11-1 Mathematical Patterns

Objectives:

- To identify mathematical patterns.
- To identify explicit or recursive patterns.
- To use a formula for finding the n^{th} term of a sequence.

Vocabulary:

sequence: a pattern of objects seperated by commas

ex 2, 4, 6, . . . x, 0, x, 0, . . .

term: one specific object in a sequence

the previous term

typical notation $t_1, t_2, t_3, \ldots t_{n-1}, t_n, t_{n+1}$

Examples: Determine the pattern. Find the next three terms.

1. 2, 4, 6, 8, ... 10, 12, 14

add 2

2. 2, 5, 8, 11, ... 14, 17, 20

add 3

3. 20, 2, 0.2, 0.02, … .002 .0002

divide by 10

mult. by $\frac{1}{10}$

.00002

4. 5, 6, 9, 14, 21, … 30 41 54

+1 +3 +5 +7

add Consecutive odds

Starting at 1

Recursive definition (think "recurring"):

Defines the terms of a sequence based on the previous term

Notation:

sequence:

$a_1, a_2, a_3, a_4, \ldots, a_{n-1}, a_n, a_{n+1}$

$$\begin{cases} a_1 = \# & \text{(starting point)} \\ a_n = 3\underbrace{a_{n-1}}_{\text{previous term}} - 2 & \text{(pattern)} \end{cases}$$

Examples: **a.** Identify the pattern.

b. Find the next 2 terms.

c. Write the recursive definition.

d. Use the recursive definition to find the 7^{th} and 8^{th} term.

5. 3, 5, 7, 9, 11, ...

a. add 2

b. 13, 15

c. $\begin{cases} a_1 = 3 \\ a_n = a_{n-1} + 2 \end{cases}$

d. $a_7 = a_6 + 2$
$= 13 + 2$
$= 15$

$a_8 = a_7 + 2$
$= 15 + 2$
$= 17$

6. 2, 6, 18, 54, 162, ...

a. multiply by 3

b. 486, 1458

c. $\begin{cases} a_1 = 2 \\ a_n = 3a_{n-1} \end{cases}$

d. $a_7 = 3a_6$
$= 3(486)$
$= 1458$

$a_8 = 3a_7$
$= 3(1458)$
$= 4374$

Examples: Use the following recursive definitions to find the first 5 terms of the sequence.

7. $\begin{cases} a_1 = 11 \\ a_n = a_{n-1} - 1 \end{cases}$

the next term

previous term

$11 \overset{-1}{,} 10 \overset{-1}{,} 9 \overset{-1}{,} 8 \overset{-1}{,} 7$

8. $\begin{cases} a_1 = -2 \\ a_n = -2a_{n-1} \end{cases}$

$-2 \overset{x\cdot -2}{,} 4 \overset{x\cdot -2}{,} -8 \overset{x\cdot -2}{,} 16 \overset{x\cdot -2}{,} -32$

9. $\begin{cases} a_1 = 3 \\ a_n = -a_{n-1} + n \end{cases}$

10. $\begin{cases} a_1 = 2 \\ a_n = 4\left(a_{n-1}\right)^2 - n \end{cases}$

9. $\begin{cases} a_1 = 3 \\ a_n = -a_{n-1} + n \end{cases}$

term no.

the next term

the previous term

$\underset{a_1}{\underline{3}}, \underset{a_2}{\underline{-1}}, \underset{a_3}{\underline{4}}, \underset{a_4}{\underline{0}}, \underset{a_5}{\underline{5}}$

$a_2 = -a_{2-1} + 2$
$= -a_1 + 2$
$= -3 + 2$
$= -1$

$a_3 = -a_2 + 3$
$= --1 + 3$
$= 1 + 3$
$= 4$

$a_4 = -a_3 + 4$
$= -4 + 4$
$= 0$

$a_5 = -a_4 + 5$
$= -0 + 5$
$= 5$

10. $\begin{cases} a_1 = 2 \\ a_n = 4(a_{n-1})^2 - n \end{cases}$

$\underset{a_1}{\underline{2}}, \underset{a_2}{\underline{14}}, \underset{a_3}{\underline{781}}, \underset{a_4}{\underline{2{,}439{,}840}}, \underset{a_5}{\underline{\approx 2.38 \times 10^{13}}}$

$a_2 = 4 \cdot a_1^2 - 2$
$= 4 \cdot 2^2 - 2$
$= 4 \cdot 4 - 2$
$= 16 - 2$
$= 14$

$a_3 = 4 \cdot a_2^2 - 3$
$= 4 \cdot 14^2 - 3$
$= 4 \cdot 196 \cdot 3$
$= 784 - 3$
$= 781$

Explicit definition:

Defines the n^{th} term (a_n) interms of n (location)

Notation:

$$\underbrace{a_n}_{\text{term}} = 3n - 5$$

$n \rightarrow$ location

235^{th} term

$$a_{235} = 3(235) - 5$$
$$= 700$$

Examples: Use the following explicit definitions to find the first 5 terms of the sequence.

