Student ID:			Signature:	
Full Name:				

Midterm

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University of Toronto

CSC418H
Computer Graphics

Fall 2014

Instructor: Luke Moore

Wednesday, October 22, 2014
Duration: 60 minutes

This is a closed book exam (no calculators, PDAs, laptops or other examination aids)

Note: It is a violation of the University's Code of Behaviour on Academic Matters to cheat on an examination. Cheating includes copying another student's work, possessing unauthorized examination aids, having someone else take your examination, or knowingly allowing your work to be copied.

Question 1: Two transformations *f1* and *f2* commute when *f1(f2(p))* = *f2(f1(p))* for all *p*. For each pair of 2D transformations below, specify whether or not they commute. If they do commute, use matrices to show that they commute in the general case. If they do not commute, use either general matrices or give a specific example.

a) [3 marks] A uniform scale in 2D and a rotation

b) [3 marks] A non-uniform scale in 2D and a rotation

c) [3 marks] A shear and a translation

Question 2: [6 marks] Consider a triangle in 2D and a point *p* that lies inside the triangle. After the triangle and the point are transformed by a general 2D homography *H*, is *p* guaranteed to lie inside the transformed triangle? Explain your answer.

Question 3:

a) [3 marks] Give the matrix to rotate points in 3D by an arbitrary angle counter-clockwise about the *y*-axis, according to the right-hand rule.

b) [1 mark] Give the specific matrix version of part a) where the angle is 90 degrees. Simplify all values in the matrix to numbers. This matrix will rotate the *z*-axis into the *x*-axis.

c) [2 marks] Give the inverse of the matrix in part b).

d) [4 marks] Suppose that, in world space, the camera is located at the origin and facing down the negative *x*-axis, with the up direction being the positive *y*-axis. Give the transformation matrix that transforms from coordinates in camera space (where the camera is facing the negative *z*-axis) into coordinates in world space.

e) [2 marks] Using the camera to world transformation from part d), give the matrix for the world to camera transformation.

f) [4 marks] Suppose the projection matrix is given by

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Give the position in image space coordinates and the pseudo-depth of the point in world space located at *(-10, 20, -30)*.

Question 4:

a) [5 marks] Consider a section of a 2D circle defined parametrically by the following:

$$f_1(t) = (x_1(t),\ y_1(t)) = \left(t,\quad (1 - t^2)^{\frac{1}{2}}\right)$$
$$where\ 0 \leq t \leq 1$$

Compute the general formula for the tangent at an arbitrary value of *t*. Express this formula in terms of $x_1(t)$ and $y_1(t)$. Evaluate the curve and the tangents at $t = 0$ and $t = \frac{1}{\sqrt{2}}$.

b) [5 marks] Now consider the same curve defined by this second parametric function:

$$f_2(t) = (x_2(t),\ y_2(t)) = \left(cos\left(\frac{\pi}{2}(1-t)\right),\quad sin\left(\frac{\pi}{2}(1-t)\right)\right)$$
$$where\ 0 \leq t \leq 1$$

Compute the general formula for the tangent at an arbitrary value of *t*. Express this formula in terms of $x_2(t)$ and $y_2(t)$. Evaluate the curve and the tangents at $t = 0$ and $t = \frac{1}{2}$.

c) [2 marks] What is the relationship between the tangents in parts a) and b)?

Question 5:

a) [2 marks] What is a backface?

b) [3 marks] What is backface culling and what advantage does it provide? Under what conditions can it be used?

c) [2 marks] Give a formula to determine if a polygon is backfacing.