Section 01I, MTWRF 10:05 – 11:30 p. m.

Signature: ____________________ Name: ______________________

ID number: ______________

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ENTER YOUR NAME AND FORM 2 (FM2) IN THE SCANTRON SHEET

DO QUESTION [I] WHICH CONSISTS OF

15 MULTIPLE CHOICE PROBLEMS FOR A TOTAL OF 75 POINTS

DO ONE OF THE TWO LONG PROBLEMS PROBLEMS [II], OR [III]

IF YOU DO BOTH LONG PROBLEMS, THE LAST ONE WILL BE IGNORED

CROSS OUT THE BOX 2 OR 3 OF THE PROBLEM YOU OMIT

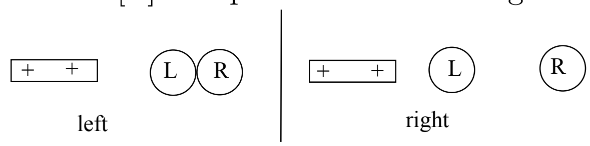
Your signature signifies that you will obey the HONOR CODE
You may be asked to show your photo ID during the exam.

THERE IS A SEPARATE FORMULA SHEET

[I] This problem has five multiple choice questions. The best answers are marked with [X].

[I.1] L and R are two uncharged (neutral) metal spheres on an insulating wood table. The spheres are in contact with each other. A positively charged rod R is brought close to L as shown in the figure to the left below. Sphere R is now moved away from L, as in the figure to the right below. What are the final charge states of L and R?

[A] Both L and R are neutral. [X] L is negative and R is positive.
[C] Both L and R are negative.
[D] Both L and R are positive. [E] L is positive and R is negative.

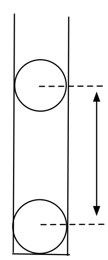


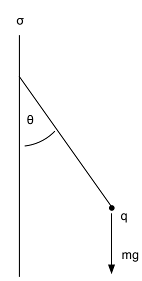
[I.2] The charge on the square plates of a parallel plate capacitor of area A and separation d is Q. The potential across the plates is maintained with constant voltage by a battery as they are pulled apart to twice their original separation.The amount of charge on the plates is now equal to

[A] 4Q. [B] 2Q. [C] Q. [X] Q/2. [E] Q/4.

[I.3] One very small uniformly charged plastic ball is located directly above another such charge in a test tube as shown in the figure to the left below. The balls of mass m are in equilibrium a distance d apart because of the weight mg of the balls, and the electrical forces. If the charge on each ball is doubled, but the masses remain the same, the distance between the balls in the test tube would become

[X] $2d$. [B] d. [C] $4d$. [D] $8d$. [E] $\sqrt{2d}$.





[I.4] A charge q hangs from a string that makes an angle θ with a large sheet with surface charge density σ as shown in the figure to the right above. At equilibrium

[X] $\tan\theta = \frac{\sigma q}{2\epsilon_0 mg}$ [B] $\tan\theta = \frac{2\epsilon_0 mg}{\sigma q}$ [C] $\tan\theta = \frac{\sigma q}{\epsilon_0 mg}$ [D] $\tan\theta = \frac{\epsilon_0 mg}{\sigma q}$
[E] The length of the string is needed to give the answer

[I.5] A charge q is at $x = 0$ and a charge $4q$ is at $x = L$. Find where on the x axis the electric field $\vec{E} = 0$.

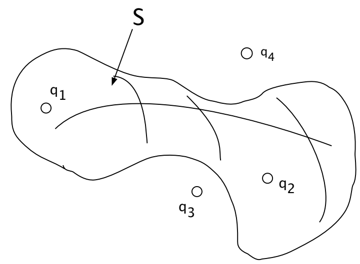
[A] At $x = \frac{L}{2}$ [B] At $x = \frac{L}{3}$, and $x = -L$ [C] At $x = -L$ [X] At $x = \frac{L}{3}$ [E] At $x = \frac{4L}{3}$

[I.6] The figure to the below shows four charges, q_1, q_2, q_3, and q_4. S is a Gaussian surface (closed surface) with q_1 and q_2 inside. The other charges are not inside S. Consider Gauss' law and choose the true statement.

