sc1: by [-3.1, 3.1] sc1: 1
 $g(1) = 2, g(1.3) = 2, g(2) = 4$
 compressed by 1/2 horizontally
- $g(x) = \lfloor -x \rfloor$

 [-4.7, 4.7] sc1: by [-3.1, 3.1] sc1: 1
 $g(1) = -1, g(1.3) = -2, g(2) = -2$
 reflected across the y-axis
- $g(x) = \lfloor x + 4 \rfloor$

 [-4.7, 4.7] sc1: by [-3.1, 3.1] sc1: 1
 $g(1) = 5, g(1.3) = 5, g(2) = 6$
 shifted left 4 units
- $g(x) = 3\lfloor x \rfloor$

 [-4.7, 4.7] sc1: by [-3.1, 3.1] sc1: 1
 $g(1) = 3, g(1.3) = 3, g(2) = 6$
 expanded by 3 vertically

2 Spreadsheet Investigation

Using Linear Equations

(Use with Lesson 2-4.)

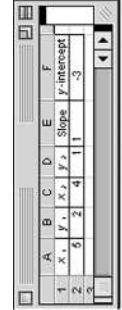
The slope intercept form of a linear equation is $y = mx + b$, where m is the slope and b is the y -intercept. Recall that the formula for the slope of a line through (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$. You can use the formula for slope and the slope-intercept form to find the value of b .

Example State the slope and y -intercept of the graph of the line through $(5, 2)$ and $(4, 1)$. Then write an equation of the line in slope-intercept form.

Step 1 Use Columns A and B to represent the first point, and Columns C and D to represent the second point on the line. Enter the formula for slope in Column E.

Step 2 Substitute one of the given points into the slope-intercept form and solve for b . Since we know the slope of the line, we can solve for b .
 $y = mx + b$
 Slope-intercept form
 $y_1 = mx_1 + b$
 Substitute (x_1, y_1) .
 $y_1 - mx_1 = b$
 Solve for b .

Enter this formula into Column F using the names of the spreadsheet cells.



The slope of the line through $(5, 2)$ and $(4, 1)$ is 1 and the y -intercept is -2 . Thus, the equation of the line slope-intercept form is $y = 1x + (-2)$ or $y = x - 2$.

EXERCISES

Use a spreadsheet to find the slope and y -intercept of the line through each pair of points. Then write an equation of the line in slope-intercept form.

- $(0, -5), (2, 5)$ 2. $(4, 2), (-3, -5)$
 5, -5; $y = -5x - 5$ 1, -2; $y = -x - 2$
- $(-1, -4), (1, 3)$ 4. $(-4, -9), (8, 3)$
 $\frac{7}{2}, -\frac{1}{2}$; $y = \frac{7}{2}x - \frac{1}{2}$ $\frac{11}{12}, -\frac{5}{3}$; $y = \frac{11}{12}x - \frac{5}{3}$
- $(12, 9), (10, 10)$ 6. $(-1.5, 3.1), (0.9, 1.9)$
 $-\frac{1}{2}, 15$; $y = -\frac{1}{2}x + 15$ -0.5, 2.35; $y = -0.5x + 2.35$

7. Does the spreadsheet work when two points have the same x -coordinates? Explain.
No; The slope is undefined.

NAME _____ DATE _____ PERIOD _____

3 Graphing Calculator Investigation

Linear Programming (Use with Lesson 3-4.)

A graphing calculator can store the x - and y -coordinates when using the **intersect** command in the [CALC] menu. This can be displayed on the home screen and used to evaluate an expression with x and y variables. This process is useful in finding the vertices of the feasible region and determining the maximum or minimum value for $f(x, y)$.

Example Graph the system $x - 3y \geq -7$, $5x + y \leq 13$, $x + 6y \geq -9$, $3x - 2y \geq -7$, and $f(x, y) = 4x - 3y$. Find the coordinates of the feasible region. Then find the maximum and minimum values for the system.

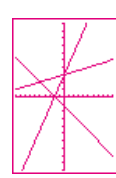
Solve each inequality for y . Enter each boundary equation in the **Y=** screen. Find the vertices of the feasible region. Then find the values of $f(x, y)$ to determine the maximum and minimum values.

Keystrokes: $\text{V}=\text{}$ $\text{C}1$ + 3 $\text{X.T.}\theta/\theta$ + $\text{C}7$ + 3 ENTER
 $\text{C}5$ $\text{X.T.}\theta/\theta$ + 13 ENTER $\text{C}1$ + 6 $\text{X.T.}\theta/\theta$ + $\text{C}3$ + 9 ENTER
 $\text{C}3$ $\text{X.T.}\theta/\theta$ + $\text{C}2$ + 2 $\text{X.T.}\theta/\theta$ + $\text{C}7$ + 2 ZOOM 6
 2nd [CALC] 5 ENTER 2nd [QUIT] 2nd [1]
 $\text{X.T.}\theta/\theta$ ALPHA [Y] + 4 $\text{X.T.}\theta/\theta$ + 3 ALPHA [Y] 2nd [1] ENTER
 GRAPH 2nd [CALC] 5 ENTER ENTER 2nd [QUIT]
 2nd [ENTRY] ENTER GRAPH 2nd [CALC] 5 ENTER ENTER
 ENTER ENTER 2nd [QUIT] 2nd [GRAPH] 2nd
 [CALC] 5 ENTER ENTER 2nd [QUIT] 2nd
 [ENTRY] ENTER .

The maximum value of the system is 18 and the minimum value is -10 .

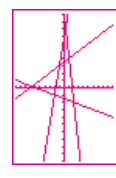
Extra disks

1. $2x + 3y \geq 6$
 $3x - 2y \geq 4$
 $5x + y \geq 15$
 $f(x, y) = x + 3y$



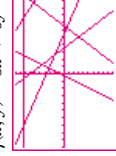
$[-10, 10]$ set by $[-10, 10]$ set 1
 $(0, 2)$, $(3, 0)$, $(2, 9)$
 min. = 3, max. = 17

2. $y \leq 4x + 6$
 $x + 4y \leq 7$
 $2x + y \leq 7$
 $x - 6y \leq 10$
 $f(x, y) = 2x - y$



$[-10, 10]$ set by $[-10, 10]$ set 1
 $(-1, 2)$, $(-2, -2)$, $(3, 1)$, $(4, -1)$
 min. = -4 , max. = 9

3. $y \leq 16 - x$
 $0 \leq 2y \leq 17$
 $2x + 3y \geq 11$
 $y \leq 3x + 1$
 $y \geq 2x - 13$
 $y \geq 7 - 2x$
 $f(x, y) = 5x + 6y$



$[-10, 10]$ set by $[-10, 10]$ set 1
 $(5.5, 0)$, $(6.5, 0)$, $(7.5, 8.5)$,
 $(1.2, 4.6)$, $(2.5, 2)$, $(9.66, 6.33)$
 min. = 24.5, max. = 88.5

Graph each system. Find the coordinates of the vertices of the feasible region. Then find the maximum and minimum values for the system.

NAME _____ DATE _____ PERIOD _____

3 Spreadsheet Investigation

Break-Even Point (Use with Lesson 3-1.)

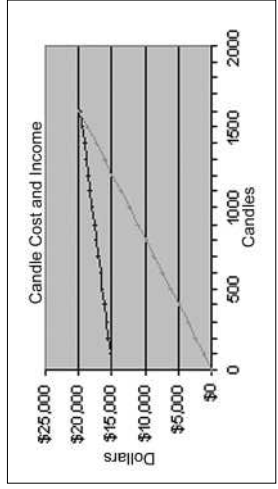
You have learned that the break-even point is the point at which the income equals the cost. You can use the formulas and charts in a spreadsheet to find a break-even point.

Example Carly Ericson is considering opening a candle business. She estimates that she will have an annual overhead of \$15,000. It costs Carly \$3.00 to make a jar candle, which she sells for \$12.50. What is Carly's break-even point?

A	B	C
1	Candles	Income
2	0	\$15,000
3	100	\$15,300
4	200	\$15,600
5	300	\$15,900
6	400	\$16,200
7	500	\$16,500
8	600	\$16,800
9	700	\$17,100
10	800	\$17,400
11	900	\$17,700
12	1000	\$18,000
13	1100	\$18,300
14	1200	\$18,600
15	1300	\$18,900
16	1400	\$19,200
17	1500	\$19,500
18	1600	\$19,800
19		\$20,000

Use Column A for the number of candles. Columns B and C are the cost and the income, respectively. Extend the rows of the spreadsheet to find the point at which the income first exceeds the cost. The break-even point occurs between this point and the previous point. In this case, the break even point occurs between 1500 and 1600 candles.

