Name________________________ Period _____

Divisibility Rules

Algebra 9.0

A **Prime Number** is a whole number whose only factors are 1 and itself. To find all of the prime numbers between 1 and 100, complete the following exercise:

1. Cross out 1 by **Shading** in the box completely.
   1 is neither prime nor composite. It has only 1 factor - itself.
2. Use a forward **Slash** \ to cross out all multiples of 2, starting with 4.
   2 is the first prime number.
3. Use a backward **Slash** / to cross out all multiples of 3 starting with 6.
4. Multiples of 4 have been crossed out already when we did #2.
5. Draw a **Square** on all multiples of 5 starting with 10. 5 is prime.
6. Multiples of 6 should be X'd already from #2 and #3.
7. **Circle** all multiples of 7 starting with 14. 7 is prime.
8. Multiples of 8 were crossed out already when we did #2.
9. Multiples of 9 were crossed out already when we did #3.
10. Multiples of 10 were crossed out when we did #2 and #5.

All of the remaining numbers are prime.

How many prime numbers are left between 1 and 100? ______

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Answer: use your chart for help.

Is 51 prime? If not, what are its factors? _____________

Is 59 prime? If not, what are its factors? _____________

Is 87 prime? If not, what are its factors? _____________

Is 91 prime? If not, what are its factors? _____________

Name________________________ Period _____

Divisibility Rules

Algebra 9.0

There are some easy tricks you can use to determine if a number is divisible by 2, 3, 4, 5, 6, 8, 9 and 10.

A number is divisible by:
2 - if it is even.
3 - if the sum of its digits is divisible by 3.
4 - if the number formed by the last 2 digits is divisible by 4.
5 - if the ones digit is 5 or 0.
6 - if it is divisible by 2 AND 3. (All even multiples of 3.)
7 - there is no good trick for 7.
8 - if the number formed by the last 3 digits is divisible by 8.
9 - if the sum of the digits is divisible by 9.
10 - if the last digit is a 0.
11: We will learn this trick separately.

Practice: Write **y**es or **n**o in each blank.

Determine whether 21,408 is divisible by:

2 - ____ 6 - ____
3 - ____ 8 - ____
4 - ____ 9 - ____
5 - ____ 10 - ____

Determine whether 1,345,866 is divisible by:

2 - ____ 6 - ____
3 - ____ 8 - ____
4 - ____ 9 - ____
5 - ____ 10 - ____

Determine whether 222,222,225 is divisible by:

2 - ____ 6 - ____
3 - ____ 8 - ____
4 - ____ 9 - ____
5 - ____ 10 - ____

Write the complete prime factorization for each number below. Use a factor tree if necessary:

Ex: 1,600 **1.** 210 **2.** 297 **3.** 192

$$= 2^6 \cdot 5^2$$

GCF and LCM

The GCF is the Greatest Common Factor between two or more numbers.

Sometimes the GCF is obvious:
Find the GCF for each pair of numbers.
1. 50 and 75 **2.** 49 and 56 **3.** 45 and 60

When the GCF is not obvious:
Ex.
Find the GCF between 405 and 585.

$$405 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 5$$

$$585 = 3 \cdot 3 \cdot 5 \cdot 13$$ Common factors are $3 \cdot 3 \cdot 5 = 45$, the GCF is 45.

<u>notes:</u>
The GCF between a pair or set of numbers is the product of their common prime factors.

Practice:
Find the GCF.

1. 108 and 126 **2.** 154 and 210 **3.** 108 and 288

The LCM is the Least Common Multiple. This means the smallest number that both numbers divide with no remainder.

The LCM is rarely obvious:
Find the LCM for each pair of numbers.
1. 5 and 7 **2.** 10 and 15 **3.** 16 and 24

When the LCM is not obvious:
Ex.
Find the LCM between 144 and 168.

$$144 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$$ Shared factors are $2 \cdot 2 \cdot 2 \cdot 3 \ldots$,

$$168 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 7$$ other factors are $2 \cdot 3 \cdot 7 \ldots$ so

GCF = $2 \cdot 2 \cdot 2 \cdot 3 \cdot 2 \cdot 3 \cdot 7 = 1{,}008$

GCF and LCM: Venn Diagrams

Algebra 9.0

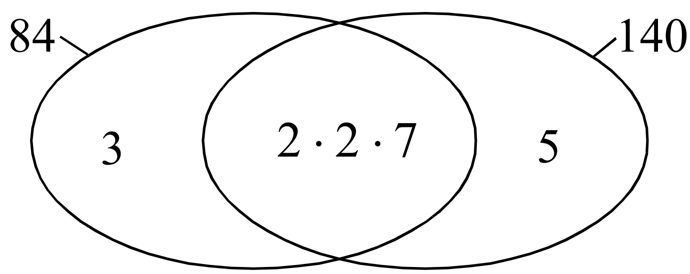
Review Practice:
Find the **GCF and LCM** for each:

1. 36 and 168 **2.** 28, 42, and 105

Venn Diagrams are a great way to solve GCF and LCM problems.

Example: Use a Venn diagram to find the GCF and LCM between 84 and 140.

$$84 = 2 \cdot 2 \cdot 3 \cdot 7$$
$$140 = 2 \cdot 2 \cdot 5 \cdot 7$$



Example: Use a Venn diagram to find the GCF and LCM for 75 and 105:

Practice: Use a Venn diagram to find the GCF and LCM for each.

1. 45 and 60 **2.** 80 and 112 **3.** 28, 42, and 105

Name________________________ Period _____

GCF and LCM

Algebra 9.0

Find the **GCF and LCM** for each pair or set of numbers:
You may use a calculator, and Venn diagrams are encouraged but not required.

1. 54 and 80

GCF ______ LCM ________

2. 88 and 136

GCF ______ LCM ________

3. 90 and 105

GCF ______ LCM ________

4. 45 and 72

GCF ______ LCM ________

5. 96 and 160

GCF ______ LCM ________

6. 153 and 180

GCF ______ LCM ________

7. 20 and 40

GCF ______ LCM ________

8. 64 and 88

GCF ______ LCM ________

9. 270 and 351

GCF ______ LCM ________

10. 143 and 221 (neither one is prime)

GCF ______ LCM ________

Factoring the GCF

You can find the GCF of expressions which include variables and exponents:

Examples:

1. Find the GCF of $72x^5y^2$ and $120x^3y^7$:

2. Find the GCF of $93a^3b^{11}$ and $124a^5b^7$:

Practice: Find the GCF for each pair or set:

1. $15x^{10}$ and $25x^{20}$
2. $78m^3n^4$ and $130m^5n$
3. $28a^2b$ and $21ab^2$ and $30a^2b^2$

We have learned to **Factor**. Factoring is like **Reverse Distribution**.

