Selective Immigration Policies, Human Capital Accumulation and Migration Duration in Infinite Horizon*

by

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Abstract

An increasing literature encourages the use of selective immigration policies as a tool to promote growth in developing countries. It is argued that, since not everybody is allowed to migrate, under these policies a poor country may well turn out with more human capital than in autarky. The implicit assumption that human capital is only useful to cross the border is clearly counterfactual: wage differentials provide for strong incentives to education. Selective policies tie-up human capital accumulation and migration duration, thus we study the joint determination of these variables. Moreover, our analisys is not restricted to a single-period, voluntary migration: there is no reason to consider a single migration spell, and our model includes an aggregate shock as a source of constrained migration. Contrary to the "brain gain with a brain drain" reasoning, we find that selective policies may be harmful for human capital accumulation. As a consequence, their effectiveness is questionable, and they may produce a "brain loss" rather than a brain gain. Additionally, border closure backfires on migration duration.

Keywords: migration duration, human capital, selective policies, brain gain, brain drain.

JEL Classification: F 200, F220.

1 Introduction

Unlike flows of capital or goods, inflows of immigrants can generate frictions with natives and xenophobia, particularly when combined to high unemployment. National governments are, therefore, highly concerned about regulation of immigration.

The eastward enlargement of the European Union is going to add approximately 50 million people to the existing labor force. Large and persistent wage diffentials support the incentives for extensive mass migration from low-wage, densely populated countries, to the developed world (Lundborg and Segerstrom 2002). After these events, the governments of destination countries are urged to raise entry barriers. According to OECD (1999, 2001), in the recent years many countries have modified their entry regulations to reinforce borders control and restrict the entry, residence and work requirements. These barriers are taking the form of selective immigration policies based on human capital requirements, and they are renewing the brain drain concerns[1].

However, according to the recent "new brain drain economics" (Mountford, 1997; Stark and Wang, 2002), selective policies may benefit the origin countries if they are sufficiently severe: on the one hand, they incentivate human capital accumulation and, on the other hand, most of it is retained within the homeland.

A remarkable assumption of these papers is that only permanent emigration is considered. A well-developed literature proves that this assumption is too restrictive (for an example, see Dustmann 2003)[2].

Unfortunately, most models of temporary emigration suffer from other simplifying assumptions: the return decision is permanent[3], and emigration is only voluntary. From a theoretical perspective, it is not evident why migration should be limited to a single spell. There exists, indeed, clear evidence that even at the end of the XIX

[1] For a recent survey about the brain drain literature we refer to Commander et al. (2003).

[2] We refer to Dustmann (2003) for further references about temporary migrations. OECD (1999, 2001) dedicates as well particular attention to this subject.

[3] See for example Galor and Stark (1990), and Dustmann (1997). See Hill (1987), instead, for a model of multiple migrations.

century repeated migration spells of 3-4 years were not uncommon (Baines 1991)[4][5]. As for the importance of constrained migration, OECD (2001) emphasizes the role of regional conflicts in increasing the migratory flows. Emigration waves from Europe in the 1840s were associated with famine and revolutions. Bonifazi and Strozza (2001) describe the huge population relocation occurred after World War II. We refer to Chiswick and Hatton (2002) for more data on the effects of wars in Africa, Vietnam, Laos, Cambodia, and the disintegration of Yugoslavia. In spite of its importance, this factor is not present in current models of migration duration.

Selective entry policies tie up the decisions about human capital accumulation and migration duration. Thus, they make it necessary to account for the joint determination of these variables. Moreover, to overcome some typical drawbacks of the literature, our model does not restrict the number of migration spells and includes the possibility of constrained emigration. This is made possible by using an infinite horizon approach, and by considering a shock affecting agents within the source country.

Our findings show that selective immigration policies may also hinder human capital accumulation ("brain loss"). The intuition behind our results is that these policies make returns to human capital uncertain with respect to a policy of open migration, therefore a risk-averse agent is likely to reduce her equilibrium human capital. As a consequence, the findings of the "new brain drain economics" can easily be reversed.

Similar concerns are present in Schiff (2005), who lists many reasons to explain why the beneficial effects of the brain gain can be overestimated, and presents some empirical evidence for the prevalence of the brain drain.

Finally, our results question the consistency of restrictive immigration policies

[4] "One reason for thinking that the emigrants *intended* to remain abroad for only a relatively short period is that many made a second emigration just after returning. For example, ten per cent of the Italian immigrants into the U. S. in 1904 were entering for the second time" (Baines, 1991, p. 36); and "As transport improved, emigration became less final. [...] The changes also favoured a relatively new kind of emigrant -one who expected to return within a relatively short period" (Baines, 1991, p. 41)

[5] According to Chiswick and Hatton (2002), over the same period the outflow of returning migrants from the U.S. grew from less than 10% up to 30% of the inflow. In more recent times, similar results are reported by Byerlee (1974) for African migrants, and by Cornelius (1978) and North and Houstoun (1976) for Mexican ones.

with the objective of reducing the immigrants' stock: entry closure biases the incentives towards longer migration spells and they increase the share of permanent migrants. Though this outcome is not novel in the literature -see, among others, Faini (1996) and Kossoudji (1992)- we have derived this result under more general conditions[6].

The paper is organised as follows: next Subsection reviews some main findings in the literature. Our model is developed in Sections 2 and 3. In Section 4 we discuss our results, and a sensitivity analysis is used to illustrate our findings in Section 5. Section 6 contains a comparison of our results to those present in the literature. Conclusions are reported in Section 7. The proofs are gathered in the Appendix.

1.1 Related Literature

The "new brain drain" literature underlines the possible benefits of a brain drain.

In an OLG framework there are several mechanisms able to generate a beneficial brain drain, and they rely basically on the existence of externalities on the human capital: Vidal (1998) points to enhanced intergenerational transmission of skills and education; Mountford (1997) and Beine et al. (2001) stress the possibility of intergenerational spillovers between skilled workers.

