KING FAHD UNIVERSITY OF PETROLEUM AND MINERAL

Department of Mathematical Sciences

Test No. I MATH - 521 Sem 111

Student #:_____________ Name:____________________

Show All Your Work. No Credits for Answers Not Supported by Work.

In this exam the symbols $\Im_F$, $\Im_C$, and $\Im_D$ will denote the topology of finite complement, the topology of countable complement and the discrete topology respectively.

Q1) (14 Points) Define each of the following:

a. Numerically equivalent sets

b. Topologically equivalent spaces

c. A basis for a topological space

d. Dense subset

e. Frontier of a set

f. Housdorff space

g. Normal space

Q2) (8 Points) Let A and B be two subsets of a set X and let $f : X \to X$ be a function. Complete each of the following:

a. $f^{-1}(A \cap B)$ [.........] $f^{-1}(A) \cap f^{-1}(B)$

b. $f^{-1}(A - B)$ [.........] $f^{-1}(A) - f^{-1}(B)$

c. $f(f^{-1}(A))$ [.........] A

d. $f^{-1}(f(A))$ [.........] A

Q3) (12 Points) Consider the space $(\mathbb{Z}_+, \Im_F)$. Let $A = \{0,1\}$ and $B = \{3n : n \in \mathbb{Z}_+\}$.

a. Is A $\Im_F$- open?

b. Is A $\Im_F$- closed?

c. Find A°

d. Find $\overline{A}$

e. Find $fr(A)$

f. Find A'

g. Is B $\Im_F$-open?

h. Is B $\Im_F$- closed?

i. Find B°

j. Find $\overline{B}$

k. Find $fr(B)$

l. Find B'

Q4) (8 Points) Let X be a set and consider the topologies $\Im_F$, $\Im_C$, and $\Im_D$ for X.

a. How are $\Im_C$ and $\Im_F$ related, if at all?

b. How are $\Im_C$ and $\Im_D$ related, if at all?

c. If $\Im_C = \Im_D$ what must be true about X?

d. If $\Im_C = \Im_F$ what must be true about X?

Q5) (8 Points) True or false. Tick as true (‖) or false (X):

a. Any two countable sets are equivalent.

b. If A is countably infinite subset of an uncountable set B, then $\boldsymbol{B \sim B\text{-}A}$

c. Every subset of a topological space is either open or closed.

d. The set of all open rays is a basis for the usual topology for $\mathbb{R}$.

e. The set of all open intervals is a subbasis for the usual topology for $\mathbb{R}$.

f. The boundary of any set is closed.

g. The set $\cup\{[1/n,\ n] \mid n \in \mathbb{Z}_+\}$ is closed set in $\mathbb{R}$ with the usual topology.

h. Each boundary point of A is a limit point of a set A.

Q6) (8 Points) Let $(X, \Im)$ be a topological space and let $A \subseteq Y \subseteq X$.

a. Briefly describe how $\Im_{rel\,Y}$ is defined.

b. How are $\overline{A}$ and $\overline{A}_{rel\,Y}$ related?

c. How are A° and $A^{\circ}_{rel\,Y}$ related?

d. How are $fr(A)$ and $fr(A)_{rel\,Y}$ related?

Q7) (8 Points) Consider $\mathbb{R}^2$ with the usual topology. Consider the subsets $A = \{(x,0) \mid -2 \leq x \leq 2\}$ and $B = \{(x,y) \mid x^2 + y^2 \geq 2\}$.

(a) Find A°

(b) Find B°

(c) Find $fr(A)$

(d) Find $fr(B)$

(e) Find A'

(f) Find B'

(g) Find $\overline{A} \cap \overline{B}$

Q8) (6 Points) Let X be uncountable set and consider the topologies $\Im_F$, $\Im_C$, and $\Im_D$ for X. Circle the property which the space has.

a. $(X, \Im_F)$ is T_0 T_1 T_2 *regular* *normal* T_3 T_4

b. $(X, \Im_C)$ is T_0 T_1 T_2 *regular* *normal* T_3 T_4

c. $(X, \Im_D)$ is T_0 T_1 T_2 *regular* *normal* T_3 T_4

Q9) (10 Points) Let $(X, \Im)$ be a topological space and let $A \subseteq X$. Prove or disprove each of the following:

a. $X - \overline{A} = (X - A)^{\circ}$

b. $A^{\circ} = \left(\overline{A}\right)^{\circ}$

Q10) (10 Points) Let X be a nonempty set and let $\{X_i\}$ be a class of topological spaces. Assume that we have a set of functions $\{f_i : X \rightarrow X_i\}$. The smallest topology on X that will make each f_i continuous is called the **weak topology for *X* generated by the** f_i's.

Assume that $X_i = \mathbb{R}$ with the usual topology $\Im_u$ for each *i*, and consider the set $\{f_i : \mathbb{R} \rightarrow X_i\}$. Describe the weak topology generated by the f_i's, where

a. $\{f_i\}$ is the set of all constant functions.

b. $\{f_i\}$ is the set of functions each of which is equal to $f(x) = \begin{cases} 0 & if \;\; x \leq 0 \\ 1 & if \;\; x > 0 \end{cases}$.

Q11) (18 Points) Let $(X, \Im)$ be a topological space and let $A \subseteq X$. Prove each of the following:

a. The set A is dense in X if its complement has empty interior.

b. If the set A has empty frontier, then it is both open and closed.