Ratios and Rates
Math Night

ACTIVITY
4.1

SUGGESTED LEARNING STRATEGIES: Summarize/Paraphrase/Retell, Use Manipulatives, Look for a Pattern, Think/Pair/Share

Ms. Yang's class wants to have a Math Night. Ms. Yang agrees to plan the activities so long as the students plan the food and drinks. The Math Night theme will be *Individual Fun Facts*.

Work with a partner to sample some of the activities Ms. Yang has planned for her students. Record your data.

Activity 1: Writing Speed
How fast can you write the word *math* five times? Try with each hand.

a. Rate for the *dominant* hand:
b. Rate for the *non-dominant* hand:

Activity 2: Reading Speed
How many words can you read in one minute? You may use your own book or the passage your teacher will give you.

a. Reading rate in minutes:
b. Reading rate in seconds:

Activity 3: Heart Rate
Measure your heart rate by counting your pulse for 30 seconds. To do this, locate your pulse by placing your index and middle fingers on the thumb side of your wrist, then count the number of beats. What is your heart rate?

Activity 4: Jumping Jacks

a. How many jumping jacks can you do in 15 seconds?
b. What is your jumping jacks rate?

Activity 5: Are You Tongue Twisted?

a. How fast can you say "Peter Piper picked a peck of pickled peppers" 5 times?
b. What is your speaking rate?

Activity 6: Are You a Fast Walker?
Look at the materials your teacher has provided.

a. How could you find your walking rate? Explain.
b. What is your walking rate?

My Notes

Because rates will vary widely due to individual differences, no sample results are provided.

CONNECT TO SCIENCE

The hand a person uses most often is called the *dominant* hand. The other hand is called the *non-dominant* hand.

TEACHER TO TEACHER Some mathematics educators differ about the relationship of ratios and rates. Sometimes they are treated as distinct terms, so that one is not a special case of the other, and both are defined as comparisons.

Ratio: a comparison of numbers with the same units, such as 3 *cups* water: 5 *cups* juice
Rate: a comparison of two measurements with different units, such as $5 per 1 *hour*

Sometimes a rate is defined simply as a special type of ratio.

Ratio: any comparison of two numbers or measurements
Rate: a special ratio in which the two terms have different units

(*Continued on next page*)

ACTIVITY 4.1 Investigative

Ratios and Rates

Activity Focus
- Writing ratios
- Equivalent ratios and proportions
- Rates, unit rates, and unit prices

Materials
- Measuring tape
- Clock with second hand or stopwatches
- Food coloring
- 2 clear cups (same size)
- Tablespoon
- Water
- BLM 3: Reading Passage (optional)

Chunking the Activity

Act. 1–6	#11–14	#29–30
#1–3	#15–16	#31–32
#4–6	#17–19	#33–37
#7–8	#20–23	#38
#9–10	#24–28	

Paragraph Summarize/Paraphrase/Retell

Activities 1–6 Use Manipulatives, Look for a Pattern, Think/Pair/Share Have students work in pairs for these activities, which actively involve and motivate them. Do not introduce the term *rate* now. Later, students will apply what they learn about rates to find their personal rates.

Before starting, check that students can measure seconds with a clock or stopwatch. Let students choose a reading passage for Activity 2 or use the passage provided in Resources at the back of this Teacher's Edition. Activity 6 is most easily done in a hallway with 1-foot tiles, or you can use masking tape to mark off feet on the floor.

ACTIVITY 4.1 *Continued*

Paragraph Marking the Text, Think Aloud, Vocabulary Organizer

1-2 Vocabulary Organizer These questions serve as review of writing ratios. Students should understand that the order of the quantities, juice concentrate and water, determines the placement of the terms. Most sources give 2 to 3, 2:3, and $\frac{2}{3}$ as the acceptable ways to write a ratio, but some sources accept $2 \div 3$ as well. A writing math and mini-lesson are provided for students new to writing ratios.

3 Guess and Check This question gives students an opportunity to think about ratio relationships before exploring them mathematically.

Whatever the students answer here is acceptable at this point, as this is merely meant as an activator. In addition, this question brings forth the common misconception that if you add the same amount to both terms in a ratio, you will have an equivalent ratio.

Ratios and Rates
Math Night

SUGGESTED LEARNING STRATEGIES: Marking the Text, Think Aloud, Guess and Check, Vocabulary Organizer

My Notes

The students must work together to plan the food and drinks. Ms. Yang already has some juice concentrate, napkins, and cups they can use. First the students need to figure out how to blend the concentrate with water to make enough juice for 26 people.

The label on the can of mix gives two mixing options.

Directions: Use one of these options
1. Add 2 cups concentrate to every 4 cups water, or
2. Add 3 cups concentrate to every 5 cups water.

You can compare two quantities, or show a relationship between them, by writing a **ratio**. The numbers that are compared are called **terms**.

WRITING MATH

Ratios can be written as fractions, with a colon (:), or using the word *to*.

For example:

$\frac{8}{9}$ or 8:9 or 8 to 9

1. The directions for mixing the juice show a relationship between what 2 quantities? concentrate and water

2. Write a ratio in 3 different ways to show this relationship for each option.

Option 1	Option 2
2 cups concentrate	3 cups concentrate
4 cups water	5 cups water
$\frac{2}{4}$	$\frac{3}{5}$
2:4	3:5
2 to 4	3 to 5

3. The students agree that they want to make juice with the most flavor. Which option do you think they will choose? Explain why you think they will make that choice.