To factor an expression:

a. Look for the GCF of all terms, including the variables.
b. Place the GCF outside of the parenthesis.
c. Divide each original term by the GCF to get the terms inside the parenthesis.

Examples: Factor each.

1. $57x^2 - 152xy$
2. $110x^5y^{10} - 132x^{15}y^5$

Practice: Factor each.

1. $160x^3y - 96x^2y^2$
2. $25x^4 - 45x^3 + 15x^2$

Practice: Factor each.

1. $65x^3y - 91x^2y + 13xy$
2. $44x^{40}y^7 - 144x^{10}y^2$

Name________________________ Period _____

Factoring the GCF

Algebra

For each polynomial, factor the GCF from the expression.
These should be easy enough to factor the GCF in your head.

1. $18x^3 - 24x$

1. ____________________________

2. $3a^3 - 6a^2b + 9ab^2$

2. ____________________________

3. $16a - 32ab + 20b$

3. ____________________________

4. $14m^5 - 28m^3 + 12m^2$

4. ____________________________

5. $10a^3 - 8a^2b^2 + 14a^2b$

5. ____________________________

6. $8x^2y - 18x^2 + 12xy^2$

6. ____________________________

7. $30xy^4 - 20xy^3 - 15xy^2$

7. ____________________________

Name________________________ Period _____

Factoring the GCF

Algebra

For each polynomial, factor the GCF from the expression.
You will likely need to find the GCF separately with these problems.

8. $80x^3 - 64x$

8. ____________________________

9. $52a^3 - 68a^2b + 60ab^2$

9. ____________________________

10. $70a^2 - 56ab + 42b^2$

10. ____________________________

11. $60m^5 - 48m^3 + 108m^2$

11. ____________________________

12. $95a^9b^8 - 70a^6b^{12}$

12. ____________________________

13. $38x^2y - 57x^2 + 76xy^2$

13. ____________________________

Polynomials

A **polynomial** is a sum of one or more terms called **monomials.**

Examples: $x + y$ $\quad 2x^3 - xy + 4y$ $\quad 12x^3$

A **monomial** is the product of variables and constants (numbers).

Examples: $2abc^3$ $\quad x^7$ $\quad -\frac{5}{7}x$

A **binomial** is the sum of two monomials.

Examples: $2a^2 + a$ $\quad x^2 - 3$ $\quad \frac{2}{3}x^4 - 5x^2$

A **trinomial** is the sum of three monomials.

Examples: $2a - b + c$ $\quad x^7 - y^5 - 3$

The **degree** of a **monomial** is the sum of the exponents of its variables.

Examples: $2a^2$ 2nd degree $\quad 5ab^6c$ = 1+6+1 = 8th degree

note: The degree of a constant is zero.

The **degree** of a **polynomial** is the largest degree of its monomial terms.

Examples: $2a^2 - 7a$ is a 2nd degree binomial

$5a^5 + 2b^2 - 3c$ is a 5th degree trinomial

Polynomials

Ordering Polynomials:
The general rule for ordering a polynomial is to write the terms in descending order by powers of a given variable:

Example: Arrange by descending powers of x: $2x + x^3 - 5 - 3x^5$

Example: Arrange by descending powers of x: $xy + xy^2 - x + y$

Practice:
Order the following polynomials by descending powers of x.

1. $5xy - 3x^2 - x^3 + 2xy^2$
2. $3x^2 + 4x^3y^2 + 5x^2y^3$
3. $ax^2 - ax^3 - a^3x + a^2$
4. $7 + 2x - x^3y + x^5$

Answer:
What degree is each of the polynomials above?

Other tiebreakers: Alphabetical order.

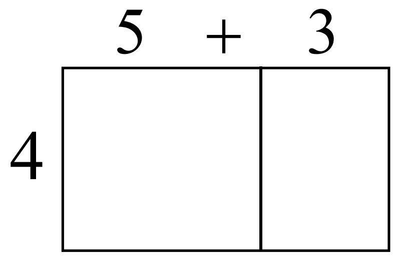
Ex: Arrange by descending powers of a: $2a^3c^2 + a^3b - 5a^3b^2 - 3a^3c$

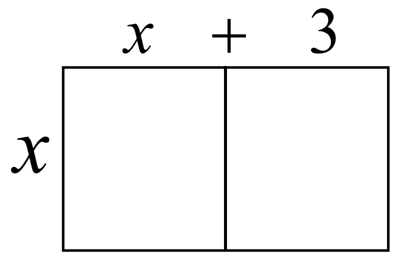
Practice:
Order the following polynomials by descending powers of a.

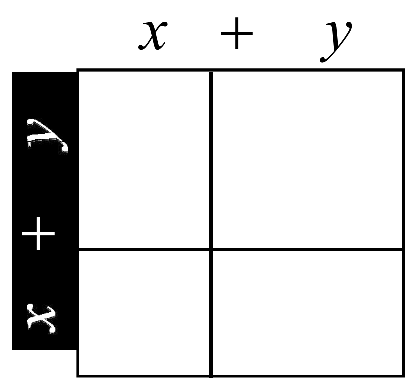
1. $-3ab + a^2 - c^2 + 2b^2$
2. $3a^2 + a^3b + 5a^2b^3 - a^3c^2$
3. $ax^2 - ax^3 - a^3y + a^2y$
4. $7az^2 + 3ax - 2ay^2 + a$

Multiplying Polynomials

Find the area of each rectangle below:







Multiplying Binomials:
Setup a grid like the one above to solve the following:

1. $(x+3)(x+5)$

2. $(2x+5)(x-2)$

The FOIL Method:
First **ac**
Outer **ad**
Inner **bc**
Last **bd**

$$(a+b)(c+d)=ac+ad+bc+bd$$

Examples: Expand each using the FOIL Method.

1. $(a+1)(a-3)$

2. $(2x-y)(3x+y)$

Practice: Expand each using the FOIL Method.