[A] Charges q_3 and q_4 outside the surface do not affect $\mathbf{E}$ on the surface.

[B] $\oint_S \mathbf{E}\cdot\mathbf{n}\,dA = \frac{q_1+q_2-q_3-q_4}{\epsilon_0}$ [X] $\oint_S \mathbf{E}\cdot\mathbf{n}\,dA = \frac{q_1+q_2}{\epsilon_0}$ [D] $\oint_S \mathbf{E}\cdot\mathbf{n}\,dA = \frac{q_1+q_2+q_3+q_4}{\epsilon_0}$

[E] none of the above because the figure is not symmetrical.



[I.7] Two very large parallel plastic plates of area A and thickness t separated by a distance d have uniformly distributed charge densities $\sigma_1, \sigma_2, \sigma_3$ and σ_4 as shown in the figures below Treat the area A as very large (∞). The electric field components E_x in regions I and III is

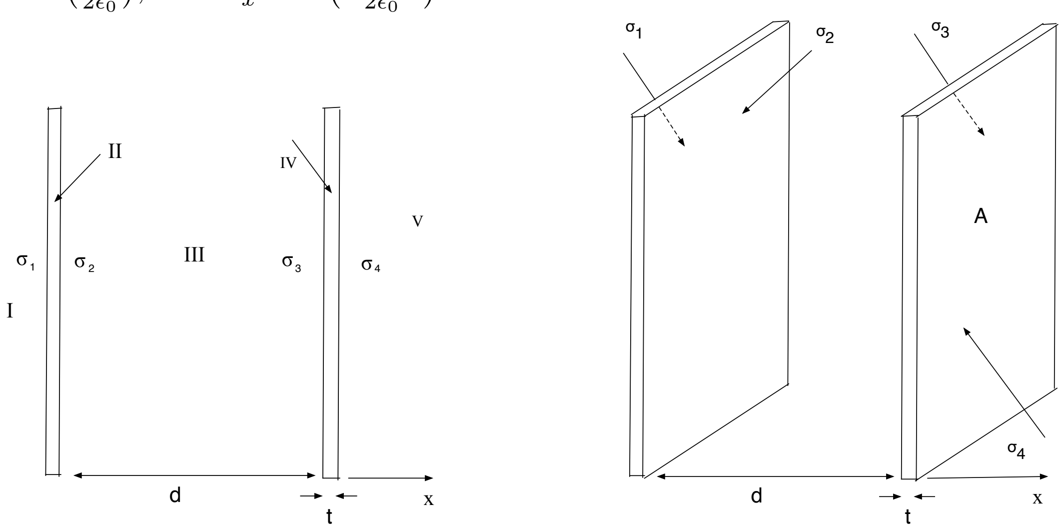
[A] $E_x^I = \left(\frac{\sigma_1+\sigma_2+\sigma_3+\sigma_4}{2\epsilon_0}\right)$, and $E_x^{III} = -\left(\frac{\sigma_1+\sigma_2+\sigma_3+\sigma_4}{2\epsilon_0}\right)$

[B] $E_x^I = \left(\frac{\sigma_1+\sigma_2+\sigma_3+\sigma_4}{2\epsilon_0}\right)$, and $E_x^{III} = -\left(\frac{\sigma_1+\sigma_2-\sigma_3-\sigma_4}{2\epsilon_0}\right)$

[C] $E_x^I = \left(\frac{\sigma_1+\sigma_2-\sigma_3-\sigma_4}{2\epsilon_0}\right)$, and $E_x^{III} = -\left(\frac{\sigma_1+\sigma_2-\sigma_3-\sigma_4}{2\epsilon_0}\right)$

[X] $E_x^I = -\left(\frac{\sigma_1+\sigma_2+\sigma_3+\sigma_4}{2\epsilon_0}\right)$, and $E_x^{III} = \left(\frac{\sigma_1+\sigma_2-\sigma_3-\sigma_4}{2\epsilon_0}\right)$

[E] $E_x^I = -\left(\frac{\sigma_1}{2\epsilon_0}\right)$, and $E_x^{III} = \left(\frac{\sigma_2-\sigma_3}{2\epsilon_0}\right)$

