Park City Mathematics Institute
Lesson Study 2012

Conditional, Converse, and Bi-Conditional Statements, Oh My!

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The lesson we developed and then taught was an introduction to conditional, converse, and bi-conditional statements. We pre-taught the first half of the lesson to a group of summer math camp students (8^{th} – 12^{th} grade) and then we taught the full lesson to a class of students at a junior high in Park City.

Grade level: 9-10

Strand: Geometry or Integrated I

Class time: 60 minute class period

Materials:

- Individual whiteboards, markers, erasers (or just notebook paper and pencils)
- Worksheets
- Exit slip
- Chart paper

Objectives:

1. Students will be able to determine whether conditional and converse statements are true or false by creating examples and providing counterexamples when necessary.
2. Students will be able to identify the original statements and switched statement using the language "conditional" and "converse".
3. Students will be able to create and evaluate bi-conditional statements.

Prerequisite: a basic understanding of geometric properties and definitions.

Lesson Study Abstract
Conditional, Converse, and Bi-Conditional Statements -- Oh My!

The purpose of our lesson study was to collaboratively craft, teach, and revise a lesson for ninth or tenth grade students. We met a total of eight hours per week for three weeks. The lesson was first taught to twelve high school students attending the Park City Mathematics Institute summer math camp. The second time the lesson was taught to a ninth grade class of seven summer school students at a local junior high school. Analysis and revisions were made after each implementation of the lesson.

The lesson objectives were for students to identify, create, and evaluate conditional, converse, and bi-conditional statements. We developed a lesson which included an introductory game that built a foundation for the students to understand and apply logical reasoning to real world and mathematical contexts. Student observations from the game led to formal definitions of conditional, converse, and bi-conditional statements. Students then practiced creating and evaluating the truth values of mathematical statements. We concluded the lesson with a short assessment.

This document includes our lesson study process, revised lessons, and reflections.

Lesson Plan for Geometry/Logic

1. **Title of the Lesson:** Logic: Conditional Statements, Converse Statements, and Bi-conditional Statements—Oh My!

2. **Goal of the Lesson:**
   Students will be able to
   - Determine whether conditional and converse statements are true or false by creating examples and providing counter-examples when necessary.
   - Identify the original statements and switched statements using the language "conditional" and "converse".
   - Create and evaluate the validity of bi-conditional statements.

3. **Prerequisites for the Lesson:**
   Previous to today's lesson students had been learning about deductive and inductive reasoning in the context of triangle congruence. The previous lessons were not required prerequisites for this lesson, but the previous day's content was referenced in the lesson.

   Students were required to apply knowledge from previous coursework about:

   - Divisibility of odd and even numbers
   - Triangle classifications
   - Area of a triangle
   - Triangle congruence/triangle congruence theorems
   - Quadrilateral properties/classification
   - Squaring numbers

4. **Relationship of the Lesson in Standards for Mathematics**
   CCSS Standards for Mathematical Practice: Construct viable arguments and critique the reasoning of others (#3)

   Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Utah State Standards: Geometry Standard #1: Use inductive and deductive reasoning to develop mathematical arguments.

a. Write conditional statements, converses, and inverses, and determine the truth-value of these statements.

b. Formulate conjectures using inductive reasoning.

c. Prove a statement false by using a counterexample.

5. **Weekly Progression**

During the week that the lesson study was done these were the topics that were covered.

- Inductive and deductive reasoning
- Proofs using SSS, SAS, ASA, AAS postulates
- Understand and use logical reasoning to make and evaluate arguments (ex. Conditional statements, converse, inverse, contrapositive, counter-example, Law of Syllogism, and "if and only if" statements (biconditional)).
- Review on Thursday
- Exam on Friday

6. Instruction of the Lesson:

Students will understand the concept of a conditional statement by determining the truth-value of non-mathematical conditional statements. New vocabulary will be introduced after the concepts are solidified, as a means for discussion. Students will then use then apply the concepts and new vocabulary to mathematical examples. Students will be able to synthesis this new information by creating their own conditional statements and determining the true-value.

7. Materials:

- Name tents
- Whiteboards, tissues, markers, white paper?
- Poster for the Launch game statements (make sure to space the statements in pairs) Two columns (one for statements and one for "truth value").
- Worksheets
- Exit tickets
- Square/rhombus/triangle straw manipulative

8. Learning Process

Activities/Tasks/Directions ***(Italics indicate anticipated answers for worksheet)***	**Teacher Support & Anticipated Student Responses**
1. Introduction Teacher introduces himself/herself and asks students to share something about them.	
2. Launch Game	(10 mins)

"We are going to start with a game. When I finish the directions, you are going to stand up and play the game in pairs. You will each have a whiteboard and I will read a series of statements. After each statement, decide with your partner if it is "True or False" and write that down. BUT, if you think it is false, write down a reason why. The statements must be either TRUE or FALSE. If you get the right answer, then you will take a step forward. Now, there are no "bigfoot" steps. Just put one foot in front of the other!" Read statements one at a time. "If you live in Alaska, then you live in the largest state in the United States." "If you live in the largest state in the United States, then you live in Alaska." "If you grew up in Park City, Utah, then you have seen snow." "If you have seen snow, then you grew up in Park City, Utah." "If you are a twin, then you have a sibling." "If you have a sibling, then you are a twin." "If I am breathing, then I am sleeping." "If I am sleeping, then I am breathing."	After going over directions, "So what are you going to write on your whiteboard?" "If you think it is false, what else do you have to write?" "When do you take a step forward?" Be sure to have students use the term "counterexample." Tell students that if a counterexample is strong enough to convince someone that the statement is false, then that will be the correct answer. The answer must be true or false for the entire group. Give students a few seconds to discuss and write down their answers on their white boards. When students appear ready, have them show their answers. Students should alternate writing on the white board in their pairs. Have students read counterexamples if they think the statement is false. Tell students if they are correct or incorrect and have them step forward if correct. Record truth value on the poster.

3. Reflection & Introduction of Vocabulary

On worksheet**:** Looking at the statements from the game, what patterns do you notice? (What is similar? What is different?)

On worksheet**,**

1. With your partner, create a definition of a **conditional statement** in your own words.

   b. Create your own **conditional statement**.

