

DEPARTMENT OF MATHEMATICS
University of Toronto at Mississauga

MAT 133Y Test #2A

November 25, 2005

Fill in the following information in INK!

Last Name: _____.

Given Name: _____.

Student #: _____.

YOUR TUTOR	A.B.	A.C.	D.K.	F.M.	J.H.	R.D.	S.A.	W.N.
YOUR TUTORIAL # OR TUTORIAL DAY/TIME								

Please Circle your Tutor's initials and enter the tutorial number or tutorial day/time

INSTRUCTIONS

1. Only simple scientific calculators are allowed. NO graphing calculators or ones that have memory and are able to manipulate matrices are allowed.
2. The time allotted is **110** minutes.
3. Questions 1 and 2 each have 10 parts. These questions have a box provided for you to put your final answer in. Make sure your answer is clear. There are NO part marks for the parts of these two questions.
4. Questions 3 and 4 require you to show all your work and write out a concluding statement for each part. A correct answer obtained with false reasoning will not receive any marks.
5. There are 9 pages to this test. Make sure you have all of them.
6. Budget your time. Good Luck!

Question #	Out of	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

QUESTION 1: For each part, clearly indicate your final short answer in the appropriate **BOX** provided. [25 marks, 2.5 marks for each part A, B, ... H₁, and H₂, with **NO part marks** for incorrect final answers.] Please note that your answers should be exact. It is safer to leave your answer in fraction form (e.g. it would be incorrect to put down 0.33 if the actual answer is 1/3)

- A) If the odds in favor of an event E are 2:7 (read as 2 to 7), find $P(E')$.
(Note: E' stands for E complement)

Answer for A:

- B) In how many ways can a basketball coach assign the five different positions to her eight-member team if all members are equally qualified at all positions?

Answer for B:

- C) How many distinguishable horizontal arrangements of all the letters in BOOKSHOPS are possible?

Answer for C:

- D) A random variable X has a distribution given by $f(0) = 0.4$, $f(1) = 0.3$, $f(2) = 0.3$. Find the expected value and the variance of X .

Answer for D:

Expected value =

Variance =

- E) If a fair red die and a fair green die are rolled, find the probability that the sum is greater than 8, given that a 4 shows on the red die.

Answer for E:

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QUESTION 1 (continued...)**F) Three fair coins are tossed.****F₁) Find the probability that three heads show;****Answer for F₁:****F₂) Find the probability that exactly two heads show;****Answer for F₂:**

G) Given that $\begin{bmatrix} a & b & 0.4 \\ 0.2 & 0.2 & b \\ 0.1 & 0.3 & b \end{bmatrix}$ is a transition matrix for a Markov chain. Find the value of a .

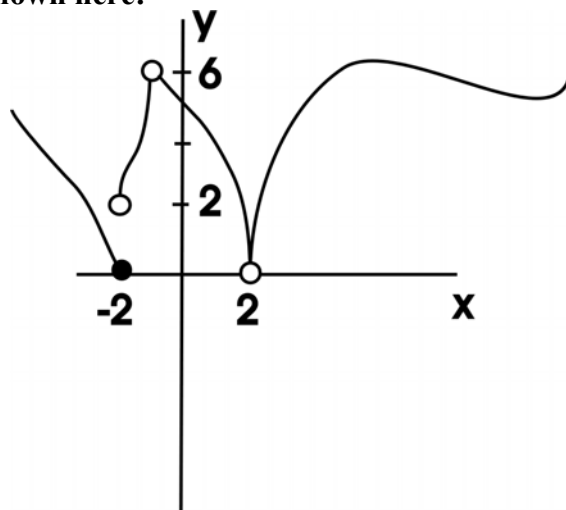
Answer to G:

H) If events E and F are independent (which also implies that E' (the complement of E) and F are independent), and if you are given that $P(E) = 0.3$ and $P(F) = 0.5$,

H₁) Find $P(E' \cap F)$:**Answer for H₁:****H₂) Find $P(E' \cup F)$:****Answer for H₂:**

QUESTION 2: For each part, clearly indicate your final short answer in the appropriate BOX provided.
 [25 marks, 2.5 marks for each part A₁, A₂, ... D₁, D₂, and E, with NO part marks for incorrect final answers.]