☐ Option 1 is more flavorful.

☑ Option 2 is more flavorful.

☐ Options 1 and 2 are equally flavorful.

Answers will vary. This question is developed in the next items. It is more important for students to explain their choice clearly than it is for them to realize that Option 2 is the better choice.

Teacher to Teacher (*Continued*)

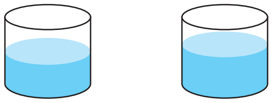
Consider the following situation:

- Would 1 ounce of food coloring per pound of dough be a rate because it compares ounces to pounds, while 1 ounce of food coloring per 16 ounces of dough would be a ratio since it relates ounces to ounces?
- Are interest rates truly rates given that they technically compare similar units, money and money?

Most mathematicians agree that *ratio* is a more general term that encompasses rates and that *ratio* is simply a relationship between two quantities. In this activity, *ratio* is used as a generic term while *rate* applies to ratios that involve two different kinds of units.

SUGGESTED LEARNING STRATEGIES: Create Representations, Quickwrite, Debriefing

The students notice that Option 2 has one more cup of concentrate and one more cup of water than Option 1. They wondered if adding one cup of concentrate and one cup of water to Option 1 changed the flavor of the juice. To help visualize the comparison between a ratio of 2 to 4 and a ratio of 3 to 5, look at the models your teacher will show you.



4. Color the drawings above to visually compare the ratios.

5. What can you conclude about the two ratios by looking at the models?

Answers may vary. Sample answer: The ratio of 3:5 would have more flavor than the ratio of 2:4.

6. The students want to determine how much water they will add to preserve the ratio of each option when they add 6 cups of the concentrate.

a. Predict how much water they will add to preserve the ratio of Option 1.

12 cups of water will be added. This is confirmed in the next question so it is important that students give a realistic prediction, but it does not have to be exact.

b. Predict how much water they will add to preserve the ratio of Option 2.

10 cups of water will be added.

My Notes

ACTIVITY 4.1 *Continued*

4 **Create Representations** You can make up a "juice concentrate" by adding food coloring to water. Then dilute that mixture with water to simulate the 2:4 and 3:5 ratios in the clear containers. You may need to experiment a bit to have a noticeable difference in the two solutions. You want the students to be able to see that the ratios are not equivalent, so the cup showing 2:4 should be lighter than the cup showing 3:5.

5 **Quickwrite** You may want to continue to add the "concentrate" to the cups of water, showing different ratios and why just adding 1 part of concentrate and 1 cup of water, or 2 parts of concentrate and 2 cups of water, and so on, does not keep the same ratio of concentrate to water.

6 **Quickwrite, Debriefing** It is important for students to understand that ratios have a multiplicative relationship, and not an additive one. Give various examples to solidify this before moving on if needed.

MINI-LESSON: Writing Ratios

Ratios can be written in three different ways. If there are 3 red marbles and 4 blue marbles, we say the ratio of red marbles to blue marbles is 3 to 4 (3 red marbles to every 4 blue marbles). This ratio can also be written as 3:4 or $\frac{3}{4}$. The first number must represent the first thing listed, in this case the red marbles. Write a ratio to represent each of the following comparisons.

1. A class has 14 boys and 11 girls
2. 24 cookies for every 1 batch
3. A bag holds 2 pens, 4 pencils, and 1 notebook. What is the ratio of notebooks to pens?

ACTIVITY 4.1 *Continued*

7-8 Create Representations (7a), Look for a Pattern (7b, c, 8), Quickwrite (7c, 8b), Think/Pair/Share (8) These questions give students a chance to explore (in tabular form) what happens when the terms of ratios are doubled, tripled, and so on. A connection should be made to tables of values for functions, with which the students should be familiar. By completing these tables they are creating equivalent ratios.

SUGGESTED LEARNING STRATEGIES: Create Representations, Look for a Pattern, Quickwrite, Think/Pair/Share

My Notes

7. Using **ratio tables** is a way to compare the ratios $\frac{2}{4}$ and $\frac{3}{5}$.

a. Complete each ratio table to show the relationship of juice concentrate to water for each mixing option if you double the recipe, triple it, and so on.

Option 1	Juice concentrate	2	4	6	8	10
	Water	4	8	12	16	20

Option 2	Juice concentrate	3	6	9	12	15
	Water	5	10	15	20	25

b. Highlight the column in each table that has the same number of cups of juice concentrate and write each ratio below.

Option 1: 6 to 12, Option 2: 6 to 10

c. What does this tell you about the strength of each mixture?

Answers may vary. Sample answer: Because the two mixtures have the same amount of juice concentrate, the one with more water is going to be less flavorful. So Option 1 is less flavorful.

8. Now use a different color to highlight the column in each table that has the same number of cups of water.

a. Write each ratio.

Option 1: 10 to 20, Option 2: 12 to 20

b. What does this tell you about the strength of each mixture?

Answers may vary. Sample answer: The two mixtures have the same amount of water, so the one with more juice concentrate will be more flavorful. Option 2 is more flavorful.

SUGGESTED LEARNING STRATEGIES: Create Representations, Look for a Pattern, Vocabulary Organizer

9. If the students increase the juice concentrate 1 cup at a time, find how much water they would need to use in each case.

a. Complete the ratio tables below.

