7

HOW STABLE AM I? TRIANGLE CONGRUENCE

I. INTRODUCTION AND FOCUS QUESTIONS

Have you ever wondered how bridges and buildings are designed? What factors are being considered in construction of buildings and bridges?.

Designing structures requires the knowledge of triangle congruence, its properties, and principles.

This module includes definition of congruent triangles, the congruence postulates and theorems, and proving congruency of triangles. These concepts and skills will equip you to investigate, formulate, communicate, analyze and solve real-life problems related to structure stability.

How are problems on structure stability solved?

Let's us investigate the answers to these questions in this module.

II. LESSONS AND COVERAGE

In this module, you will examine these questions when you study the topics below:

Lesson 1	Definition of Congruent Triangles
Lesson 2	Congruence Postulates
Lesson 3	Proving Congruence of Triangles
Lesson 4	Applications of Triangle Congruence

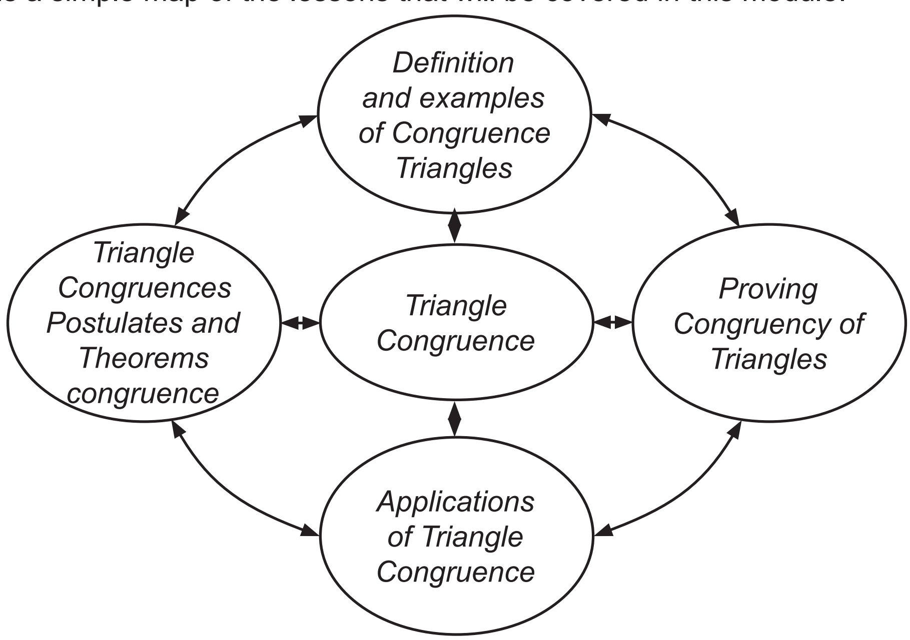
OBJECTIVES:

In these lessons you will learn to:

Lesson 1	Define and illustrate congruent triangles.
Lesson 2	State and illustrate the SAS, ASA, and SSS Congruence Postulates.
Lesson 3	Apply the postulates and theorems on triangle congruence to prove statements on congruences.
Lesson 4	Apply triangle congruences to perpendicular bisector and angle bisector.

Module Map

Here is a simple map of the lessons that will be covered in this module.



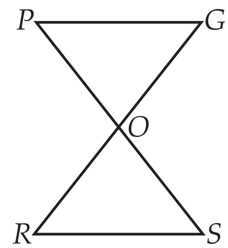
Learning Goals and Targets
To do well in this module, you need to remember and do the following.

1. Define the terms that are unfamiliar to you.
2. Explore the websites which will help you to better understand the lessons.
3 Make a portfolio of your output
4. Answer and complete the exercises provided.
5. Collaborate with the teacher and peers.

Find out how much you already know about this module. Please answer all items. Take note of the items that you were not able to answer correctly and look for the right answer as you go through this module.

III. PRE-ASSESSMENT

1. In the figure $\Delta POG \cong \Delta SOR$, what is the side corresponding to $\overline{PO}$?



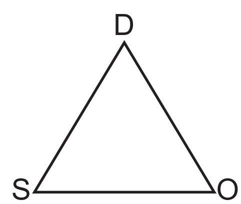
   a. $\overline{OS}$
   b. $\overline{RD}$
   c. $\overline{RS}$
   d. $\overline{SO}$

2. Listed below are the six pairs of corresponding parts of congruent triangles. Name the congruent triangles.

$\overline{SA} \cong \overline{JO}$	$\angle D \cong \angle Y$
$\overline{AD} \cong \overline{OY}$	$\angle A \cong \angle O$
$\overline{SD} \cong \overline{JY}$	$\angle S \cong \angle J$

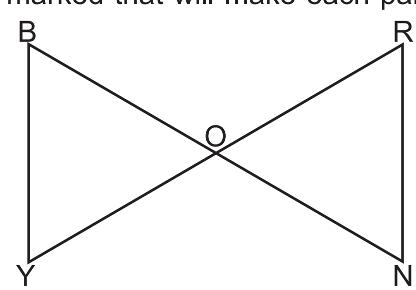
   a. $\Delta ASD \cong \Delta JOY$
   b. $\Delta ADS \cong \Delta YJO$
   c. $\Delta SAD \cong \Delta JOY$
   d. $\Delta SAD \cong \Delta JYO$

3. In ΔDOS, what side is included between $\angle D$ and $\angle O$?



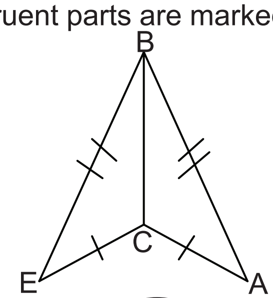
   a. $\overline{DO}$
   b. $\overline{DS}$
   c. $\overline{SD}$
   d. $\overline{SO}$

4. Name the corresponding congruent parts as marked that will make each pair of triangles congruent by SAS.



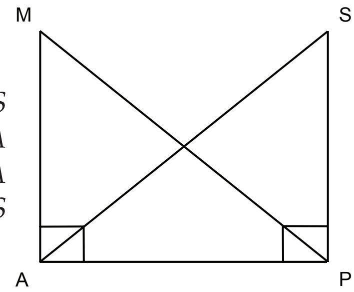
   a. $\overline{BY} \cong \overline{NR}$, $\angle BOY \cong \angle NOR$, $\overline{BO} \cong \overline{NO}$
   b. $\overline{BO} \cong \overline{NO}$, $\angle BOY \cong \angle NOR$, $\overline{RO} \cong \overline{YO}$
   c. $\overline{YO} \cong \overline{OR}$, $\overline{BO} \cong \overline{ON}$, $\angle BOY \cong \angle NOR$
   d. $\angle B \cong \angle N, BO \cong \overline{NO}$, $\overline{OY} \cong \overline{OR}$

5. If corresponding congruent parts are marked, how can you prove $\Delta BEC \cong \Delta BAC$?



   a. ASA
   b. LL
   c. SAS
   d. SSS

6. Identify the pairs of congruent right triangles and tell the congruence theorem used.

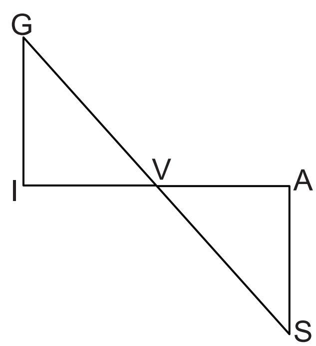


a. $\Delta PMA \cong \Delta APS$
b. $\Delta MAP \cong \Delta SPA$
c. $\Delta MPA \cong \Delta SPA$
d. $\Delta AMP \cong \Delta PAS$

7. What property of congruence is illustrated in the statement? If $\overline{AB} \cong \overline{DE}$, $\overline{EF} \cong \overline{DE}$ then $\overline{AB} \cong \overline{EF}$.