1. $(x+3)(x-5)$

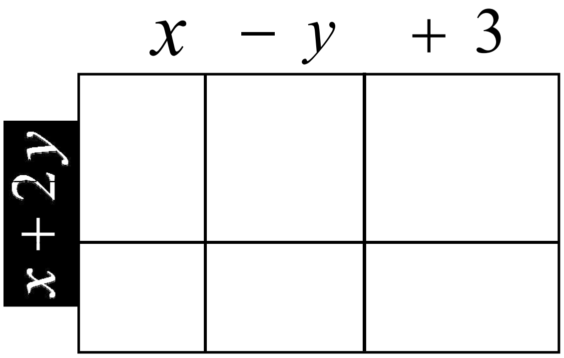
2. $(2a-c)(3a+c)$

3. $(1-2a)(3-a)$

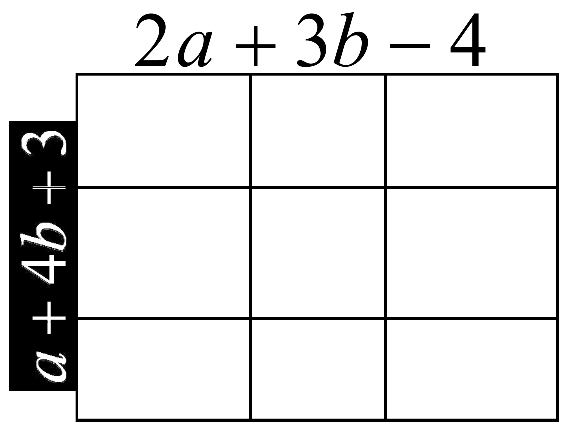
4. $(x^2-y^2)(x+4)$

Multiplying Polynomials

Find the area of each rectangle below:



$(x+2y)(x-y+3)$



$(a+4b+3)(2a+3b-4)$

Multiplying longer polynomials is easier using the grid method:
Setup a grid like the one above to solve the following:
Remember to combine like terms and place answers in descending order.

1. $(x+y+3)(y+5)$

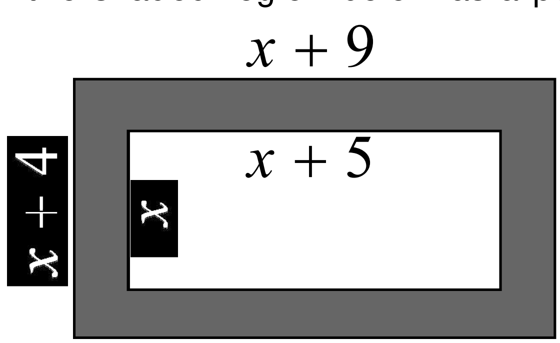
2. $(2a+5b-3)(a-b+2)$

Practice:
Expand each.

1. $(x+y+3)(y+5)$

2. $(2a+5b-3)(a-b+2)$

Practice:
Express the area of the shaded region below as a polynomial in simplest form:



Beyond the Grid

Algebra

Once you have learned the grid and FOIL methods, you should begin to see multiplying polynomials is just distribution.

Examples:

1. $(x+y)(2x-y+5)$
2. $(5a-4b)(2a-3b+1)$

Practice:

Multiply each.

1. $(x-2y)(x+y+5)$
2. $(a-2b)(2a+3b-4)$

Now, multiply each using the Distributive Property.
You should notice something about the answers.

1. $(3x-2y)(x+5y)$
2. $a(2a-b)-6b(2a-b)$
3. $3x(x+5y)-2y(x+5y)$
4. $(a-6b)(2a-b)$

Work Backwards: write each as a product of binomials.

1. $x(2x+3)-y(2x+3)$
2. $a(2a+3b)-2b(2a+3b)$
3. $2x(x+3y)-5(x+3y)$
4. $2a(a-7)-5b(a-7)$

Use the grid method when problems get more complex:

1. $(x-2y+5)(2x+3y-2)$
2. $(a^3-2a^2+5a)(a^2+3a-2)$

Special Cases

Multiply each pair of binomials using the FOIL Method.
Simplify answers.

1. $(x+3)(x+5)$
2. $(a+5b)(2a-3b)$
3. $(x+3)(x-3)$
4. $(a+5b)(a-5b)$
5. $(x+3)(x+3)$
6. $(a-5b)^2$

#1 and #2 are typical trinomials.
#3 and #4 are called DIFFERENCE OF SQUARES. Why?
#5 and #6 are called PERFECT SQUARE trinomials.

More practice: Difference of Squares.
Solve each using FOIL, try to recognize a shortcut.

1. $(2x-3)(2x+3)$
2. $(2a+3b)(2a-3b)$
3. $(x^2+5x)(x^2-5x)$
4. $(a^3+3)(a^3-3)$

How do you recognize a difference of squares?

More practice: Perfect Squares.
Solve each using FOIL, try to recognize a shortcut.

1. $(2x-3)^2$
2. $(2a+3b)^2$
3. $(x^2+5x)^2$
4. $(a^3-3)^2$

Challenge: Expand $(a-b)(a-b)(a+b)(a+b)$ **in 1 minute.**

Quiz Review

Algebra 9.4

Factor out the GCF for each trinomial.

100. $15x^3y^2 - 20x^2y^2 + 10xy^3$

200. $108a^4b^2 + 135a^4b - 36a^2b^2$

300. $136a^3x - 72ax^2 + 48ax$

400. $119x^3 - 68x^2 + 187x$

Multiply each:
Order your answers by descending powers of x or a.

100. $2xy(x - 3y) + 3x(x^2 - xy)$

200. $(2a^2 + b)(b^2 - a)$

300. $(x^2 + 5x - 3)(x^2 - 4x)$

400. $(a - 3b + c)(2a + 2b - c)$

Multiply each.
Order your answers by descending powers of x or a.

100. $(30+3)(30-3)$

200. $(a + 5b)^2$

300. $(x^4 - 3x^3)(x^4 + 3x^3)$

400. $(a^2 - 3)^2(a^2 + 3)^2$

Name________________________ Period _____

Factoring and FOIL Practice Quiz

Algebra 9.4

Factor each expression (Reverse distribution):

1. $42x^3y^3 - 21x^2y^4 + 30xy^5$

1. ______________________

2. $48a^5b^3 - 80a^4b^2$

2. ______________________

3. $57x^6y^2 - 95x^5y + 152x^4$

3. ______________________

Simplify each
(Distribute, combine like terms, and then reorder the terms by descending powers of x or a)**:**

4. $4(x^2y - xy) + x(xy - 3y)$

4. ______________________

5. $ab(a+3) - 2a(4 - b - 5ab)$

5. ______________________

6. $x^2(2xy - x^2) - 2(x - 7)$

6. ______________________

Multiply
(FOIL or Grid method)

7. $(x^2 - 4)(x + 3)$

7. ______________________

8. $(a - 3)(2 - a)$

8. ______________________

9. $(3x - 2)(x - y)$

9. ______________________

Name________________________ Period _____

Factoring and FOIL Practice Quiz

Algebra 9.4

Multiply each (Look for perfect squares and difference of squares):