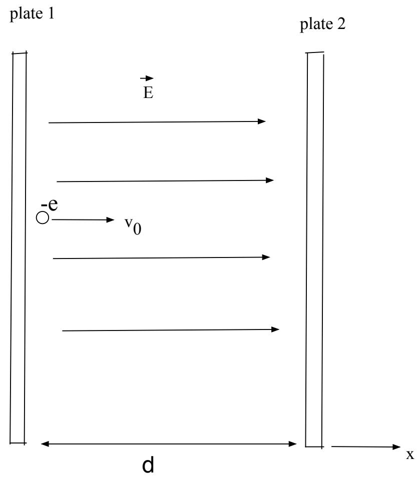


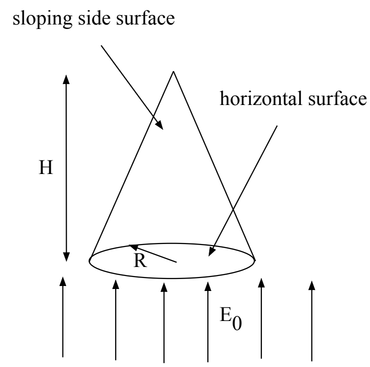
[I.8] A particle of charge q is fixed at the origin so it cannot move. A second particle of charge q and mass m is initially at $x = a$ and released from rest. Find the speed v of the second particle, for $v \ll c$, when it has moved to a very large ($x \to \infty$) distance.

[X] $v \to \sqrt{\frac{q^2}{2\pi\epsilon_0 am}}$ [B] $v \to \sqrt{\frac{q}{2\pi\epsilon_0 am}}$ [C] $v \to 0$ [D] $v \to \frac{q^2}{4\pi\epsilon_0 a^2 m}$ [E] $v \to \sqrt{\frac{q^2}{2\pi\epsilon_0 a^2 m}}$

[I.9] Two large charged parallel metal plates are separated by a distance d as shown in the figure to the left below. The electric field between the plates is $\vec{\mathbf{E}} = E\hat{\mathbf{x}}$ with $E > 0$. An electron of mass m and charge $-e$ is released at plate 1 with speed v_0 as shown in the figure to the left below. When it reaches plate 2 its speed neglecting gravity will be

[X] $\sqrt{v_0^2 - \frac{2eEd}{m}}$ [B] $\sqrt{v_0^2 + \frac{2eE}{md}}$ [C] $\sqrt{v_0^2 - \frac{2eE}{md}}$ [D] $\sqrt{v_0^2 + \frac{2eEd}{m}}$ [E] v_0





[I.10] A cone has a circular horizontal base of radius R and height H as shown in the figure to the right above. It is immersed in a uniform vertical electric filed of magnitude E_0. The electric filed flux through the sloping side surface is

[A] zero [X] $E_0(\pi R^2)$ [C] $E_0(\pi r\sqrt{H^2 + R^2})$ [D] $E_0(\pi R^2 + \pi r\sqrt{H^2 + R^2})$ [E] $E_0(RH)$

[I.11] A uniform electric field is directed along the x-axis from right to left, so $E_x = -500$ V/m. If the electric potential at $x = 0$ m is $V(x = 0) = 0$ V, what is the electric potential at $x = 2$ m?

[A] 2000 V [X] 1000 V [C] 500 V [D] 0 V [E] -1000 V

[I.12] The electric potential is given as $V(x, y, z) = x^2y - z^3$. Find the electric field.

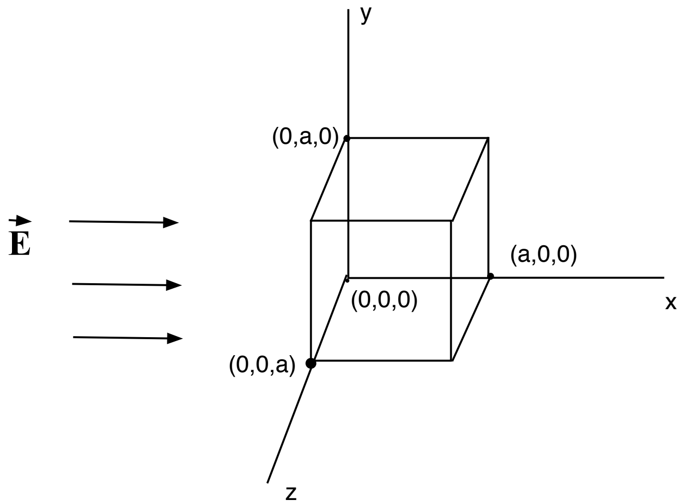
[X] $\vec{E} = -2xy\hat{\imath} - x^2\hat{\jmath} + 3z^2\hat{k}$ [B] $\vec{E} = \frac{x^3y}{3}\hat{\imath} + \frac{x^2y^2}{2}\hat{\jmath} - \frac{z^4}{4}\hat{k}$ [C] $\vec{E} = \frac{-x^3y}{3}\hat{\imath} - \frac{x^2y^2}{2}\hat{\jmath} + \frac{z^4}{4}\hat{k}$