The chart tool of the spreadsheet allows you to graph the data. The graph verifies the solution.



Extra disks

- If Carly could decrease her annual overhead to \$14,000, what would the break-even point be? **between 1400 and 1500 candles**
- Suppose Carly decreases her annual overhead to \$14,000 and increases the price of a candle to \$14.00. What is the new break-even point? **between 1200 and 1300 candles**

NAME _____ DATE _____ PERIOD _____

4 Spreadsheet Investigation

Cramer's Rule (Use with Lesson 4-5.)

You have learned to solve systems of linear equations by using matrix equations and the inverse matrix. Another way to solve systems is to use *Cramer's Rule*. Study the spreadsheet below to discover Cramer's Rule.

	A	B	C
1	6	3	-12
2	5	1	8
3			
4	=A1*B2-B1*A2		
5	=C1*B2-B1*C2		
7	=A1*C2-C1*A2		
9	=(A6/A4)		
11	=(A8/A4)		
12			

To use the spreadsheet to solve a system of equations, write each equation in the form below.

$$ax + by = c$$

The values for the system $6x + 3y = -12$ and $5x + y = 8$ are shown. In the spreadsheet, the values of a , b , and c for the first equation are entered in cells A1, B1, and C1, respectively. The values of a , b , and c for the second equation are entered in cells A2, B2, and C2, respectively.

The values in cells B10 and B11 represent the solution for the system.

Now Class

- Study the formula in cell A4. Write a matrix whose determinant is found using this formula.

$$\begin{vmatrix} A1 & B1 \\ A2 & B2 \end{vmatrix}$$
- Write matrices whose determinants are found using the formulas in cells A6 and A8.

$$\begin{vmatrix} C1 & B1 \\ C2 & B2 \end{vmatrix}; \begin{vmatrix} A1 & C1 \\ A2 & C2 \end{vmatrix}$$
- Explain how the values of x and y are found using Cramer's rule.

$$x = \frac{\begin{vmatrix} C1 & B1 \\ C2 & B2 \end{vmatrix}}{\begin{vmatrix} A1 & B1 \\ A2 & B2 \end{vmatrix}}; y = \frac{\begin{vmatrix} A1 & C1 \\ A2 & B2 \end{vmatrix}}{\begin{vmatrix} A1 & B1 \\ A2 & B2 \end{vmatrix}}$$

Use the spreadsheet to solve each system of equations.

- $6x + 3y = -12$
 $5x + y = 8$
(4, -12)
- $8x - 3y = 11$
 $6x + 9y = 15$
(1.6, 0.6)
- $3y = 4x + 28$
 $5x + 7y = 8$
(-4, 4)
- $0.3x + 1.6y = 0.44$
 $0.4x + 2.5y = 0.66$
(0.4, 0.2)
- $9x = -0.5x + 4$
 $y = 4x - 5$
(2, 3)

NAME _____ DATE _____ PERIOD _____

4 Graphing Calculator Investigations

Matrices for 30°, 45°, 60° Rotations (Use with Lesson 4-4.)

The rotation matrix for 90° counterclockwise about the origin is $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. The general rotation matrix for any angle θ counterclockwise about the origin is $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

Example 1 Find the coordinates of the image of $\triangle ABC$ with vertices $A(0, 0)$, $B(6, 0)$ and $C(3, 4)$ after a counterclockwise rotation of 30° about the origin.

Enter the coordinates of the vertices of the in vertex matrix [A] and the rotation matrix in matrix [B]. Be sure the calculator is set in Degree mode. Keystrokes: 2nd [MATRIX] \blacktriangleright [ENTER] 2 [ENTER] 3 [ENTER] 0 [ENTER] 3 [ENTER] 0 [ENTER] 4 [ENTER] 2 [ENTER] [MATRIX] \blacktriangleright 2 2 [ENTER] 2 [ENTER] COS 30 \blacktriangleright [ENTER] 2 [ENTER] SIN 30 \blacktriangleright [ENTER] 30 \blacktriangleright [ENTER] 2 [ENTER] [QUIT] \blacktriangleright 2 [ENTER] [MATRIX] 2 \blacktriangleright 2 [ENTER] [MATRIX] 1 [ENTER] . Hold \blacktriangleright to scroll across to see the other coordinates.

The coordinates of the image of $\triangle ABC$ are $A'(0, 0)$, $B'(5.2, 3)$, and $C'(0.6, 5.0)$.

Example 2 Find the coordinates of the image of $\triangle ABC$ with vertices $A(0, 0)$, $B(6, 0)$ and $C(3, 4)$ after a two rotations of 45° counterclockwise about the origin.

In order to rotate the image twice, store the vertex matrix of the first image.

Keystrokes: 2nd [MATRIX] \blacktriangleright 2 [ENTER] [ENTER] COS 45 \blacktriangleright [ENTER] \blacktriangleleft [SIN 45 \blacktriangleright [ENTER] SIN 45 \blacktriangleright COS 45 \blacktriangleright [ENTER] 2nd [QUIT] 2nd [MATRIX] 2 2nd [MATRIX] 1 [ENTER] 2 [ENTER] [ENTRY] [STO] 2nd [MATRIX] 6 [ENTER] 2 [ENTER] [ENTRY] 2nd [MATRIX] 6 [ENTER] 2nd [MATRIX] 6 [ENTER].

The vertices of $\triangle A'B'C'$ are $A'(0, 0)$, $B'(0, 6)$, and $C'(-4, 3)$.

Exercises

Quadrilateral $ABCD$ has vertices $A(0, 0)$, $B(4, 0)$, $C(4, 6)$, and $D(0, 6)$. Find the coordinates of the vertices of the image after each counterclockwise rotation. Round to the nearest tenth.

- 45°
 $A'(0, 0)$, $B'(2.8, 2.8)$, $A'(0, 0)$, $B'(3.5, 2)$, $C'(-1.4, 7.1)$, $D'(-4.2, 4.2)$ $C'(0.5, 7.2)$, $D'(-3, 5.2)$
- 30°
 $A'(0, 0)$, $B'(2.3, 3.5)$, $C'(-3.2, 6.5)$, $D'(-5.2, 3)$
- 60°
 $A'(0, 0)$, $B'(-2.8, 2.8)$, $A'(0, 0)$, $B'(1, 3.9)$, $C'(-4.8, 5.4)$, $D'(-5.8, 1.6)$
- 120°
 $A'(0, 0)$, $B'(-2, 3.5)$, $C'(-7.2, 0.5)$, $D'(-5.2, -3)$
- 75°
 $A'(0, 0)$, $B'(-2.8, 2.8)$, $C'(-7.1, -1.4)$, $D'(-4.2, -4.2)$

NAME _____

DATE _____

PERIOD _____

5

Graphing Calculator Investigation

Using Tables to Factor by Grouping (Use with Lesson 5-4.)

The **TABLE** feature of a graphing calculator can be used to help factor a polynomial of the form $ax^2 + bx + c$.

Example 1 Factor $10x^2 - 43x + 28$ by grouping.

Make a table of the negative factors of $10 \cdot 28$ or 280. Look for a pair of factors whose sum is -43 .
Enter the equation $y = \frac{280}{x}$ in **Y1** to find the factors of 280. Then, find the sum of the factors using $y = \frac{280}{x} + x$ in **Y2**. Set up the table to display the negative factors of 280 by setting $\Delta Tbl = -1$. Examine the results.

Keystrokes: $\boxed{Y=}$ $\boxed{280}$ $\boxed{\div}$ $\boxed{XT\theta/n}$ \boxed{ENTER} \boxed{VARS} $\boxed{\blacktriangleright}$ \boxed{ENTER} \boxed{ENTER} $\boxed{+}$ $\boxed{XT\theta/n}$ \boxed{ENTER} $\boxed{2nd}$ $\boxed{[TBLSET]}$ $\boxed{\leftarrow}$ $\boxed{1}$ \boxed{ENTER} $\boxed{\leftarrow}$ $\boxed{1}$ \boxed{ENTER} $\boxed{2nd}$ $\boxed{[TABLE]}$.