   Challenge! (yes, you have to try this.) Write a statement that is NOT a conditional

Term	Definition	Example
Conditional	*An if….then…. statement* *If* *p, then q* *Label hypothesis and conclusion* $p \rightarrow q$ *(ASSUME THE HYPOTHESIS IS TRUE)*	*Have a student share their example from*
Converse *Feet switching back and forth*	*Switch the p and q* $q \rightarrow p$	*Have a student say the converse of the class example*
Bi-conditional	*Combine two statements with the same truth value using "if and only if"* $p \leftrightarrow q$	*You live in Alaska if and only if you live the largest state.*

2. a. Write the **converse** of your statement

(4 mins)
Hand out reflection and notes worksheet. Have a student read the reflection question. Have students share responses with the class and record observations.

Anticipated Student Responses:

- *If, then*
- *Two parts*
- *2nd part flips*
- *sometimes both true, sometimes both false, sometimes one true, one false*

(3 mins)
Have students answer all parts of question 1. For part a, explain that they should create a definition that would tell someone how to create a conditional if they had not played the game. Have each pair put their definition on the board.

(5 mins)
Have a class discussion about the definition and fill in the vocabulary chart for the "conditional" row, coming up with a class example. Introduce language of "hypothesis" and "conclusion" and write structure: "If p, then q" on vocabulary chart.
"What do we call the 'if' statement in science class?"

Share non-conditional statements.
"Why is that not a conditional?"

(5 mins)
Refer back to student responses in the Reflection. They probably noticed that alternating statements switched the order of the hypothesis and conclusion. Introduce term "converse" and fill in definition on the vocabulary chart.
"What would be the converse of the class example?"

Have students complete question 2. Share answers.

"How did you create the converse?" Push students to use vocabulary "conditional," "hypothesis" and "conclusion."

(5 mins)

from question 2a. b. Is a converse a conditional statement? Why or why not? 3. Which conditional and converse pairs from the game were both **true**?	Have students answer question 3. Describe that when a conditional and its converse have the same truth value, they can be combined to form a valid bi-conditional using the phrase "if and only if." "How can you combine the two statements from the game using 'if and only if'"? Have students write answers in the example column of the vocabulary chart.
4. Practice with Math Examples Have someone read the directions. Students can work individually or in pairs.	(16 mins) Students will work individually or in pairs to complete the practice worksheet. If they are struggling, stop the class and share answers to number 1. Pushing questions for individuals or pairs of students during work time for each question: 1. "What word does divisible sound like?" "What numbers are divisible by 3? Which of those are not odd?" 2. "What type of angle has 90 degrees?" If students put a counter-example, redirect them to Direction C. 3. "What do the words 'equilateral' and 'congruent' mean?" 4. Have students draw a picture. "Are we convinced? Is one picture enough?" Show students a square created from straws and ribbon. "What makes this shape a square?" "Can we do anything to it and keep the four sides the same lengths?" 5. "How can we tell if two triangles have the same area?" "What is the formula for area of a triangle?"

	6. "Just like in the game, are there any other numbers that makes this equation true?" "How can we multiply two numbers to get a positive number?"

On worksheet:

1.		True or False?	Counterexample?
Original Statement	**If a number is divisible by 3, then it is odd.**	*False*	*6, 12, 18, etc.*
Converse:	**If** *a number is odd,* **Then** *it is divisible by 3*.	*False*	*5, 7, 11, etc.*
Could these statements be rewritten as a bi-conditional? (Circle **YES** or **NO**.)			

2.		True or False?	Counterexample?
Original Statement	**If a triangle has a 90 degree angle, then it is a right triangle.**	*True*	
Converse:	**If** *a triangle is a right triangle* **then** *it has 90 degrees.*	*True*	
Could these statements be rewritten as a bi-conditional? (Circle **YES** or **NO**.)			

3.		True or False?	Counterexample?
Original Statement	**If a triangle has three congruent sides, then the triangle is equilateral.**	*True*	
Converse:	*If a triangle is equilateral, then the triangle has three congruent sides.*	*True*	
Could these statements be rewritten as a bi-conditional? (Circle **YES** or **NO**.)			

4.		True or False?	Counterexample?
Original Statement	**If a quadrilateral has sides of equal**		

	length, then the quadrilateral is a square.	*False*	*Rhombus*
Converse:	*If a quadrilateral is a square, then it has sides of equal length.*	*True*	
Could these statements be rewritten as a bi-conditional? (Circle **YES** or **NO**.)			

5.		True or False?	Counterexample?
Original Statement	**If two triangles are congruent, then the areas of the two triangles are equal.**	*True*	
Converse:	*If the areas of two triangles are equal, then the two triangles are congruent.*	*False*	*12 as area:* *possible b and h: 6 and 4, 12 and 2, 8 and 3*
Could these statements be rewritten as a bi-conditional? (Circle **YES** or **NO**.)			

6.		True or False?	Counterexample?
Original Statement	**If x = 3, then x^2 = 9.**	*True*	
Converse:	*If $x^2 = 9$, then $x = 3$.*	*False*	*$x = -3$*
Could these statements be rewritten as bi-conditionals? (Circle **YES** or **NO**.)			

5. Closing Discussion Ask a pair to share one of their bi-conditional statements. Conduct short discussion connecting the lesson back to previous days' work and "Remember at the beginning of the lesson, I said we were going to come back to what you had learned in the previous days regarding triangle congruence."	(4 mins) Ask students when they share their answers: "How did you rewrite the statements as a bi-conditional?" "How did you know that the bi-conditional would be valid?" (4 mins) What is one of the postulates or theorems for triangle congruence? If a student responds with a valid explanation of one, but does not say it as a true conditional, then inform students that all postulates

<table>
<tr><td></td><td>and theorems can be written as TRUE conditional statements. How can we write SSS as a true conditional statement? What do we need for a conditional statement?</td></tr>
<tr><td>
6. Exit Slip
"The exit slip is a way for us to know how well we taught this lesson. It looks similar to your worksheet, but this time you will fill in all the parts. Be sure to use a math example."