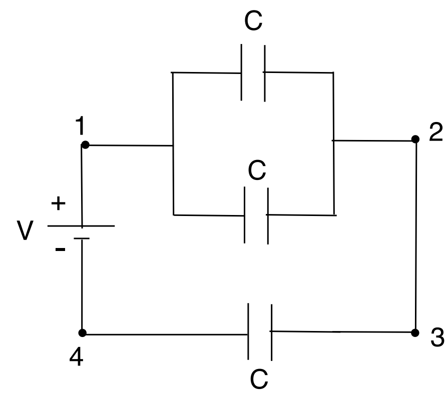
[D] $\vec{E} = -xy\hat{\imath} - x^2\hat{\jmath} + z^2\hat{k}$ [E] $\vec{E} = 2xy\hat{\imath} + x^2\hat{\jmath} - 3z^2\hat{k}$

[I.13] An electric field in space is given by $\vec{\mathbf{E}} = (E_0 + E_1 \frac{x}{a})\hat{\mathbf{x}}$. Calculate the outward electric flux Φ_E through the surface of cube shown in the figure to the left below.

[A] $\Phi_E = (E_0 + E_1)a^2$ [B] $\Phi_E = E_0 a^2$ [C] $\Phi_E = (2E_0 + E_1)a^2$ [X] $\Phi_E = E_1 a^2$

[E] $\Phi_E = 0$





[I.14] In the circuit shown to the right the three capacitors are equal. The voltages $V_1 - V_2$ and $V_3 - V_4$ are

[A] $V_1 - V_2 = \frac{V}{2}$, and $V_3 - V_4 = \frac{V}{2}$ [B] $V_1 - V_2 = \frac{V}{2}$, and $V_3 - V_4 = \frac{-V}{2}$

[X] $V_1 - V_2 = \frac{V}{3}$, and $V_3 - V_4 = \frac{2V}{3}$ [D] $V_1 - V_2 = \frac{2V}{3}$, and $V_3 - V_4 = \frac{V}{3}$

[E] $V_1 - V_2 = \frac{V}{3}$, and $V_3 - V_4 = \frac{-2V}{3}$

[I.15] When two or more capacitors are connected in series across a potential difference

[A] the potential difference across the combination is the algebraic sum of the potential differences across the individual capacitors.

[B] each capacitor carries the same amount of charge.

[C] the equivalent capacitance of the combination is less than the capacitance of any of the capacitors.

[X] All of the above choices are correct.

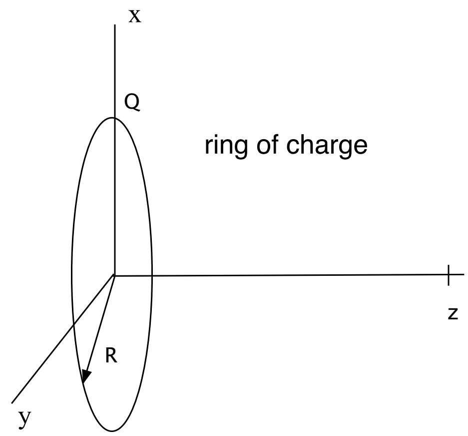
[E] [A] and [B] are correct but [C] is false.

[II] A ring of radius R has a uniformly distributed positive charge Q as shown in the figure.

[a] Find the electric potential $V(0,0,z)$ at a distance z along the axis of the ring.