The possibility of migration increases the expected returns to human capital, and thus the incentive to education. Stark et al. (1997) distinguish between education and ability: productivity depends on the latter. Whith asymmetric information about the worker's ability, the incentive to invest in education and migrating is even stronger for low-ability individuals; however, after their ability (productivity) is observed, they will find convenient to return. Stark and Wang (2002) use a static model to state some conditions under which a restrictive immigration policy in the *destination* country increases the welfare of the *sending* country: the idea is that entry rationing

[6] This outcome is also known by demographers. Bonifazi and Strozza (2001) consider the introduction of entry barriers in Germany after the oil shocks. After 1975, inflow was reduced, but new entries occurred mainly through family reunification. See King (1993) for similar results. Family reunification indicates that migration is becoming permanent: the costs of returning may be too high to permit an easy reversal. Currently, family reunifications account for at least one half of the *legal* inflow into the E.U. (OECD, 2001; 2004). For further references on the effect of the post-oil shocks frontier closure on family reunification we refer to Venturini (2001, p. 217-221 and the authors quoted therein).

in developed countries can keep most human capital at home.

Schiff (2005) questions the approach of the "new brain drain" literature, and argues that the benefits of a brain drain are overestimated. He gives a simple example of his idea: since in the steady state all variables are constant, this must be true for the number of educated individuals as well. Thus, after emigration, there exist no net brain gain.

This result in Schiff (2005) relies on three assumptions: 1) there exist a single unit of education: individuals choice only whether or not being educated; 2) entry abroad is based on quotas; 3) emigration is permanent. When education is treated as a continuous variable, it may well be that the number of educated individuals is constant, but the *level* of their education is higher. Moreover, selective immigration policies are increasingly based on human capital requirements, rather than on quotas. In such a case, it it is not true that the probability of entry decreases as the number of skilled workers increases. Finally, overlooking the possibility of return migration may in its turn overestimate the real extent of the brain drain.

The literature about return migration adopts mainly life-cycle models.

An early contribution to the study of migration duration it given by Djajic and Milbourne (1988). They develop a two-period model to study the effect of wage differentials in determining migration flows and their final effect on the equilibrium wages, but they are aware that more research is needed to understand why "some migrants make several trips, some stay longer than others, and some never return". Hill (1987) stresses, interestingly, the importance of "the repetitive character of contemporary labor migration"; in spite of that, his assumption of an identical duration for each migration spell can be deceptive. Temporary migration is studied in several contributions of Dustmann (for example, 1997; 2003).

2 Migration in infinite horizon

The life-cycle model is the basis of the literature surveyed in the previous Section. However, the use of only two periods hides "the repetitive character of contemporary labor migration" (Hill, 1987), and this is why the return decision is generally considered permanent. In this Section, we try to overcome this restriction by considering an infinitely lived migrant. When we use an infinite horizon model, it appears immediately that considering only one stay abroad is arbitrary.

There exist an Origin country (O) and a Destination country (D). O is populated by n potential migrants. Any native of O has to choose how long to stay abroad, and how much human capital to build.

In both countries there exists a unique consumption good produced using only capital by means of a linear technology:

$$c_t^i = k_t \qquad i = O, D \quad t = 0, 1, 2.... \tag{1}$$

For simplicity, country O is not endowed with capital, that can only be imported from D. As a consequence, the lifetime utility of a permanent resident of O would be zero. This creates the incetive to migrate, and it is equivalent to an incentive based on the wage differential.

2.1 Destination country

In D labour can be used to accumulate physical capital. More precisely, any agent supplies a unit of human-capital augmented labour per period. As a consequence, physical capital accumulation is faster for skilled individuals. This feature of our model enables us to generate very easily returns proportional to the human capital. Indeed, we are particularly concerned in describing not only the role of the human capital in crossing the border, but also its importance to get higher benefits from migration. Thus we depict the physical capital accumulation in D as follows:

$$k_{t+1} = k_t + \lambda(1 + h) \tag{2}$$

where h is human capital, and $\lambda \in (0, 1)$ defines the efficiency of the process transforming labor and human capital into physical capital. Remarkably, unskilled workers are still able to accumulate capital by using only work: as $h = 0$, we have $[k_{t+1} = k_t + \lambda]$. This mirrors the difference in productivity among different agents, and ensures the existence of an incentive to migrate for any individual.

With respect to the utility, we adopt a simple quadratic specification. The per-period utility in D is

$$u^D\left(c_t^D\right) = c_t^D - \left(c_t^D\right)^2 \tag{3}$$

By substituting (1) into (3), we can write

$$u^D\left(c_t^D\right) = k_t - k_t^2 \tag{4}$$

2.2 Origin country

Migration can be defined as a technology to import the physical capital required for producing in O. However, the risk of a crisis is a key difference between destination countries and developing countries. Indicators of risk are widely used in the business community and in the academic literature (Beine et al., 2003; Easterly and Levine, 1997)[7]. We assume that in O the physical capital owned by an individual is confiscated with probability $p \in (0,1)$. This shock is not correlated across individuals. Such an assumption enables us to account for constrained migration, which is not modelled in the current literature. The magnitude of the constrained migration flow depends on how many individuals are hit by the shock. Examples of crises include economic crunches, political turmoils, natural disasters[8]. After a capital confiscation, the individual is forced to re-migrate.

Therefore, the capital stock in O is

$$k_{t+1} = II \cdot k_t \tag{5}$$

where II is the following indicator function:

$$\begin{aligned} II &= \begin{cases} 0 \text{ with probability } p \\ 1 \text{ with probability } (1-p) \end{cases} \\ j &= 1...n \end{aligned} \tag{6}$$

Confiscation of k_T is ruled out in D, where the economic and political environment is comparatively highly stable.

A key assumption of the literature on temporary migration is that the marginal utility of consumption is higher at home: the per-period utility of consuming in O is

[7] Typically, the rankings combine measures of political risk (such as the threat of war) and economic risk (such as the size of fiscal deficits). They may also include measures that affect a country's liquidity and solvency (eg, its debt structure and foreign-exchange reserves).