<u>On worksheet:</u>

Directions. Fill in the blanks for each conditional statement.
<table>
<tr><td colspan="2">Exit Slip</td><td>True or False?</td><td>Counterexample?</td></tr>
<tr><td>Original Statement</td><td></td><td>True</td><td></td></tr>
<tr><td>Converse:</td><td></td><td></td><td></td></tr>
<tr><td colspan="4">Could these statements be re-written as a <u>bi-conditional?</u> (Circle YES or NO.)</td></tr>
</table>
</td><td>(4 mins)</td></tr>
</table>

Teaching and Reflecting on the Lesson

The focus of the lesson study process is on planning, teaching, revising and re-teaching a lesson, followed by a reflection on how well the lesson met the designated mathematical objectives. Given that we were going to plan a lesson for a geometry class, we chose the following objectives:

1. Students will be able to determine whether conditional and converse statements are true or false by creating examples and providing counterexamples when necessary.
2. Students will be able to identify the original statements and switched statement using the language "conditional" and "converse".
3. Students will be able to create and evaluate bi-conditional statements.

Pre-teaching

After developing the initial version of the lesson plan, we tried It out with student volunteers from West Lake High School in the greater Salt Lake area. Twelve students from the high school math camp at the Park City Mathematics Institute participated in the trial lesson taught by Sousada Chidthachack, while the remainder of the lesson study team observed.
The lesson plan was intended for an hour-long session, but we only had thirty minutes to practice with these students. Because of the time restraint, Sousada was only able to cover the launch game and the first question on the student reflection worksheet. Although we did not have a chance to finish the entire lesson, we were able to reflect on and anticipate student misconceptions of the remaining parts of the lesson and adjust the lesson accordingly.

After this first teaching, we learned that the launch game was an engaging way to start the lesson. However, we decided that we needed to be more specific when giving directions for the game so that there would be more consistency among the student groups. For example, if a statement was proven false by a counterexample, then only students who answered "false" could receive a point. We also needed to address how large a step could be taken when a group answered correctly.

Although we only got through the reflection portion of the student task, it was enough to have a discussion and make quality changes. During the lesson run-through, we had students create their own definitions of conditional statements and share their definitions with the class, but students still felt confused about the exact definition we wanted them to leave with. Based on the way the students shared their reflections and based on the feedback we got from the students after the lesson was over, we agreed that it would be better if the class came up with a common definition and example of each vocabulary term so as to maintain consistency and accuracy.

We also decided vocabulary should be structured with a graphic organizer so that students can easily reference the terms throughout the lesson. The graphic organizer was formatted with three columns: term, definition, and example. The three terms were conditional, converse, and bi-conditional. When the conditional was defined during the lesson, we did not specify that the hypothesis should be assumed to be true. We made a note of that for the actual lesson. Additionally, we changed the wording of our question about bi-conditional statements to be more mathematically accurate.

Based on what we anticipated would happen in the actual lesson, we made other changes to the task. In terms of the order in which the lesson was delivered, we decided that it would be better to teach all three of the vocabulary words together before the student task. One reason for this is the way they were presented in the graphic organizer. In doing this, we omitted some of the questions on the original student reflection worksheet, as some of them strayed from our objective and others were redundant to the created graphic organizer.

We added pushing and probing questions to the task, including, "Do you have a different example?" "Do you agree?" and "Do you have a different counterexample?" We also anticipated some possible student misconceptions for one of the task questions and decided that we could encourage them to draw a picture and provide a manipulative to help them better understand the concept.

Teaching to Students

After revising the original lesson, we went to Treasure Mountain Junior High School to teach a group of seven students preparing for Honors Integrated Two (integrated concepts from algebra and geometry). Dave Herron taught the lesson, while the remainder of the lesson study team observed. Because the teacher informed us about the class prior to this day, we had name cards prepared for the students and pre-grouped them according to the teacher's suggestions. We learned that there is one visually impaired student, the three boys are friends, and the four girls are comfortable working together. The students were seated in a u-shaped desk configuration at the front of the room, with the boys grouped together and the girls paired on either side of the boys.

During the hour-long session, six of the seven questions on the task worksheet were covered. Many of the misconceptions we had anticipated were handled. The lesson could have challenged them even more, because they seemed to have learned the material quite quickly. Of the three objectives, the first two were met by all students, and the third was not met (the seventh question on the worksheet), partially due to time constraints.

After the lesson was taught, we watched a video of the lesson, discussed it, and refined the lesson plan. We made changes to all three parts of the lesson.

Launch game: We decided to incorporate the use of the term *counterexample,* since they were required to provide one each time they decided a statement was false. We thought that students should read aloud what they write and use the word *counterexample* in their sentence - "My counterexample is..." We also agreed to add a false-false bi-conditional example so that we could reference it later in the lesson.

Student task: We learned that students picked up the concepts quickly but had some difficulty distinguishing the parts of the concept. We decided that we would need to emphasize that each statement was a conditional statement and that each statement needed to have its own truth value. The column in the graphic organizer titled 'definition' should be changed to 'structure' because what we had them fill in was not a technical definition, but rather the structure of the statement. We also considered introducing the term "bi-conditional" in a way that is more connected to the other vocabulary words. Student thought would flow better and be more organized if we used the same example from the two terms prior.

Exit slip: According to the exit slips we collected, it was evident that seven out of seven students were able to successfully create their own true conditional statement, write its converse, and evaluate its truth-value. Students were able to answer correctly when asked if each statement could be rewritten as a bi-conditional.

In reviewing the lesson and student work, we changed the third objective from "...create bi-conditionals…" to "...create and evaluate bi-conditionals…" Although we did not get to this objective, we refined the lesson in a way that promised deeper understanding of this term and allowed enough time to teach it. We decided that when teaching bi-conditional statements, we will emphasize that any statement can be re-written as a bi-conditional, but only conditional and their converse statements that have the same truth value are valid bi-conditional statements. To allow for more practice of this concept, we will include an example that provides students with a bi-conditional and asks students to deconstruct the statement into two conditional statements while evaluating each of the truth-values and providing counterexamples if necessary.

In order to foster more student interaction and participation, we need to be sure to call on all students. We could also revise the practice activity to make it more interactive and challenging. One consideration is to have them write a math conditional statement and circulate it around the class. At each rotation, students will evaluate its truth-value, rewrite it in a different form, or offer feedback on the work, depending on the round.