A) Given $y = f(x)$ has a graph as shown here:



A₁) Determine $f(2)$

Answer for A₁:

A₂) Determine $\lim_{x \rightarrow 2} f(x)$

Answer for A₂:

A₃) Determine $\lim_{x \rightarrow -2^-} f(x)$

Answer for A₃:

A₄) Determine $\lim_{x \rightarrow -2} f(x)$

Answer for A₄:

A₅) Is the given function continuous at $x = 2$?

Answer for A₅:

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QUESTION 2 (continued...)

B) Evaluate the following 2 limits. If the limit does not exist state that it does not exist, use the symbol ∞ or $-\infty$ where appropriate.

$$\text{B}_1) \lim_{x \rightarrow -3} \left(\frac{x^2 - 9}{3 + x} \right)$$

Answer for B₁:

$$\text{B}_2) \lim_{x \rightarrow -\infty} \left(\frac{x^3 + 1}{2 + x^2} \right)$$

Answer for B₂:

In parts C and D where a set of points is indicated, express these values in interval form.

{e.g. to indicate values of x such that $x \leq 1$ or $x > 3$, you would have to write: $x \in (-\infty, 1] \cup (3, \infty)$ }

C) Find the value(s) of x for which $k(x) = \frac{x-2}{4x^4+1}$ is continuous.

Answer for C:

D) Given the following 2 inequalities, in each case, solve for x . Express your answer in interval form.

$$\text{D}_1) x^2 - 5x - 36 \leq 0$$

Answer for D₁:

$$\text{D}_2) \frac{x^2 - 5x - 36}{x + 4} \leq 0$$

Answer for D₂:

QUESTION 3: [6 + 6 + 6 + 7 = 25]

3a) A person deposits \$1000 in a savings account that pays an interest rate of 4.75% compounded continuously.

- i) Find the balance in the account at the end of 3.5 years.
(Express your answer rounded UP to the nearest dollar) [6 marks]

- ii) Find the annual rate r compounded semiannually, equivalent to the interest rate given above.
(Express your answer as a percentage with 3 decimals) [6 marks]

3b) For a family of at least 2 children, let E and F be the events given by:

$E = \{\text{at most one boy}\}$

$F = \{\text{at least one child of each gender}\}$

Assume that a child of either gender is equally likely and that, for example, having a girl first and a boy second is just as likely as having a boy first and a girl second.

For EACH of the following TWO situations A and B:

Write down the sample space S and the 2 sets E and F , calculate the appropriate probabilities, and determine whether E and F are independent.

Justify your answer!

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QUESTION 3b (continued...)

Situation A: The family has exactly 2 children.

[6 marks]

Situation B: The family has exactly 3 children.

[7 marks]

QUESTION 4: [3 + 10 + 12 = 25]

4. In a certain region, cola drinking customers have a choice of buying Pepsi-Cola (Pepsi) or Coca-Cola (Coke). It has been observed that people tend to get 'hooked' on the cola they buy: if a customer buys Pepsi on one day, then the probability that he/she will switch to Coke on the next day is 0.3; if a customer buys Coke one day then the probability that he/she will stick with Coke the next day is 0.8. (Express all answers accurately. Do not round up or down)

- a) Write the transition matrix for this Markov process. [3 marks]
- b) Find what percentage of the population buys Pepsi and what percentage buys Coke after 2 days, if on "day 0" (initially) 30% buy Pepsi and 70% buy Coke. [10 marks]

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QUESTION 4 (continued...)

- c) Explain what a “steady state vector” is, and find it for the given “Pepsi/Coke” problem.
[12 marks]