[8] Consider that temporary migration is often used to build homes or to start a business (See Dustmann and Kirchkamp 2002, Mesnard 2004 and the references quoted therein). For some evidence on the role of crises, see Chiuri, de Arcangelis and Ferri (2004).

then

$$\begin{aligned} u^O\left(c_t^O\right) &= \alpha_j\left[c_t^O - \left(c_t^O\right)^2\right]; \\ \alpha_j &> 1 \; and \; finite \\ \alpha_j &\sim g(\alpha); \\ j &= 1...n \end{aligned} \tag{7}$$

α_j is a coefficient used to depict the preference for consuming at home, and k_T is the capital imported after T periods of migration. As a consequence, for a given c, consumption in O dominates consumption in D. Therefore we have

$$u^O\left(c_t^O\right) = \alpha_j\left(k_T - k_T^2\right) \tag{8}$$

In spite of poor economic conditions, in O it is possible to build up human capital, as it is often the case for developing countries. An individual can build up human capital by means of an effort $e(h)$:

$$e\left(h\right) = \theta_j h^2 \tag{9}$$

the parameter θ_j enables us to introduce different abilities to build human capital:

$$\begin{aligned} \theta_j &> 0; \\ \theta_j &\sim f(\theta) \\ j &= 1...n \end{aligned} \tag{10}$$

Human capital accumulation occurs only in $t = 0$, when agents decide their human capital level. Since consumption is produced through capital, the per-period utility is $\alpha_j\left(k_T - k_T^2\right)$ if the shock does not occur, and 0 if the shock occurs:

$$u^O(c_t^O) = \begin{cases} 0 \text{ with probability } p \\ \alpha_j\left(k_T - k_T^2\right) \text{ with probability } (1-p) \end{cases} \tag{11}$$

Strictly speaking, the parameters describing the individual charachteristics are α, β, θ. In addition, h and T are the choice variables determining k_T and $\pi(h)$. All of them should be indexed, and this would make our notation quite heavy. In order to keep things as simple as possible, in what follows we drop the j index since it can be done unambigously. In Section 5 we are going to present some numerical examples for different agents.

3 Migration duration and human capital accumulation

Now we are going to describe the individual optimization problem over infinite horizon. To make things clearer, we are going to write the program for a permanent migrant, then for a temporary migrant when entry to D is open and, finally, for a temporary emigrant when entry to D is restricted.

3.1 Permanent migration

Let us suppose first that the agent is leaving forever her country with h human capital, and that there is no entry restriction to D. The first T periods are devoted to the accumulation of physical capital according to (2). Let $V^D(h,T)$ and $V^D(k_T)$ be the value functions describing, respectively, the present value of the utility in D from 0 to $(T-1)$, and the present value of the utility in D from period T onwards. We have thus

$$V^D(h,T) = \sum_{t=0}^{T-1} \beta^t u^D(c_t^D) = \sum_{t=0}^{T-1} \beta^t (k_t - k_t^2) \tag{12}$$

and

$$V^D(k_T) = \sum_{t=T}^{\infty} \beta^t u^D(c_t^D) = \sum_{t=T}^{\infty} \beta^t (k_T - k_T^2) \tag{13}$$

where $0 < \beta < 1$ is the intertemporal discount factor. The lifetime expected utility of a permanent migrant is

$$U_{PM}(h,T) = V^D(h,T) + V^D(k_T) - \theta h^2 \tag{14}$$

by substituting the values for $V^D(h,T)$ and $V^D(k_T)$[9], we get finally

$$U_{PM}(h,T) = \frac{(h+1)T\lambda((h+1)T\lambda - 1)\beta^T}{\beta - 1} - h^2\theta +$$

$$+\frac{-(h+1)\lambda\left((T(\beta-1)-\beta)(\beta-1)\beta^T + (\beta + h\lambda + \lambda - 1)\beta\right)}{(\beta-1)^3}+$$

$$+\frac{\left(\beta^T\left(2T\beta(h+1)^2 + \beta(h+1)^2 + T^2\left((h+1)^2 - 2\beta\right)\right) - (h+1)^2\beta^2\right)\lambda^2}{(\beta-1)^3}+$$

[9] $V^D(h,T)$ and $V^D(k_T)$ are computed in the Appendix.

$$+\frac{\beta^{T+1}\left((h+1)^2(T-1)^2\beta-2h(h+2)T^2\right)\lambda^2}{(\beta-1)^3} \tag{15}$$

Any individual has to compare her expected lifetime utility under permanent migration and under temporary migration. To make this comparison easier, it is useful to introduce the following lemma:

Lemma 1 *The utility of a permanent migration is upper bounded.*

Proof. See the Appendix ■

This lemma will be used in order to prove that there always exist agents who prefer temporary migration to permanent migration.

3.2 Temporary migration

3.2.1 Open entry We are now introducing temporary migration. While $V^D(h,T)$ is still the present value of staying abroad for T periods, let $V^O(k_T)$ be the value function describing the present value of returning to O with the accumulated capital k_T. Since in O the crisis may force a new migration, infinitely many spells of migration are possible. The sequence of migrations and returns gives the following utility:

$$\tilde{U}_{TM}(h,T)=V^D(h,T)+\beta^T\{(1-p)V^O(k_T)+p\tilde{U}_{TM}(h,T)\} \tag{16}$$

Obviously, $V^O(k_T)$ is weighted for the probability of the good state of the world $(1-p)$. On the other hand, capital confiscation occurs with probability p and the agent has to re-migrate. In this case, the migration process restarts, and the agent expects $\tilde{U}_{TM}(h,T)$ again.

3.2.2 Restricted entry When selective policies are used, human capital is quite important in determining the probability of entering D.

In our model this probability, denoted by $q(\pi(h),\Psi)$, is a function of individual and institutional characteristics. Human capital-based screening gives everybody a probability of entering $\pi(h)$, while the parameter $\Psi\in[0,1]$ depicts the weight that the immigration policy places on the human capital: when Ψ is close to 1, immigrants are screened according to the human capital they bring into the country[10]. When Ψ is close to 0, entry is open to anybody. This is a convenient way to represent the mix of individual and institutional characteristics enabling an individual to cross the border. The overall probability $q(\pi(h), \Psi)$ of entering D is therefore given by

$$q(\pi(h), \Psi) = \Psi\pi(h) + (1 - \Psi) \tag{17}$$

$\pi(h)$ has the following properties:

$$\begin{aligned} \pi'(h) &> 0 \quad \textit{and bounded} \\ \pi(0) &= \pi_0 > 0 \\ \lim_{h\to\infty} \pi(h) &= 1 \end{aligned} \tag{18}$$

Quite intuitively, $\pi(h)$ is increasing in h. $\pi(0) = \pi_0 > 0$ is the probability of entering as a totally unskilled worker (it may well be the probability of entering illegally). $\lim_{h\to\infty} \pi(h) = 1$ means that the probability of admission approaches unity as human capital approaches infinity[11].