A. Symmetric
B. Transitive
C. Reflexive
D. Multiplication

8. $\Delta GIV \cong SAV$ deduce a statement about point V.

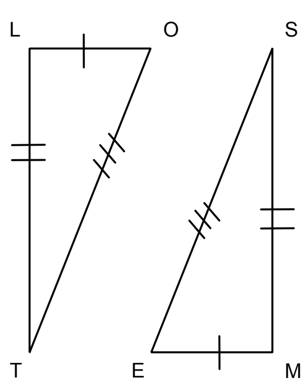


a. V is in the interior of ΔGIV.
b. V is in the exterior of ΔSAV.
c. V is in the midpoint of $\overline{GS}$.
d. V is collinear with G and I.

9. Is the statement "corresponding parts of congruent triangles are congruent" based on

a. Definition
b. Postulate
c. Theorem
d. Axiom

10. Use the marked triangles to write proper congruence statement.



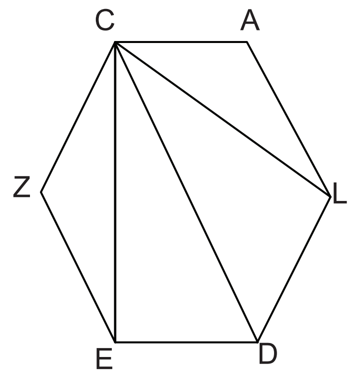
A. $\overline{LT} \cong \overline{MS}$
$\overline{LO} \cong \overline{ME}$
$\overline{OT} \cong \overline{MS}$
$\Delta LOT \cong \Delta MES$

B. $\overline{LT} \cong \overline{SM}$
$\overline{LO} \cong \overline{ME}$
$\overline{OT} \cong \overline{ES}$
$\Delta LOT \cong \Delta SME$

C. $\overline{LT} \cong \overline{MS}$
$\overline{OL} \cong \overline{ME}$
$\overline{OT} \cong \overline{SE}$
$\Delta LOT \cong \Delta MSE$

D. $\overline{TL} \cong \overline{MS}$
$\overline{LO} \cong \overline{ME}$
$\overline{OT} \cong \overline{ME}$
$\Delta TOL \cong \Delta SME$

11. Hexagon CALDEZ has six congruent sides. $\overline{CE}$, $\overline{CD}$, $\overline{CL}$ are drawn on the hexagon forming 4 triangles.
Which triangles can you prove congruent?



a. $\Delta CEZ \cong \Delta CDE$
$\Delta CDE \cong \Delta CAL$
b. $\Delta CEZ \cong \Delta CAL$
$\Delta CED \cong \Delta CLD$
c. $\Delta CED \cong \Delta CEZ$
$\Delta CLA \cong \Delta CLD$
d. $\Delta CZE \cong \Delta CED$
$\Delta DEC \cong \Delta LCD$

12. $\Delta ABC \cong \Delta DEF$, which segment is congruent to $\overline{AB}$:

a. $\overline{BC}$
b. $\overline{AC}$
c. $\overline{DE}$
d. $\overline{EB}$

13. $\Delta SUM \cong \Delta PRO$, which angle is congruent to $\angle M$?

a. $\angle S$
b. $\angle P$
c. $\angle R$
d. $\angle O$

14. $\Delta TIN \cong \Delta CAN$, then ΔNAC is congruent to ____.

a. ΔITN
b. ΔNIT
c. ΔTNI
d. ΔINT

15. Jancent knows that $AB = XY$ and $AC = XZ$. What other information must he know to prove $\Delta ABC \cong \Delta XYZ$ by SAS postulate?

a. $\angle B \cong \angle Y$
b. $\angle C \cong \angle Z$
c. $\angle A \cong \angle X$
d. $\angle C \cong \angle X$

16. Miguel knows that in ΔMIG and ΔJAN, $MI = JA$, $IG = AN$, and $MG = JN$. Which postulate or theorem can he use to prove the triangles congruent?

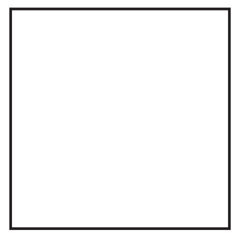
   a. ASA
   b. AAS
   c. ASA
   d. SSS

17. In ΔABC, $AB = AC$. If $m\angle B = 80$, find the measure of $\angle A$.

   a. 20
   b. 80
   c. 100
   d. 180

18. You are tasked to make a design of the flooring of a chapel using triangles. The available materials are square tiles. How are you going to make the design?

   a. Applying triangle congruence by ASA
   b. Applying triangle congruence by SAS.
   c. Applying triangle congruence by SSS
   d. Applying triangle congruence by AAS



For items 19 to 20
Complete the proof. Fill in the blank with the letter of the correct answer.

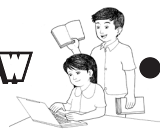
   a. $\overline{CO} \cong \overline{CO}$
   b. ASA
   c. SAS
   d. $\angle BCO \cong \angle ACO$

In ΔABC, let O be a point in AB such that CO bisects $\angle ACB$, if $\overline{AC} \cong \overline{BC}$.
Prove that $\Delta ACO \cong \Delta BCO$.

Statements	Reasons
1. $\overline{AC} \cong \overline{BC}$	1. Given
2. $\overline{CO}$ bisects $\angle ACB$	2. Given
3. ____(19)_____	3. Definition of angle bisector
4. $\overline{CO} \cong \overline{CO}$	4. Reflexive Property of Congruence
5. $\Delta ACD \cong \Delta BCO$	5. ____(20)_______

Lesson 1 Definition of Congruent Triangles

What to Know



Let's begin this lesson by finding out what congruent triangles are. As you go over the activities, keep this question in mind. "**When are two triangles congruent?**"

Activating Prior Knowledge

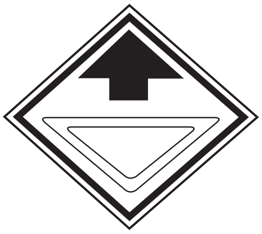
1. What is the symbol for congruence?
2. If $\Delta ABC \cong \Delta XYZ$, what are the six pairs of corresponding congruent parts?
3. How do we measure an angle?
4. How can you draw an angle of specified measure?
5. What is the sum of the measures of the angles of a triangle?