11. $a_n = 2n - 1$

$a_1 = 2 \cdot 1 - 1 = 1$

$a_2 = 2 \cdot 2 - 1 = 3$

$a_3 = 2 \cdot 3 - 1 = 5$

$a_4 = 2 \cdot 4 - 1 = 7$

$a_5 = 2 \cdot 5 - 1 = 9$

12. $a_n = 2(3)^n$

$a_1 = 2 \cdot 3^1 = 2 \cdot 3 = 6$ $*3$

$a_2 = 2 \cdot 3^2 = 2 \cdot 9 = 18$ $*3$

$a_3 = 2 \cdot 3^3 = 2 \cdot 27 = 54$ $*3$

$a_4 = 2 \cdot 3^4 = 2 \cdot 81 = 162$

$a_5 = 2 \cdot 3^5 = 2 \cdot 243 = 486$

13. $a_n = n^2 - 1$

$a_1 = 1^2 - 1 = 0$

$a_2 = 2^2 - 1 = 3$

$a_3 = 3^2 - 1 = 8$

$a_4 = 4^2 - 1 = 15$

$a_5 = 5^2 - 1 = 24$

$0, 3, 8, 15, 24, \ldots$

14. $a_n = \dfrac{n+3}{n^2}$

$a_1 = \dfrac{1+3}{1^2} = \dfrac{4}{1} = 4$

$a_2 = \dfrac{2+3}{2^2} = \dfrac{5}{4}$

$a_3 = \dfrac{3+3}{3^2} = \dfrac{6}{9} = \dfrac{2}{3}$

$a_4 = \dfrac{4+3}{4^2} = \dfrac{7}{16}$

$a_5 = \dfrac{5+3}{5^2} = \dfrac{8}{25}$

$4, \dfrac{5}{4}, \dfrac{2}{3}, \dfrac{7}{16}, \dfrac{8}{25}, \ldots$

11-1 Sequences Name ______________________ Date ______

a. Find the next two terms of each sequence.
b. Write a recursive formula for each sequence.

Please complete part "a" on both sides.

1. $-14, -8, -2, 4, 10, \ldots$ 16, 22

+6

$a_1 = -14$

$a_n = a_{n-1} + 6$

2. $6, 5.7, 5.4, 5.1, 4.8, \ldots$ 4.5, 4.2

$-.3$

$a_1 = 6$

$a_n = a_{n-1} - .3$

3. $1, -2, 4, -8, 16, \ldots$ -32, 64

$* -2$

$a_1 = 1$

$a_n = a_{n-1} * -2$

4. $1, 3, 9, 27, \ldots$ 81, 243

$* 3$

$a_1 = 1$

$a_n = a_{n-1} * 3$

5. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

$\frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}$

$* \frac{1}{2}$

$a_1 = 1$

$a_n = a_{n-1} * \frac{1}{2}$

6. $\frac{2}{3}, 1, 1\frac{1}{3}, 1\frac{2}{3}, 2, \ldots$

$\frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}, \frac{6}{3}, \frac{7}{3}, \frac{8}{3}$

$= 2\frac{1}{3}$, $= 2\frac{2}{3}$

$a_1 = 2/3$

$a_n = a_{n-1} + \frac{1}{3}$

7. $36, 39, 42, 45, 48, \ldots$ 51, 54

+3

$a_1 = 36$

$a_n = a_{n-1} + 3$

8. $36, 30, 24, 18, 12, \ldots$ 6, 0

-6

$a_1 = 36$

$a_n = a_{n-1} - 6$

9. $9.6, 4.8, 2.4, 1.2, 0.6, \ldots$ .3, .15

$\div 2$

$\times \frac{1}{2}$

$a_1 = 9.6$

$a_n = a_{n-1} * \frac{1}{2}$

a. Label each formula as explicit, recursive or neither.
b. Find the first 4 terms of the sequence.