10. $(x+y)(x-y)$

10. ______________________

11. $(2a+3)^2$

11. ______________________

12. $(3+2a)(3-2a)$

12. ______________________

13. $(2x+y^3)(2x-y^3)$

13. ______________________

14. $(2a+b)(2a+b)(2a-b)$

14. ______________________

15. $(3+2x)^2$

15. ______________________

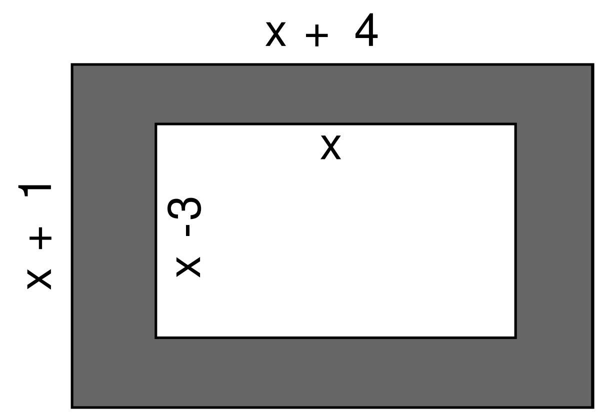
16. $\left[(x-1)(x+1)\right]^2$

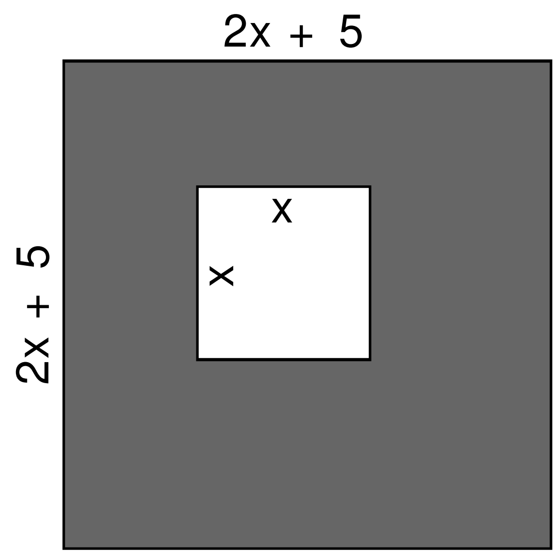
16. ______________________

Polynomial Applications

A common use for multiplying polynomials involves finding area.

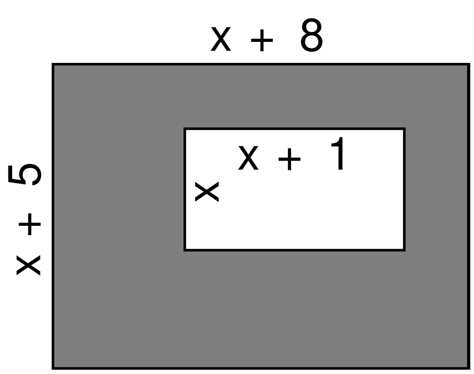
Example: Express the area of the shaded regions in terms of x.

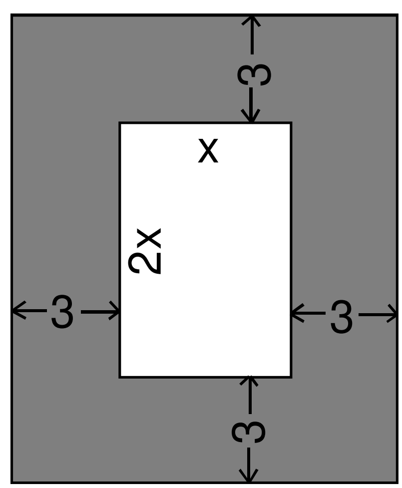




I will call these 'frame' problems because the diagrams usually look like frames.

Practice: Express the area of the shaded regions in terms of x.





Polynomial Applications

Word problems can involve similar area problems, but the diagrams must be given.

Example:
You are matting a photograph that is twice as tall as it is wide. You want to have five inches of matting around the entire photograph. Express the area of matting you will need based on the width (w) of the photograph.

Example:
Barry bought a new rectangular rug for his rectangular dining room. The rug is three feet longer than it is wide. The room is six feet wider than his rug, and seven feet longer than the rug. Express the area of bare floor that will be showing in terms of the rug's width (w).

Answer: If there are 190 square feet of bare floor showing, what ar the dimensions of the rug?

Practice:
Jeremy has a backyard pool surrounded by a tiled walkway that is two yards wide. The pool is 5 yards longer than it is wide. Express the area of the walkway in terms of the width (w) of the pool.

Answer: If the walkway is 196 square yards, how long is the pool?

Practice:
A painting has a frame that is 7 inches wider and 8 inches taller than the artwork it surrounds. The artwork is 5 inches taller than it is wide. Express the area of the frame in terms of the painting's width (w).

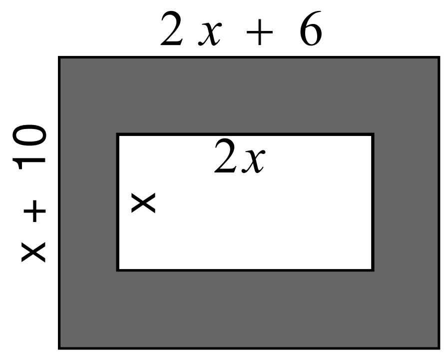
Answer: If the area of the frame is 196 square inches, what is the height of the painting?

Name________________________ Period _____

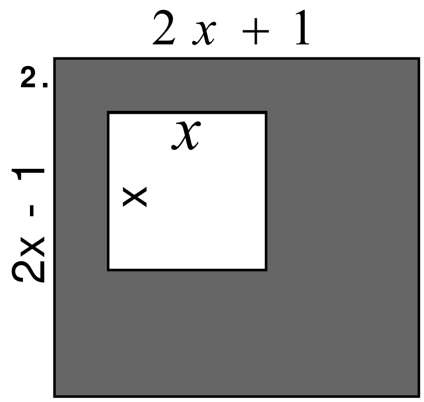
Polynomial Applications

Algebra 9.4

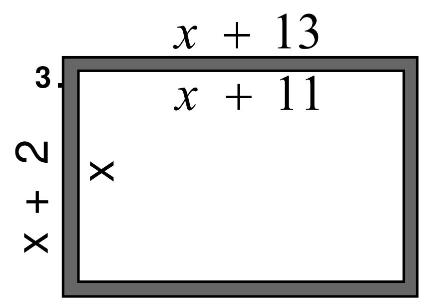
Express the area of each shaded region in terms of x.



2.