Option 1	Concentrate (m)	1	2	3	4	5
	Water (w)	2	4	6	8	10

Option 2	Concentrate (m)	1	2	3	4	5
	Water (w)	$1\frac{2}{3}$	$3\frac{1}{3}$	5	$6\frac{2}{3}$	$8\frac{1}{3}$

b. What patterns do you notice in the table for Option 1?
Answers will vary. Every time one cup of mix is added, two cups of water are added.

c. What is the rule for the Option 1 table?
$m = \frac{1}{2}w$ or $w = 2m$

d. What patterns do you notice in the table for Option 2?
Answers will vary. For every cup of mix, you add $1\frac{2}{3}$ cups of water.

e. What is the rule for the Option 2 table?
$m = \frac{3}{5}w$ or $w = \frac{5}{3}m$

Relationships that have the same ratio are called **proportional relationships** and can be represented with the algebraic rule $y = mx$. The amount, m, is the factor by which y increases each time. It represents a *constant rate of change*.

10. Graph the data and compare the graphs.

a. List the ratio pairs from each ratio table in Question 9 as ordered pairs. **Option 1: (1, 2), (2, 4), (3, 6), (4, 8), (5, 10); Option 2: points $\left(1, 1\frac{2}{3}\right)$, $\left(2, 3\frac{1}{3}\right)$, (3, 5), $\left(4, 6\frac{2}{3}\right)$, $\left(5, 8\frac{1}{3}\right)$**

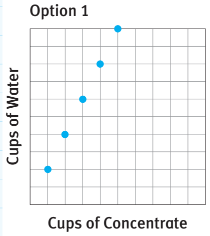
b. Graph the ordered pairs on the grids in the My Notes space.

c. What do you notice about the graph of the data in each ratio table? **Answers may vary. Sample answer: The points on each graph seem be in a straight line.**

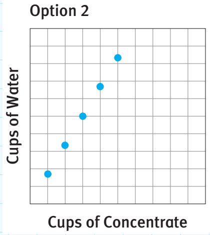
My Notes

Relationship of Concentrate to Water

Option 1



Option 2



ACTIVITY 4.1 *Continued*

9 Create Representations, Look for a Pattern Students must simplify the ratios in order to find the amount of water that gets mixed with 1 cup juice concentrate. This helps them see that as the number of cups of juice concentrate increases by 1 cup, the number of cups of water actually increases by 2 for Option 1 and by $1\frac{2}{3}$ for Option 2. Writing rules for the ratio tables highlights the fact that proportional relationships are multiplicative.

TEACHER TO TEACHER Learning to express linear relationships with the general equation $y = mx$ helps prepare students for algebra where they will learn about direct variation, usually written $y = kx$. In algebra, students will also learn about the more general equation of a line, $y = mx + b$.

10 Graphing each ratio pair allows students to visualize that the ratios from each table form a line, thus there is a linear relationship among these equivalent ratios.

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 194, #1–3a

UNIT 4 PRACTICE
p. 235, #1–2

11-12 Think/Pair/Share
Question 11 emphasizes the part-to-part relationship of this ratio, forcing students to consider the whole, which is 8 cups of juice. Realistically, this is not enough juice for the class, as they need enough juice for 26 people. In Question 12, the students are led through doubling the recipe, or terms of the ratio. This problem is intentionally not written as equivalent fractions in order to stress the proportional reasoning of doubling each term.

13-14 Think/Pair/Share (14), Create Representations (13c), Vocabulary Organizer (14) As the 6:10 ratio only makes 16 cups of juice, the students now discover in Question 13 that they will need to multiply each term of the ratio by 7. $\frac{3}{5} \times \frac{7}{7} = \frac{21}{35}$, which makes a total of 56 cups. By setting $\frac{3}{5}$ equal to $\frac{21}{35}$ they have written a proportion in Question 14.

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Create Representations, Vocabulary Organizer

My Notes

READING MATH

$\frac{\text{3 cups concentrate}}{\text{5 cups water}}$ is read, "3 cups concentrate to every 5 cups water" or "3 cups concentrate per 5 cups water."

11. Ms. Yang's class decides to make the juice using the concentrate and water from Option 2.

a. Explain how to determine the number of cups of juice one batch of this mixture will make.
Add the amount of concentrate and the amount of water. $3 + 5 = 8$ cups

b. To give each of the 26 people attending Math Night 2 cups of juice, how many cups do they need to make? Explain how you determined your answer. 52. I multiplied 26 by 2.

12. To make more juice, the students double their recipe.

a. Express the ratio of concentrate to water when they revise the recipe.
$\frac{\text{6 cups mix}}{\text{10 cups water}}$

b. How many cups of juice will that make? Explain why this is or is not enough juice for the party.
16 cups; it is not enough because the students need 52 cups, and doubling the recipe gives only 16 cups.

13. The students need to increase their mixture to make enough juice to serve two cups each to the 26 people attending.

a. Find a ratio equal to 3 cups concentrate per 5 cups water that provides enough juice, with as little extra juice as possible. Explain how you determined your answer. 21 to 35

b. Write a number sentence that shows the original ratio is equal to your new ratio. $\frac{3}{5} = \frac{21}{35}$

The equation you just wrote shows a proportional relationship, and is called a **proportion**.

MATH TERMS

A **proportion** is a mathematical statement describing two ratios that are equal to each other.

14. By writing an equivalent ratio, the students find out how to make more juice while keeping the same relationship between mix and water.

a. How many cups of juice will the students make using the ratio you found in Question 13b? 56

b. How many extra cups of juice will there be? 4

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Marking the Text, Discussion Group, Summarize/Paraphrase/Retell, Vocabulary Organizer

Now the students must decide *what kind* of pizza to order. They decided on pepperoni and extra cheese. They found that 16 people want pepperoni and 10 people want extra cheese.