When entry is made uncertain, the sequence of migrations and returns gives the following utility:

$$\tilde{U}_{TM}(h,T) = V^D(h,T) + \beta^T \left\{ (1-p) V^O(k_T) + pq\tilde{U}_{TM}(h,T) + p(1-q) V^O(0) \right\} \tag{19}$$

The above expression has to be interpreted as follows: $V^D(h,T)$ is again the discounted utility of the first T periods spent abroad. After T periods, the agent returns to O with the accumulated capital k_T. At home, her utility will be $V^O(k_T)$ when capital is conserved (with probability $(1-p)$). Conversely (with probability p), her

[10] We think that this function depicts selective policies better than the usual threshold used in the literature: since D can be considered as the "rest of the world", and since thresholds differ across countries and over time, the agent is aware that a marginal incresase in human capital implies a marginal increase in his probability of migrating successfully. For more information about entry requirements in different countries, we refer to Magris and Russo (2005).

[11] This feature of the policy is used for simplicity. Indeed, we can hypothesize that there exists H such that $\pi(h) = 1$, i.e. that the government establishes a threshold level of human capital (H) which entitles to free mobility. Obviously, when $h \geq H$, $\pi'(H) = 0$ because any unit of human capital beyond the threshold can not increase further the probability of entering D. Notice that $h^* > H$ could still be possible because h still affects the accumulation of physical capital k.

capital is confiscated and she attempts to re-migrate. Then, either she will succeed and re-build her capital stock getting $\tilde{U}_{TM}(h,T)$ (with probability q), or she won't, and she will simply get the utility of living in O without capital ($V^O(0)$), (with probability $(1-q)$).

Finally, the lifetime utility of a temporary migrant $U_{TM}(h,T)$ is given by (19) minus the disutility of the effort needed to build the human capital stock:

$$U_{TM}(h,T) = \tilde{U}_{TM}(h,T) - \theta h^2 \tag{20}$$

Constructing the expressions for $V^D(h,T)$, $V^O(k_T)$ and $V^O(0)$ (see the Appendix) and substituting them into (19) and (20) gives the expected lifetime utility associated to T periods of migration and to h human capital:

$$U_{TM}(h,T) = \frac{\frac{(h+1)(1-p)T\alpha\beta^T\lambda(1-(h+1)T\lambda)}{1-(1-p)\beta}}{1-\frac{p\beta^T(\pi(h)\Psi-\Psi+1)}{(1-(1-p)\beta)(1-\beta(\Psi-\Psi\pi(h)))}} -$$

$$-\frac{\frac{(h+1)\lambda\left(\beta^T\left((h+1)\left((-\beta T+T+\beta)^2+\beta\right)\lambda-(T(\beta-1)-\beta)(\beta-1)\right)-\beta(\beta+(h+1)(\beta+1)\lambda-1)\right)}{(\beta-1)^3}}{1-\frac{p\beta^T(\pi(h)\Psi-\Psi+1)}{(1-(1-p)\beta)(1-\beta(\Psi-\Psi\pi(h)))}} - \theta h^2 \tag{21}$$

The emigrant has to maximize (21) with respect to T and h. This optimization program does not admit a closed-form solution. However, it is easy to obtain some sufficient conditions for the existence of a solution (T^*, h^*) with $h^* \geq 0$.

Proposition 2 *(Human capital accumulation and migration duration under uncertainty). There exist an open set of parameters such that the problem of maximizing (21) admits at least a solution. In particular, this open set contains intervals described through inequalities in the Appendix.*

Proof. See the Appendix. ■

The above Proposition is quite intuitive: temporary migrants accumulate abroad the physical capital that is going to produce consumption in O. It is worth to remark that the incentive to accumulate abroad and return exists for unskilled workers as well ($h^* \geq 0$). Notice that we are reporting *sufficient* conditions. This means that a solution exists under quite general conditions.

In Figure 3.1, we show a plot of (21)[12].

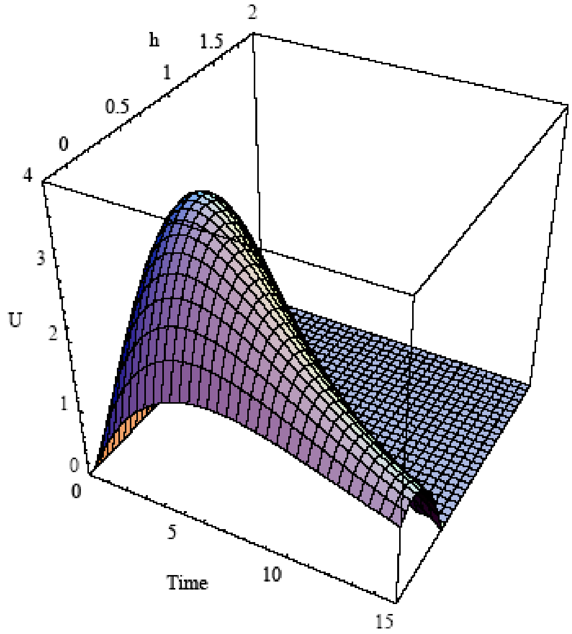

Figure 3.1: a plot of the utility function

3.3 The choice between temporary and permanent migration

Ceteris paribus, the decision to migrate permanently or temporarily depends on the extent of the preference for consuming at home, that is, on the magnitude of α. Thus we can write the following proposition:

Proposition 3 *(temporary migration and permanent migration): There always exist α^* such that temporary migration is preferred to permanent migration for any $\alpha \geq \alpha^*$.*

Proof. See the Appendix. ■

The intuition for this result is quite simple: remark that α is on the numerator of (21). Maximum utility is increasing in α, and α can be arbitrarily high. Since (15) is upper bounded, it is always possible to find α^* such that temporary emigration dominates permanent emigration.

[12] The plot is obtained with the following parameter values: $\alpha = 8$; $\beta = .85$; $\Psi = 1$; $p = .1$; $\theta = 3$; $\lambda = .05$, $\pi(h) = 1 - e^{-\frac{h}{2}}$.

It is also important to stress that any policy that increases the utility of migrating forever (15) increases α^* and thus the share of permanent emigrants, given by $\int_{\underline{\alpha}}^{\alpha^*} g(\alpha)d\alpha$. As α denotes the importance of cultural and ethnic factors, one may think that it is close to one when O and D are homogeneous in language, culture and traditions. This means that the incentive to return is more important when O and D are different. When the share of permanent emigrants grows agents who experience a more difficult assimilation are incentivated to stay abroad[13]. In the long run, this might exacerbate the problems related to the cultural assimilation.

Finally, it is worth to stress that the denominator of (21) is minimum for $\Psi = 0$, thus, in terms of individual utility, free entry always dominates entry rationing.