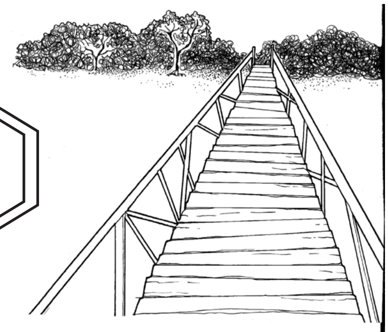
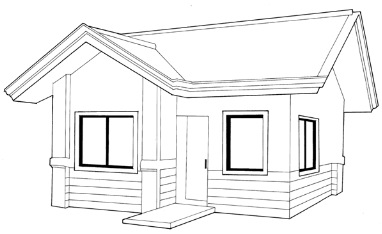
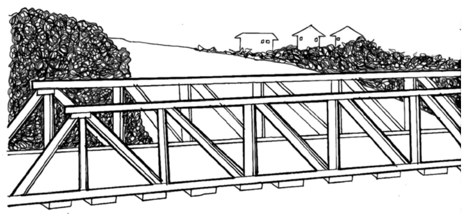
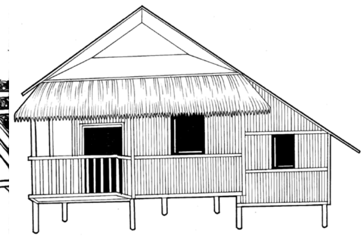
For numbers 6 to 10 define or Illustrate each of the following:

6. Midpoint
7. Vertical angles
8. Right Triangle
9. Hypotenuse
10. Isosceles Triangle

The wonders of Geometry are present everywhere, in nature and in structures. Designs and patterns having the same size and same shape play important roles especially on the stability of buildings and bridges. What ensures the stability of any structures?

Hook: In coming to school, have you met Polygon? Name it and indicate where you met it. (Answers vary, Rectangles windows, 20 peso bill from my pocket, triangles from bridges, and buildings and houses etc.)



Activity 1 PICTURE ANALYSIS

Form a group. Answer the following questions based on the pictures above.

1. How will you relate the picture to your ambition?
2. If you were an architect or an engineer, what is your dream project?
3. What can you say about the long bridge in the picture? How about the tall building? (presence of congruent triangles, its stability, uses of bridges for economic progress,
4. Why are there triangles in the structures? Are the triangles congruent? When are two triangles congruent?
5. Why are bridges and buildings stable?

You gave your initial ideas on congruent triangles and the stability of bridges and buildings.

Let's now find out how others would answer the question and compare their ideas to our own, We will start by doing the next activity.!

What to Process

Let's begin by finding out what congruent triangles are.

Activity 2 FIND YOUR PARTNER

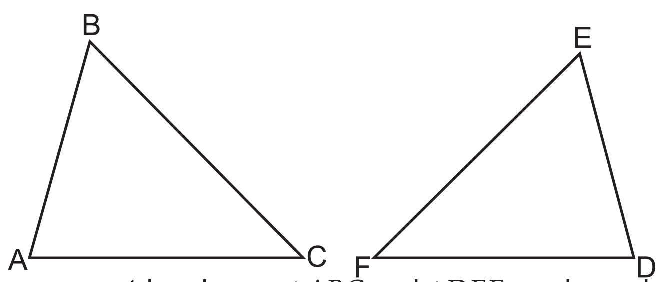
Instruction

Your group (with 10 members) will be given five pairs of congruent figures, each shape for each member. At the count of three, you will find your partner whose holding the same shape as yours.

QUESTIONS ?

1. Why/How did you choose your partner?
2. Describe the two figures you have.
3. What can you say about the size and shape of the two figures?
4. We say that congruent figures have the same size and the same shape. Verify that you have congruent figures.

..For each group pick up a pair of congruent triangles



Name your triangles as ΔABC and ΔDEF as shown in the figure.

Investigate: Matching vertices of the two triangles

First Match: $ABC \leftrightarrow EDF$ (A corresponds to E, B corresponds to D, C corresponds to F)

Second Match: $ABC \leftrightarrow EFD$

Third Match: $ABC \leftrightarrow DEF$

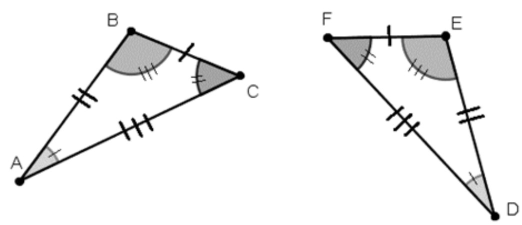
In which of the above pairings are the two triangles congruent? Fill up the activity sheet on the next page.

Group No.___________

Match	Corresponding sides	Congruent or not congruent?	Corresponding Angles	Congruent Or not congruent?
First				
Second				
Third				

> ***Two triangles are congruent** if their vertices can be paired so that corresponding sides are congruent and corresponding angles are congruent.*

$\Delta ABC \cong \Delta DEF$ Read as "triangle ABC is congruent to triangle DEF."
$\cong$ symbol for congruency
Δ symbol for triangle.

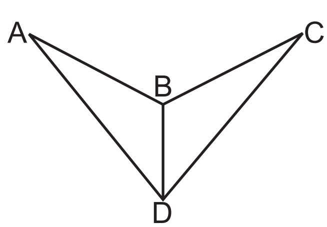


The congruent corresponding parts are marked identically.
Can you name the corresponding congruent sides? Corresponding congruent angles?

Answer the questions below write your answers in your journal:

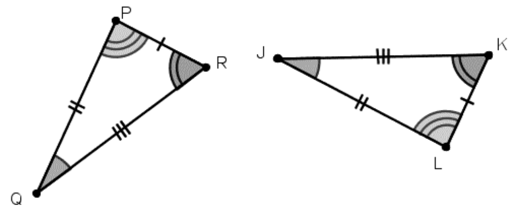
- ✓ What are congruent triangles?
- ✓ How many pairs of corresponding parts are congruent if two triangles are congruent?
- ✓ Illustrate $\Delta TNX \cong \Delta HOP$ Put identical markings on congruent corresponding parts.
- ✓ Where do you see congruent triangles?

Exercise 1



1. ΔABD ≅ ΔCBD, Write down the six pairs of congruent corresponding parts
2. Which triangles are congruent if $\overline{MA} \cong \overline{KF}$, $\overline{AX} \cong \overline{FC}$, $\overline{MX} \cong \overline{KC}$; $\angle M \cong \angle K$, $\angle A \cong \angle F$, $\angle X \cong \angle C$. Draw the triangles.

3. Which of the following shows the correct congruence statement for the figure below?



a. $\Delta PQR \cong \Delta KJL$
b. $\Delta PQR \cong \Delta LJK$
c. $\Delta PQR \cong \Delta LKJ$
d. $\Delta PQR \cong \Delta JLK$

You can now define what congruent triangles are .In order to say that the two triangles are congruent, we must show that all six pairs of corresponding parts of the two triangles are congruent.

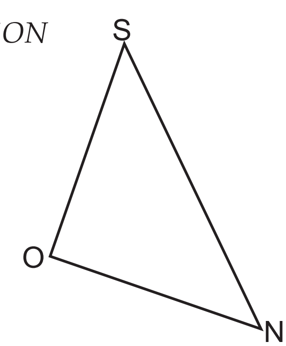
Let us see how can we verify two triangles congruent using fewer pairs of congruent corresponding parts.

Topic 2: Triangle Congruence Postulates

Before we study the postulates that give some ways to show that the two triangles are congruent given less number of corresponding congruent parts, let us first identify the parts of a triangle in terms of their relative positions..