15. Write a ratio in fraction form that shows the relationship of pepperoni slices to cheese slices. $\frac{16}{10}$

16. The students found that the average number of slices each person would eat was 2 slices. Write a ratio equivalent to the one you wrote for Question 15 that shows the relationship of pepperoni to cheese slices, assuming each person will eat 2 slices. $\frac{32}{20}$

17. Use this ratio to determine the total number of pizza slices they need. Show your work.

52 slices; $32 + 20 = 52$

18. Another way to figure out the total slices needed is to write a ratio comparing pizza slices to people. Write the average number of slices per 1 person as a ratio in fraction form. $\frac{2}{1}$

What you have just written is a special type of ratio known as a rate. This rate shows a relationship between quantities measured with different units (slices and people). Earlier, when pepperoni slices were compared to cheese slices, you compared different toppings (pepperoni and cheese), which had the same unit (slices). This type of ratio is also a rate.

When the rate is *per 1 unit,* such as slices per 1 student, it is called a **unit rate**. Unit rates are easy to spot because they are often written with the word *per* or with a slash (/) (for example, *slices* ***per*** *person* or *slices/person*).

19. Name at least 2 other situations where you have noticed a relationship expressed with the word *per*.

Answers may vary. Sample answers: miles per hour, heart beats per minute, $10 per hour, 10 cents per minute, $3.15 per gallon

My Notes

ACADEMIC VOCABULARY

A **rate** is a comparison of two different units, such as miles per hour, or two different things measured with the same unit, such as cups of concentrate per cups of water.

ACADEMIC VOCABULARY

Rates are called **unit rates** when they are expressed in terms of 1 unit.

Examples of unit rates are 60 miles per hour or 12 words per second.

TEACHER TO TEACHER Students are moving from their understanding of ratios to learning about rates and unit rates as they make decisions about purchasing pizza for the party. Until now, they were relating quantities with the same units, such as *cups* concentrate and *cups* water, while now they will relate quantities with *different* units. Students learn to use equivalent fractions, unit rates, and proportions to solve missing value problems and numerical comparison problems.

Paragraph Marking the Text

15-16 These questions give students more practice with writing ratios. Students should be encouraged to include labels so that they can see they are comparing similar units, slices to slices, as well as to emphasize its property as a ratio, versus the fractional property of part to whole: 16 pepperoni slices/10 cheese slices. While some ratios have fractional meaning, there is no part to whole meaning shown by pepperoni to cheese slices. However, it is important that students are comfortable using the fraction form for ratios, as it makes computation with ratios easier.

17 Think/Pair/Share This question gives students more practice with doubling the terms in a ratio and considering the whole.

18 Think/Pair/Share In Question 16, students learned that Ms. Yang's class will eat an average of 2 slices per person. In this question they write this in mathematical terms as a rate: 2 slices/1 student. This helps them make a connection with prior knowledge.

Paragraph Marking the Text, Summarize/Paraphrase/Retell, Vocabulary Organizer Students learn that a rate like the one they just wrote is called a *unit* rate.

19 Students connect this new terminology to real-life instances where they have used rates.

20-21 Activating Prior Knowledge (21) Students see how to set up a proportion in order to solve for a missing value. Although solving for the missing value as if they were finding equivalent fractions is a procedure students have used before, they may need review. Guide students to remember that to write equivalent fractions, they multiply the given fraction by 1 in any fractional form. This is different conceptually than telling students to multiply the numerator and denominator by the same number. Remind students that multiplying by 1 does not change the value of a number, because of the multiplicative identity.

22 Identify a Subtask This question returns students to the real-life context and reviews how to interpret remainders in division. Students reason that an answer of 6 R 4 means the class will eat 6 whole pizzas (48 slices), and 4 slices of another pizza. They must buy 7 pizzas in order to have enough pizza.

23 Think/Pair/Share This question gives students more practice with the equivalent fraction strategy, and introduces them to working backward. Instead of multiplying by 1 to find a missing value in an equivalent ratio, they divide by 1 to find unit rate. Using proportions here introduces students to simplifying ratios.

24 Students learn how to find the unit rate. They may reason that if 2 pizzas cost $19.98, then 1 pizza costs half that.

My Notes

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Activating Prior Knowledge, Identify a Subtask, Think/Pair/Share

20. Use the unit rate to find the total number of slices needed. Set up a proportion. Fill in the values you know.

Unit Rate Slices/Person		Rate for Total Total Slices/Total People
$\dfrac{\boxed{2}\ \textit{slices}}{\boxed{1}\ \textit{person}}$	=	$\dfrac{?\ \textit{slices}}{\boxed{26}\ \textit{people}}$

21. Finding equivalent rates is just like finding equivalent fractions. Rewrite the proportion and use the Property of One to solve. Think of this as *finding an equivalent fraction.*

$\dfrac{2\text{ slices}}{1\text{ person}} \times \dfrac{26}{26} = \dfrac{52\text{ slices}}{26\text{ people}}$

22. If a large pizza is cut into 8 slices, how many pizzas must the students buy? 7 pizzas

Two pizza places have free delivery to the school.

Mama T's Pizza
Get **2** large 1-topping pizzas for just **$19.98!**
1 large 1-topping pizza is **$10.99**

TONI'S PIZZA
1 large 1-topping pizza for **$10.99**
3 large 1-topping pizzas: only **$29.70**

23. Is $19.98 for 2 pizzas at *Mama T's Pizza* a good deal? To determine this, you must find the price per pizza. Find the price per pizza by finding an equivalent fraction. This time you will *divide* by 1 in the form of $\frac{2}{2}$.