4 Brain gain or brain loss?

A new immigration policy is a change in Ψ. Generally, a government may affect π_0 and the shape of $\pi(h)$ as well; however, varying Ψ is the most direct intervention to make entry rationing more -or less- severe[14]. We are now going to examine the impact of Ψ on T^* and h^*. By means of the implicit function theorem it is easy to show (see the Appendix) that the sign of $\frac{\partial h^*}{\partial \Psi}$ is given by the sign of

$$\frac{\partial}{\partial \Psi}\left(\frac{\partial U_{TM}(h^*, T^*)}{\partial h}\right) \tag{22}$$

Studying the properties of (22) enables us to write the following Proposition:

Proposition 4 *(Brain Gain and Brain Loss). When $\frac{\partial h^*}{\partial \Psi} > 0$, selective policies cause a brain gain. When $\frac{\partial h^*}{\partial \Psi} < 0$, instead, they cause a brain loss. Moreover, as we show in Section 5, the possibility of a brain loss is not limited to a neighbourhood of the actual Ψ -as proved by the Implicit Function Theorem- but it can occur over a large interval.*

Proof. See the Appendix. ■

Proposition 2 states that the argument for "a brain gain with a brain drain" can easily be reversed, since it depends on the sign of $\frac{\partial h^*}{\partial \Psi}$. After running several

[13] For an enlightening analysis of the assimilation problem, see the seminal Lazear's (1999) article.

[14] It is useful to recall that $\Psi = 1$ indicates that entry is totally screening-based.

simulations, we can argue that a brain loss occurs easily when the following conditions are present:

1) h^* is sufficiently small;

2) λ is sufficiently small;

3) $\pi'(h)$ is sufficiently small

Condition 1) means that the brain loss is likely to occur for individuals with low human capital -that is, those with high values of θ. Condition 2) means that tranforming human capital into physical capital should be "difficult". Condition 3) means that the selective policy has to be "severe", i.e. the probability of entering the destination country increases slowly as h increases.

In other words, in order to avoid the risk of a brain loss, the probability of entering should increase rapidly with human capital. Unfortunately, this would be inconsistent with a severe entry rationing based on human capital requirements. Notice that the "brain loss" of our model differs from the "brain drain", which stems from permanent migration[15]. The brain loss occurs via a reduction of the equilibrium level of human capital, and it is not due to emigration.

Finally, we can get an important insight from our model: if we interpret θ_j as a country-specific parameter rather than an individual charachteristic, we can apply the above reasoning on a country scale: a brain loss may damage the economies turned up with $\frac{\partial h^*}{\partial \Psi} < 0$.

Remark 5 *Since the sign of* $\frac{\partial h^*}{\partial \Psi}$ *may be different for different agents, applying a uniform policy towards different individuals or countries can increase the world human capital dispersion.*

This finding casts some serious doubts on the effects the widespread adoption of point schemes may have in the long run: such policies may benefit the receiving countries, but they are not a panacea and there exists a possibility that they exacerbate inequality. Beine et al. (2003) report evidence consistent with the hypothesis that the possibility of migrating is a powerful incentive to build human capital. This incentive effect may well depend more on wage differentials than on the selective policy.

[15] Remark that our definition of brain loss differs from the one used in Schiff (2005) as well. The latter concerns the increased brain drain due to selective policies.

5 Numerical examples

5.1 Optimal migration duration

Since it is not possible to use the implicit function theorem to evaluate the effect of Ψ and p on T^*, we have to use a series of simulations.

The model has been simulated for different parameter values and different functional forms of $\pi(h)$[16]. When searching the effect of Ψ over T^*, we have computed T^* when Ψ ranges from 0 to 1, in steps of 0.01. Intuitively, T^* increases with Ψ especially for individuals with low human capital. This is quite intuitive: for skilled workers the probability of entering approaches 1, thus they are entitled to free mobility. Figure 5.1 below gives an example, where the solid line is traced for $\theta = 3$, and the dotted line for $\theta = 1$.[17]:

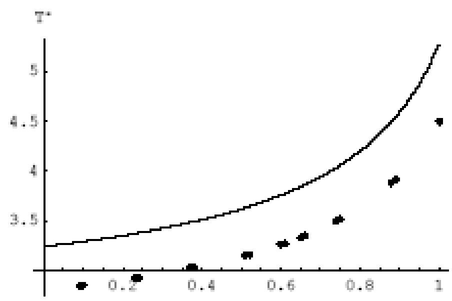

Figure 5.1: selective policy and migration duration

In Figure 5.2 we have plotted T^* against p, for $p \in (0, 1)$[18]. The solid line is obtained when $\Psi = 1$, and the dotted line when $\Psi = 0$. Again, we can observe that the selective policy biases our results towards a longer migration spell.

[16] Our simulations are available upon request.

[17] The values used are $\alpha = 8$; $\beta = .85$; $\lambda = .05$; $p = .25$; $\pi(h) = (1 - e^{-\frac{h}{20}})$.

[18] The values used are $\alpha = 8$; $\beta = .85$; $\theta = 3$; $\lambda = .05$; $\pi(h) = (1 - e^{-\frac{h}{20}})$.

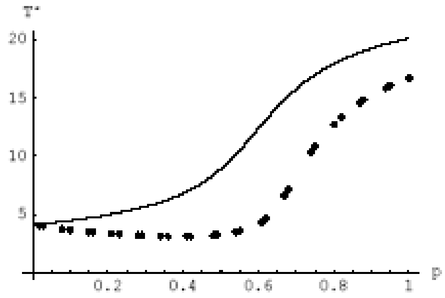

Figure 5.2: risk and migration duration

Summarizing, our simulations show a trade-off between entry rationing and migration duration: for a given θ, border closure tends to increase the time spent abroad and reduces total utility. This can be seen as a simple result of the Lucas critique.

5.2 Brain loss

In Figure 5.3, we show two examples of brain loss. Notice that, for the chosen parameters[19], h^* is monotonically decreasing with Ψ. Intuitively, we find that the lower h^*, the higher the negative effect of the selective policy. In the graph on the left, plotted for $\theta = 2$, h^* decreases from .45 ($\Psi = 0$) to .35 ($\Psi = 1$). In the graph on the right, plotted for $\theta = 1$, h^* decreases from .78 ($\Psi = 0$) to .65 ($\Psi = 1$). The relative reduction is, respectively, of 28% and of 20%. Thus, the effect is stronger for the more disadvantaged individual.