The last line of the table shows that $-43x$ may be replaced with $-8x + (-35x)$.

$$10x^2 - 43x + 28 = 10x^2 - 8x + (-35x) + 28$$

$$= 2x(5x - 4) + (-7)(5x - 4)$$

$$= (5x - 4)(2x - 7)$$

Thus, $10x^2 - 43x + 28 = (5x - 4)(2x - 7)$.

Example 2 Factor $12x^2 - 7x - 12$.

Look at the factors of $12 \cdot -12$ or -144 for a pair whose sum is -7 . Enter an equation to determine the factors in **Y1** and an equation to find the sum of factors in **Y2**. Examine the table to find a sum of -7 .

Keystrokes: $\boxed{Y=}$ $\boxed{\leftarrow}$ $\boxed{144}$ $\boxed{\div}$ $\boxed{XT\theta/n}$ \boxed{ENTER} \boxed{VARS} $\boxed{\blacktriangleright}$ \boxed{ENTER} \boxed{ENTER} $\boxed{+}$ $\boxed{XT\theta/n}$ \boxed{ENTER} $\boxed{2nd}$ $\boxed{[TBLSET]}$ $\boxed{1}$ \boxed{ENTER} $\boxed{2nd}$ $\boxed{[TABLE]}$.

$$12x^2 - 7x - 12 = 12x^2 + 9x + (-16x) - 12$$

$$= 3x(4x + 3) - 4(4x + 3)$$

$$= (4x + 3)(3x - 4)$$

Thus, $12x^2 - 7x - 12 = (4x + 3)(3x - 4)$.

Extra Checks

Factor each polynomial.

- $y^2 - 20y - 96$ $2. 4z^2 - 33z + 35$ $3. 4y^2 + y - 18$ $4. 6a^2 + 2a - 15$
- $(y + 4)(y - 24)$ $(4z - 5)(z - 7)$ $(4y - 9)(y - 2)$ **prime**
- $6m^2 + 17m + 12$ $6. 24z^2 - 46z + 15$ $7. 36y^2 + 84y + 49$ $8. 4b^2 + 36b - 403$
- $(2m + 3)(3m + 4)$ $(12z - 5)(2z + 3)$ $(6y + 7)^2$ $(2b + 31)(2b - 13)$

NAME _____

DATE _____

PERIOD _____

5

Spreadsheet Investigation

Appreciation and Depreciation (Use with Lesson 5-7.)

When an asset such as a house increases in value over time, it is said to *appreciate*. If the value increases by a fixed percent each year, or other period of time, the amount y of that quantity after t years is given by

$$y = a(1 + r)^t$$

where a is the initial amount and r is the percent of increase expressed as a decimal. You can use a spreadsheet to investigate future values of an asset.

Example

Michael Blackstock is considering buying a piece of investment property for \$95,000. The homes in the area are appreciating at an average rate of 4% per year. Find the expected value of the home in 1 year, 1 year and 6 months, 4 years, and 6 years and 9 months.

Use rows 1 and 2 to enter the initial amount and the rate of increase. Then use Column A to enter the amounts of time. Enter the numbers of months as a fraction of a year since t is measured in years. Column B contains the formulas for the value of the home.

Format the cells containing the values as currency so that they are displayed as dollars and cents. The expected value of the home after each amount of time is shown in the spreadsheet.

A	B
1	Initial value = \$95,000.00
2	Rate = 0.04
3	YEAR
4	1
5	1.5
6	4
7	4.5
8	6
9	6.75
10	
Value	
1	\$99,900.00
2	\$104,796.93
3	\$110,792.73
4	\$117,997.73
5	\$125,432.73
6	\$133,199.73
7	\$141,309.73
8	\$149,772.73
9	\$158,609.73
10	\$167,840.73

Extra Checks

- If Mr. Blackstock chooses another property in the neighborhood that costs \$99,900, what are the expected values of that home in the same periods of time? **\$103,896.00, \$105,953.55, \$116,968.87, \$130,178.88**
- What would Mr. Blackstock's profit be on the \$99,900 home if he sold it after 9 years and 3 months? **\$143,589.89**
- If an antique chair worth \$165.00 increases in value an average of $3\frac{1}{2}\%$ every year, how much will it be worth next year? **\$170.78**
- Often assets like cars decrease in value over time. This asset is said to *depreciate*. If the value decreases by a fixed percent each year, or other period of time, the amount y of that quantity after t years is given by $y = a(1 - r)^t$, where a is the initial amount and r is the percent of decrease expressed as a decimal. Use a spreadsheet to find the value of a car purchased for \$18,500 after 2 years, 2 years and 6 months, and 4 years and 3 months if the car depreciates at a rate of 12% per year. **\$14,326.40, \$13,439.35, \$10,745.41**

NAME _____ DATE _____ PERIOD _____

6 Spreadsheet Investigation

Approximating the Real Zeros of Polynomials

(Use with Lesson 6-5.)

You have learned the Location Principle, which can be used to approximate the real zeros of a polynomial.

1	A	B	C	D	E	F	G	H	I	J
2	-5	5	x	-5	-3.3333333	-1.6666667	0	1.6666667	3.3333333	5
3		$f(x)$	23	9.1111111	0.7777778	-2	9.1111111	0.7777778	23	
4										

In the spreadsheet above, the positive real zero of $f(x) = x^2 - 2$ can be approximated in the following way. Set the spreadsheet preference to manual calculation. The values in A2 and B2 are the endpoints of a range of values. The values in D2 through J2 are values equally in the interval from A2 to B2. The formulas for these values are $A2, A2 + (B2 - A2)/6, A2 + 2*(B2 - A2)/6, A2 + 3*(B2 - A2)/6, A2 + 4*(B2 - A2)/6, A2 + 5*(B2 - A2)/6$, and B2, respectively.

Row 3 gives the function values at these points. The function $f(x) = x^2 - 2$ is entered into the spreadsheet in Cell D3 as $D2^2 - 2$. This function is then copied to the remaining cells in the row.

You can use this spreadsheet to study the function values at the points in cells D2 through J2. The value in cell F3 is positive and the value in cell G3 is negative, so there must be a zero between -1.6667 and 0. Enter these values in cells A2 and B2, respectively, and recalculate the spreadsheet. (You will have to recalculate a number of times.) The result is a new table from which you can see that there is a zero between 1.41414 and 1.414306.

Because these values agree to three decimal places, the zero is about 1.414.

This can be verified by using algebra.

EXERCISES

By solving $x^2 - 2 = 0$, we obtain $x = \pm\sqrt{2}$. The positive root is $x = \pm\sqrt{2} = 1.414213...$, which verifies the result.

1. Use a spreadsheet like the one above to approximate the zero of $f(x) = 3x - 2$ to three decimal places. Then verify your answer by using algebra to find the exact value of the root. **The spreadsheet gives $x = 0.667$. By solving for x algebraically, $x = \frac{2}{3}$. So, the approximation is correct.**
2. Use a spreadsheet like the one above to approximate the real zeros of $f(x) = x^2 + 2x + 0.5$. Round your answer to four decimal places. Then, verify your answer by using the quadratic formula. **The process gives -1.7071 and -0.2929 to the nearest ten-thousandth. The quadratic formula gives $x = -1 \pm \frac{\sqrt{2}}{2}$. $-1 - \frac{\sqrt{2}}{2} \approx -1.7071$ and $-1 + \frac{\sqrt{2}}{2} \approx -0.2929$.**

3. Use a spreadsheet like the one above to approximate the real zero of $f(x) = x^3 - \frac{3}{2}x^2 - 6x - 2$ between -0.4 and -0.3 . **-0.3781 to the nearest ten-thousandth**

Glencoe Algebra 2

38

© Glencoe/McGraw-Hill

NAME _____ DATE _____ PERIOD _____

6 Graphing Calculator Investigation

Quadratic Inequalities and the Test Menu

(Use with Lesson 6-7.)

The inequality symbols, called *relational operators*, in the TEST menu can be used to display the solution of a quadratic inequality. Another method that can be used to find the solution set of a quadratic inequality is to graph each side of an inequality separately. Examine the graphs and use the **intersect** function to determine the range of values for which the inequality is true.

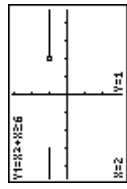
Example 1 Solve $x^2 + x \geq 6$.