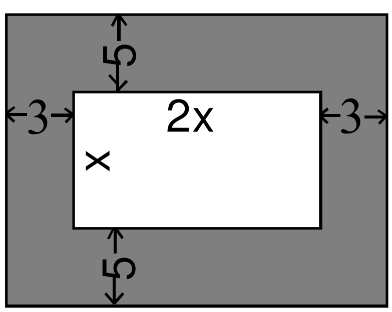


3.

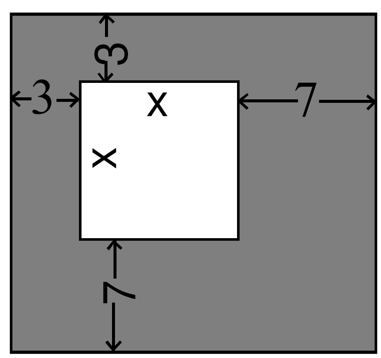


Express the area of each shaded region in terms of x.

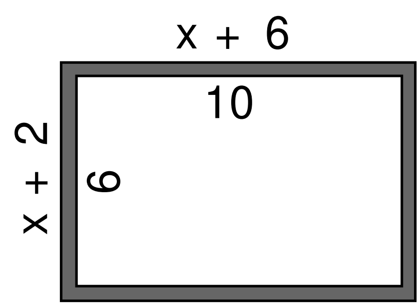
4.



5.



6.



Name________________________ Period _____

Polynomial Applications

Algebra 9.4

Solve each. Include a sketch for each.

7. Connor is planting a garden surrounded by 1-foot square concrete blocks. The garden will be 10 feet longer than it is wide. Express the number of square blocks he will need based on the width (w) of the garden.

If he uses 56 blocks, how many square feet is the area enclosed by the blocks? ______

8. Kerry takes a sheet of paper that is 3 inches shorter than it is wide. He cuts a hole out of the paper that leaves 2 inches of paper on all sides of the hole. Express the area of the *remaining* paper rectangle in terms of w, the width of the original sheet.

If there are $52in^2$ of paper remaining, what were the dimensions of the cut-out hole? ______

9. A company manufactures windows that are 30 inches taller than they are wide. The window comes with an aluminum frame that is 6 inches wide on three sides, and 10 inches wide at the bottom. Express the area of the aluminum frame in terms of the window's width (w).

If the area of the frame is $1,392in^2$, what is the height of the window? ______

Standard Form and Factoring

A Quadratic Equation written as a function looks like this:

$$y = Ax^2 + Bx + C$$ We will call this Standard Form.

Examples: List values for A, B, and C:

$$y = 2x^2 + 3x - 5 \qquad y = x^2 + 5x$$

When you multiply a pair of (1st degree) binomials,
you get a quadratic expression.

$$(x+3)(x-5) = x^2 - 2x - 15$$

Think!
In the equation above, what are the A, B, and C values?
How did we get the values for B and C?

Factoring: Easy ones.
Today we will learn to factor simple quadratics by reversing the FOIL method.

Review: Multiply $(x+2)(x+4)$

$$x^2 + 6x + 8$$

Factoring:
Find two numbers which can be added to get 6 and multiplied to get 8.

More Examples: Factor.

1. $x^2 - 5x + 6$

2. $x^2 - 9x - 10$

Standard Form and Factoring

Algebra 9.5

Practice: Factor. Write Prime for any that cannot be factored.

1. $x^2 + 8x + 12$
2. $x^2 + 8x + 15$
3. $x^2 + 10x + 24$
4. $x^2 + 10x + 9$
5. $x^2 + 8x - 33$
6. $x^2 - 4x + 5$
7. $x^2 + 16x - 36$
8. $x^2 - 5x + 4$

Practice: Factor. Write Prime for any that cannot be factored.

1. $x^2 - 8x + 7$
2. $x^2 - 7x - 30$
3. $x^2 + x - 30$
4. $x^2 + 19x - 42$
5. $x^2 - 2x + 1$
6. $x^2 - 3x + 4$
7. $x^2 + 7x + 10$
8. $x^2 - x - 2$

Factoring: GCF with 'Easy Ones'

Examples: Factor completely.
Begin by factoring out the GCF.
Finish by using reverse FOIL.

$2x^3 - 10x^2 + 8x$

$2x^2y - 14xy + 24y$

Practice: Factor Completely.

1. $5x^2 + 40x + 60$
2. $x^5 + 8x^4 + 15x^3$
3. $ax^2 + 10ax - 24a$
4. $3x^2 + 39x + 90$
5. $2x^3 + 10x^2 - 72x$
6. $-x^2y^2 - 13xy^2 + 30y^2$

Practice: Factor. Write Prime for any that cannot be factored.

1. $3x^2 - 24x + 21$
2. $x^3 - 2x^2 + x$
5. $-14x^2 + 28x - 14$
6. $5x^2 - 65x - 150$
7. $9x^2 + 63x + 90$
8. $-24x^2y + 24xy + 48y$

Name________________________ Period _____

Factoring 'Easy Ones' with GCFs

Algebra 9.5

Factor eact expression by first factoring the GCF and then using reverse FOIL. Write Prime for any that cannot be factored.

1. $4x^2 - 4x - 24$

2. $2x^2 - 28x + 98$

3. $-5x^2 - 35x - 60$

4. $x^2y - 18xy + 17y$

5. $9x^2 + 36x - 108$

6. $6x^2y - 12xy - 48y$

7. $10x^3 - 30x^2 - 100x$

8. $2x^2 + 4x - 70$

9. $7x^2 + 49x + 42$

10. $-2x^2 - 6x + 20$

11. $x^2y^2 - xy^2 - 72y^2$

12. $9x^2 + 27x - 36$

13. $-3x^2 + 6x - 3$

14. $ax^2 + 32ax + 31a$

Factoring: Difference of Squares

Algebra 9.7

Examples: Factor completely.

$x^2 - 9$ $\quad 25x^2 - 49$ $\quad x^4 - 36$

Practice: Factor Completely.

1. $x^2 - 1$
2. $9x^2 - 121$
3. $x^8 - 25$
4. $100x^2 - 169$
5. $9x^4 - 1$
6. $2x^2 - 50$

You can factor out the GCF first.

Examples: Factor completely.

$x^3 - 25x$ $\quad 16x^2 - 100$ $\quad 3x^4 - 27$

Practice: Factor Completely.

1. $5x^2 - 45$
2. $4xy^2 - 36x$
3. $x^7 - x$
4. $x^3y^3 - 4xy$
5. $6x^5 - 6x$
6. $-4x^2 + 64$

Factoring: Perfect Squares

Examples: Factor completely.