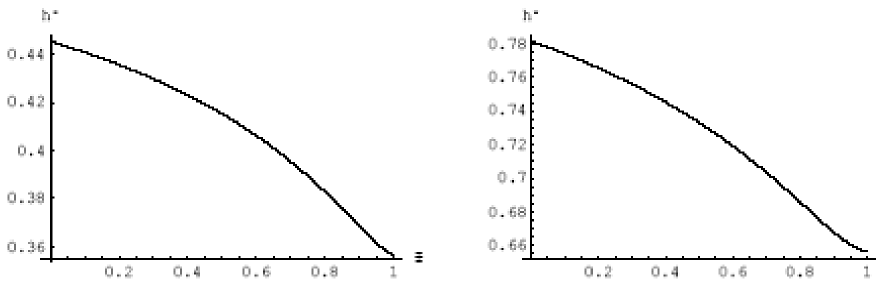

Figure 5.3: brain loss

[19]The parameters used are: $\alpha = 8$; $\beta = .85$; $\theta = 1$; $\lambda = .05$; $p = .2$; $\pi(h) = (1 - e^{-\frac{h}{10}})$.

Remarkably, the plot in Figure 5.3 yields a stronger result than Proposition 3: the Implicit Function Theorem proves the brain loss only in a neighborhood of the equilibrium, while in our example we show that the result can be valid over the entire range of Ψ.

5.3 Risk and optimal human capital accumulation

Using the Implicit Function Theorem[20], it is easy to see that the sign of the derivative $\frac{\partial h^*}{\partial p}$ is the sign of

$$\frac{\partial}{\partial p}\left(\frac{\partial U(T,h)}{\partial h}\right). \tag{23}$$

In principle, this sign is ambiguous: while risk makes it less attractive the effort to acquire human capital, at the same time a skilled migrant can recover abroad easier. From this point of view, human capital has an insurance effect. Which effect prevails is in principle undetermined. In figure 5.4.[21] we give an example where h^* is decreasing with p. The dotted curve is obtained with $\Psi = 0$, and the solid one with $\Psi = 1$.

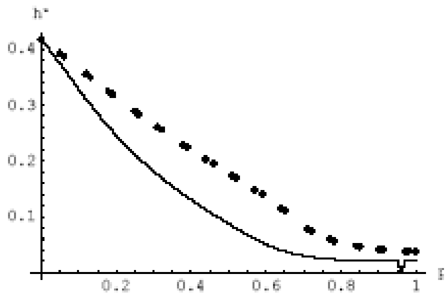

Figure 5.4: risk and opimal human capital

We can see again that the equilibrium human capital tends to be higher under the open-immigration policy. By varying the parameter values it is also possible to obtain a non-monotonic relatioship between h^* and p, but the result that the dotted line lies above the solid one seems quite robust.

[20] It is done as in the proof of Proposition 4.

[21] The parameter values are $\alpha = 8$, $\beta = .85$, $\theta = 3$; $\lambda = .05$, $\pi(h) = 1 - e^{\frac{h}{10}}$.

6 Brain drain Vs. brain gain

The contributions mentioned in Section 1.1 state some conditions under which a restrictive immigration policy in the *destination* country enhances the *origin* country's welfare. These results are obtained under two crucial assumptions: migration is permanent, and human capital is only useful to cross the border[22]. Both these assumptions are counterfactual.

When they are dropped, results can be quite different. First of all, we reproduce the well-known conclusion that, as long as migration is temporary, there is no brain drain. More importantly, in our setting, total freedom of emigration does not generate *per se* a brain drain: both incentives to return and incentives to build up human capital exist *independently of selective immigration policies.* The brain drain is due not only to wage differentials, but also to individual preferences and to immigration policies. Highly skilled individuals have $\pi(h)$ close to 1, and they can decide their location according to their preferences: if $\alpha \leq 1$ there is no reason to stay in O.

Our work does not deal with the growth problem associated to the brain drain -a related paper is that of Reichlin and Rustichini (1998)- however, it is possible to draw some conclusions about the stock of human capital in O.

First, any admission restriction increases the α^* necessary to return, therefore the share of permanent migrants. This is a brain drain effect. Second, if things get bad, agents with the "wrong" sign of $\frac{\partial h^*}{\partial \Psi}$ reduce their education. As a result, O can easily be worse off than under an open migration policy. When, instead, a brain gain occurs, it would be necessary to compare the loss due to increased permanent emigration to the gain of the unsuccessful migrants. In an endogenous growth framework à la Reichlin and Rustichini (1998) switching from a brain gain to a brain loss (and viceversa) can have dramatic effects, by deepening divergence or by speeding up convergence. This suggests that selective policies are a powerful, but double-edged, tool: their use should be carefully evaluated.

[22]It is important to mention the result in Mountford (1997): he clearly shows that the equilibrium human capital is increasing with the probability of migrating. Nonetheless, in his model, he can't provide for free migration without causing a complete human capital depletion in the source country.

7 Conclusions

The study of migration duration is receiving increasing attention, as well as the effect of the chances of migration on human capital accumulation. In our work, we have attempted to connect these streams of literature.

We think that we have carefully modelled this major point of our paper by studying the joint decision about emigration and human capital accumulation in an infinite-horizon framework. Our findings question the effectiveness of human-capital based immigration policies. First, we reproduce in a more general model the well-known result that closed-door policies backfire on migration duration. For example, Kossoudji (1992) found that attempts to enforce the U.S.-Mexican border eventually "alter lengths of spells of future trips to the U.S.". Second, we have stressed the impact of the macroeconomic risk on the decision to accumulate human capital and to stay longer abroad. Third, selective policies are a double-edged weapon: they can both foster and harm the equilibrium level of human capital.

With respect to an open immigration policy they make the returns to human capital uncertain, and they can have ambiguous effects on the incentives to education. This may cause a "brain loss", rather than a brain gain. At least, this result suggest that selective immigration policies should not be used unconditionally, and that the logic behind the "brain gain with a brain drain" is not always correct.

The policy implications of this finding are quite important. If the sign of $\frac{\partial h^*}{\partial \Psi}$ differs across a country's population, selective policies are likely to increase the human capital dispersion within the country. If, instead, the sign of $\frac{\partial h^*}{\partial \Psi}$ differs according to the nationality of origin we should observe more human capital dispersion across countries. This can have worrisome effects on the possibility of convergence in the long run. We hope to develop our future research in this direction.

Finally, it is worth to recall that economic policies can affect p : plans aimed to reduce the risk in the developing countries reinforce by themselves the incentive to return. Though it may be difficult to modify these properties of an economy, there are no theoretical reasons why international, co-ordinated development policies should be less effective or more costly than enforcing strict frontiers closure. It is also important to mention the result that trade liberalization can be the best option for an incentive-compatible immigration reduction (Trefler, 1998).