Lessons Learned

After debriefing the second teaching of the lesson, we learned both about the value of crafting a lesson as a group and reflecting as a group to improve our craft. Student achievement was our main focus. Through collaboration and planning, we learned:

- The objective should guide us through each part of the lesson, from questioning to independent work.
- Even carefully planned lessons cannot always predict student responses accurately.
- Questions can advance or hinder a lesson. We need to aim for questions that push and probe student thinking to encourage participation.
- Visuals and manipulative guide student understanding of difficult concepts.
- Games are great motivators for student engagement.
- Posters made ahead of time with objectives help the flow of class and provide a visual for what was missed when it was verbally shared.
- Every teacher has valuable ideas and input that dramatically improve a lesson.
- It is alright to start over or edit our objectives to support student learning within time constraints.
- Clearly stated directions and carefully written questions help clarify student understanding.

Reflection on Teaching the Lesson to Volunteer Students: ***Sousada Chidthachack***

As part of our lesson study, I tested the first part of our lesson to a group of twelve students who were asked to "think" like freshmen while I was teaching. Before teaching the lesson I realized that I needed to make sure all parts of our lesson supported our objectives. I only had half an hour to teach the lesson, half of the allotted time for the actual lesson. I was only able to cover the launch game and a reflection on conditional statements. This made it really difficult for the group of teachers and me to anticipate student responses and misconceptions. On a positive note, we left with some helpful suggestions from students about the clarity of our objectives and asked for improvement ideas for the first objective that was covered.

Teaching students who are highly motivated and above average math students was not a challenge. In a perfect world, the lesson would have zero classroom management issues, just as I had during my lesson. The only surprise for me was that some students were shy and some students were a bit hesitant to share their original answers. However, I quickly redirected the class by changing the question from 'Do we have any volunteers to share their example?' to 'How many of you completed the example?' The new question changed the tone of the classroom immediately, since everyone raised their hands to the second question. Also, instead of asking one student to fulfill a task, I asked for several volunteers to build off of one student or to clarify their peers' responses, so that we could combine ideas to make it less intimidating. It worked well. Three students contributed verbally, but we did not write out a class definition. At the end, a student suggested that as a class we write the "final answer" together instead of just having the teacher give the definitions. We made changes accordingly.

At the end of thirty minutes, I collected the reflections. After studying the student reflections of the launch, I noticed that some were still unclear about the structure of a conditional statement. Some were more focused on the truth-values than on the required parts. The time constraint did not allow time for more clarification, but as a group, we were able to anticipate some of the possible misconceptions and come up with more meaningful questions, ones that pushed or probed student thinking.

Overall, I had a positive experience despite several new teachers, veteran teachers and a video camera watching closely. I would highly recommend lesson study to teachers who are interested in improving their craft. I also see the value of videotaping a lesson and going through it with my peers.

Reflection on Teaching Our Lesson: *Dave Herron*

We planned our lesson collectively, but I was chosen to teach the day prior to our lesson. To allow for effective reflection on our lesson, I attempted to stick faithfully to our lesson plan. Managerially, my job was easy—the students were well-behaved, obedient, and willing to participate.

After teaching, I learned that: 1) The wording of questions matters; 2) Group-work does not necessarily lead to good results, and 3) Lessons should be adaptable for fast and slow learners.

The wording of questions matters. In our first lesson, we asked students to define a conditional statement. Rather than focusing on the structure of conditional statements, however, students wanted to explain that the order of the words affects the truth value and that you can switch the hypothesis and conclusion. To encourage students to focus on the structure, we worded the question differently the second time, asking students to <u>write a sentence that tells a student not here today how to write a conditional statement.</u> We were looking for the same answer both times, but only the second question elicited our anticipated response.

Group-work does not necessarily lead to good results. When planning our lesson, we wanted to vary the format and let students talk and participate with each other. Therefore, when it was time to establish a common definition of a conditional statement, we had the students write down definitions in small groups first. In retrospect, this activity did not lead to new insights and made other parts of the lesson rushed. Perhaps we should have continued our large group discussion and established a definition together. The lesson is: group-work just for the sake of group-work is unnecessary. The task should lend itself naturally to group-work, instead of having group-work thrust upon the task.

Lessons should be adaptable for fast and slow learners. Conceptually, the students were fast learners. In fact, students caught on so quickly that I may have missed an opportunity to push the students to a deeper level of understanding than our lesson plan allowed. We could have anticipated an opportunity to discuss valid and invalid bi-conditional statements, to compare different counterexamples, or to prepare their own examples and try to stump their peers.

Name: Date: Favorite Sport:

Logic: Conditional, Converse, and Bi-Conditional Statements —Oh My!

Objectives: 1. Students will be able to determine whether conditional and converse statements are true or false by creating examples and providing counterexamples when necessary.

2. Students will be able to identify the original statements and switched statement using the language "conditional" and "converse".

3. Students will be able to create bi-conditional statements.

Introduction: *Hi guys, as your teacher probably mentioned, we are a group of teachers from around the country who are here in Park City to become better teachers. So thank you for letting us come teach today! We have been working on this lesson for a few days. My name is Mr. Herron, and I teach Algebra in Texas. Since I am not from here, can we go around and say one thing that you think I should do, see, or eat in Par City before I leave??*

Game Directions*: We are going to start with a game. When I finish the directions, you are going to stand up and play the game in pairs. You will each have a whiteboard and I will read a series of statements. After each statement, decide with your partner if it is "True or False" and write that down. BUT, if you think it is false, write down a reason why. The statements must be either TRUE or FALSE. If you get the right answer, then you will take a step forward. Now, there are no "bigfoot" steps. Just put one foot in front of the other! Are there any questions? OK, so what are you going to write on your whiteboard? If you think it is false, what else do you have to write? When do you take a step forward?*

Statements for the game:

If you live in Alaska, then you live in the largest state in the United States.
If you live in the largest state in the United States, then you live in Alaska.

If you grew up in Park City, Utah, then you have seen snow.
If you have seen snow, then you grew up in Park City, Utah.

If you are a twin, then you have a sibling.
If you have a sibling, then you are a twin.

If I am breathing, then I am sleeping.
If I am sleeping, then I am breathing.

Reflection: Looking at the statements from the game, what patterns do you notice? (What is similar? What is different?)

- *If, then*
- *Two parts*
- *2^{nd} part flips*
- *sometimes both true, sometimes both false, sometimes one true, one false*

1 a. With your partner, create a definition of a **conditional statement** in your own words.

c. Create your own **conditional statement**.

c. **Challenge**! (yes, you have to try this.) Write a statement that is NOT a conditional statement.