Included angle is the angle between two sides of a triangle.
Included side is the side common to two angles of a triangle.

In ΔSON



$\angle S$ is included between $\overline{SN}$ and $\overline{SO}$.
$\angle O$ is included between $\overline{OS}$ and $\overline{ON}$.
$\angle N$ is included between $\overline{NS}$ and $\overline{NO}$.
$\overline{SO}$ is included between $\angle S$ and $\angle O$.
$\overline{ON}$ is included between $\angle O$ and $\angle N$.
$\overline{SN}$ is included between $\angle S$ and $\angle N$.

Exercise 2

Given ΔFOR, can you answer the following questions even without the figure?

1. What is the included angle between $\overline{FO}$ and $\overline{OR}$?
2. What is the Included angle between $\overline{FR}$ and $\overline{FO}$?
3. What is the included angle between $\overline{FR}$ and $\overline{RO}$?
4. What is the included side between $\angle F$ and $\angle R$?
5. What is the included side between $\angle O$ and $\angle R$?
6. What is the included side between $\angle F$ and $\angle O$?

Activity 3 LESS IS MORE

SAS (Side-Angle-Side) Congruence Postulate

1. Prepare a ruler, a protractor ,a pencil and a bond paper.
2. Work in group of four.
3. Follow the demonstration by the teacher.
   a. Draw a 7- inch segment.
   b. Name it $\overline{BE}$.
   c. Using your protractor make angle B equal to 70° degrees.
   d. From the vertex draw $\overline{BL}$ measuring 8 inches long.
   e. How many triangles can be formed?
   f. Draw ΔBEL
   g. Compare your triangle with the triangles of the other members of the group. Do you have congruent triangles?
   h. Lay one triangle on top of the others. Are all the corresponding sides congruent? How about the corresponding angles?
   i. What can you say about any pair of congruent triangles ?

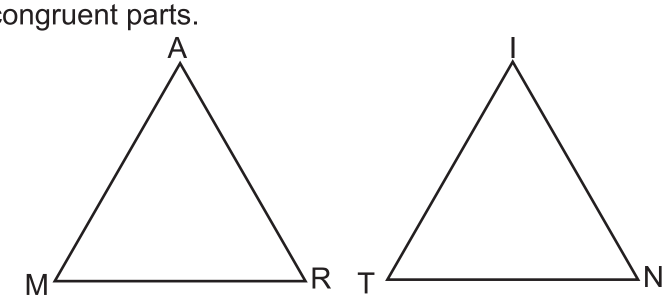
SAS (Side-Angle-Side) Congruence Postulate

If the two sides and an included angle of one triangle are congruent to the corresponding two sides and the included angle of another triangle, then the triangles are congruent.

If $\overline{MA} \cong \overline{TI}$, $\angle M \cong \angle T$, $\overline{MR} \cong \overline{TN}$

Then $\Delta MAR \cong \Delta TIN$ by SAS Congruence Postulate

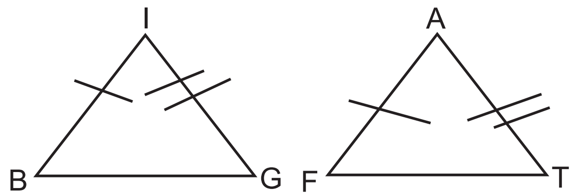
Mark the congruent parts.



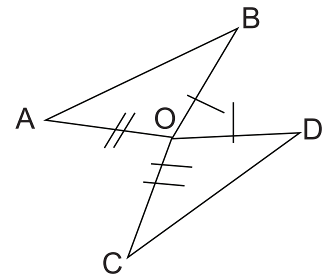
Exercise 3

Complete the congruence statement using the SAS congruence postulate.

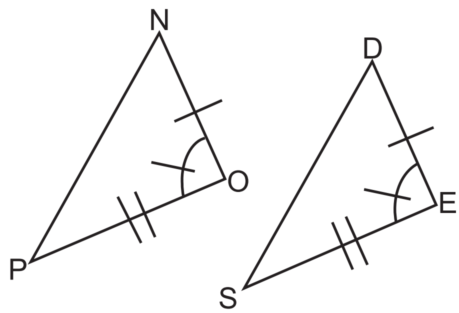
1. $\Delta BIG \cong \Delta$_____



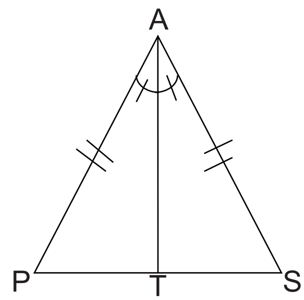
3. $\Delta ABO \cong \Delta$_____



2. $\Delta PON \cong \Delta$____



4. $\Delta PAT \cong \Delta$_____



After showing that the two triangles are congruent showing only two sides and the included angle of one triangle to two sides and included angle of another triangle, you try another way by doing activity 4

Activity 4 TRY MORE

ASA (Angle-Side Angle) Congruence

Prepare the following materials; pencil, ruler, protractor, a pair of scissors

Working independently, use a ruler and a protractor to draw ΔBOY with two angles and the included side having the following measures: $m\angle B$ = 50, $m\angle O$ = 70 and $\overline{BO}$ =18 cm

1. Draw $\overline{BO}$ measuring 18 cm
2. With B as vertex draw angle B measuring 50,
3. With O as vertex draw angle O measuring 70,
4. Name the intersection as Y.
5. Cut out the triangle and compare it with four of your classmates.
6. Describe the triangles.
7. Put identical marks on the congruent corresponding sides and angles.
8. Identify the parts of the triangles which are given congruent.

ASA (Angle-Side-Angle) Congruence Postulate

If the two angles and the included side of one triangle are congruent to the corresponding two angles and an included side of another triangle, then the triangles are congruent.

If $\angle A \cong \angle E$, $\overline{JA} \cong \overline{ME}$, $\angle J \cong \angle M$, ***then*** $\Delta JAY \cong \Delta MEL$

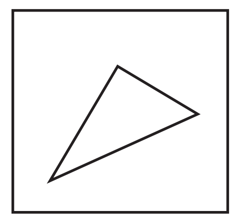
Draw the triangles and mark the congruent parts.

Activity 5 SIDE UP

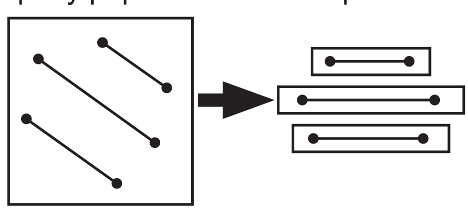
SSS (Side-Side-Side) Congruence Postulate

You need patty papers, pencil, a pair of scissors

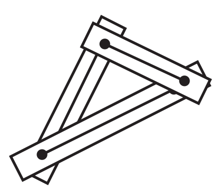
1. Draw a large scalene triangle on your patty paper.



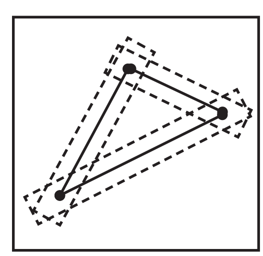
2. Copy the three sides separately onto another patty paper and mark a dot at each endpoint. Cut the patty paper into three strips with one side on each strip.