$x^2 - 6x + 9$

$25x^2 - 70x + 49$

Practice: Factor Completely.

1. $x^2 - 10x + 25$
2. $x^2 + 16x + 64$
3. $4x^2 + 12x + 9$
4. $x^4 + 14x^2 + 49$

Recognizing Perfect Squares:
Don't be fooled by these imposters!
Only one is a perfect square. Can you find it? **Factor and Check!**

1. $x^2 - 14x - 49$
2. $4x^2 + 18x + 81$
3. $25x^2 - 20x + 4$
4. $9x^4 - 6x + 1$

Easy Ones, Perfect Squares, and Difference of Squares:
Put it all together. Try to recognize how to factor each.

1. $2x^2 - 16x - 40$
2. $a^2 - 81$
3. $x^2 - 6x + 5$
4. $17x^2 - 34x - 51$

Name________________________ Period _____

Factoring and FOIL Review

Algebra

Multiply

(FOIL or Grid method)

1. $(x^2 - 3x + 1)(x + 5)$

1. ______________________

2. $(4a - 3)^2$

2. ______________________

3. $(3x - 2)(x - y)$

3. ______________________

4. $(12x^3 - 7)(12x^3 + 7)$

4. ______________________

Factor Each Completely

(Look for GCFs, Perfect Squares, and Difference of Squares. Write PRIME for any that cannot be factored.)

5. $x^2 - 13x - 30$

5. ______________________

6. $x^4 - 16$

6. ______________________

7. $x^2 - 9x + 20$

7. ______________________

8. $x^3 - 3x^2 + 18x$

8. ______________________

9. $x^2 + 17x + 42$

9. ______________________

10. $3x^2 - 27$

10. ______________________

11. $9x^2 - 30x + 25$

11. ______________________

12. $3x^4 - 39x^2 + 108$

12. ______________________

Name________________________ Period _____

Factoring Practice

Algebra

Challenge 1:
Factor <u>Completely.</u>

$$256x^8y - y^9$$

Ch. 1. ____

Challenge 2:
The number 65,535 is equal to $2^{16} - 1$. Use what you know about a difference of squares to find the four prime factors of 65,535 without a calculator (be ready to explain how this can be done).

Ch. 2. ________________________

Name________________________ Period _____

Factoring and FOIL Practice Quiz

Algebra

Multiply

(FOIL or Grid method)

1. $(a^2 - b)(a + 2b - 3)$

1. ________________________

2. $(2x - 5)^2$

2. ________________________

3. $(3x - y)(3x + y)$

3. ________________________

4. $(x^3 - 2)(2x + 3)$

4. ________________________

Factor Each Completely

(Look for GCFs, Perfect Squares, and Difference of Squares. Write PRIME for any that cannot be factored.)

5. $x^2 - 13x - 30$

5. ________________________

6. $121a^2 - b^2$

6. ________________________

7. $x^2 - 19x + 48$

7. ________________________

Name________________________ Period _____

Factoring and FOIL Practice Quiz

Algebra

Factor Each Completely

(Look for GCFs, Perfect Squares, and Difference of Squares. Write PRIME for any that cannot be factored.)

8. $x^2y - 8xy + 15y$

8. ______________________

9. $x^2 - 12x + 11$

9. ______________________

10. $5x^2 + 15x$

10. ______________________

11. $25x^2 - 40x + 16$

11. ______________________

12. $2x^2 + 2x - 40$

12. ______________________

13. $x^4 - 81$

13. ______________________

14. $144x^2 - 24x + 1$

14. ______________________

Name________________________ Period _____

Factoring and FOIL Self-Check

Algebra 9.7

Factor each (Look for perfect squares and difference of squares, GCF, and easy ones).

1. $x^2 - 6x + 9$

2. $x^2y - 5xy + 6y$

3. $9x^2 - 49$

4. $x^2 - 7x - 30$

5. $4x^2 - 28x + 49$

6. $4x^2 - 64$

Name________________________ Period _____

Factoring and FOIL Self-Check

Algebra 9.7

Factor each (Look for perfect squares and difference of squares, GCF, and easy ones).

1. $x^2 - 6x + 9$

2. $x^2y - 5xy + 6y$

3. $9x^2 - 49$

4. $x^2 - 7x - 30$

5. $4x^2 - 28x + 49$

6. $4x^2 - 64$

Factoring Review.

Algebra 9.7

Easy Ones: Factor completely. Write *PRIME* for any that cannot be factored.

Ex.: $x^2 - 9x + 20$

1. $x^2 + 6x - 16$

2. $x^2 - 3x - 28$

3. $x^2 - 25x - 54$

4. $3x^2y + 12xy - 15y$

Difference of Squares: Factor completely. Write *PRIME* where applicable.

Ex.: $16x^2 - 9$

1. $49x^2 - 144$

2. $x^2 - 100$

3. $ax^2 - ay^2$

4. $4x^2 + 36$

Perfect Squares: Factor completely. Write *PRIME* where applicable.

Ex.: $x^2 - 10x + 25$

1. $x^2 + 8x + 16$

2. $4x^2 - 40x + 100$

3. $x^2 - 2x - 1$

4. $25x^2 + 60xy + 36y^2$

Hard Ones 'Magic Number'

Look at the trinomial below.
Is there a GCF to be factored?
Is it an 'Easy One', a Perfect Square, or a Difference of Squares?

$$4x^2 - 16x + 15$$

The answer to all of these questions is "No." We will call this type of factoring the 'Magic Number' Method.

Example: Factor $4x^2 - 16x + 15$

1. Find/Factor the Magic Number.
2. Rewrite the middle term.
3. Regroup.
4. Factor out the GCFs.
5. Finish (____)(____)

Two more examples. Watch Carefully!

1. $3x^2 + 11x - 20$

2. $10x^2 - 9x + 2$

Practice: Factor each completely.

1. $9x^2 - 3x - 2$

2. $4x^2 + 13x + 10$

Practice: Factor each completely.

1. $25x^2 + 20x + 4$

2. $3x^2 - 30x + 27$

Hard Ones 'Magic Number'

Algebra 9.7

Look at each trinomial below. DO NOT TRY TO FACTOR THEM.
Label each with: EASY ONE
DIFFERENCE OF SQUARES
PERFECT SQUARE
HARD ONE (MAGIC NUMBER)
(hint: there are two of each)

1. $x^2 + 4x - 5$
2. $81x^2 - 72x + 16$
3. $x^2 - 144$
4. $5x^2 - 46x + 9$
5. $x^2 + 22x + 40$
6. $x^2y^2 - 9$
7. $16x^2 + 56x + 49$
8. $3x^2 + 25x - 18$

Now, try to factor them.