When you have written your definition, write it on the board. Come to a consensus – an "if," "then" statement.

Notes:

Term	Definition	Example
Conditional	*An if….then…. statement* *If p, then q* *Label hypothesis and conclusion* *p → q* *(ASSUME THE HYPOTHESIS IS TRUE)*	*Have a student share their example from*
Converse *Feet switching bac and forth*	*Switch the p and q* *q → p*	*Have a student say the converse of the class example*
Bi-conditional	*Combine two statements with the same truth value using "if and only if"* *p ←→ q*	*You live in Alaska if and only if you live in the largest state.*

Introduce converse and fill in chart.

2. a. Write the **converse** of your statement from question 2a.

   b. Is a converse a conditional statement? Why or why not?

3. a. Which conditional and converse pairs from the game were both **true**?

Introduce term "bi-conditional" and fill in chart.
Have someone read the directions. Work on the first one with your partner. We'll go over it in about 2-3 minutes.

DIRECTIONS. For each conditional statement, complete the following:

A. Write the **converse** statement.
B. Decide if each statement is **true** or **false**.
C. If the statement is false, provide a **counterexample** (a reason that proves it is false).

1.		True or False?	Counterexample?
Original Statement	**If a number is divisible by 3, then it is odd.** *What word does divisible sound like? Divisible means it's evenly divided by. List numbers divisible by 3, then find numbers that are not odd.*	*F*	*6, 12, 18, etc.*
Converse:	**If** *a number is odd,* **Then** *it is divisible by 3.*	*F*	*5, 7, 11, etc.*

Call on a pair of students to share. Do you agree with the counterexample? Do you have another counterexample?

2.		True or False?	Counterexample?
Original Statement	**If a triangle has a 90 degree angle, then it is a right triangle.**	*True*	*Redirect to direction C.*
Converse:	**If** *a triangle is a right triangle* **then** *it has 90 degrees.*	*True*	
Are these statements bi-conditional? (Circle **YES** or **NO**.)			

3.		True or False?	Counterexample?
Original Statement	**If a triangle has three congruent sides, then the triangle is equilateral.**	*True*	
Converse:	*If a triangle is equilateral, then the triangle has three congruent sides.*	*True*	
Are these statements bi-conditional? (Circle **YES** or **NO**.)			

4.		True or False?	Counterexample?
Original Statement	**If a quadrilateral has sides of equal length, then the quadrilateral is a square.** *Draw a picture. Are we convinced? Is one picture enough?*	*False*	*rhombus*
Converse:	*If a quadrilateral is a square, then it has sides of equal length.*	*True*	
Are these statements bi-conditional? (Circle **YES** or **NO**.)			

5.		True or False?	Counterexample?
Original Statement	**If two triangles are congruent, then the areas of the two triangles are equal.**	*True*	
Converse:	*If the areas of two triangles are equal, then the two triangles are congruent.*	*False*	*12 as area:* *possible b and h: 6 and 4, 12 and 2, 8 and 3*
Are these statements bi-conditional? (Circle **YES** or **NO**.)			

6.		True or False?	Counterexample?
Original Statement	**If $x = 3$, then $x^2 = 9$.**	*True*	
Converse:	*If $x^2 = 9$, then $x = 3$.*	*False*	*$x = -3$* *Just like in the game, are there any other numbers that makes this equation true? How can we multiply two numbers to get a positive number?*
Are these statements bi-conditional? (Circle **YES** or **NO**.)			

Bi-conditional statements can be re-written in just one sentence by using **"if and only if."**

Example: If a triangle has three sides of different lengths, then it is a scalene triangle. ***True!***

If a triangle is scalene, then it has three sides of different lengths. ***True!***

A triangle is scalene **if and only if** it has three sides of different lengths.

7. Now, go back and write each **bi-conditional** statement using **"if and only if."**

Review/Share:
Ask a pair to share one of their bi-conditional statements. How did you rewrite the statements as a bi-conditional?

Concluding Discussion:
Remember at the beginning of the lesson, I said we were going to come back to what you had learned in the previous days regarding triangle congruence. What is one of the postulates or theorems for triangle congruence? If a student responds with a valid explanation of one, but does not say it as a true conditional, then inform students that all postulates and theorems can be written as TRUE conditional statements. How can we write SSS as a true conditional statement? What do we need for a conditional statement?

<u>Exit Slip:</u>

Have students complete exit slip.

Directions. Fill in the blanks for each conditional statement.

Exit Slip		True or False?	Counterexample?
Original Statement		**True**	
Converse:			
Are these statements <u>bi-conditional?</u> (Circle **YES** or **NO**.)			

Name: Date: Favorite Sport:

Logic: Conditional, Converse, and Bi-Conditional Statements —Oh My!

Objectives: 1. Students will be able to determine whether conditional and converse statements are true or false by creating examples and providing counterexamples when necessary.

2. Students will be able to identify the original statements and switched statement using the language “conditional” and “converse”.

3. Students will be able to create bi-conditional statements.

Reflection: Looking at the statements from the game, what patterns do you notice? (What is similar? What is different?)

1. a. With your partner, create a definition of a **conditional statement** in your own words.

   b. Create your own **conditional statement**.

   d. **Challenge**! (yes, you have to try this.) Write a statement that is NOT a conditional statement.

Notes:

Term	Definition	Example
Conditional		
Converse		
Bi-conditional		

2. a. Write the **converse** of your statement from question 1b.

b. Is a converse a conditional statement? Why or why not?

3. a. Which conditional and converse pairs from the game were both **true**?

DIRECTIONS. For each conditional statement, complete the following:

A. Write the **converse** statement.
B. Decide if each statement is **true** or **false**.
C. If the statement is false, provide a **counterexample** (a reason that proves it is false).

1.		True or False?	Counterexample?
Original Statement	**If a number is divisible by 3, then it is odd.**		
Converse:	**If** ____, **then**___________________________________		
Could these statements be re-written as a bi-conditional? (Circle **YES** or **NO**.)			

2.		True or False?	Counterexample?
Original Statement	**If a triangle has a 90 degree angle, then it is a right triangle.**		
Converse:	**If** ___________________________________, **then**___________________________________		
Could these statements be re-written as a bi-conditional? (Circle **YES** or **NO**.)			

3.		True or False?	Counterexample?
Original Statement	**If a triangle has three congruent sides, then the triangle is equilateral.**		
Converse:			
Could these statements be re-written as a bi-conditional? (Circle **YES** or **NO**.)			