3. Arrange the three segments into a triangle by placing one endpoint on top of the another.



4. With a third patty paper, trace the triangle formed. Compare the new triangle with the original triangle. Are they congruent?



5. Try rearranging the three segments into another triangle. Can you make a triangle not congruent to the original triangle? Compare your results with the results of your classmates.

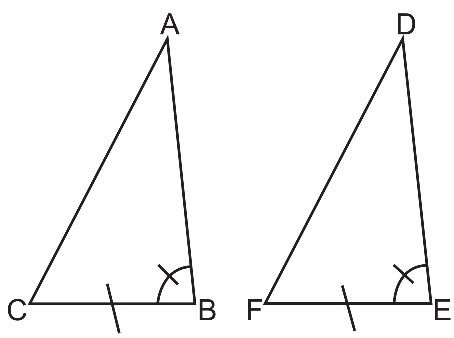
SSS (Side-Side-Side) Congruence Postulate

If the three sides of one triangle are congruent to the three sides of another triangle, then the triangles are congruent.

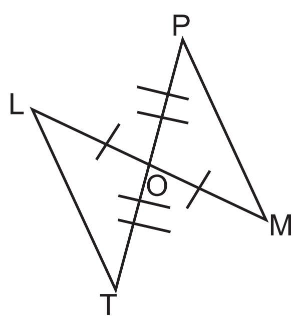
If $\overline{EC} \cong \overline{BP}$, $\overline{ES} \cong \overline{BJ}$, $\overline{CS} \cong \overline{PJ}$, then $\Delta ESC \cong \Delta BJP$, draw the triangles and mark the congruent parts., then answer exercise 4.

Exercise 4

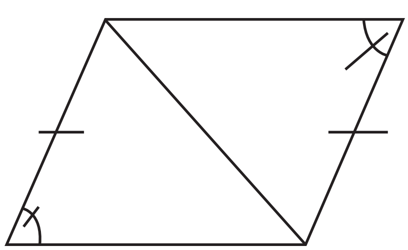
Corresponding congruent parts are marked. Indicate the additional corresponding parts needed to make the triangles congruent by using the specified congruence postulates.



a. ASA _______
b. SAS _______



a. SAS ______
b. SSS ______



a. SAS ______
b. ASA ______

Now that you can show triangles congruent with

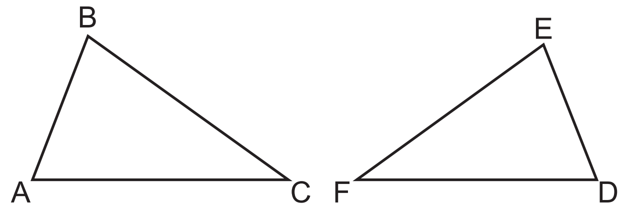
- two corresponding sides and an included angle;
- two angles and an included side;
- three pairs of corresponding sides *congruent, you are now ready to prove two triangles congruent deductively.*

Topic 3: Proving Triangle Congruence

Activity 6 LET'S DO IT

Let's find out how we can apply the Congruence Postulates to prove two triangles congruent. Study the following examples and answer exercise 5.

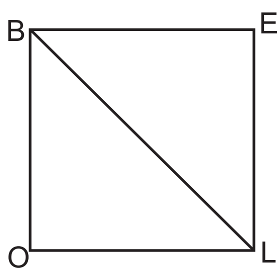
Given: $\overline{AB} \cong \overline{DE}$
$\angle B \cong \angle E$
$\overline{BC} \cong \overline{EF}$
Prove: $\Delta ABC \cong \Delta DEF$



Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$	1. Given
2. $\angle B \cong \angle E$	2. Given
3. $\overline{BC} \cong \overline{EF}$	3. Given
4. $\Delta ABC \cong \Delta DEF$	4. SAS Postulate

Exercise 5

Try this



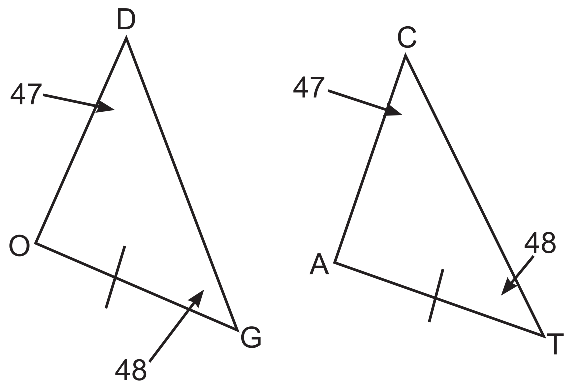
Given: $\overline{BE} \cong \overline{LO}$, $\overline{BO} \cong \overline{LE}$
Prove: $\Delta BEL \cong \Delta LOB$

Complete the Proof:

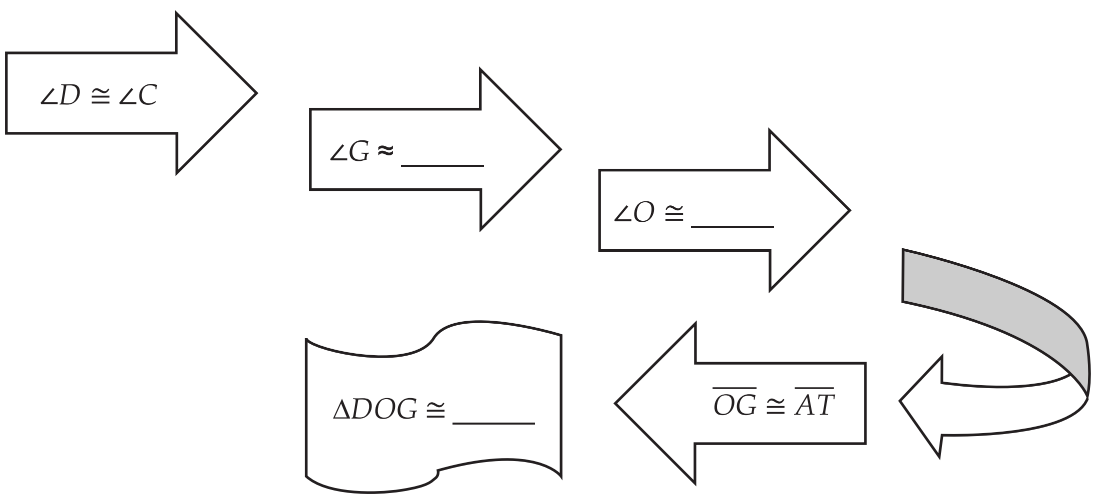
Statements	Reasons
1.	1. Given
2. $\overline{BO} \cong \overline{LE}$	2.
3	3.
4. $\Delta BEL \cong \Delta LOB$	4.

Let's try to prove a theorem on congruence,

Given the triangles below, a pair of corresponding sides are congruent, and two pairs of corresponding angles have the same measure.