1. $x^2 + 4x - 5$
2. $81x^2 - 72x + 16$
3. $x^2 - 144$
4. $5x^2 - 46x + 9$
5. $x^2 + 22x + 40$
6. $x^2y^2 - 9$
7. $16x^2 + 56x + 49$
8. $3x^2 + 25x - 18$

Name________________________ Period _____

Factoring Review

Algebra 9.7

Factor each: Write Prime for any that cannot be factored.

1. $x^2 - 14x + 49$

2. $x^2 + 3x - 28$

3. $10x^2 + 11x - 6$

4. $25x^2 - y^2$

5. $9x^2 - 12x + 4$

6. $9 - x^2$

7. $x^2 - 4x + 3$

8. $8x^2 - 22x - 21$

Name________________________ Period _____

Factoring Review

Algebra 9.7

Factor each: Write Prime for any that cannot be factored.

9. $x^2 - 21x + 54$

10. $x^2 - 18x + 77$

11. $4x^2 - 14x + 49$

12. $81x^2 - 121y^2$

13. $12x^2 - 11x - 5$

14. $4x^2y^2 - 81$

15. $2y^2 - 5xy + 2x^2$

16. $x^4 - 2x^2y^2 + y^4$

Factoring Quiz Review

Algebra 9.7

Factor each: Write Prime for any that cannot be factored.

100. $x^2 - 16$

200. $x^2 - 3x - 54$

300. $12x^2 - 25x + 12$

400. $6x^2 - 21xy + 9y^2$

Factor each: Write Prime for any that cannot be factored.

100. $169 - x^2$

200. $x^2 - 19x + 48$

300. $x^6 - x^2$

400. $2x^2y^2 - 9xy + 9$

Factor each: Write Prime for any that cannot be factored.

100. $7x - 49$

200. $25x^2 - 10x + 1$

300. $10x^2 + 11x - 8$

400. $x^4 - 5x^2 + 4$

Name________________________ Period _____

Factoring and FOIL Practice Quiz

Algebra 9.8

Multiply each (Look for perfect squares and difference of squares, order the terms by descending powers of x):

1. $(x-3)^2$

1. ______________________

2. $(3a+1)(3a-1)$

2. ______________________

3. $(3x+y)(2y-x)$

3. ______________________

Factor each COMPLETELY

NONE OF THE PROBLEMS BELOW ARE PRIME.

(Look for perfect squares and difference of squares, easy ones and hard ones).

4. $9x^2-y^2$

4. ______________________

5. a^3-9a

5. ______________________

6. $x^2-17x+60$

6. ______________________

7. $2x^2-10x+12$

7. ______________________

Name________________________ Period _____

Factoring and FOIL Practice Quiz

Algebra 9.8

Factor each COMPLETELY

Write PRIME for any that cannot be factored.

(Look for perfect squares and difference of squares, easy ones and hard ones).

8. $25x^2 - 100y^2$

8. ______________________________

9. $4x^2 - 36x + 81$

9. ______________________________

10. $x^2 - 3x + 40$

10. ______________________________

11. $10x^2 - 39x + 14$

11. ______________________________

12. $x^4 - 2x^2y^2 + y^4$

12. ______________________________

Simplifying Expressions

Algebra

Practice: Factor each.

1. $x^2 + 3x$ 2. $x^2 - 9$ 3. $2x^2 + x - 15$

Now, try to simplify the following:

1. $\dfrac{x^2 + 3x}{x^2 - 9}$ 2. $\dfrac{x^2 - 9}{2x^2 + x - 15}$ 3. $\dfrac{2x^2 + x - 15}{x^2 + 3x}$

Practice: Simplify each expression.

1. $\dfrac{x(x+7)}{(x+7)(x+2)}$ 2. $\dfrac{x^2 - 9x + 20}{x^2 + x - 20}$

Practice: Simplify each expression.

1. $\dfrac{x^2 - 25}{x^2 - 10x + 25}$ 2. $\dfrac{x^2 - 5x + 4}{2x^2 - 9x + 4}$

Practice: Simplify each expression.

1. $\dfrac{2x(x-3)(x+3)}{x(x+3)(x+3)}$ 2. $\dfrac{5x^3 - 10x^2}{3x^2 - 6x + 2}$

3. $\dfrac{6x^2 - 23x + 20}{4x^2 - 20x + 25}$ 4. $\dfrac{x^4 - 18x^2 + 81}{x^2 + 6x + 9}$

Name________________________ Period _____

Simplify by Factoring

Algebra 9.7

Factor each and simplify where possible.

1. $\dfrac{x(x-7)}{(x-7)^2}$

2. $\dfrac{x^2+5x+6}{x^2+x-6}$

3. $\dfrac{x^2+12x-13}{x^2-2x+1}$

4. $\dfrac{x^2-25}{x^2-10x+25}$

5. $\dfrac{3x^2-15x+12}{3x^2+15x+12}$

6. $\dfrac{5x^2-40x+35}{x^2-8x+7}$

7. $\dfrac{x^3-9x^2+14x}{x^3-4x^2+4x}$

8. $\dfrac{x^4-5x^2+4}{x^2-3x+2}$

Solving Equations by Factoring

Practice: Solve each.

1. $3x + 5 = 5$
2. $3(x+5) = 15$
3. $x(x+3) = 0$

If $ab = 0$ **then either** $a = 0$ **or** $b = 0$.

If $(x-3)(x+5) = 0$ **then either** $(x-3) = 0$ **or** $(x+5) = 0$.

Examples: Solve each for x. Each will have two solutions.

1. $x(x+7) = 0$
2. $(x-9)(x+5) = 0$
3. $x^2 - 6x - 16 = 0$
4. $2x^2 - 7x - 15 = 0$

Practice: Solve for x. Each will have two solutions.

1. $x^2 - 3x = 0$
2. $x^2 + 9x + 20 = 0$
3. $x^2 - 16 = 0$
4. $6x^2 + 7x - 10 = 0$

Tricky Examples: Solve each for x.

1. $x^2 - x = 2$
2. $\frac{1}{5}x^2 + 5 = 2x$

Tricky Practice: Solve each for x.

3. $x^2 - 3x = 10$
4. $\frac{1}{2}x^2 - x = 12$

Name________________________ Period _____

Solving Quadratics by Factoring

Algebra 9.7

Factor each and simplify where possible.