$\dfrac{\$19.98}{2\text{ pizzas}} \div \dfrac{2}{2} = \dfrac{\$\boxed{9.99}}{1\text{ pizza}}$

24. Notice, by setting up the ratio $\frac{\$19.98}{2\ \textit{pizzas}}$, you treat it as a fraction and divide 19.98 by 2. You also divide the unit *dollars* (or *price*) by the unit *pizzas* to get "dollars / pizzas" or "price per pizza".

SUGGESTED LEARNING STRATEGIES: Identify a Subtask, Question the Text, Create Representations, Quickwrite, Group Presentation

My Notes

25. Another way to find the cost of each pizza is to reason this way: "If $19.98 is the cost of 2 pizzas, then how many dollars does 1 pizza cost?"

$\frac{\$19.98}{2 \text{ pizzas}} \longrightarrow \frac{\boxed{\$9.99}}{1 \text{ pizza}}$

For this way of reasoning, you think about *how much for one*. Think of this as *finding the unit rate*. When a problem involves working with money, the unit rate is called the **unit price**.

26. How much do the students save by using this deal instead of buying 2 pizzas at regular price?
$1.00 per pizza, or $2 for 2 pizzas

27. What is the price per pizza for the deal at *Toni's Pizza*?
$9.90

28. To decide where they will get the better deal, the students cannot simply compare rates. They need a specific number of pizzas, so the better deal may depend on how many pizzas they are buying.

a. Determine how much it would cost to buy 7 pizzas from *Mama T's Pizza*. The students can use the deal for every 2 pizzas they buy, but the seventh pizza will be at regular price. Show how to use a proportion to help determine your answer.
$\frac{\$19.98}{2 \text{ pizzas}} \times \frac{3}{3} = \frac{\$59.94}{6 \text{ pizzas}}$; $\$59.94 + \$10.99 = \$70.93$

b. Determine how much it would cost to buy 7 pizzas from *Toni's Pizza*. Show your work.
$\frac{\$29.70}{3 \text{ pizzas}} \times \frac{2}{2} = \frac{\$59.40}{6 \text{ pizzas}}$; $\$59.40 + \$10.99 = \$70.39$

c. Where should the students buy their pizza? Explain.
Explanations may vary. Sample answer: Toni's Pizza; $70.93 − $70.39 = $0.54 cheaper.

ACTIVITY 4.1 *Continued*

25-28 Identify a Subtask (25) Question 25 relates unit rate to the context. Students may solve this using the unit rate or by finding the price of 2 pizzas at regular price.

28 Question the Text, Create Representations (a, b), **Quickwrite** (c), **Group Presentation** This problem brings up the issue of "better deal". This is a big discussion point when discussing smart shopping.

The group of students in the problem must understand that the unit price only tells them how good the deal is, but that it cannot be applied to their total number of pizzas. For instance, if the students buy 3 pizzas from Mama T's Pizza, they only get 2 pizzas with the unit price of $9.99 (the 2-for deal), but the third pizza will cost the regular price, $10.99. Although Toni's Pizza has the better deal when considering unit price, it is not necessarily the better deal for these students who need 7 pizzas.

Guide the class to look at various possibilities for buying the pizza. If the students only needed to buy 4 pizzas, it would be smarter to use the deal of $19.98 for 2 pizzas, as their total would be $19.98 × 2 = $39.96. Using the Toni's Pizza deal for $29.70 for 3 pizzas + 1 pizza at regular price would cost a total of: $29.70 + $10.99 = $40.69.

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 194, #3b–5

UNIT 4 PRACTICE
p. 235, #3–5

ACTIVITY 4.1 *Continued*

TEACHER TO TEACHER The next series of questions moves students from using the fraction strategy and factor-of-change strategy to find total pizza cost, to using the unit rate strategy to compare rates in numerical comparison problems as they plan to purchase paper plates. This time "better deal" is based upon unit rate, as they are not purchasing exact amounts.

29 Students should answer Question 29a and 29b using only logic, without doing any written work. Since Part c cannot be done in the same way, it gives students a need to use unit rates.

30 Look for a Pattern, Quickwrite (b), Think/Pair/Share, Self Revision/Peer Revision This problem guides students through using unit rates to solve numerical comparison problems. After writing each rate in fractional form students see that they cannot easily multiply the terms of one ratio to have common terms with the other. Thus, they must find unit prices.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Quickwrite, Think/Pair/Share, Self Revision/Peer Revision

My Notes

Now that the juice and pizza are figured out, the students must purchase paper plates. They are not concerned about buying a specific number of each this time, because they will use the extra plates in the future. They are searching for the best deals.

29. The table shows rates for the cost of paper plates at three different stores. Each store has two options.

Paper Supplies	Party!	Local Grocer
$10.99/100 plates	$3.39/15 plates	$3.39/15 plates
$6.90/100 plates	$3.39/25 plates	$6.39/24 plates

a. Decide which is the better rate at *Paper Supplies*. Explain your thinking.

Answers may vary. Sample answer: Both rates at *Paper Supplies* have the same number of plates, but that the first is more expensive. So $6.90/100 plates is the better deal.

b. Decide which is the better rate at *Party!* Explain your thinking.

Answers may vary. Sample answer: Both rates at *Party!* have the same cost, but $3.39/25 plates gives more plates and therefore is the better deal.

c. Can you easily figure out the better rate at *Local Grocer?* Why or why not?