Work in Pairs to discuss the proof of the theorem by completing the flow chart



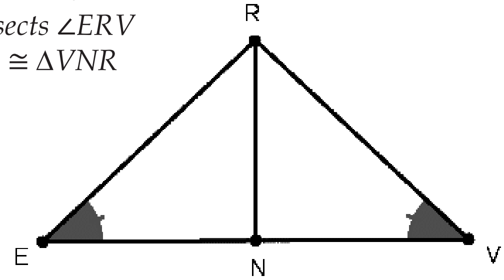
Supply the reason for each

> When you completed the proof, review the parts of the two triangles which are given congruent.
> Have you realized that you have just proved the AAS congruence Theorem?

AAS (Angle-Angle-Side) Congruence Theorem

If two angles and a non-included side of one triangle are congruent to the corresponding two angles and a non-included side of another triangle, then the triangles are congruent.

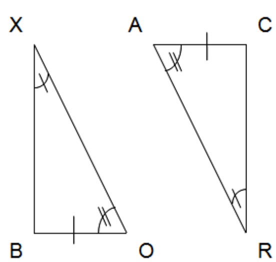
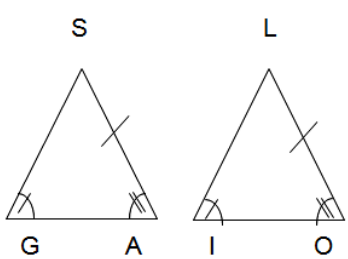
Example:
Given: $\angle NER \cong \angle NVR$
$\overline{RN}$ *bisects* $\angle ERV$
Prove: $\Delta ENR \cong \Delta VNR$

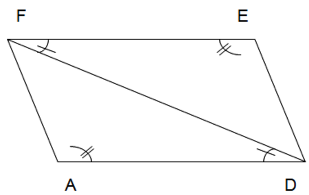
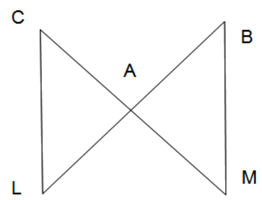


Statements	Reasons
1. $\angle NER \cong \angle NVR$	1. Given
2. $\overline{RN}$ *bisects* $\angle ERV$	2. Given
3. $\angle NER \cong \angle NVR$	3. Definition of angle bisector
4. $\overline{RN} \cong \overline{RN}$	4. Reflexive Property
5. $\Delta ENR \cong \Delta VNR$	5. AAS Postulate

Exercise 6

Complete the congruence statement by AAS congruence.

Figure	Congruence Statement
	$\Delta BOX \cong$ _________
	$\Delta GAS \cong$ _________

<table>
<tr>
<td>
</td>
<td>$\Delta FED \cong$ __________</td>
</tr>
<tr>
<td>

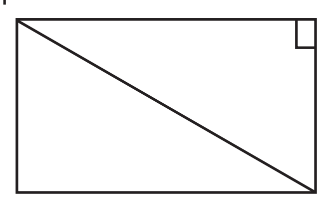
$\overline{CM}$ bisects $\overline{BL}$ at A
$\angle L \cong \angle B$</td>
<td>$\Delta BAM \cong$ __________</td>
</tr>
</table>

How are we going to apply the congruence postulates and theorems.
In right triangles? Let us now consider the test for proving two right triangles congruent.

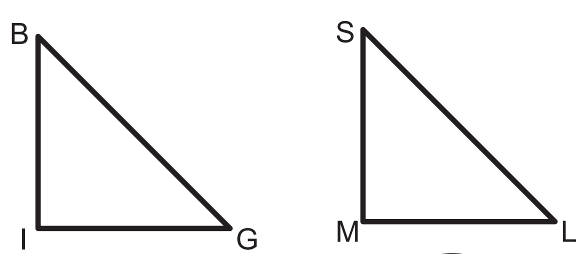
Activity 7 KEEP RIGHT

Recall the parts of a right triangle with your groupmates.

1. Get a rectangular sheet of paper.
2. Divide the rectangle diagonally as shown.



3. Discuss with your group and illustrate the the sides and angles of a right triangle using your cut outs
   - What do you call the side opposite the right angle?
   - What do you call the perpendicular sides?
   - How many acute angles are there in a right triangle?
4. Name your triangles as shown below



5 . If ΔBIG and ΔSML are right triangles, $\angle \boldsymbol{I}$ and $\angle \boldsymbol{M}$ are right, $\overline{\boldsymbol{BI}} \cong \overline{\boldsymbol{SM}}$, $\overline{\boldsymbol{IG}} \cong \overline{\boldsymbol{ML}}$ prove $\Delta \boldsymbol{BIG} \cong \Delta \boldsymbol{SML}$.

6. Discuss the proof with your group.

7. Answer the following questions:
   - What kind of triangles did you prove congruent?
   - What parts of the right triangles are given congruent?
   - Complete the statement: If the _______ of one right triangle are congruent to the corresponding ____ of another right triangle, then the triangles are ______.

Since all right angles are congruent you can now use only two pairs of corresponding parts congruent in order to prove two triangles congruent, The proof you have shown is the proof of the LL Congruence Theorem .

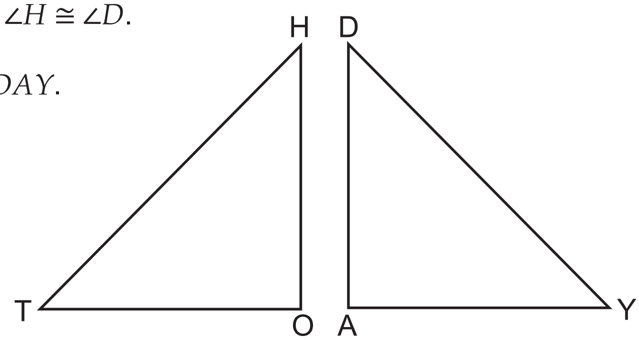
LL Congruence Theorem

If the legs of one right triangle are congruent to the legs of another right triangle, then the triangles are congruent.

The LL Congruence Theorem was deduced from SAS Congruence Postulate.

Consider the right triangles HOT and DAY with right angles at O and A, respectively, such that $\overline{HO} \cong \overline{DA}$, and $\angle H \cong \angle D$.

Prove: $\Delta HOT \cong \Delta DAY$.



Each group will present the proof to the class either by two column form or using flow chart or paragraph form to deduce the theorem:

LA (leg-acute angle) Congruence Theorem

If a leg and an acute angle of one right triangle are congruent to a leg and an acute angle of another right triangle, then the triangles are congruent.

Now it's your turn to prove the other two theorems on the congruence of right triangles.

Activity 8 IT'S MY TURN

Each group will make a power point presentation using flowchart to prove the following theorems.

HyL (Hypotenuse-leg) Congruence Theorem

If the hypotenuse and a leg of one right triangle are congruent to the corresponding hypotenuse and a leg of another triangle, then the triangles are congruent.

HyA (Hypotenuse-Acute angle) Congruence Theorem

If the hypotenuse and an acute angle of one right triangle are congruent to the corresponding hypotenuse and an acute angle of another right triangle, then the triangles are congruent.