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Appendix

Derivation of $V^D(h,T)$:

we have to find

$$V^D(h,T) = \sum_{t=0}^{T-1} \beta^t \left[c_t - c_t^2\right] \tag{A.1}$$

subject to

$$\begin{aligned} c_t &= k_t \\ k_{t+1} &= k_t + \lambda(1+h) \\ k_0 &= 0. \end{aligned}$$

Integrating (A.1) for T periods, we get

$$\begin{aligned} V^D(h,T) = & \frac{-\lambda(1+h)\left[-\beta(\beta+(h+1)(\beta+1)\lambda-1)\right]}{(\beta-1)^3} - \\ & -\frac{\lambda(1+h)\left[\beta^T\left((\beta-1)(\beta-T(\beta-1))+(h+1)\left((-\beta T+T+\beta)^2+\beta\right)\lambda\right)\right]}{(\beta-1)^3} \end{aligned} \tag{A.2}$$

Derivation of $V^O(0)$:

in the current period consumption is zero, and the agent is going to re-migrate in the following period with probability q. If she succeeds, her utility will be $\tilde{U}_{TM}(h,T)$, otherwise she will get again $V^O(0)$. Therefore, we have

$$V^O(0) = 0 + \beta\left\{q\tilde{U}_{TM}(h,T) + (1-q)V^O(0)\right\} \tag{A.3}$$

from which it is easy to get the expression for $V^O(0)$:

$$V^O(0) = \frac{\beta q \tilde{U}_{TM}(h,T)}{1-\beta(1-q)}. \tag{A.4}$$

Derivation of $V^O(k_T)$:

the computation of $V^O(k_T)$ is less straightforward: the utility of k_T in the first period is $\alpha(k_T - k_T^2)$. It is easy to compute

$$k_T = (1+h)\lambda T \tag{A.5}$$

In the following period, with a probability $(1-p)$ the shock does not occur and therefore utility is still $V^O(k_T)$. Conversely, the individual will re-migrate with probability q or get $V^O(0)$ with probability $(1-q)$. We have thus the following expression for $V^O(k_T)$:

$$V^O(k_T) = \alpha\left[k_T - k_T^2\right] + \beta\left\{(1-p)V^O(k_T) + pq\tilde{U}_{TM}(h,T) + p(1-q)V^O(0)\right\} \tag{A.6}$$

Solving (A.6) with respect to $V^O(k_T)$, and using (A.4) and (A.5), we get:

$$V^O(k_T) = \frac{\alpha T\lambda(1+h)(1-(1+h)\lambda T) + \beta\left\{pq\tilde{U}_{TM}(h,T) + p(1-q)\frac{\beta q\tilde{U}_{TM}(h,T)}{1-\beta(1-q)}\right\}}{1-\beta(1-p)} \tag{A.7}$$

Now, substituting (A.4) and (A.7) into (19):

$$\tilde{U}_{TM}(h,T) = V^D(h,T) + \beta^T\left\{\frac{pq\tilde{U}_{TM}(h,T)}{[1-\beta(1-q)]}\right\} +$$
$$+\beta^T\left\{\frac{\alpha T\lambda(1-p)(1+h)[1-\beta(1-q)][1-T\lambda(1+h)] + (1-p)\beta pq\tilde{U}_{TM}(h,T)}{[1-\beta(1-q)][1-\beta(1-p)]}\right\} \tag{A.8}$$

by rearranging the above expression, and by using (A.2), we obtain $\tilde{U}_{TM}(h,T)$. Finally, plugging $\tilde{U}_{TM}(h,T)$ into (20), we get (21).

Proof of Lemma 1)

By direct inspection of eq. (15), we immediately verify that the denominator of $U_{PM}(h,T)$ is bounded away from zero. Thus, we should check that h or T never drive the numerator to $+\infty$. This can be done by computing first the coefficient of h^2 that is given by

$$\left[-\frac{((-\beta T+T+\beta)^2+\beta)\lambda^2\beta^T}{(\beta-1)^3} + \frac{T^2\lambda^2\beta^T}{\beta-1} + \frac{(\beta+1)\lambda^2\beta}{(\beta-1)^3}\right] \tag{A.9}$$

.

It is easy to verify that this coefficient is always negative for any $T \geq 1$[23]. On the other hand, we have

$$\lim_{T\to\infty} U_{PM}(h,T) = \frac{(h+1)\beta\lambda(\beta+(h+1)(\beta+1)\lambda-1)}{(\beta-1)^3} - h^2\theta, \tag{A.10}$$

[23] Indeed, the only positive term is $\left[-\frac{((-\beta T+T+\beta)^2+\beta)\lambda^2\beta^T}{(\beta-1)^3}\right]$ which is max for $T=1$. In such a case, the coeffcient reduces to $\left[\frac{T^2\lambda^2\beta^T}{\beta-1}\right] < 0$.

that is upper bounded because the coefficient of h^2 is still negative.

Proof of Proposition 2)

Consider eq. (21): α enters the numerator, in the expression $\frac{\beta^T\alpha(1-p)(h+1)\lambda T(1-(h+1)T\lambda)}{1-(1-p)\beta} \equiv \frac{\beta^T\alpha(1-p)(k_T-k_T^2)}{1-(1-p)\beta}$.

Since the utility $(k_T - k_T^2)$ cannot be negative in the max of (21), it follows that the max lifetime expected utility is increasing with α. In Proposition 1) we have proved that $U_{PM}(h,T)$ is bounded. Since α can be arbitarily high, it is always possible to find α^* such that $U_{TM}(h,T) > U_{PM}(h,T)$.

Proof of Proposition 3)

It is useful to recall that $U_{TM}(h,T) = \tilde{U}_{TM}(h,T) - \theta_j h^2$. Notice first that $U_{TM}(h,T)$ is zero in the origin, and it is continuous. Our aim is to obtain some sufficient conditions to prove that $U_{TM}(h,T)$ admits an interior max. We proceed in two stages: first, we obtain the conditions under which $U_{TM}(h,T)$ is non-positive for h,T sufficiently large; then we prove that there exists a point where $U_{TM}(h,T)$ is positive. This implies the existence of at least an interior max.