Place the calculator in Dot mode. Enter the inequality into Y1. Then trace the graph and describe the solution as an inequality.

Keystrokes: $\sqrt{\square}$ $\boxed{X_1}$ $\boxed{0}$ $\boxed{+}$ $\boxed{X_2}$ $\boxed{0}$ $\boxed{+}$ $\boxed{2}$ nd $\boxed{[TEST]}$ $\boxed{4}$ $\boxed{6}$ $\boxed{[ZOOM]}$ $\boxed{4}$.

Use **TRACE** to determine the endpoints of the segments.

These values are used to express the solution of the inequality, $\{x \mid x \geq -3 \text{ or } x \geq 2\}$.



$[-4.7, 4.7]$ set:1 by $[-3.1, 3.1]$ set:1

Example 2 Solve $2x^2 + 4x - 5 \leq 3$.

Place the left side of the inequality in Y1 and the right side in Y2.

Determine the points of intersection. Use the intersection points to express the solution set of the inequality. Be sure to set the calculator to **Connected** mode.

Keystrokes: $\sqrt{\square}$ $\boxed{2}$ $\boxed{X_1}$ $\boxed{0}$ $\boxed{+}$ $\boxed{4}$ $\boxed{X_2}$ $\boxed{0}$ $\boxed{-}$ $\boxed{5}$ $\boxed{[ENTER]}$ $\boxed{3}$ $\boxed{[ENTER]}$ \boxed{ZOOM} $\boxed{6}$.

Press $\boxed{2}$ nd $\boxed{[CALC]}$ $\boxed{5}$ and use the \blacktriangleleft key to move the cursor to the left of the first intersection point. Press $\boxed{[ENTER]}$. Then move the cursor to the right of the intersection point and press $\boxed{[ENTER]}$ $\boxed{[ENTER]}$. One of the values used in the solution set is displayed. Repeat the procedure on the other intersection point.

The solution is $\{x \mid -3.24 \leq x \leq 1.24\}$.

EXERCISES

Solve each inequality.

1. $-x^2 - 10x - 21 < 0$ **$\{x \mid x < -7 \text{ or } x > -3\}$**
2. $x^2 - 9 < 0$ **$\{x \mid -3 < x < 3\}$**
3. $x^2 + 10x + 25 \leq 0$ **$\{x \mid x = -5\}$**
4. $x^2 + 3x \leq 28$ **$\{x \mid x \leq -1.5 \text{ or } x \geq 1\}$**
5. $2x^2 + x \geq 3$ **$\{x \mid x < -1.5 \text{ or } x > -1.5\}$**
6. $4x^2 + 12x + 9 > 0$ **$\{x \mid x < -1.5 \text{ or } x > -1.5\}$**
7. $23 > -x^2 + 10x$ **$\{x \mid x \leq 3.58 \text{ or } x \geq 6.41\}$**
8. $x^2 - 4x - 13 \leq 0$ **$\{x \mid -2.12 \leq x \leq 6.12\}$**
9. $(x + 1)(x - 3) > 0$ **$\{x \mid x < -1 \text{ or } x > 3\}$**

© Glencoe/McGraw-Hill

Glencoe Algebra 2

37

7 Graphing Calculator Investigation

Rational Root Theorem (Use with Lesson 7-6.)

The following program performs synthetic division and displays the depressed polynomial coefficients in rational form. The program will allow the testing of possible rational zeros of a polynomial function.

```
PROGRAM:SYNTHDIV
Disp "DEGREE OF DIVIDEND" P+1→P
Input M
Disp "COEFFICIENTS?"
A→L1(P)
If P<M+1
Goto 1
Lbl 2
Input U
Disp "POSSIBLE ROOT"
1→P
0→S
Input R
If U=0
Goto 2
If U=1
Goto 4
Goto 4
Disp Q►Frac
Pause
RQ→S
Lbl 1
Q→L2(P)
P+1→P
If P≤M+1
Goto 3
A→L1(P)
If P<M+1
Goto 1
Lbl 4
0→P
1+P→P
Lbl 5
L2(P)→L1(P)
If P<M+1
Goto 5
Goto 2
RQ→S
```

Example Find all of the rational zeros of $f(x) = 2x^3 - 11x^2 + 12x + 9$.

Use the program to test possible zeros.

Keystrokes: `PRGM` `SYNTHDIV` `ENTER` `3` `ENTER` `2`
`ENTER` `1` `ENTER` `2` `ENTER` `11` `ENTER` `12` `ENTER` `9` `ENTER`.

Press `ENTER` until the screen displays **Done**.

The column of numbers are the coefficients of the depressed polynomial. Since the last number is not zero, press `ENTER` `3` `ENTER`. Choose 0 for the same coefficients. Press `ENTER` `0` `ENTER` until finished. Repeat this until a zero is found. Then press `ENTER` `2` for the degree of the depressed polynomial and `ENTER` `1` for the quotient.

The zeros are 3, 3, and $\frac{1}{2}$.

ixt: cises

Find all the zeros of each function.

- $f(x) = x^3 - 8x^2 - 23x + 30$ **1, -3, 10**
- $f(x) = x^3 - 7x^2 + 2x + 40$ **-2, 4, 5**
- $f(x) = 2x^3 - x^2 - 32x + 16$ **4, -4, $\frac{1}{2}$**
- $f(x) = x^4 + x^3 - 11x^2 - 9x + 18$ **1, -2, 3, -3**
- $p(x) = 3x^4 + 11x^3 + 11x^2 + x - 2$ **-1, -2, $\frac{2}{3}$** **6. $p(x) = x^4 - 2x^3 + x^2 - 8x - 12$ **-1, 3, ±2i****
- $p(x) = 3x^5 + x^4 - 243x - 81$ **3, -3, - $\frac{1}{3}$** **8. $p(x) = 3x^4 + 13x^3 + 15x^2 - 4$ **-2, $-\frac{-1 \pm \sqrt{13}}{6}$****

7 Spreadsheet Investigation

Operations on Functions (Use with Lesson 7-7.)

It is possible to perform operations on functions such as addition, subtraction, multiplication and division. You can use a spreadsheet to investigate the relationships among functions.

Consider the functions $f(x) = 3x + 2$, $g(x) = x^2 - 2x$, and $h(x) = x^2 + x + 2$. Find the function values of each function for several values of x . Does it appear that $f(x) + g(x) = h(x)$?

	A	B	C	D	E
1	x	f(x)	g(x)	f(x)+g(x)	h(x)
2		-4	-10	-14	14
3		-2.5	-5.5	-8	5.75
4		-1	-4	-5	2
5		0	-2	-2	2
6		1	-1	0	4
7		4	4	8	22
8		12	38	50	158

Use Column A for the chosen values of x . Columns B, C, and E are $f(x)$, $g(x)$, and $h(x)$ respectively. Use Column D for $f(x) + g(x)$. For every value of x , $f(x) + g(x) = h(x)$.

ixt: cises

Study and use the spreadsheet above.

- Find $h(x) = (3x + 2) + (x^2 - 2x)$. How does it compare to $h(x)$?
 $h(x) = x^2 + x + 2 = h(x)$
- Change the functions in the spreadsheet to $f(x) = \frac{x}{2}$, $g(x) = 1 - x^2$, and $h(x) = 1 + \frac{x}{2} - x^2$. How are these functions related? Is it true that $f(x) + g(x) = h(x)$? **$f(x) + g(x) = h(x)$; yes**
- Make a conjecture about $(f + g)(x)$ for any functions $f(x)$ and $g(x)$.
 $(f + g)(x) = f(x) + g(x)$
- Make a conjecture about $(f - g)(x)$ for any functions $f(x)$ and $g(x)$. Use the spreadsheet to test your conjecture. Does it appear to be true? Explain your answer. **$(f - g)(x) = f(x) - g(x)$; See students' work.**

Find $(f + g)(x)$, $(f - g)(x)$, for each $f(x)$ and $g(x)$. Use the spreadsheet to find function values to verify your solutions. 5-7. See students' spreadsheets.

- $f(x) = 6x + 8$
 $g(x) = 9 + x$
 $7x + 17$; $5x - 1$
- $f(x) = x^2 + 1$
 $g(x) = 3x - 4$
 $x^2 + 3x - 3$; $x^2 - 3x + 5$
- $f(x) = 10x^2$
 $g(x) = 6 - x^2$
 $9x^2 + 6$; $11x^2 - 6$

NAME _____ DATE _____ PERIOD _____

8 Graphing Calculator Investigation

Matrices and Equations of Circles (Use with Lesson 8-3.)