1. $(x-3)(x-5)=0$

2. $2x^2-5x+2=0$

3. $x^2+12x+36=0$

4. $25x^2-1=0$

5. $x^2-x-12=0$

6. $6x^2-19x+10=0$

7. $9x^2+3x-2=0$

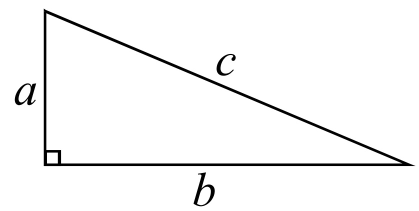
8. $x^2+6x=7$

9. $8x^2+1=6x$

Challenge: $x^4+9=10x^2$ **(4 solutions)**

Factoring Problems

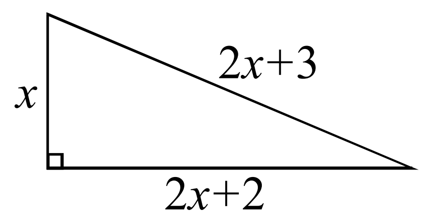
For the problems below, you must know the Pythagorean Theorem:



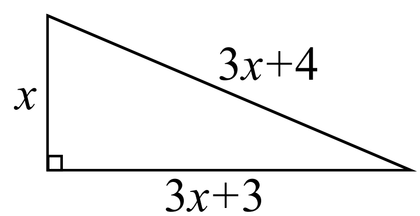
In any right triangle:

$$a^2 + b^2 = c^2$$

Example:
Find the lengths of the sides of the right triangle below.



Practice:
Find the lengths of the sides of the right triangle below.



Practice:

1. In a right triangle, the hypotenuse is 9 inches longer than the shortest side. The length of the medium side is just one inch longer than the length of the shortest side. What is the perimeter in inches of the triangle?

2. The hypotenuse of a right triangle is 1cm longer than the long leg. The short leg is 1cm shorter than half the long leg. What is the triangle's area?

Name________________________ Period _____

Solving Quadratics by Factoring

Algebra

Factor each and simplify where possible.

1. $x^2 - 10x + 21 = 0$

2. $16x^2 - 9 = 0$

3. $x^2 - x = 56$

4. $25x^2 + 3 = 20x$

5. $9x^2 + 9x = 10$

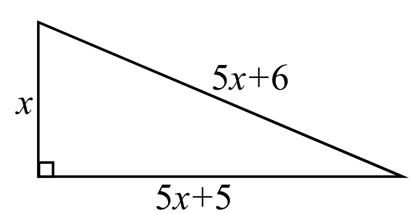
6. $x^2 = 3x + 10$

7. $8x^2 = 18x + 5$

8. $x^3 - 4x = 0$ **(3 solutions)**

9. The equation $x^2 + kx + 36 = 0$ has only one solution for positive integer k. What is k ?

10. Find the perimeter of the triangle below.



Clever Factoring:

Some tricks and more difficult problems:

Example:

One of the solutions to the equation $x^2 + a = 6x$ is 5.

a. What is the value of a?
b. What is the other solution?

Practice:

1. The equation $3x^2 - ax = 8$ has $x = 4$ as a solution.

a. What is the value of a?
b. What is the other solution?

2. The equation $ax^2 - 5x = 2$ has $x = 1$ as a solution.

a. What is the value of a?
b. What is the other solution?

Example:

How can the polynomial $(x+y)^2 - 9$ can be factored into the product of two trinomials?

Practice:

1. Factor the following into a product of trinomials: $(x-2)^2 - y^2$.

2. Factor the following into a product of trinomials: $x^2 - y^2 - 10x + 25$.

Solving Trickier Equations Practice:

1. Solve for x: $3(x^2 - 4) - x(x^2 - 4) = 0$.

Hint: Where have you seen something similar to this before?

2. Solve for x: $\dfrac{10}{x^2} - \dfrac{9}{x} + 2 = 0$.

Hint: use a common denominator.

3. Solve for x: $\dfrac{x-6}{14-3x} = \dfrac{3}{x-2}$.

Factoring Test Review

Algebra 9.8

Perfect Squares and Difference of Squares: Factor each.

100. $9 - x^2$

200. $6x^2 - 24$

300. $2x^2y^2 + 12xy + 18$

400. $x^4 - 81$

Easy Ones and Magic Number:
Write Prime for any that cannot be factored.

100. $x^2 - 6x + 8$

200. $x^2 - 22x + 72$

300. $12x^2 + 5x - 3$

400. $6x^2 + xy - 2y^2$

Solve each: Write Prime for any that cannot be factored.

100. $\frac{1}{3}x^2 - 30x = 0$

200. $x^2 + 24 = 11x$

300. $\frac{4}{7}x^2 + 7 = 4x$

400. $6x^3 = 25x^2 - 14x$

Name________________________ Period _____

Factoring and FOIL Practice Test

Algebra 9.8

Factor each COMPLETELY

Write PRIME for any that cannot be factored.

(Look for perfect squares and difference of squares, easy ones and hard ones, and GCF problems).

1. $x^2 - 4y^2$

1. ______________________

2. $34x^2 - 85x$

2. ______________________

3. $x^2 - 13x - 30$

3. ______________________

4. $3x^2 + x + 10$

4. ______________________

5. $a^3 - 4a^2 + 4a$

5. ______________________

6. $2x^2 + 19x + 24$

6. ______________________

7. $x^4 - 11x^2 - 80$

7. ______________________

Name________________________ Period _____

Factoring & FOIL Practice Test (4)

Algebra 9.8

Solve for x:

Some problems may have more than one solution. List **all** solutions in the blank provided.

8. $3x(x-7)=0$

8. x=________________________

9. $x^2-6x+9=0$

9. x=________________________

10. $10x^2+6=17x$

10. x=________________________

11. $x^2+72=18x$

11. x=________________________

Multiply:

12. $(5x-4)(x-5)$

12. ________________________

13. $(3x^2-5x)^2$

13. ________________________

14. $(x-2)(x-3)(x+2)$

14. ________________________

Name________________________ Period _____

Factoring and FOIL Practice Test

Algebra 9.8

Solve for x:

Some problems may have more than one solution. List **all** solutions in the blank provided.

8. $3x(x-7)=0$

8. x=________________________

9. $x^2-6x+9=0$

9. x=________________________

10. $10x^2+6=17x$

10. x=________________________

11. $\frac{1}{6}x^2+12=3x$

11. x=________________________

Simplify each:

12. $\frac{x^2-5x+6}{x^2-x-6}$

12. ________________________

13. $\frac{4x^2-36x+81}{4x^2-81}$

13. ________________________

14. $\frac{4x^3-10x^2}{5x^2-20}$

14. ________________________