4.		True or False?	Counterexample?
Original Statement	**If a quadrilateral has sides of equal length, then the quadrilateral is a square.**		
Converse:			
Could these statements be re-written as a bi-conditional? (Circle **YES** or **NO**.)			

5.		True or False?	Counterexample?
Original Statement	**If two triangles are congruent, then the areas of the two triangles are equal.**		
Converse:			
Could these statements be re-written as a bi-conditional? (Circle **YES** or **NO**.)			

6.		True or False?	Counterexample?
Original Statement	**If $x = 3$, then $x^2 = 9$.**		
Converse:			
Could these statements be re-written as a bi-conditional? (Circle **YES** or **NO**.)			

Bi-conditional statements are written in just one sentence by using **"if and only if."**

Example: If a triangle has three sides of different lengths, then it is a scalene triangle. ***True!***

If a triangle is scalene, then it has three sides of different lengths. ***True!***

A triangle is scalene **if and only if** it has three sides of different lengths.

7. Now, go back and write each **bi-conditional** statement using **"if and only if"** in the box next to the **YES** or **NO.**

DIRECTIONS. Fill in the blanks for each conditional statement.

Exit Slip		True or False?	Counterexample?
Original Statement		**True**	
Converse:			
Could these statements be re-written as a bi-conditional? (Circle **YES** or **NO**.)			

Wednesday, July 11, 2012

Lesson Study 8:00AM to 9:10 AM Timeline

Teacher: Dave Herron (referred to as DH)

Topic: Logic: Conditional Statements, Converse Statements, and Bi-Conditionals---Oh My!

Reporter: Sousada Chidthachack

Students: Alena, Perla, John, Theo, Kade, Kristy and Sam

Seating arrangement is u-shaped with teacher located in the center.

Objectives:

1. Students will be able to determine whether conditional and converse statements are true or false by creating examples and providing counterexamples when necessary.
2. Students will be able to identify the original statements and switched statement using the language 'conditional' and 'converse.'
3. Students will be able to create bi-conditional statements.

8:05 Objective described with connection to earlier this week (yesterday)

DH: What have you been learning about so far this week?

Student 1: proofs

DH: Good. What else? What did you learn on Monday?

Student 2: inductive and deductive reasoning

DH: OK, nice. So you have been learning about logic. Logic is a way to argue, but like in math. Well today we are going to continue talking about logic, just like you have been studying for the last two days. And we are going to start off with a game. In a minute, I am going to have you stand up, but first I will explain the game. You are going to work in pairs, and each pair will get a whiteboard. I will read a series of statements. For each statement, you are going to decide if it is True or False. But, if you think it is false, then you have to give a reason why. You can't just say False, you have to convince me. Otherwise, we will assume that it is true. If you are right, then you will take a step forward. Now, no bigfoot steps, just put one foot in front of the other and move forward. At the end of the game, the team that has taken the most steps forward will win. Are those directions pretty clear? So Kade, what do you need to write down on the whiteboard?

S1: The answer.

DH: Which is what?

S2: Either true or false.

DH: Good. And if it is false, what do you have to write?

S3: A reason why.

DH: Good. OK, let's play.

Launch/Game directions described.

Students reiterate directions

8:08 *Students line up. Game Begins.*

DH*: hands out supplies.*

8:09 a student arrives tardy…

Statements and Answers:

*1**)*** **If you live in Alaska, then you live in the largest state in the United States**. *Each team answers true*

2) **If you live in the largest state in the United States, then you live in Alaska**. *Each team answers true*

DH: OK, that was a simple example. Let me trying something a bit more tricky.

3) **If you grew up in Park City, Utah, then you have seen snow**.

Kristy and Theo provide a counterexample for whether "If you grew up in Park City, then you have seen snow" is true. DH asks Theo if someone who moved away as a baby actually 'grew up' in Park City. Theo answers that maybe they only come here for the summer. Teacher responds that they are on the right track, but he is not convinced yet and allows only two of the groups to take a step forward.

*4**)*** **If you have seen snow, then you grew up in Park City, Utah**. *Each team answers false. They all provide a counterexample along the lines of 'it snows in other states.'*

*5**)*** **If you are a twin, then you have a sibling**. *Each team answers true*

*6**)*** **If you have a sibling, then you are a twin**. *Each team answers false. They all provide a counterexample along the lines of "the sibling could be older or younger than you."*

7) **If you are sleeping, then you are breathing**. *Each team answers true*

8) **If you are breathing, then you are sleeping**. *Each team answers false. They all provide a counterexample along the lines of "you are always breathing."*

8:13 teacher passes out lesson worksheets

8:14 game ends and Sam arrives

8:15 John reads reflection instructions

(Teacher familiar with promethean board)

Reflection 2:30 minutes student independent-----

Student comment: Write something a bit differently but (it?) all means the same thing.

Vocab......opposite----what did we flip----opposite or flipping

Discussion of reflection was 4minutes-----820 ends

***8:15** DH hands out reflection while students are answering last question on launch

8:16 A second student arrives tardy

8:20 *definition of conditional statement*

2 minutes John writes and Alena writes

minutes Sam joins writing on the board

DH Collects observations

S: You can write something a bit differently but mean the same thing

S2: They had the same idea… one is true and other is false.

S3: Each question was kind of similar, but opposite.

DH: So we kept a lot of it the same, we switched what?

S3: I don't know

DH: How should we write this down?

Writes: "Statements were a bit different" (opposite)

What about something basic? You may not think to write it down; something obvious…

S4: You use the same words (if, then)

DH: Have you seen this before… it came up a lot… where?

S: Science class

DH: That's right!

S: The leading is some type of opposite

DH writes "leading question" and "following" = 2 parts

DH: I wrote something there (2 parts)…. You wouldn't say "If I go to the movies" to determine if it was true or false.

You covered all the parts. We call this a conditional statement.

8:26 Directions to task (worksheet)…. Write a definition for a conditional statement for someone who wasn't here (w a partner) that <u>tells them how to write a conditional statement.</u>

8:26 students coming up with own conditional statements.

S: (inaudible)…. It has two parts!

8: 27 Students write definition on white board one at a time by groups/partners

8:30 DH checks student work… comments, "That's really good. You're getting the hang of this, we'll move more quickly.