[b] Find the z component $E_z(0,0,z)$ of the electric field at a distance z along the axis of the ring.

[c] Show that an electron located at a distance $z \ll R$ performs simple harmonic motion and find the frequency ω.



ANSWERS:

[a] Divide the ring into small elements of charge dQ. Every element of charge is at the same distance $r = \sqrt{z^2+R^2}$ from the point $(0,0,z)$. An element of charge produces a potential $dV(0,0,z)$, and the entire ring produces a potential $V(0,0,z)$ at that point given by

$$dV = \frac{dQ}{4\pi\epsilon_0 r}, \text{ and } V = \int_{ring} dV = \int_{ring} \frac{dQ}{4\pi\epsilon_0 r} = \int_{ring} \frac{dQ}{4\pi\epsilon_0\sqrt{z^2+R^2}}$$

but z and R are constant in the integral so

$$V(0,0,z) = \frac{1}{4\pi\epsilon_0\sqrt{z^2+R^2}} \int_{ring} dQ = \frac{Q}{4\pi\epsilon_0\sqrt{z^2+R^2}}$$

[b] We have that

$$E_z(0,0,z) = -\frac{\partial V(0,0,z)}{\partial z} = \frac{-Q}{4\pi\epsilon_0}\frac{\partial}{\partial z}\left(\frac{1}{\sqrt{z^2+R^2}}\right) = \frac{-Q}{4\pi\epsilon_0}\left(\frac{(-1/2)2z}{(z^2+R^2)^{3/2}}\right) = \frac{Qz}{4\pi\epsilon_0}\left(\frac{1}{(z^2+R^2)^{3/2}}\right)$$

[c] The force on an electron of charge $-e$ is $F_x = -eE_z$. For $z \ll R$ we have that

$$E_z \approx \frac{Qz}{4\pi\epsilon_0 R^3}, \text{ so } F_z = -\frac{Qe}{4\pi\epsilon_0 R^3} z = -Kz,$$

therefore the electron, of mass m_e performs simple harmonic motion with frequency

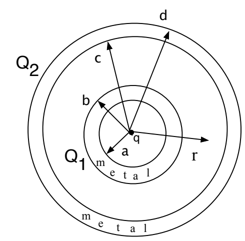
$$\omega = \sqrt{\frac{K}{m_e}} = \sqrt{\frac{Qe}{4\pi\epsilon_0 R^3 m_e}}$$

[III] A conducting (metal) spherical shell of inner and outer radii a and b respectively is inside another conducting concentric shell of inner and outer radii c and d as shown. There is vacuum between the shells and in the cavity inside the small shell. The inner shell carries no charge $Q_1 = 0$ and the outer one a charge of $Q_2 = -2q$. There is also a point charge q at the center of the cavity inside the small shell.

[a] Find the electrostatic electric field for $r < a, a < r < b, b < r < c, c < r < d$, and $d < r$.

[b] Find the surface charge densities σ at the four surfaces $r = a, r = b, r = c$, and $r = d$..

[c] Integrate the electric field to find the potential difference $V(r = b) - V(r = c)$.



Answers:

[a]

$$E_r(r) = \begin{cases} \frac{q}{4\pi\epsilon_0 r^2} & \text{if } r \leq a \text{ because } q_{\text{inside}} = q; \\ 0 & \text{if } a < r < b \text{ because } \vec{E} = 0 \text{ inside conductor;} \\ \frac{(Q_1+q)}{4\pi\epsilon_0 r^2} = \frac{q}{4\pi\epsilon_0 r^2} & \text{if } b < r \leq c \text{ because } q_{\text{inside}} = q; \\ 0 & \text{if } c < r \leq c \text{ because } \vec{E} = 0 \text{ inside conductor;} \\ \frac{(Q_1+Q_2+q)}{4\pi\epsilon_0 r^2} = \frac{-q}{4\pi\epsilon_0 r^2} & \text{if } d < r \text{ because } q_{\text{inside}} = -q; \end{cases}$$

[b] At the surface of a metal $\sigma = \epsilon_0 E_n$, where $\hat{n}$ is the normal outward from the metal.