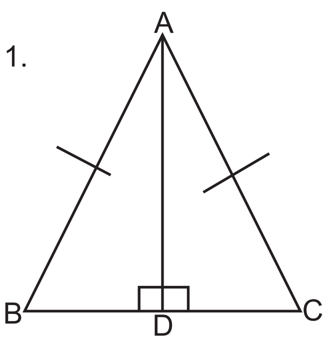
Guide:

1. Draw the figure.
2. What is given and what is to be proved?
3. Write the proof in two-column form.

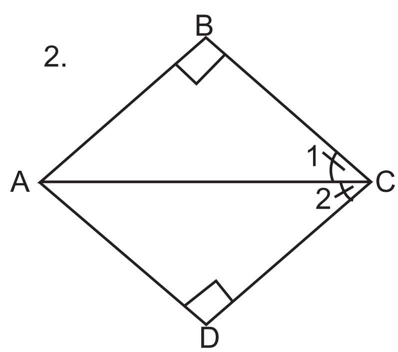
Exercise 7

In each figure, congruent parts are marked. Give additional congruent parts to prove that the right triangles are congruent and state the congruence theorem that .justifies your answer.

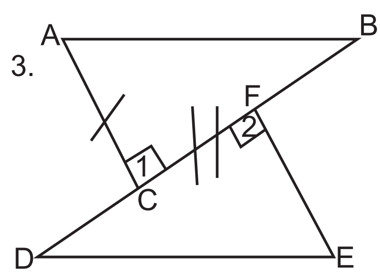
1.



2.

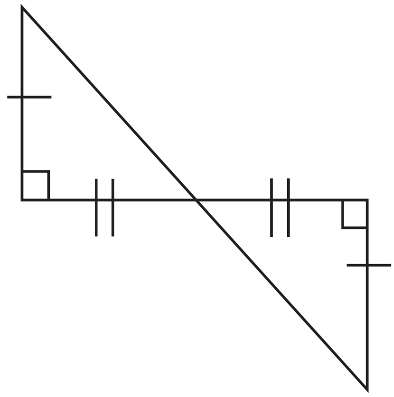


3.

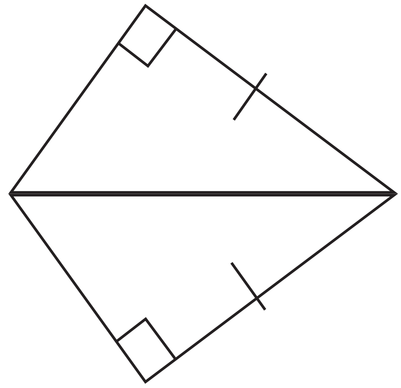


State a congruence theorem. on right triangles.

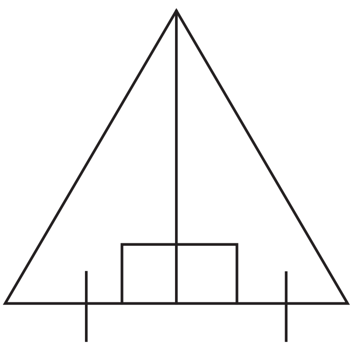
4. __________



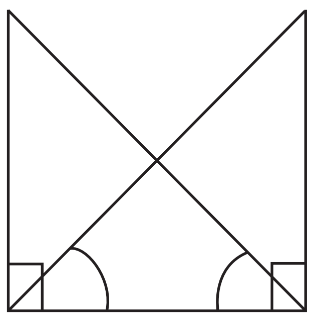
5. __________



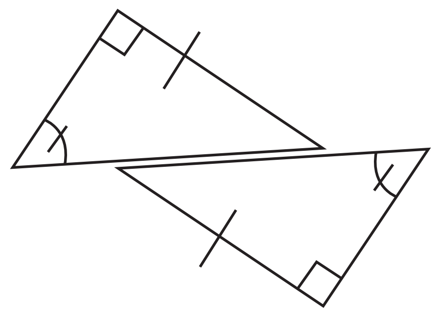
6.



7.



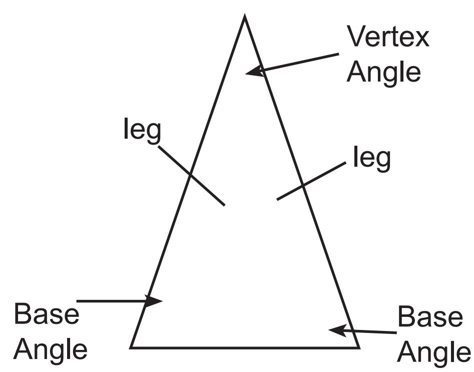
8.



Topic 4: Application of Triangle Congruence

> *After studying the congruence postulates and theorems you are now ready to apply them.*
> *How can you prove that two angles or two segments are congruent?*
> *If they are parts of congruent triangles we can conclude that they are congruent. Let us see how.*

Activity 9 WHAT ELSE?



Do you still remember what an isosceles triangle is?

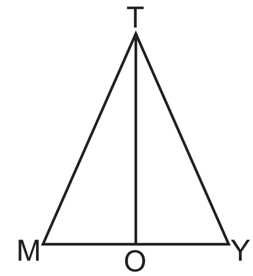
A triangle is isosceles if two of its sides are congruent. The congruent sides are its legs; the third side is the base; the angles opposite the congruent sides are the base angles; and the angle included by the legs is the vertex angle.

Consider ΔTMY with $\overline{TM} \cong \overline{TY}$

Is $\angle M \cong \angle Y$?

You find out by completing the proof.

Remember that if they are corresponding parts of congruent triangles then they are congruent.



1. Draw the bisector TO of $\angle T$ which intersects $\overline{MY}$ at O.
2. ________ ≅ _________ by definition of a bisector
3. ________ ≅ _________ given
4. ________ ≅ _________ (Why) ______________
5. ________ ≅ _________ SAS
6. $\angle M \cong \angle Y$ ____________________

Isosceles Triangle Theorem:

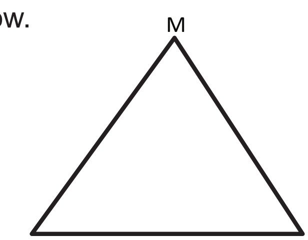
If two sides of a triangle are congruent then the angles opposite these sides are congruent.

How about the converse of isosceles triangle theorem? **If two angles of a triangle are congruent then the sides opposite these angles are congruent.**

Exercise 8

Prove the converse of the isosceles triangle theorem with your group.

Discuss with your group the proof of the statement: **An equilateral triangle is equiangular.** Use the figure and be guided by the questions below.



Given: ΔMIS is equilateral

Prove: ΔMIS is equiangular

In order to prove that ΔMIS is equiangular you must prove first that $\angle M \cong \angle I \cong \angle S$

1. $\overline{MI} \cong \overline{MS}$ Why?
2. What kind of triangle is ΔMIS?.
3. What angles are congruent? Why?
4. $\overline{MI} \cong \overline{MS}$ Why?
5. What angles are congruent? Why?
6 $\angle M \cong \angle I \cong \angle S$ Why?

How will you show that each angle of an equilateral triangle measures 60°?

Guide Questions:

a. What is the sum of the measures of the angles of a triangle?

b. What is true about equilateral triangle?

Exercise 9

1. What is the difference between an equilateral triangle and isosceles triangle?
2. One angle of an isosceles triangle measures 60°. What are the measures of the other two angles?
3. An angle of an isosceles triangle is 50°. What are the measures of the other two angles? Is there another possible triangle?

Discuss the proof of: **The bisector of the vertex angle of an isosceles triangle is perpendicular to the base at its midpoint.** Do this with your group.