Explanations may vary. Sample answer: No, neither the price nor the number of plates is the same, and while one has a greater number of plates, the cost is also higher, so you cannot use simple reasoning.

30. Now consider only the rates at *Local Grocer*.

a. Write each rate at *Local Grocer* in fractional form.

$\frac{\$3.39}{15 \text{ plates}}$ and $\frac{\$6.39}{24 \text{ plates}}$

b. Can you multiply 15 plates by something to get 24, or $3.39 by something to get $6.39? How does this affect your ability to compare these two rates as you did for the *Paper Supplies* and *Party!* stores?

Answers may vary. Sample answer: There is no whole number by which I can multiply $3.39 to get $6.39 or 15 to get 24.

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Think Aloud, Simplify the Problem, Look for a Pattern, Quickwrite

My Notes

c. When it is not easy to find an equivalent fraction to compare quantities, find the unit rate for each deal to find the unit price (price per plate).

$$\frac{\$3.39}{15 \text{ plates}} = \frac{\boxed{\$0.226}}{1 \text{ plate}}$$

\$ 0.23 per plate or \$ 0.23 /plate

d. Use this unit price (price per plate) to find the cost for 24 plates. Is that more or less than the other rate of \$6.39/24 plates?

$$\frac{\$0.23}{1 \text{ plates}} = \frac{\boxed{\$5.52}}{24 \text{ plate}}$$

\$5.52/24 plates is less than \$6.39/24 plates.

31. Another way to find the cost of a pack of 24 plates that has the same rate as \$3.39/15 plates is to write a proportion. Let *c* represent the unknown cost of the 24 plates.

$$\frac{\$3.39}{15 \textit{ plates}} = \frac{c}{24 \textit{ plates}}$$

To figure out a rule you can use to solve for *c*, think about some procedures you already know. Finish solving the equations for *c*, but do not simplify the terms. Explain what steps you use.

Equation	Solve for *c*. Do not simplify.	What are the steps?
a. $3 = \frac{c}{5}$	$5 \times 3 = c$	Multiply both sides by 5.
b. $\frac{c}{7} = 4$	$c = 7 \times 4$	Multiply both sides by 7.
c. $\frac{3}{5} = \frac{c}{9}$	$\frac{3}{5} \times 9 = c$ or $\frac{3 \times 9}{5} = c$	Multiply both sides by 9.
d. $\frac{c}{4} = \frac{2}{7}$	$c = \frac{4 \times 2}{7}$ or $c = \frac{2}{7} \times 4$	Multiply both sides by 4.

e. What are the similarities in Parts a–d?

Answers may vary. Sample answer: I always multiplied both sides of the equation by denominator of the term containing the variable

ACTIVITY 4.1 *Continued*

30 (*Continued*) **Graphic Organizer** (c, d) By reasoning that 15 plates cost \$3.39, students use prior knowledge to determine that 1 plate would cost $\frac{3.39}{15} = 0.226$, or about \$0.23, giving \$0.23 per plate, or \$0.23/plate. This is a good time to review rounding money. Remind students to round to the nearest cent when finding unit price since they are working with money.

The purpose of these questions is to lead students to understanding the algorithm for solving proportions.

31 **Think Aloud, Simplify the Problem** (a–d), **Quickwrite** (c), **Look for a Pattern** (e) Students use prior knowledge to solve for c in simple equations. They simply undo operations as learned in Unit 3. Students SHOULD NOT use the cross-multiply and divide algorithm in Parts c and d. That is what they are exploring. In order to apply what they learn in these problems to the algorithm, students are asked not to simplify the terms.

Paragraph Look for a Pattern, Discussion Group Students study three worked-out solutions to proportions and find that they can multiply cross terms, and then divide by the other term, which is diagonally across from the *a*. This is sometimes called: cross multiply and divide.

32 Look for a Pattern (c), Quickwrite (c, d), Discussion Group Encourage discussion of the solutions so that you can determine whether students understand and can apply the algorithm.

33 Quickwrite, Discussion Group This problem allows students to simplify the terms to find the total cost of the 24 plates. They find that their answer is not the same as when they used a unit rate in Question 30d, because in Question 30d students rounded 0.226 to \$0.23, and then multiplied it by 24. Using the algorithm in this question, students did not round. If students are unable to determine this reason, use guided questioning to help them figure it out, but do not tell them the reason. By having students go back to Question 30d and use 0.226 × 24 they will see that their answer now matches this question. They should not change their answer to Question 30d. Explain that they have compared the answers to verify that the processes are the same, and it is only rounding that may make the final answers different.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Quickwrite, Discussion Group

My Notes

MATH TERMS

An **algorithm** is a mathematical rule or procedure for carrying out a computation.

You can also use an **algorithm** to solve a proportion. Look at the following worked-out proportions.

Proportion 1: $\frac{2}{5} = \frac{a}{7}$	Steps:
$(7 \times 5)\frac{2}{5} = (7 \times 5)\frac{a}{7}$	Multiply both sides by 7 × 5.
$7 \times 2 = 5 \times a$	Simplify.
$\frac{7 \times 2}{5} = \frac{5 \times a}{5}$	Divide both sides by 5.
$\frac{7 \times 2}{5} = a$	Simplify.
Proportion 2: $\frac{3}{11} = \frac{a}{2}$	Steps:
$3 \times 2 = 11 \times a$	Multiply both sides of the equation by 11 × 2.
$\frac{3 \times 2}{11} = a$	Divide both sides of the equation by 11.
Proportion 3: $\frac{a}{9} = \frac{5}{8}$	Steps:
$a \times 8 = 9 \times 5$	Cross-multiply the fractions.
$a = \frac{9 \times 5}{8}$	Divide to isolate the variable.