A graphing calculator can be used to write the equation of a circle in the form $x^2 + y^2 + Dx + Ey + F = 0$ given any three points on the circle.

Example Write the equation of the circle that passes through the given points. Identify the center and radius of each circle.

a. $A(5, 3)$, $B(-2, 2)$, and $C(-1, -5)$

Substitute each ordered pair for (x, y) in $x^2 + y^2 + Dx + Ey + F = 0$ to form the a system of equations. $-2D + 2E + F = -8$ $-D - 5E + F = -26$
 $5D + 3E + F = -34$
 Solve the system using a matrix equation to find D , E , and F . Replace the coefficients in the expanded form. Then, complete the square to write the equation in standard form to identify the center and radius.

Keystrokes: $\boxed{2nd} \boxed{[MATRIX]} \boxed{\blacktriangleright} \boxed{ENTER} \boxed{3} \boxed{ENTER} \boxed{5}$
 $\boxed{ENTER} \boxed{3} \boxed{ENTER} \boxed{1} \boxed{ENTER} \boxed{(-)} \boxed{2} \boxed{ENTER} \boxed{2} \boxed{ENTER} \boxed{1} \boxed{ENTER} \boxed{(-)} \boxed{1}$
 $\boxed{ENTER} \boxed{5} \boxed{ENTER} \boxed{1} \boxed{ENTER} \boxed{2nd} \boxed{[MATRIX]} \boxed{\blacktriangleright} \boxed{ENTER} \boxed{EDIT} \boxed{2}$
 $\boxed{ENTER} \boxed{3} \boxed{ENTER} \boxed{1} \boxed{ENTER} \boxed{(-)} \boxed{34} \boxed{ENTER} \boxed{(-)} \boxed{8} \boxed{ENTER} \boxed{(-)} \boxed{26} \boxed{ENTER}$
 $\boxed{2nd} \boxed{[QUIT]} \boxed{2nd} \boxed{[MATRIX]} \boxed{ENTER} \boxed{x^2} \boxed{2nd} \boxed{[MATRIX]} \boxed{2}$
 \boxed{ENTER} .

Thus, $D = -4$, $E = 2$, and $F = -20$.
 The expanded form is $x^2 + y^2 - 4x + 2y - 20 = 0$.
 After completing the square, the standard form is $(x - 2)^2 + (y + 1)^2 = 25$.
 The center is $(2, -1)$, and the radius is 5.

b. $A(-2, 3)$, $B(6, -5)$, and $C(0, 7)$

Find a system of equations. Then enter the equations into an augmented matrix. Reduce the matrix to row reduced echelon form using the **rref** command. The row reduced echelon form of an augmented matrix will display the solution to the system.

$-2D + 3E + F = -13$ $6D - 5E + F = -61$ $7E + F = -49$
 Keystrokes: Enter the system of equations as $[A]$, a 3×4 augmented matrix. Then use the reduced row echelon form by pressing $\boxed{2nd} \boxed{[MATRIX]} \boxed{\blacktriangleright} \boxed{ALPHA} \boxed{[B]} \boxed{2nd} \boxed{[MATRIX]} \boxed{ENTER} \boxed{ENTER}$.
 The solution is $D = -10$, $E = -4$, and $F = -21$. The expanded form is $x^2 + y^2 - 10x - 4y - 21 = 0$, standard form is $(x - 5)^2 + (y - 2)^2 = 50$. The center is $(5, 2)$ and the radius is $5\sqrt{2}$.

Exercises

Write the equation of the circle that passes through the given points. Identify the center and radius of each circle.

- $(0, -1)$, $(-3, -2)$, and $(-6, -1)$ **2.** $(7, -1)$, $(11, -5)$, and $(3, -5)$ **3.** $(-2, 7)$, $(-9, 0)$, and $(-10, -5)$
 $x^2 + y^2 + 6x - 6y - 7 = 0$; $x^2 + y^2 - 14x - 6x + 10y - 135 = 0$;
 $C(-3, 3)$, $R = 5$ $C(7, -5)$, $R = 4$ $C(3, -5)$, $R = 13$

NAME _____ DATE _____ PERIOD _____

8 Spreadsheet Investigation

Parabolas (Use with Lesson 8-2.)

You have learned many of the characteristics of parabolas with vertical and horizontal axes of symmetry. The information is summarized in the table at the right. You can use what you know to create a spreadsheet to analyze given equations of parabolas.

form of equation	$y = a(x - h)^2 + k$	$x = a(y - k)^2 + h$
vertex	(h, k)	(h, k)
axis of symmetry	$x = h$	$y = k$
focus	$(h, k + \frac{1}{4a})$	$(h + \frac{1}{4a}, k)$
directrix	$y = k - \frac{1}{4a}$	$x = h - \frac{1}{4a}$
direction of opening	upward if $a > 0$, downward if $a < 0$	right if $a > 0$, left if $a < 0$
length of latus rectum	$ \frac{1}{a} $ units	$ \frac{1}{a} $ units

IN-CLASS

The spreadsheet below uses the equation of a parabola in the form $y = a(x - h)^2 + k$ or $x = a(y - k)^2 + h$ to find information about the parabola. x or y is entered in Column D and the values of a , h , and k are entered into Columns A, B, and C respectively.

	A	B	C	D	E	F	G	H	J
	a	h	k	y or x	Vertex	Length of Latus Rectum (units)	Axis of Symmetry	Focus	Direction of Opening
1	0.25	3.00	-1	x	3, -1	4.00	y = -1	4, -1	x = 2
2	3.00	-4.00	2.00	y	-4, 2	0.33	x = -4	-4, 2.08	y = 1.92
3	-1.00	3.00	2.00	y	3, 2	1.00	x = 3	3, 1.75	y = 2.25

1. Which row represents the equation $y = 3x^2 + 24x + 50$? **ROW 3**

2. Write the standard form of the equation represented by row 2.

$x = \frac{1}{4}(y + 1)^2 + 3$

3. What formula should be used in cell F2? **1/ABS(A2)**

4. Find the vertex, length of latus rectum, axis of symmetry, focus, directrix, and direction of opening of a parabola with equation $(y - 8)^2 = -4(x - 4)$.
(8, 4); 4; y = 4; (7, 4); x = 9; left

NAME _____ DATE _____ PERIOD _____

9 Graphing Calculator Investigation

Horizontal Asymptotes and Tables

(Use with Lesson 9-3.)

The line $y = b$ is a horizontal asymptote for the rational function $f(x)$ if $f(x) \rightarrow b$ as $x \rightarrow \infty$ or as $x \rightarrow -\infty$. The horizontal asymptote can be found by using the **TABLE** feature of the graphing calculator.

Example Find the horizontal asymptote for each function.

a. $f(x) = \frac{1}{x^2 + 4x - 5}$

Enter the function into **Y1**. Place **[TblSet]** in the Ask mode. Enter the numbers 10,000, 100,000, 1,000,000, and 5,000,000 and their opposites in the x -list.

Keystrokes: $Y=$ 1 $\frac{1}{x^2 + 4x - 5}$ \rightarrow 4 \rightarrow **[X,T,θ,n]** \rightarrow 5 \rightarrow **[2nd]** **[TblSET]** \rightarrow **[ENTER]** **[2nd]** **[TABLE]**. Then enter the values for x .

X	Y1
10000	1E-19
100000	1E-18
1000000	1E-17
5000000	1E-16
10000000	1E-15
100000000	1E-14
500000000	1E-13
1000000000	1E-12
Y1=	3.999999968E-14

Notice that as x increases, y approaches 0. Thus, when $y = 0$ is the horizontal asymptote.

b. $f(x) = \frac{3x^2}{2x^2 + 5x - 6}$

Enter the equation into **Y1**. Enter the numbers 10,000, 100,000, 1,000,000, and 5,000,000 and their opposites in the x -list. Note the pattern. As x increases, y approaches 1.5. Thus, $y = 1.5$ is the horizontal asymptote.

X	Y1
10000	1.4986
100000	1.5
1000000	1.5
5000000	1.5004
10000000	1.5004
100000000	1.5
500000000	1.5
1000000000	1.5
Y1=	1.499999925

Exercises

Find the horizontal asymptote for each function.