S: Reads the definition (#3) on board

DH: Since we live in a democracy, let's vote. We can also combine definitions

S: Student chooses " A statement that states something and then the complete opposite after, changing the statement to either true or false."

DH: We don't need to rewrite everything.

Writes on smart board: "If p, then q." (under p: hypothesis; under q: conclusion)

Let's read aloud the examples that you wrote.

Class examples are read…..

S: If you surf, then you live by the ocean.

S2: If you are running, then you are not walking.

S3: If you are protected from the weather, then you are in a house.

S4: If you are in a house, then you are protected from the weather.

Class selects the surfing example for entire group

DH: What is the hypothesis?

S: you surf

DH: What is the conclusion?

S: you live by the ocean

DH: When you hear the word 'converse;' what do you think of?

S: Conversation

DH: What if you are not in school?

S: Converse shoes!

DH: Good. Like walking (?)

DH: We take the q and switch it with p

If q, then p

What word am I going to write under the q?

S: conclusion

DH: What do you think, do you agree?

S:

DH: Am I trying to trick you?

If q, then p. Q is the hypothesis even if you switch it, p is the conclusion.

S: If the sky is blue, is that a conditional statement?

DH: What does it need?

S: The second part.

DH: (Silence…) It can be more than one answer

S: Then the grass is green

DH: Good! Two parts!

8:39 We have 20 minutes left, let's do the next part. We can skip 2 a. You already did this….

DH: Counterexample is a reason why it isn't true; then write false. Using the surfing example: the counter example would be you live in Utah and not near the ocean.

8:41 DH: Flip the page

Biconditional… bicycle…. Binary numbers…. 2! Which statements were both true?

S: Alaska one.

DH: Bi-conditional statements have the same truth values: either both true or both false. When a statement is bi-conditional, we don't have to write 'if' and 'then' anymore. We can write "(first part, p) if and only if (second part, q)." Take a minute to re-write the Alaska example by connecting the two parts using 'if and only if.'

8:45 DH circles the room

S: You live in Alaska if and only if you live in the largest state

8:47 DH: You can write it either way. You live in the largest state if and only if you live in Alaska.

I'm going to pass out a worksheet. Some of you are working ahead. You can work alone. Let's go over directions. You can work at your own pace. If you have difficulty then I can assist you. You can raise your hand, don't be afraid. You can talk to a partner as long as it's about math.

8: 50 Student on board to do #1

8: 53 #1 is done

DH: They can both be true or both be false. It's kind of weird. I usually think of biconditional as both true, but they can both be false, I'm going to add this (writes on smart board and adds to definition of biconditional)

8:55 DH circles room… says "I like the bicondtional statements you are writing. 30 more seconds to finish up. If you didn't finish that's okay. We're going over #4 and #5 together if you didn't get them that's okay.

Five more minutes.

DH to S: Can you read the original statements?

S: reads #4: "If a quadrilateral has sides of equal length, then the quadrilateral is a square."

DH: Good. And what did you write as the converse?

S: "If a quadrilateral is a square, then it has sides of equal length."

DH to S: Good. Was the original statement true or false?

S1: True.

DH: Do you agree, John?

S2: Yes.

DH: So everyone agrees. So a square is the only shape? Hmmmm… (Teacher pulls out a square made of straws. The square can be transformed so that the angles are no longer 90 degrees.) I have a square. What makes it a square? (*Brief pause; no response.)* What has to be true about the angles?

S: They have to be 90 degrees.

DH: Yes. But can I change this so the angles are no longer 90 degrees? (*Teacher begins to push the square into a rhombus*.) Alena, you even drew a picture. So this statement is false. We just found a counterexample. And all you have to do is draw a picture---that is a good enough explanation. What do we call this shape? I heard someone say parallelogram. That is a good word, but it has a more specific name. (*Pause—no response*). I always forget the shape names too. This starts with an 'r'

Choral Response: Rhombus

DH: Good. OK, let's move on to #5. Christy, did you make it to #5?

Christy: Yes.

DH: Good. Would you mind reading your statement?

Christy: If two triangles are congruent, then their areas are equal?

DH: And back to Christy again—do you think this is true or false?

Christy: True.

DH: All right. Kade, what is the converse of #5?

Kade: False.

DH: Oh, so you jumped to the truth evaluation. Why do you say false?

Kade: You could have an isosceles and an equilateral triangle with the same area.

DH: Wow, did you draw it at all?

Kade: No. (Laughter).

DH: Did anyone draw a picture?

Alena: I did.

DH: Great. Can you draw your picture on the board.

Alena draws an equilateral and an isosceles triangle with similar sides.

DH: Good. We have two triangles that look different. This triangle is….

Choral Response: Equilateral.

DH: And this one is…

Choral Response: Isosceles.

DH: Exact. And what is the formula for the area?

Christy: ½ base times height.

DH: Great. So we could go through and try to prove that these two triangles have the exact same area using our formula for the area of a triangle. But, we are just about out of time, so we are going to move on to our exit slip. Before we do, though, can anyone remember the theorems from yesterday?

S: Not exactly

DH: They had a lot of different letters in them right.?

Theo: CPCTC

DH: Which means…?

Theo: Congruent parts of congruent triangles are congruent?

DH: Pretty much. I thing there is a "corresponding" in there somewhere, but you are just about right. How about any other theorems that use the letters S and A?

Alena: SSA

DH: SSA, what do you mean?

Alena: Side, side, angle.

DH: What does that mean? Can anyone restate that as a conditional?

Theo: If a triangle is congruent, then it is proven by Side-Side-Angle.

Alena: Not always.

DH: What do you mean?

Alena: It could be proven by something else.

DH: True. But wouldn't it also be proven by SSA?

Alena: I guess.

DH: I see what you mean though. Theo, maybe we should say, "If a triangle is congruent, then it **could** be proven by Side-Side-Angle.

Theo: OK.

DH: All the theorems that you learned yesterday were originally conditional statements before they were proven. Now we state them as facts, but it wasn't always like that. So using conditional statements is a good way to argue and prove your arguments. OK, I want you to turn over your papers. We are going to do an exit ticket. The exit ticket is very similar to the problems on your worksheet, except that now all the boxes are blank. You will come up with your own conditional statement. But, it has to be true. That part is already on there. Then, you will write the converse and decide if it is true or false. You must decide. Finally, you will provide a counterexample if necessary. OK, and these must all be math-related. But that can't be just like the ones we did in class. So, I am looking for an original math conditional statement.