Procedure:

a. Draw an Isosceles ΔABC.
b. Draw the bisector BE of the vertex $\angle B$ which intersects $\overline{AC}$ at E.
c. Prove that the two triangles BEA and BEC are congruent.
d. Show that E is the midpoint $\overline{AC}$.
e. Show $\overline{BE}$ is perpendicular to $\overline{AC}$ at E. (Remember that segments are perpendicular if they form right angles.)

Your work will be presented in class.

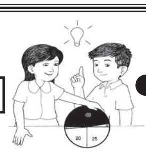
Theorem: The bisector of the vertex angle of an isosceles triangle is perpendicular to the base at its midpoint.

In this section, the discussion was on Congruent Triangles.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let's go deeper by moving on to the next section.

What to Understand



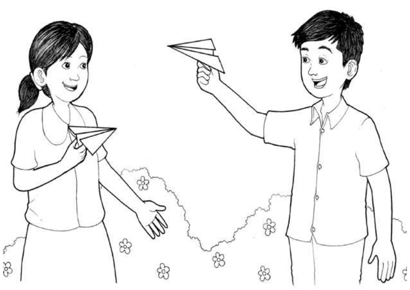
Your goal in this section is to take a closer look at some aspects of the topic.

Keep in your mind the question: **"How does knowledge in triangle congruence will help you to solve real life problems?"**

Questions:

- When are two triangles congruent?
- What are the conditions for triangle congruence.?
- How can we show congruent triangles through paper folding?
- Say something about Isosceles triangle.
- Is equilateral triangle isosceles?
- Is equilateral triangle equiangular?
- What can you say about the bisector of the vertex angle of an isosceles triangle?

Activity 10 FLY FLY FLY



During the Math Fair, one of the activities is a symposium in which the delegates will report on an inquiry about an important concept in Math. You will report on how congruent triangles are applied in real-life. Your query revolves around this situation;

1. Design at most 5 different paper planes using congruent triangles.
2. Fly the paper planes one at a time. Record the flying time of each plane. Then, choose the most stable one.
3. Point out the factors that affect the stability of the plane.
4. Explain why such principle works.
5. Draw out conclusion and make recommendations.

Procedure:

1. Each group will prepare 5 paper planes
2. Apply your knowledge on triangle congruence.
3. Follow steps 2 to 5.
4. What is the importance of congruent triangles in making paper planes?

Activity 11 SARANGOLA NI PEPE

Another application of congruent triangles is on stability of your kites.

Show us how triangle congruence works.

In the upcoming City Festival, there will be a kite flying. You are to submit a certain design of kite and an instruction guide of how it operates. The designer who can come up with a kite which can fly the longest wins a prize.

Present the mechanics on how you come up with such a design.

Activity 12 3 – 2 – 1 CARD

Since you are done with the concepts and activities about triangle congruence, now let us summarize it by completing the table below:

3	things you have learned
2	things which are interesting
1	question you still have in mind

What to Transfer

Your goal in this task is to apply your real-life situations. You will be given a practical task which will demonstrate your understanding of triangle congruence.

Performance
GRASPS TASK

(S) One of the moves of the City Council for economic development is to connect a nearby island to the mainland with a suspension bridge for easy accessibility of the people. Those from the island can deliver their produce and those from the mainland can enjoy the beautiful scenery and beaches of the island.

(R) As one of the engineers of the DPWH who is commissioned by the Special Project Committee, (G) you are tasked to present (P) a design/blueprint (P) of a suspension bridge to the (A) City Council together with the City Engineers. (S) Your presentation will be evaluated according to its accuracy, practicality, stability and mathematical reasoning.

Now that you are done check your work with the rubric below.

CRITERIA	Outstanding 4	Satisfactory 3	Developing 2	Beginning 1	RATING
Accuracy	The computations are accurate and show a wise use of the concepts of triangle congruence.	The computations are accurate and show the use of the concepts of triangle congruence.	Some computations are erroneous and show the use of some concepts of triangle congruence.	The computations are erroneous and do not show the use of the concepts of triangle congruence.	
Creativity	The design is comprehensive and displays the aesthetic aspects of the mathematical concepts learned.	The design is presentable and makes use of the concepts of geometric representations.	The design makes use of the geometric representations but not presentable.	The design doesn't use geometric representations and not presentable.	
Stability	The design is stable, comprehensive and displays the aesthetic aspects of the principles of triangle congruence.	The design is stable, presentable and makes use of congruent triangles.	The design makes use of triangles but not stable.	The design does not use triangles and is not stable.	
Mathematical reasoning	The explanation is clear, exhaustive or thorough and coherent. It includes interesting facts and principles.	The explanation is clear and coherent. It covers the important concepts.	The explanation is understandable but not logical.	The explanation is incomplete and inconsistent.	
				OVERALL RATING	

Another challenge to you is this task for you to accomplish at home

Submit a journal on how you proved two triangles congruent. Did you enjoy the lesson on triangle congruence?

Take a picture of triangles in the house. Identify how each of these congruences could help a builder to construct a furniture. Make a portfolio of these pictures and discussion.

SUMMARY/SYNTHESIS/GENERALIZATION

Designs and patterns having the same size and the same shape are seen in almost all places. You can see them in bridges, buildings, towers, in furniture even in handicrafts and fabrics

Congruence of triangles has many applications in real world. Architects and engineers use triangles when they build structures because they are considered to be the most stable of all geometric figures. Triangles are oftentimes used as frameworks, supports for many construction works. They need to be congruent.

In this module you have learned that:

- Two triangles are congruent if their vertices can be paired such that corresponding sides are congruent and corresponding angles are congruent.
- The three postulates for triangle congruence are:
  a. *SAS Congruence* – if two sides and the included angle of one triangle are congruent respectively two sides and the included angle of another triangle then the triangles are congruent.
  b. *ASA Congruence* – if two angles and the included side of one triangle are congruent respectively two angles and the included side of another triangle then the triangles are congruent.
  c. *SSS Congruence* – if the three sides of one triangle are congruent respectively three sides of another triangles then the triangles are congruent.
- AAS Congruence Theorem – if the two angles and the non-included side of one triangle are congruent to the two angles and the non-included side of another triangle than the triangles are congruent.
- The congruence theorems for right triangles are:
  a. *LL Congruence* – if the legs of one right triangle are congruent respectively to the legs of another right triangle, then the triangles are congruent.
  b. *LA Congruence* – if a leg and an acute angle of one triangle are congruent respectively to a leg and an acute angle of another right triangle, then the triangles are congruent.
  c. *HyL Congruence* – if the hypotenuse and a leg of one right triangle are congruent respectively to the hypotenuse and a leg of another right triangle, the triangles are congruent.
  d. *HyA Congruence* – if the hypotenuse and an acute angle of one right triangle are congruent respectively to the hypotenuse and an acute angle of another right triangle, then the triangles are congruent.
- Isosceles Triangle Theorem – If two sides of a triangle are congruent then the angles opposite these sides are congruent.
- Converse of Isosceles Triangle Theorem – if two angles of a triangle are congruent then the sides opposite these angles are congruent.
- An equilateral triangle is equiangular.
- The measure of each angle of an equilateral triangle is 60°.