- $f(x) = \frac{2x}{x+1}$ **y = 2**
- $f(x) = \frac{x^2 - 1}{2x^2 - 7x + 12}$ **y = $\frac{1}{2}$**
- $f(x) = \frac{6x^3}{2x^3 - 2x^2 - 2}$ **y = 3**
- $f(x) = \frac{2x}{3x^2 - 5x - 1}$ **y = 0**
- $f(x) = \frac{15x^2 - 3x + 7}{x^3}$ **y = 0**
- $f(x) = \frac{x^2 - 8x^2 - 4x + 11}{x^4 - 3x^3 - 4x - 6}$ **y = 0**
- $f(x) = \frac{5x^2 - 3}{x - 2}$ **none**
- $f(x) = \frac{6x^3}{2x^2 - 3x + 6}$ **none**
- $f(x) = \frac{2x - 4}{2}$ **none**

NAME _____ DATE _____ PERIOD _____

9 Spreadsheet Investigation

Variation

(Use with Lesson 9-4.)

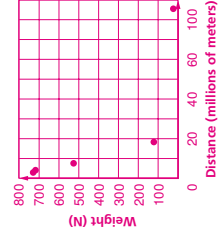
You have learned to solve problems involving direct, inverse, and joint variation. Many physical situations involve at least one of these types of variation. For example, according to Newton's law of universal gravitation, the weight of a mass near Earth depends on the distance between the mass and the center of Earth. Study the spreadsheet below to determine the type of variation that exists between the quantity of an astronaut's weight and the distance of the astronaut from the center of Earth.

In the spreadsheet, the values for the astronaut's weight in newtons are entered in the cells in column A, and the values for the astronaut's distance in meters from the center of Earth are entered in cells in column B. Column C contains the astronaut's distance from Earth's surface.

	A	B	C
1	Astronaut's Weight (N)	Distance from Earth's Center (m)	Distance from Earth's Surface (km)
2	734.5643	6,380,000	0
3	712.0675	6,480,000	100
4	548.9825	7,380,000	1000
5	111.4406	16,380,000	10,000
6	2.642112	106,380,000	100,000
7			

Exercises

- Use the values in the spreadsheet to make a graph of the astronaut's weight plotted against the astronaut's distance from Earth's center.
- Based on your graph, is this an inverse or direct variation? **inverse**



3. Write an equation that represents this situation. Let w represent the astronaut's weight, k the constant of variation, and R the distance from Earth's center.
 $w = \frac{k}{R^2}$

- Use the equation to find the weight of the astronaut at these distances from Earth's surface. (*Hint:* Remember to add these values to the value in cell B2 to find the distance from Earth's center.)
 - 145,300,000 m **1.299615 N**
 - 65 m **734.5494 N**
 - 25,600 m **728.7047 N**
 - 300,800,700 m **0.316872 N**
 - 180,560 m **694.6873 N**
 - 44 **44**

NAME _____ DATE _____ PERIOD _____

10 Graphing Calculator Investigation Regression Equation Lab (Use with Lesson 10-1.)

A graphing calculator can be used to determine a regression equation that best fits a set of data. This activity requires tiles labeled on one side, and a container.

Collect the Data

- Step 1** Place the tiles on the desktop and count the total number. Record the total number. Then place the tiles in the container and gently shake.
- Step 2** Pour the tiles onto the desktop, remove all the tiles with a label showing, and set these aside. Count the remaining tiles without the labels showing and return them to the container.
- Step 3** Record the data in a table like this one.

Trials		Number of tiles without label showing
x	1	y
x	2	y

- Step 4** Repeat step 2 and 3 until the number of tiles without labels is zero or the number remains constant.
- Step 5** Take the tiles that were set aside in Step 2 and pour them out of the container onto the desktop. Remove the tiles without the label showing and count the tiles with the label showing. Repeat this process until all the tiles have been removed.

- Step 6** Record the data in a table like this one.

Trials		Number of tiles with the label showing
x	1	y
x	2	y

Analyze the Data 1-6. Answers will vary.

- 1. Enter trials in **L1** and number of tiles without label showing in **L2**. Enter trials in **L3** and number of tiles with the label showing in **L4**.
- 2. Use [STATPLOT] to make a scatter plot. Make a graph on paper for each plot. Record the window used. Describe the pattern of the points.
- 3. From the [STAT] [CALC] menu find the regression equation that best fits the data. Record the two closest equations, rounding values to the nearest hundredths. List and discuss the r and/or r^2 values. Also include the graphs in determining the best-fitting regression equation.
- 4. Sketch your best-fit regression equation choice for each scatter-plot on paper.
- 5. Describe any problems with the data or the regression equations.
- 6. Insert (0, total number of tiles) in the tables and the lists. Describe the effect on the graphs. What happens with [PwrReg] and [ExpReg] when this ordered pair is inserted? Explain why this occurs?

© Glencoe/McGraw-Hill

Glencoe Algebra 2

NAME _____ DATE _____ PERIOD _____

10 Spreadsheet Investigation Net Present Value (Use after Lesson 10-6.)

You have learned how to use exponential and logarithmic functions to perform a number of financial analyses. Spreadsheets can be used to perform many types of analyses, such as calculating the Net Present Value of expenditures or investments. For example, when a business owner is considering a major purchase, it is a good idea to find out whether the investment will be profitable in the future. Consider the example of a local restaurant-delivery service that is debating whether to buy a van for \$8000. The owners of the company estimate that the van will bring in \$2500 per year over four years. They can use the following formula to find the present value of the future cash flow to find the Net Present Value (NPV), that is, how much the profits would be worth in today's dollars. $NPV = \frac{CF_n}{(1+r)^n}$, where CF_n = the cash flow in period n and r = the cost of capital, which is either the interest that will be paid on a loan or the interest that the money would earn if it were invested.

Think Aloud

- 1. If the NPV is greater than the cost, the investment will pay for itself. Based on the spreadsheet shown above, would it be cost-effective for the company to buy the van? Explain. **The cost is actually about \$75 greater than the NPV, so it would not be cost-effective to buy the van.**
- 2. Four times a year, Josey and Drew publish a magazine. They want to buy a color printer that costs \$1750. The cost of capital for this purchase would be 6%. They are planning to raise the price of their magazine from \$1 to \$2. Create a spreadsheet to determine the NPV for this purchase.
 - a. The last issue of the magazine sold 500 copies. If each issue of the magazine printed in color sells 100 copies more than the previous issue, is the printer a good investment after one year? Explain. **No, after one year the NPV is only about \$1682.14.**
 - b. If the sales of the magazine continue to rise at the same rate, is the printer a good investment after two years? **Yes, after two years the NPV is about \$5210.28. The NPV is about \$3460.28 greater than the cost.**
- 3. a. Calculate the NPV for an investment over a period of six years if the cost of capital is 4.5% and the investment will bring a cash flow of \$750 every year. **The NPV would be about \$3868.40.**
 - b. Would this be a good investment of \$3000? Explain? **Yes, the NPV is \$1131.60 greater than the cost.**

© Glencoe/McGraw-Hill

Glencoe Algebra 2

NAME _____ DATE _____ PERIOD _____

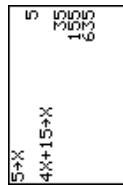
11 Graphing Calculator Investigation

Recursion and Iteration (Use with Lesson 11-6.)

A graphing calculator can be used to perform iterations and recursions.

Example 1 Find the first 3 iterates of $f(x) = 4x + 15$ if $x_0 = 5$.

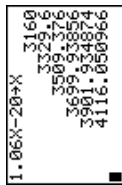
Store x_0 in X. Then enter the expression on the home screen. Store the result to X. Repeat the calculation for each iterate.
 Keystrokes: 5 [STO] [X,T,θ,n] [ENTER] 4 [X,T,θ,n] [+] 15 [STO] [X,T,θ,n] [ENTER] [ENTER] [ENTER]



$x_1 = 35$, $x_2 = 155$, and $x_3 = 635$

Example 2 A savings account has an initial balance of \$3000.00. At the end of each year, the bank pays 6% interest and charges a \$20 annual fee. Find the account balance after 6 years.

Store the initial value and enter an expression to calculate the balance at the end of a year.
 Keystrokes: 3000 [STO] [X,T,θ,n] [ENTER] 1.06 [X,T,θ,n] [=] 20 [STO] [X,T,θ,n] [ENTER] [ENTER] [ENTER] [ENTER] [ENTER] [ENTER]



At the end of six years, the account has a balance of \$4116.05.