32. What patterns do you see in the solutions to these proportions?

Answers may vary. Sample answer: Multiplying both sides of the equation by the product of the denominators is the same as multiplying the first numerator by the second denominator and the second numerator by the first denominator.

The process shown in solving these proportions is the cross products algorithm for solving proportions.

33. In question 31, you wanted to solve $\frac{\$3.39}{15\ plates} = \frac{c}{24\ plates}$.

a. Use the cross products algorithm to find the cost of 24 plates. \$5.42

b. How does this compare to the answer when you used the unit price in Question 30d?

Answers may vary. Sample answer: With the algorithm I got \$5.42 and using the unit price I got \$5.52.

The answer to Question 33b will vary depending on whether the students used a rounded value to answer Question 30d. If students did not round, both methods give the same answer

MINI-LESSON: Understanding the Algorithm

If students struggle with seeing patterns as they undo operations, help them to see relationships using equivalent fractions, similar to the algorithm for comparing fractions discovered in Unit 1.

To solve for c: $\frac{3.39}{15} = \frac{c}{24}$

$\frac{\quad}{360} = \frac{3.39}{15} = \frac{c}{24} = \frac{\quad}{360}$ $\qquad \frac{3.39 \times 24}{360} = \frac{3.39}{15} = \frac{c}{24} = \frac{c \times 15}{360}$

Considering only the numerators now that the denominators are common:

$3.39 \times 24 = c \times 15$, so $\frac{3.39 \times 24}{15} = c$, which is the same as cross-multiplying and dividing.

SUGGESTED LEARNING STRATEGIES: Create Representations, Think/Pair/Share

34. The second rate at *Local Grocer* is $6.39/24 plates.

a. Find the unit price for the rate of $6.39/24 plates.
0.26625 is about $0.27.

b. Which is the better rate at *Local Grocer*? Explain.
Answers may vary. Sample answer: $3.39/15 plates is the better deal; $0.23 is less than $0.27.

35. The students decide to buy the packages of plates priced $3.39/15 plates. They set up the proportion $\frac{3.39}{15} = \frac{c}{30}$.

a. What do the numbers and the variable in the proportion represent?
$3.39 is the cost of 15 plates; *c* is the cost of 30 plates.

b. Solve the proportion for *c*. $c = \$6.78$

36. The students want to know what a pack of 26 plates would cost if the unit price were the same as 15 plates for $3.39.

a. Use the unit price to find the cost of a pack of 26 plates at that rate.
$\frac{3.39}{15 \text{ plates}} = \frac{\$0.23}{1 \text{ plate}}; \frac{\$0.23}{1 \text{ plate}} = \frac{\$5.98}{26 \text{ plates}}$

b. Write a proportion to find the cost of a pack of 26 plates at that rate. Let *c* represent the cost.
$\frac{3.39}{15} = \frac{c}{26}$

c. Use the algorithm to determine the total cost.
$c = 5.876$, about $5.88

d. Did you get the same answers for Parts a and c? Explain.
Answers may vary. Sample answer: No; I rounded 0.226 to 0.23 in Part a so I got different answers.

My Notes

Answer to Question 36b if students do not round the unit rate:
$\frac{3.39}{15 \text{ plates}} = \frac{\$0.226}{1 \text{plate}}$,
$\frac{\$0.226}{1 \text{ plate}} = \frac{\$5.88}{26 \text{ plates}}$

The answer to Question 36d will vary depending on whether the students used a rounded value to answer Question 36a. If students did not round, both methods give the same answer.

ACTIVITY 4.1 *Continued*

34-35 Think/Pair/Share, Create Representations (35b)
Question 34 brings students back to the context of the problem, giving them the opportunity to practice finding unit price using the other rate at Local Grocer.

36 Create Representations, Think/Pair/Share This question gives students practice solving a missing-value rate problem that cannot be easily solved by finding an equivalent fraction. Students see again that these solutions are not the same due to rounding.

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37 Think/Pair/Share This question brings the pieces of the activity back together. They have now decided on the best deals for each purchase, and must find the cost per person.

Activities 1–6 Create Representations (1–6), Group Presentation (1–6) Students apply all the strategies they have learned for working with rates in order to find their personal rates for the activities they sampled at the start of the unit. They should use the data they collected during the introduction.

SUGGESTED LEARNING STRATEGIES: Summarize/Paraphrase/Retell, Quickwrite, Discussion Group, Create Representations, Think/Pair/Share, Group Presentation

My Notes

Because rates will vary widely due to individual differences, no sample rates or answers are provided.

37. The students figure the party is going to cost $77.71 If all 26 people are going to share the cost equally, how much do they each need to contribute?

about $2.99 each

Now that you know about rates and unit rates, use what you have learned and your results from the first page of activities to find your personal rates for each of the tasks!

38. For each activity, use your data to set up a proportion. Next use one of the three methods you have learned to solve the proportion. Then enter your rate for that activity. As you do this, use each method at least once.

Activity 1: Writing Speed

a. How many times can you write *math* in 1 minute with your dominant hand?

- Set up a proportion:
- Solve it:
- Express your result using a complete sentence.

b. What is your rate per minute with your non-dominant hand?

c. About how many times faster are you with your dominant hand?