We will take this with us to see if we did a good job or if our lesson was confusing….

We'll make photo copies of your work…

Thanks for your hard work today.

Teacher Words and Actions	***Student Words and Actions***
Q*: Are the directions pretty clear?*	*No response.*
Q: *So what do you actually have to write on the whiteboard?*	***A:*** *"The answer."*
Q*: And the answer is either…*	***A:*** *"True or False."*
Q*: "Good--and Kade, if you think it is false, what do you have to write?"*	***A:*** *"A reason why."*
Observation*: I read out all the answers to the Park*	***Observation****: Perla was not participating actively*

City example instead of having students read their answers. ***Suggestion:*** *Always have students read their answers. Require them to start out with 'My counterexample is…"* ***Suggestion****: Next time, include a 'false-false' bi-conditional example.*	*with Alena. She was mostly standing off to the side.* ***Suggestion:*** *Next time, require students to alternate writing on the white board.*
	During initial reflection time, the male students took longer to start writing and did not write as much as the female students.
Q: Can anyone build off of Perla's example? What does she mean by 'a bit different,' and what does she mean by 'they will mean the same thing.'	
Q: What did you say, ***John****?*	*If the first one was true, than the other one was true.*
Q: Kristi, what was something very simple that all the statements had in common?	*A: The same wording.*
Q: What was that wording?	*A: If…*
Q: If…and then what?	*A: then.*
Comment: Good. "If…, then…"	
Q: ***Sam****, do you have anything to add to that?*	
Q: ***Alena****, what did you write down?*	
Directions: With your partner, I want you to try and combine these into one sentence that would tell someone who was not here today how to write their own conditional statement. You can work with your partner or in a group of 3. As soon as you are done, feel free to send one person up to write your definition up on the white board.	

Suggestion: *Students thought of the 'pairs' as conditional statements. Next time, emphasize that each one of the 8 sentences is a conditional statement. Ask, what do numbers 1 and 2 have in common? [which are a conditional and a converse]? Then ask, what do numbers [have students choose 2 random numbers between 1 and 8] have in common? This way, students will see the similarities between two very disconnected statements.*	
Suggestion*: Consider changing the word 'definition' into 'structure' in the box under notes. B*	
Suggestion*: Consider using the same example from the conditional and converse in our bi-conditional box. Emphasize that any conditional can be rewritten as a bi-conditional, but only phrases that have the same truth value in either direction are* ***valid bi-conditionals.***	
Suggestion*: As one of the last boxes on the worksheet, fill in the bi-conditional and have students fill in the conditional, the converse, the truth values, and counterexamples.*	
Suggestion*: Change objective from "create bi-conditionals" to "create and evaluate bi-conditionals."*	
	Change
	• Description of Bi-conditional: Include a 'false-false' example during the game.
	• Don't say, "This is a simple example." It makes students feel bad if they did not find it simple.

Lesson Study PCMI 2012
Student Participation Sheet

Student Name	**Volunteered**		**Called on**	
	Closed* Question	Open* Question	Closed Question	Open Question

Kristy (F)	1		3	8
Sam (F)			4	1
Kade (M)			5	2
Theo (M)		1	5	6
John (M)	1		7	5
Perla (F)			6	2
Alena (F)	3	7	0	3
Totals	**5**	**8**	**30**	**27**

*Closed questions were defined as questions that had one intended response. (Examples: "Can you read the directions?", "What do we call this term in science?", "Which statements were both true in the game?")

*Open questions were defined as questions that had multiple possible responses. (Ex. "Read the conditional statement that you created?", "Can you explain why this is a conditional statement?", "Why do you disagree?")

Name: **Date:** **Favorite Sports:**

Logic: Conditional Statements, Converse Statements, and Bi-conditionals—Oh My! (first draft)

Reflection: Looking at the statements from the game, what patterns do you notice? (What is similar? What is different?)

Notes:

1. a. In your own words, what is a **conditional statement**?

   b. Create your own **conditional statement**.

   c. **Challenge**! (yes, you have to try this.) Write a statement that is NOT a conditional statement.

2. Write the **converse** of your statement from question 1b.

3. a. Which conditional and converse pairs from the game were both **true**?

   b. Which conditional and converse pairs from the game were both **false**?

   c. Which conditional and converse pairs from the game had different truth values?

DIRECTIONS. For each conditional statement, complete the following:

D. Write the **converse** statement.
E. Decide if each statement is **true** or **false**.
F. If the statement is false, provide a **counterexample**.

1.		True or False?	Counterexample?
Original Statement	**If a number is divisible by 3, then it is odd.**		
Converse:	**If** ___________________________________, **then**______________________________.		

2.		True or False?	Counterexample?
Original Statement	**If a triangle has a 90 degree angle, then it is a right triangle.**		
Converse:	**If** ___________________________________, **then**______________________________.		
Are these statements bi-conditional? (Circle **YES** or **NO**.)			

3.		True or False?	Counterexample?
Original Statement	**If a triangle has three congruent angles, then the triangle is equilateral.**		
Converse:			
Are these statements bi-conditional? (Circle **YES** or **NO**.)			

[Type text]

4.		True or False?	Counterexample?
Original Statement	**If a quadrilateral has sides of equal length, then the quadrilateral is a square.**		
Converse:			
Are these statements bi-conditional? (Circle **YES** or **NO**.)			

5.		True or False?	Counterexample?
Original Statement	**If two triangles are congruent, then the areas of the two triangles are equal.**		
Converse:			
Are these statements bi-conditional? (Circle **YES** or **NO**.)			

6.		True or False?	Counterexample?
Original Statement	**If $x = 3$, then $x^2 = 9$.**		
Converse:			
Are these statements bi-conditional? (Circle **YES** or **NO**.)			

How can you tell if a statement if **bi-conditional?**

____

____

____

Bi-conditional statements can be re-written in just one sentence by using **"if and only if."**

Example: If a triangle has three sides of different lengths, then it is a scalene triangle. ***True!***

If a triangle is scalene, then it has three sides of different lengths. ***True!***

A triangle is scalene **if and only if** it has three sides of different lengths.

Now, go back and write each **bi-conditional** statement using **"if and only if."**