

CHAPTER
5**Project: Constructing Points of Interest**

Objective Discover *Euler's Line* and *Feuerbach's Nine-Point Circle*.

Materials ruler, protractor, compass, colored pens or pencils

Investigation Two men credited with finding interesting relationships between special lines and points are Leonard Euler (1707-1783) and Karl Feuerbach (1800-1834). Euler discovered that of the four points—incenter, circumcenter, centroid, and orthocenter—three must be collinear for a given triangle. Feuerbach discovered that, for any triangle, when nine special points are constructed, they all lie on one circle.

Discover Euler's Line You should use a sharp pencil.

1. Draw an acute scalene triangle, $\triangle ABC$, with sides approximately 11 cm, 15 cm, and 16 cm in the center of your paper.
2. Find the circumcenter of $\triangle ABC$ and label it point Q .
3. Find the incenter of $\triangle ABC$ and label it point R .
4. Find the centroid of $\triangle ABC$ and label it point S .
5. Find the orthocenter of $\triangle ABC$ and label it point T .
6. **Analyze** Which of the four points you constructed is not on *Euler's Line*? Draw the line that passes through the three collinear points and label it l .

Construct Feuerbach's Nine-Point Circle Your work must be exact.

7. Draw an acute scalene triangle, $\triangle PQR$, with sides approximately 11 cm, 15 cm, and 16 cm in the center of your paper.
8. Find the midpoints of the sides of $\triangle PQR$. Label them A , B , and C .
9. Construct the altitudes of $\triangle PQR$. Label the vertices of the right angles D , E , and F .
10. Find the midpoints of each segment drawn from each vertex to the orthocenter. Label the midpoints G , H , and J .
11. Construct the perpendicular bisectors of a pair of segments drawn between two sets of points. Do not use points P , Q , or R . The point where the two perpendicular bisectors intersect is the center of the nine-point circle.
12. Draw the circle that contains points A , B , C , D , E , F , G , H , and J .
13. **Analyze** Would the nine points for *Feuerbach's Nine-Point Circle* all be different points if an equilateral triangle were used? Explain what happens. Show the construction to support your answer.

Present Your Results Your report should contain all the constructions above and your answers to Exercises 6 and 13.