$$\sigma = \begin{cases} \frac{-q}{4\pi a^2} & \text{at the surface } r = a \text{ because } E_n(a) = -E_r(a) = \frac{-q}{4\pi\epsilon_0 a^2}. \\ \frac{q}{4\pi b^2} & \text{at the surface } r = b \text{ because } E_n(b) = E_r(b) = \frac{q}{4\pi\epsilon_0 b^2}. \\ \frac{-q}{4\pi c^2} & \text{at the surface } r = c \text{ because } E_n = -E_r(c) = \frac{-q}{4\pi\epsilon_0 c^2}. \\ \frac{-q}{4\pi d^2} & \text{at the surface } r = d \text{ because } E_n = E_r(d) = \frac{-q}{4\pi\epsilon_0 d^2}. \end{cases}$$

[c]

$$V = V(b) - V(c) = \int_b^c \vec{E} \cdot d\ell = \int_b^c E_r dr = \int_b^c \frac{q}{4\pi\epsilon_0 r^2} dr = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{b} - \frac{1}{c}\right)$$

FORMULAE SHEET TEST I

$$g = 10\,\mathrm{m/s^2},\ e = 1.6\times 10^{-19}\ C$$
$$m_e = 9.1\times 10^{-31}\ kg,\ m_p = 1.67\times 10^{-27}\ kg,\ k = \frac{1}{4\pi\epsilon_0} = 9\times 10^9\,\frac{N\cdot m^2}{C^2}.$$

Coulomb's Law, Electric Fields (N/C)

$$\left|\vec{F}\right| = \frac{\left|q_1 q_2\right|}{4\pi\epsilon_0\, r^2} \quad \vec{F}_q = q\vec{E}, \quad \vec{E}_Q = \frac{kQ}{r^2}\hat{r},$$

Electric Field Flux Φ, and Gauss' Law

$$\Phi_{\text{closed surface}} = \oint_S \vec{E}\cdot\hat{n}\,dA = \frac{q_{\text{inside}}}{\epsilon_0}$$
$$\text{with}\quad q_{\text{inside}} = \int_V \rho\,dV, \quad\text{or}\quad q_{\text{inside}} = \int_A \sigma\,dA, \quad\text{or}\quad q_{\text{inside}} = \int_L \lambda\,dl$$
$$\text{Spherical symmetry}: E_r = \frac{q_{inside}}{4\pi\epsilon_o r^2}, \quad \text{Cylindrical symmetry}: E_r = \frac{\lambda_{inside}}{2\pi\epsilon_0 r}$$

Electric Field and sheets of surface charge density σ:
On each side of an infinite sheet $E_n = \frac{\sigma}{2\epsilon_0}$. However, outside the surface of a conductor, $E_n = \frac{\sigma}{\epsilon_0}$.

Force, Potential Energy and Torque for an Electric Dipole
The Force, torque, and potential energy of an electric dipole $\vec{p}$ immersed in uniform electric field $\vec{E}$ are $\vec{F}_{total} = 0$, $U = -\vec{p}\cdot\vec{E}$, $\vec{\tau} = \vec{p}\times\vec{E}$.

Harmonic Oscillator: $F_x = -kx$**,** $\frac{d^2x}{dt^2} + \omega^2 x = 0$**,** $\omega = \sqrt{k/m}$

Electric Potential and Energy: $W^{agent}_{i\to f} = (K+U)_f - (K+U)_i$, $W^{field}_{i\to f} = U_i - U_f$

$$V_Q(r) = \frac{Q}{4\pi\epsilon_0 r}, \quad U_q(\vec{r}) = qV(\vec{r}), \quad V_i - V_f = \int_i^f \vec{E}\cdot d\vec{s}, \quad \text{when E field is uniform } V = Ed$$
$$\vec{E} = -\frac{\partial V}{\partial x}\hat{\imath} - \frac{\partial V}{\partial y}\hat{\jmath} - \frac{\partial V}{\partial z}\hat{k}, \quad E_r = -\frac{\partial V}{\partial r} \quad \text{when E field is uniform } E = \frac{V}{d}$$

Capacitance $Q = CV$, Energy in capacitor $U = \frac{CV^2}{2}$

$$C = \frac{\kappa\epsilon_o A}{d}, \quad \text{series} \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}, \quad \text{parallel} \quad C_{eq} = C_1 + C_2$$