EXERCISES

Find the first three iterates of each function.

- $f(x) = 6x + 12$ if $x_0 = 5$
 $x_1 = 42$, $x_2 = 264$, $x_3 = 1596$
- $f(x) = 2x^2 - 3$ if $x_0 = -1$
 $x_1 = -1$, $x_2 = -1$, $x_3 = -1$
- $f(x) = x^2 - 4x + 5$ if $x_0 = 1$
 $x_1 = 2$, $x_2 = 1$, $x_3 = 2$
- $f(x) = 2x^2 + 2x + 1$ if $x_0 = \frac{1}{2}$
 $x_1 = \frac{5}{2}$, $x_2 = \frac{37}{2}$, $x_3 = \frac{1445}{2}$

A bank account has an initial balance of \$11,250.00. Interest is paid at the end of each year. Find the account balance under the given interest rate after the stated time period.

- 3.8%, 2 years **\$12,121.25**
- 4.75%, 5 years **\$14,188.05**
- 6.05%, 10 years **\$20,242.27**
- 7.44%, 15 years **\$33,009.77**

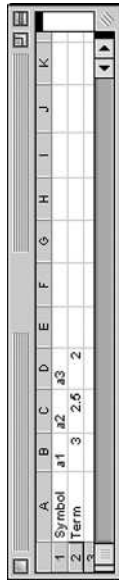
NAME _____ DATE _____ PERIOD _____

11 Spreadsheet Investigation

Sequences and Series (Use after Lesson 11-2.)

You have learned about the characteristics of numbers in a sequence. A spreadsheet can calculate a sequence and enable you to find the sum of terms in the series.

Example Create a spreadsheet like the one below and enter the first three terms of a sequence. Find the first ten terms of the sequence. Then find the sum of the first ten terms of the series.



Highlight cells B2 through D2 and move your cursor to any corner of the highlighted cells until a black cross appears. Drag across the row and release it at cell K2. The next values in the sequence will appear in the cells.

To find the sum of the first 10 terms in the series, highlight the cells containing the terms, then click the Σ symbol on the toolbar. The sum will appear in the next cell. Note that this will work for arithmetic series only. The sum of the first ten terms of this series is 7.5

EXERCISES

- Create a spreadsheet like the one in the example above. Record the initial sequence as -4, -1, and 2. Repeat the process you followed in the example. What are the next six numbers in the sequence?
5, 8, 11, 14, 17, and 20
- Describe the steps the spreadsheet program completes to find the next term in the sequence. **First, the program calculates the common difference by subtracting any term from its succeeding term. Then, it adds the common difference to the last term to find the next term in the sequence.**
- Use the spreadsheet to find the value for the 16th term in the sequence.
41
- Find the sum of the 3rd through 13th terms in the sequence.
187

NAME _____ DATE _____ PERIOD _____

12 Graphing Calculator Investigation

Probabilities (Use with Lesson 12-5.)

A graphing calculator can be used to perform calculations involving permutations, combinations, and probability.

Example 1

There are 5 girls and 3 boys on a class committee. A subcommittee of 3 people is being chosen at random. What is the probability that the subcommittee will have at least 2 girls?

$P(\text{at least 2 girls}) = P(2 \text{ girls}) + P(3 \text{ girls})$. Each probability is the product of the combinations of girls and boys divided by the combinations of all the students taken 3 at a time.

Keystrokes: $\boxed{5} \boxed{MC} \boxed{3} \boxed{1} \boxed{+} \boxed{5} \boxed{MATH}$ $\boxed{3} \boxed{1} \boxed{+} \boxed{5} \boxed{MATH}$ $\boxed{3} \boxed{0} \boxed{+} \boxed{8} \boxed{MATH}$ $\boxed{3} \boxed{3} \boxed{MATH}$ \boxed{ENTER}

The probability that the subcommittee has at least 2 girls is $\frac{5}{7}$.

Example 2

Two cards are randomly selected from a standard deck of cards. Find the probability that both cards are kings or that both cards are red.

Since these events are mutually inclusive find the combinations of 4 kings taken 2 at a time plus 26 red cards taken 2 at a time minus 2 red kings taken 2 at a time divided by the combinations of 52 cards taken 2 at a time.

Keystrokes: $\boxed{4} \boxed{MATH}$ $\boxed{3} \boxed{2} \boxed{+} \boxed{26} \boxed{MATH}$ $\boxed{3} \boxed{2} \boxed{-} \boxed{2} \boxed{MATH}$ $\boxed{3} \boxed{2} \boxed{+} \boxed{52} \boxed{MATH}$ $\boxed{3} \boxed{2} \boxed{MATH}$ \boxed{ENTER} \boxed{ENTER} .
The probability of choosing 2 kings or two red cards is $\frac{55}{221}$.

Example 3

Find each probability.

- There are 5 girls and 4 boys on the school publications committee. A group of 5 members is being chosen at random to attend a workshop on school newspapers. Find each probability.
 - at least 3 girls $\frac{10}{21}$
 - 4 girls or 4 boys $\frac{25}{126}$
 - at least 2 boys $\frac{10}{21}$
- Two cards are drawn from a standard deck of cards. Find each probability.
 - both queens or both black $\frac{55}{221}$
 - both kings or both aces $\frac{2}{221}$
 - both face cards or both black $\frac{188}{663}$
- Find the probability that a committee of 6 U.S. Representatives selected at random from 7 Democrats and 7 Republicans will have at least 3 Republicans on the committee. $\frac{302}{409}$
- Three CDs are randomly selected from a collection of 6 rock and 5 rap CDs. Find the probability that at least 2 are rock. $\frac{19}{33}$

NAME _____ DATE _____ PERIOD _____

12 Spreadsheet Investigation

Permutations and Combinations (Use after Lesson 12-2.)

You have learned the formulas for the number of permutations of n objects taken r at a time, $P(n, r)$, and the number of combinations of n objects taken r at a time, $C(n, r)$. You are going to set up a spreadsheet like the one shown below to perform analyses of these functions.

A	B	C	D	E	F
1	n	5	5	5	5
2	r	0	1	2	3
3	$P(n, r)$	1	5	20	60
4	$C(n, r)$	1	5	10	10
5					

In the spreadsheet, the values in row 1 represent n , the values in row 2 represent r , and the formulas for $P(n, r)$ and $C(n, r)$ are in rows 3 and 4, respectively.

The formula to calculate $P(n, r)$ is =FACT(B1)/FACT(B1-B2).

FACT is a special function from the function list and should not be entered from the letters on the keyboard. Enter the formula in B3. Then drag the cursor across the row to copy the formula into cells C3 through G3.

The formula for $C(n, r)$ is =FACT(B1)/FACT(B1-B2)*FACT(B2)) and should be entered in cell B4. Copy the formula into cells C4 through G4.

Check Your Understanding

- Compare the values of $P(n, r)$ and $C(n, r)$ for $n = 5$ and $r = 0$ through 5, as well as for two other choices of n and r . **Most of the values of $P(n, r)$ are much larger than the corresponding values of $C(n, r)$. The values of $P(n, r)$ tend to increase, while the values of $C(n, r)$ tend to increase and then decrease.**
 - $P(n, n) = P(n, n - 1)$
 - $C(n + 1, r) = C(n, r - 1) + C(n, r)$
 - $C(n, 0) + C(n, 1) + C(n, 2) + \dots + C(n, n) = 2^n$
- Several identities hold for $P(n, r)$ and $C(n, r)$. Use the spreadsheet to verify the following identities by finding three examples of each. **2a-2c. See students' work.**
 - $P(n, n) = P(n, n - 1)$
 - $C(n + 1, r) = C(n, r - 1) + C(n, r)$
 - $C(n, 0) + C(n, 1) + C(n, 2) + \dots + C(n, n) = 2^n$

13 Graphing Calculator Investigation

Law of Sines: Ambiguous Case (Use with Lesson 13-4.)

A graphing calculator can be used to illustrate the Ambiguous Case for the Law of Sines. This program constructs a visual representation of given information. From the drawing, the number of solutions can be determined.