Activity 2: Reading Speed
If there are about 250 words on a page, how many pages could you read in an hour based on the data you collected earlier?

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SUGGESTED LEARNING STRATEGIES: Create Representations, Group Presentation

My Notes

Activity 3: Heart Rate
Heart rate is calculated in beats per minute. Express your heart rate in beats per minute.

Activity 4: Jumping Jacks
How many jumping jacks can you do in 90 seconds?

Activity 5: Are You Tongue Twisted?
About how many times can you say, "Peter Piper picked a peck of pickled peppers" in a minute?

Activity 6: Are You a Fast Walker?
How many feet can you walk in an hour? How many *miles* per hour can you walk? (There are 5,280 ft in a mile.)

Summary: How Do You Solve Proportions?
Look back at the ways you used to find the rates for the six activities. Find the method you used most often and explain why you chose this method.

TEACHER TO TEACHER As a possible follow-up activity, allow students to collect class data on what kind of pizza they eat and how many slices. If possible, students may use this data to plan their own class party.

Suggested Assignment

CHECK YOUR UNDERSTANDING
p. 194, #6–8

UNIT 4 PRACTICE
p. 235, #6–8

Connect to AP

Including units when computing provides an additional layer of sense-making when solving problems. For example, when finding a total distance traveled, the result should be units of length.

How far did Bob travel in two hours if he averaged 60 miles per hour?

The units in this result make sense.

$60\frac{\text{mi}}{\text{hr}} \cdot 2 \text{ hr} = 120 \text{ mi}$

The units in this result do not.

$60\frac{\text{mi}}{\text{hr}} \div 2 \text{ hr} = 30 \frac{\text{mi}}{\text{hr}}/\text{hr}$

In AP mathematics, it is always important to pay attention to the units in a problem. Leaving units off an answer or giving incorrect units in an answer can cost students a valuable point on an AP examination.

ACTIVITY 4.1 *Continued*

CHECK YOUR UNDERSTANDING

1. 15 to 12, 15:12, $\frac{15}{12}$

2a. $\frac{310\text{ heartbeats}}{5\text{ minutes}}$ or $\frac{62\text{ heartbeats}}{1\text{ minute}}$

b. $\frac{\$68}{8\text{ hours}}$ or $\frac{\$8.50}{\text{hour}}$

c. $\frac{40\text{ hours}}{5\text{ days}}$ or $\frac{8\text{ hours}}{1\text{ day}}$

3a. $\frac{14\text{ yellow snacks}}{24\text{ blue snacks}}$

b. $\frac{13}{100} = \frac{65}{500}$; 65

4a. $\frac{4\text{ scoops}}{1\text{ gallon}} = \frac{12\text{ scoops}}{n\text{ gallons}}$; 3 gallons

b. It will not taste as strong.

5a. $\frac{520\text{ miles}}{8\text{ hours}} = \frac{n\text{ miles}}{1\text{ hour}}$; 65 m/h

b. Answers may vary. Sample answer: $r \times t = d$ would be $65\,\frac{m}{h} \times 8$ hours which give the 520 miles.

6. about 36 hours

7. Explanations may vary. Sample answer: 900 minutes for $89.99; the unit rate for 450 minutes for $69.99 is about $0.16 per minute, but the unit rate for 900 minutes for $89.99 is about $0.10/minute.

8. Answers may vary. Sample answer: Undoing operations or using equivalent fractions are both very much like cross-multiplying and dividing. For example, to solve the proportion: $\frac{2}{5} = \frac{x}{6}$:

   By undoing operations: $\frac{2}{5} = \frac{x}{6}$, $\frac{2 \times 6}{5} = x$ (multiply both sides by 6.

   With equivalent fractions: $\frac{2 \times 6}{5 \times 6} = \frac{2}{5} = \frac{x}{6} = \frac{x \times 5}{6 \times 5}$, so $2 \times 6 = x \times 5$, and $\frac{2 \times 6}{5} = x$

   By cross-multiplying and dividing: $\frac{2 \times 6}{5} = x$

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

1. Write a ratio in three different ways to represent the number of boys to the number of girls in the class.

Girls	Boys
12	15

2. Write a ratio for each situation.
   a. 310 heartbeats per 5 minutes
   b. $68 for 8 hours of work
   c. Work 40 hours in 5 days

3. A recent study shows that out of 100 pieces of a popular multicolored snack, there will usually be the following number of pieces of each color.

Brown	Yellow	Red	Blue	Orange	Green
13	14	13	24	20	16

   a. The numbers for two colors form a ratio that is equal to $\frac{7}{12}$. What are the colors? What is their ratio?
   b. If there were 500 pieces, about how many would be red?

4. Kate made lemonade using a powder mix. She used 4 scoops of mix to a gallon of water.
   a. Write a proportion to determine the amount of water to mix with 12 scoops of mix.
   b. If Kate mixes less lemonade powder with more water, how will her mixture be affected?

5. Jaden travels 520 miles in 8 hours.
   a. Use a proportion to find his average rate per hour.
   b. Show why the formula $d = rt$ is actually a rate problem.

6. It is about 2508 miles from Orange County in California to Orange County in Florida. With an average speed of 70 miles per hour, about how long will it take to drive from one to the other?

7. Which is the better cell phone deal if you consider only cost per minute: 450 minutes for $69.99 or 900 minutes for $89.99? Show how you know.

8. MATHEMATICAL REFLECTION Why does the cross-products method work when solving a proportion? Use an example to explain your reasoning.