Let us begin with the boundary along h. For simplicity, consider the case $\Psi = 0$. In such a case, the denominator of $\tilde{U}_{TM}(h,T)$ is positive, it depends only on T, and returns to human capital are higher for any T with respect to the case $0 < \Psi \leq 1$. Thus any conclusion that holds for $\Psi = 0$ has to hold for $0 < \Psi \leq 1$. Since the term $-\theta_j h^2$ is always negative, in studying this boundary of $U_{TM}(h,T)$ we can focus on the numerator of $\tilde{U}_{TM}(h,T)$. The coefficient of h^2 on the numerator is

$$\left(-\frac{((-\beta T + T + \beta)^2 + \beta)\,\lambda^2\beta^T}{(\beta-1)^3} + \frac{(p-1)T^2\alpha\lambda^2\beta^T}{1-(1-p)\beta} + \frac{(\beta+1)\lambda^2\beta}{(\beta-1)^3}\right) \tag{A.11}$$

. This coefficient is negative for any $T \geq 1$. Therefore, $\lim_{h\to\infty} U_{TM}(h,T) = -\infty$ for any $T \geq 1$: the function is always negative along the h boundary.

Consider now the boundary along T. We have

$$\lim_{T\to\infty} U_{TM}(h,T) = \frac{(h+1)\beta\lambda(\beta + (h+1)(\beta+1)\lambda - 1)}{(\beta-1)^3} - h^2\theta. \tag{A.12}$$

This limit is a function of h, and it has a parabolic shape. We need that it is non-positive in its max. Thus, we find the value of h that maximizes the limit, we substitute it into the limit, which becomes

$$\max_h \left(\lim_{T\to\infty} U_{TM}(h,T)\right) = \frac{\beta\lambda(\beta\lambda + 4(\beta-1)\theta(\lambda\beta+\beta+\lambda-1))}{4(\beta-1)\left((\beta-1)^3\theta - \beta(\beta+1)\lambda^2\right)} \tag{A.13}$$

then we solve

$$\max_h \left(\lim_{T\to\infty} U_{TM}(h,T) \right) \leq 0 \tag{A.14}$$

This inequality holds for

$$\begin{aligned} \theta &> \frac{1}{8(1-\beta)} \\ \frac{-4\theta(1-\beta)^2}{-4\theta(1-\beta^2)+\beta} &\leq \lambda < 1. \end{aligned} \tag{A.15}$$

When the constraints on θ and λ hold we know that $U_{TM}(h,T)$ is never positive on its boundaries. Therefore, to prove that $U_{TM}(h,T)$ admits at least an interior max, we only need to prove that there exist at least a pair (h,T) for which it is positive. We choose the pair $(1,1)$, and obtain

$$U_{TM}(1,1) = \frac{2(1-p)\alpha\beta(1-2\lambda)\lambda}{(1-(1-p)\beta)\left(1-\frac{p\beta(\pi(1)\Psi-\Psi+1)}{(1-(1-p)\beta)(1-\beta(\Psi-\Psi\pi(1)))}\right)} - \theta \tag{A.16}$$

Finally, we can write the following system of inequalities:

$$\begin{cases} U(1,1) > 0 \\ 0 < \beta < 1 \\ 0 < \pi(1) \leq 1 \\ 0 \leq \Psi \leq 1 \\ 0 < p < 1 \\ \alpha > 1 \\ \lambda = \frac{-4\theta(1-\beta)^2}{-4\theta(1-\beta^2)+\beta} \\ \theta > \frac{1}{8(1-\beta)} \end{cases} \tag{A.17}$$

we have given λ the lowest value of the interval $(\frac{-4\theta(1-\beta)^2}{-4\theta(1-\beta^2)+\beta}, 1)$ in order to speed up the computations. This system admits several sets of solutions. For brevity, we report just one: [24]

$$
\begin{aligned}
0.333 &< \beta < 0.777 \qquad \text{(A.18)} \\
0 &< \pi(1) \leq 1 \\
0 &\leq \Psi \leq 1 \\
0 &< p < 1 \\
\theta &> \frac{-\beta}{4 - 16\beta + 12\beta^2} \\
\alpha &> \frac{\left(\beta + 4\left(\beta^2 - 1\right)\theta\right)^2 ((\pi(1) - 1)(p - 1)\beta\Psi - 1)}{8(p-1)(\beta-1)\beta(\beta + 4(\beta-1)(3\beta-1)\theta)((\pi(1)-1)\beta\Psi + 1)}
\end{aligned}
$$

Proof of Proposition 4)

We apply the Implicit Function Theorem to prove that the effect of Ψ over h^* is given by the sign of $\frac{\partial}{\partial\Psi}\left(\frac{\partial U_{TM}(h,T)}{\partial h}\right)$. To simplify the notation, we indicate the utility in its max $U_{TM}(T^*, h^*)$ as $U(T^*(\Psi), h^*(\Psi), \Psi)$, and its partial derivatives withU_{ij} $i, j = h, \Psi$. Remark that, since T is discrete, in a neighborhood of (h^*), T^* does not change. The derivative $\frac{\partial h^*}{\partial\Psi}$ is thus $\frac{\partial h^*}{\partial\Psi} = -\left[\frac{U_{h\Psi}}{U_{hh}}\right]$. Notice that the denominator of $(\frac{U_{h\Psi}}{U_{hh}})$ is negative because U_{hh} is the second derivative of the utility in its max. Therefore, the sign of $(\frac{U_{h\Psi}}{U_{hh}})$ in h^* is the sign of $U_{h\Psi}$.

To prove the Proposition, it is sufficient to provide an example where $U_{h\Psi}(h^*, T^*) < 0$. This can be easily done numerically. Let us assign the following values to the parameters: $\alpha = 8$; $\beta = .85$; $\theta = 2$; $\lambda = .05$; $p = .2$. Then, let $\pi(h) = (1 - e^{-\frac{h}{10}})$. In this case, $h^* = 0.435762$ and $T^* = 3.28492$. It is possible to verify that $U_{h\Psi}(0.435, 3.284) < 0$ for a wide range of values for Ψ. For example, we have $U_{h\Psi}(0.435, 3.284) = -0.116972$ for $\Psi = 0$; $U_{h\Psi}(0.435, 3.284) = -0.204683$ for $\Psi = .5$; $U_{h\Psi}(0.435, 3.284) = -0.0150481$ for $\Psi = .9$. If we consider the integer value $T^* = 3$, the results do not change.

[24] Notice that there exist also solutions for $.777 < \beta < 1$. All the solutions and the routines written to solve the system are available upon request.