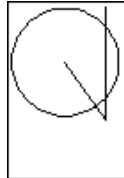
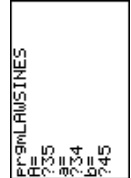
LAW SINES:

```

bcos(A→D)bsin(A→E)
max(1,int((D+a+.999))→Xmax
min(-1,D-int((D+a+.999))→Xmin
int((E+a+.999)→Ymax
min(-1,Ymax-2(Xmax-Xmin)/3)→Ymin
Zsquare
Line(0,0,D,E)
Line(0,0,Xmax,0)
Circle(D,E,a)
AxesOff:ClrDraw
    
```

Example In $\triangle ABC$, $A = 35^\circ$, $a = 34$, and $b = 45$. Determine whether $\triangle ABC$ has one, two, or no solutions.

Run the program and enter the given information. Examine the resulting figure for intersection points.
 Keystrokes: **PRGM** \blacktriangledown to highlight the **LAW SINES** then press **ENTER** **ENTER**. Follow the prompts. $A = 35$ **ENTER** $a = 34$ **ENTER** $b = 45$ **ENTER**. Notice that the circle whose radius is a units intersects the horizontal segment twice. This indicates there are two solutions or two triangles are possible.



EXERCISES

Determine whether each triangle has one, two, or no possible solutions.

1. $A = 44.3^\circ$, $a = 22$, and $b = 20.1$ **1** 2. $A = 126^\circ$, $a = 12$, and $b = 7$ **2**
3. $A = 21^\circ$, $a = 2$, and $b = 3$ **2** 4. $A = 55^\circ$, $a = 11$, and $b = 15$ **0**
5. $A = 112^\circ$, $a = 5$, and $b = 7$ **0** 6. $B = 38.6^\circ$, $b = 22.9$, and $c = 33.7$ **2**
7. $C = 30^\circ$, $c = 20.2$, and $b = 40.4$ **1** 8. $B = 50^\circ$, $b = 13$, and $c = 15$ **2**

13 Spreadsheet Investigation

Cofunctions (Use with Lesson 13-1.)

The functions of sine and cosine are *cofunctions*. Set up a spreadsheet like the one shown below to investigate the relationships between cofunctions.

	A	B	C	D	E	F	G	H
1	4°	0	30	45	60	75	90	120
2	$\sin A$	0	0.5	0.7071068	0.8660254	0.9849256	1	0.8660254
3	$\cos A$	1	0.8660254	0.7071068	0.5	0.259819	0.123E-17	-0.5

In the spreadsheet, the values in row 1 are the angle values in degrees. The values in rows 2 and 3 are the calculated values for the sine and cosine for each angle, respectively. To use the spreadsheet to find the functions for any angle, first enter each function into the spreadsheet in the form shown below.

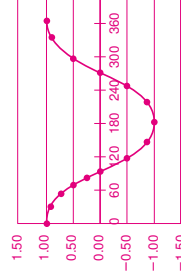
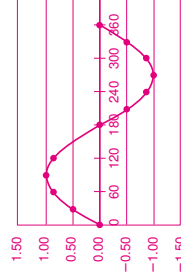
$$= \text{SIN}(B1 * \text{PI}() / 180) \quad = \text{COS}(B1 * \text{PI}() / 180)$$

This form of the formula contains additional information that is necessary for the spreadsheet to use degrees to calculate the answer. Without it, the spreadsheet cannot recognize that the angle is measured in degrees and will return the wrong answer.

Then, complete the spreadsheet by entering the angles up to 360° whose measures are multiples of 30° and 45° .

EXERCISES

1. Use the spreadsheet to make a graph of the sine values for the angles from 0° to 360° . Then make a graph of the cosine values.



2. If $f(x)$ and $g(x)$ are cofunctions, then $f(x) = g(90^\circ - x)$. Compare the shapes of the graphs. How can you tell that sine and cosine are cofunctions by their shapes? **The graphs have a similar shape, but the cosine graph is shifted 90° compared with the sine graph because they are cofunctions with $\sin \theta = \cos(90^\circ - \theta)$.**

NAME _____ DATE _____ PERIOD _____

14 Graphing Calculator Investigation

Sinusoidal Equations (Use with Lesson 14-2.)

A graphing calculator can be used to verify a sinusoidal regression equation in the form $y = a \sin (bx + c) + d$ given four data points. The sinusoidal regression is found under **STAT** **[CALC]** **[C]**.

Example As a person rides a Ferris wheel, the person's distance from the ground varies sinusoidally with time. Let t be the number of seconds that have elapsed since the Ferris wheel started. The rider's position when the last seat is filled and the Ferris wheel starts is when $t = 0$. Suppose it takes 3 seconds to reach the top of the Ferris wheel, 43 feet above the ground. The diameter of the wheel is 40 feet, and it makes a revolution every 8 seconds. Create a table of values and write the sinusoidal equation.

t sec.	1	3	5	7	9	11
$h(t)$ ft.	23	43	23	3	23	43

Keystrokes: Enter the data in **L1** and **L2**. Choose an appropriate window. Use **[STATPLOT]** to graph the points. **[STAT]** **[ALPHA]** **[C]** **[2nd]** **[L1]** **[,]** **[2nd]** **[L2]** **[,]** **[VARS]** **[ENTER]** **[ENTER]** **[ENTER]** **[ENTER]**.

```
SinReg
y=a sin(bx+c)+d
a=.7853981634
b=.7853981634
c=-.7853981634
d=23
```

$a = 20, b = \frac{\pi}{4}, c = 1, \text{ and } d = 23$
 $h(t) = 20 \sin \frac{\pi}{4}(t - 1) + 23$

EXERCISES

As the paddlewheel of a steamboat turns, a point on the paddle blade moves so that its distance, h , from the water's surface is a sinusoidal function of time. The wheel's diameter is 18 feet, and it completes a revolution every 10 seconds. The height of the point at various times is shown in the table.

t (seconds)	1.5	4	6.5	9	11.5	14
$h(t)$ (feet)	7	16	7	-2	7	16

- Why is the height the same after 1.4 seconds as it is after 4 seconds?
The wheel completes a revolution every 10 seconds.
- What are the values of a , b , c , and d ?
 $a = 9, b = \frac{\pi}{5}, c = \frac{3}{2}, \text{ and } d = 7$
- Write a regression equation.
 $h(t) = 9 \sin \left[\frac{\pi}{5} \left(t - \frac{3}{2} \right) \right] + 7$

NAME _____ DATE _____ PERIOD _____

14 Spreadsheet Investigation

Trigonometric Identities (Use after Lesson 14-3.)

A trigonometric identity holds for all values of θ where each expression is defined. For example, $\sin \theta \div \cos \theta = \tan \theta$. You have learned to prove algebraically that an equation is an identity. You can use a spreadsheet to test equations for specific values to see if an equation might be an identity.

Use the spreadsheet to test the values of expressions for different angles, enter the angle measures in the cells in Column A, and enter the expressions from the equations you want to test in the columns to the right. First, enter the formula =RADIANS(A) in Column B to convert degrees to radians. (Recall that you can do this by entering the formula RADIANS(A2) in cell B2, copying cell B2, and pasting to fill the rest of Column B.) In the spreadsheet shown, the formula SIN(B)/COS(B) is in the cells in Column C. The cells in Column D contain the formula TAN(B). Column E contains the formula TAN(-B). Notice that the values in Columns C and D agree with the identity stated above.

	A	B	C	D	E
1	Angle in degrees	Angle in radians	$\sin B / \cos B$	$\tan B$	$\tan (-B)$
2	30	0.523598776	0.577350269	0.57735027	-0.57735027
3	45	0.785398163	1	1	-1
4	60	1.047197551	1.732050808	1.73205081	-1.73205081
5	90	1.570796327	1.63246E+16	1.63246E+16	-1.63246E+16
6	120	2.094395102	-1.732050808	-1.73205081	1.73205081
7	135	2.356194489	-1	-1	1
8	150	2.617993878	-0.577350269	-0.57735027	0.57735027
9	180	3.141592654	-1.225145E-16	-1.225145E-16	1.225145E-16

EXERCISES

- Study the values in Columns D and E. What identity seems to be possible from this pattern? **$\tan (-B) = -\tan B$**
- Enter the formula SIN(B) in Column F. Then enter the formula COS(B)*TAN(B) in Column G. What identity do these two new columns suggest? **$\sin B = \cos B \tan B$**
- Make a column with the formula SIN(PI()-B). What identity do you discover? **$\sin (\pi - B) = \sin B$ or $\sin (\pi - B) = \cos B \tan B$**