25. $a_n = \frac{1}{3}n$

26. $a_n = n^2 - 6$

27. $a_1 = 5, a_n = 3a_{n-1} - 7$ R

$a_1 = 5$

$a_n = 3a_{n-1} - 7$ previous term

5, 8, 17, 44

$3 \cdot 5 - 7$
$15 - 7$
8

$3 \cdot 8 - 7$
$24 - 7$
17

28. $a_n = \frac{1}{2}(n - 1)$

29. $a_1 = 5, a_n = 3 - a_{n-1}$ R previous term

5, −2, 5, −2

$3 - 5$ $\quad$ $3 - -2$ $\quad$ $3 - 5$

30. $a_1 = -4, a_n = 2a_{n-1}$ R previous term

−4, −8, −16, −32

$-4 \cdot 2$ $\quad$ $-8 \cdot 2$ $\quad$ $-16 \cdot 2$

Name________________________ Date___________ **Practice 11-1**

a. Find the next two terms of each sequence.
b. Write a recursive formula for each sequence.

answer key

1. $-14, -8, -2, 4, 10, \ldots$ 16, 22

$\begin{cases} t_1 = -14 \\ t_n = t_{n-1} + 6 \end{cases}$

2. $6, 5.7, 5.4, 5.1, 4.8, \ldots$ 4.5, 4.2

$\begin{cases} t_1 = 6 \\ t_n = t_{n-1} - .3 \end{cases}$

3. $1, -2, 4, -8, 16, \ldots$

$\begin{cases} t_1 = 1 \\ t_n = t_{n-1} * -2 \end{cases}$

4. $1, 3, 9, 27, \ldots$ 81, 243

$\begin{cases} t_1 = 1 \\ t_n = t_{n-1} * 3 \end{cases}$

5. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$ $\frac{1}{32}$ $\frac{1}{64}$

$t_1 = 1$

$t_n = t_{n-1} * \frac{1}{2}$

6. $\frac{2}{3}, 1, 1\frac{1}{3}, 1\frac{2}{3}, 2, \ldots$ $2\frac{1}{3}$ $2\frac{2}{3}$

$t_1 = \frac{2}{3}$

$t_n = t_{n-1} + \frac{1}{3}$

7. $36, 39, 42, 45, 48, \ldots$ 51, 54

$\begin{cases} t_1 = 36 \\ t_n = t_{n-1} + 3 \end{cases}$

8. $36, 30, 24, 18, 12, \ldots$

$t_1 = 36$

$t_n = t_{n-1} - 6$

9. $9.6, 4.8, 2.4, 1.2, 0.6, \ldots$

.3

.15

$\begin{cases} t_1 = 9.6 \\ t_n = t_{n-1} \div 2 \end{cases}$

25. $a_n = \frac{1}{3}n$

$a_1 = \frac{1}{3} \cdot 1 = \frac{1}{3}$

$a_2 = \frac{1}{3} \cdot 2 = \frac{2}{3}$

a) exp.

b) $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{3}$, $\frac{4}{3}$

26. $a_n = n^2 - 6$

$a_1 = 1^2 - 6 = -5$

a) exp.

b) -5, -2, 3, 10

28. $a_n = \frac{1}{2}(n - 1)$

a) exp

b) 0, $\frac{1}{2}$, 1, $1\frac{1}{2}$

27. $a_1 = 5, a_n = 3a_{n-1} - 7$

a) rec.

b) 5^{*3-7}, 8^{*3-7}, 17^{*3-7}, 44

29. $a_1 = 5, a_n = 3 - a_{n-1}$

a) rec

b) 5^{3-5}, -2^{3--2}, 5^{3-5}, -2

30. $a_n = 2a_{n-1}$

a) Neither

b) Impossible

$a_n = 2a_{n-1}$

$a_1 = 2 \cdot a_{1-1}$

$a_1 = 2 \cdot a_0$

not given, so cannot find terms

Examples: Applications

15. Suppose you drop a handball from a height of 10 feet. After the ball hits the floor, it rebounds to 85% of its previous height. How high will the ball rebound after its fourth bounce?

original height: a_1

10 ft.

after 1st bounce: a_2

$10(.85) = 8.5$ ft

after 2nd bounce: a_3

$8.5(.85) = 7.225$

after 3rd bounce: a_4

$7.225(.85) = 6.14125$ ft

sequence:

10, 8.5, 7.225, 6.141, ...

recursive definition:

$$\begin{cases} a_1 = 10 \\ a_n = .85a_{n-1} \end{cases}$$

next 2 terms:

5.22, 4.44

5.22 ft.

explicit definition:

$y = ab^x$

$y = (10)(.85)^x$

$a_n = (10)(.85)^n$

10th term:

$a_{10} = 10(.85)^{10}$

$= 1.969$ ft.

How can you tell the difference between an explicit definition and a recursive definition?

Explicit

usually (n)
for location

Recursive

$\{$

2 parts

$a_1 = \#$

a_{n-1}