EDITORIAL

The issues of Affect in Mathematics from several corners of the world are presented in this Volume 6 Number 3 of MTRJ on line. Lynn Columba with her collaborators Robin Hojnoski and Joy Polignano investigates the impact of book reading to pre-kindergarten children by their parents or caring adults on children mathematics learning, while Sergiy Klymchuk with Cheng Chun Chor Litwin from Hong Kong compares the role of attention in the assessment of teachers' solutions of provocative questions in the sense that they looked like routine ones but in fact they had some catch. "The results from the test were startling – the vast majority of the teachers did not notice any catch and gave incorrect answers to most questions in the test."

As an interesting comparison of styles, approaches and philosophical frameworks we publish two papers of Chinese teachers of mathematics from Fuzhou, participants of the CTRAS 5 (Conference of Teaching-Research for All Students), which takes place once a year in China through the US-China collaboration organized by Prof. An Suhua from UC in Long Beach. Teaching-Research is mandated in all schools across China, and it's fascinating to observe, listen and read Chinese approaches to known to us teaching problems. It is interesting to observe the poetic classroom language they use: *intelligence is a typical character produced and expressed in education situations, and it is directed by* _virtue_ *and* _creativity., the class will be_ _wise and brilliant._

At the same time TR in China seem to rely much more on teacher's intuition, metaphors and the like, with decisive absence of measurement techniques characteristic for western approaches.

The papers of Chinese teachers are linked with their ppt presentations collected in Fuzhou, China at the CTRAS 5 conference.

LIST OF CONTENT

THE CREATION OF INTELLIGENT AND WISE CLASSES
IN HIGH SCHOOL MATHEMATICS TEACHING

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Abstract: Intelligent and wise classes in high school mathematics teaching aim to improve students' personality and to promote the development of students' intelligence. The teacher should play the role of guide in creating intelligent and wise classes. Therefore, in teaching and learning activities, teachers should give full play to their educational wisdom to focus on leading students to develop their thinking, and on creating a harmonious and active class. The art of creating intelligent and wise classes is reflected in the intelligence the teacher used in teaching. The purpose of this paper is to provide vivid teaching cases to present how to construct an intelligent and wise class through creating teaching scenarios, capturing teaching information, guiding students by creative generation in teaching and transforming academic form.

Key words：intelligent and wise class construction guidance

INTRODUCTION

Professor Cheng Shangrong synthesized research about intelligence in and out of China, and came to the conclusion that intelligence is a typical character produced and expressed in education situations, and it is directed by virtue and creativity. The purpose of obtaining intelligence is to cultivate and develop the students' capability, mathematics sensitivity and sudden enlightenment. Having the quick wit to respond in class is the primary expression of intelligence. The combination of scientific literacy and humanistic quality provide its foundation and tension. Based on these classroom teaching models, intelligent and wise classes in high school mathematics teaching are aimed at improving students' personality and promoting their intelligence. The teachers play the role of the guide, so they must put into full use their educational wisdom concerning students' development and meanwhile train students' thinking and create a harmonious and efficient classroom instruction model. The art of the creation of intelligent and wise classes in high school mathematics teaching is established by teachers' intelligence to lead the class. This article is to provide vivid teaching cases to present how to construct an intelligent and wise class through creating teaching scenarios, capturing teaching information, guiding students by creative generation in teaching and transforming academic form.

ESTABLISHING TEACHING SCENARIO IS A DRIVING FORCE AND A RICH SOURCE OF LEARNING.

What is the charm of TV series? It lies in the suspense TV series create, which is designed in different scenarios. It fires audiences' desire to explore. A German scholar once said, "You can't eat 15g of salt if it is just salt. But if it is mixed up in a bowl of delicious soup, you can easily get it in the process of enjoying the meal." Teaching

scenarios are just like the soup. The knowledge cannot be absorbed unless it is dissolved in suitable scenarios. How to inspire students' curiosity to probe and how to create an active class? That's the question raised in mathematics teaching. Uninteresting knowledge has a hard time attracting students' attention, while good questions can stimulate their main and their desire to take part in the class activities. Zeng Rong is a Master Teacher in Fudan University. Once he asked a question about the summation of infinite decreasing geometric series in his class. He asked:" What do you think when you see 0.9=0.999? How is it changed by the fraction method? What is the value? Is it equal to 1? Can you prove it? Why? " A series of questions firmly capture the students' curiosity, and lay the foundation to further explore the summation of infinite decreasing geometric series. Without doubt, such interesting and challenging questions inspire students' interest to think more, and scenarios created by these questions become a driving force and a rich source to create Intelligent and wise classes in high school mathematics teaching.

GENERATIVE TEACHING IS AN IMPORTANT WAY TO CONSTRUCT INTELLIGENT AND WISE CLASSES.

All intelligence comes from creativity. Generative teaching under the teachers' guidance is an important expression of classroom creativity. Generation includes predictable generation and unpredictable generation. Predictable generation means that after making full preparations for the classes, including carefully reviewing materials and understanding the students, the teacher leads students to carry on creative activities.

There is an example in the high school mathematics book IV *Coordinates Representation of Plane Vector Collinear,* published by People's Education Press. Suppose there is a point on the segment P_1P_2. The coordinates of P_1 and P_2 are (x_1, y_1) and (x_2, y_2) respectively.

(1)Find the coordinate of point P if point P is the middle point of the segment P_1P_2.

(2)Find the coordinates of point P if P is one of the points that trisect the segment P_1P_2 .

Thoroughly understanding the mathematical thought process behind the example and finding different ways to solve it is a good resource for generation. In the teaching of this example, the teacher should make good use of predictable generation to lead students to solve the problem: if you know the midpoint of the segment as well as the abscissa and ordinate of two points that trisect the segment, can you find the coordinates of 3 points that quadrisect the segment? What about the coordinates of 4 points that divide the segment into 5 parts equally? What about the coordinates of the points that divide the segment into n parts equally? Can you deduce and prove the conclusion you come to? Following this pattern, students can develop and exercise their own intelligence under the guidance of the teacher.

Unpredictable generation is about valuable and creative activities produced during the communication between students and teachers, or among students when they are learning. For instance, the students in the third year encounter the following example when reviewing: if $s_n = \dfrac{5}{4}n^2 + \dfrac{7}{12}n$ is the sum of the antecedent of the arithmetic progression $\{a_n\}$, how do you find the general term formula for a_n ?

Such an example is very common, but it's not easy to solve. If students know $a_n = \begin{cases} s_1(n=1) \\ s_n - s_{n-1}(n \geq 2) \end{cases}$, they will arrive quickly at the result of $a_n = \dfrac{5}{2}n - \dfrac{2}{3}$. In the process of solving this question, one student finds there is a special relationship between $\dfrac{5}{4}n^2$ and $\dfrac{5}{2}n$. So he wonders if we can find the value of a_n when s_n is differentiated.

To this unexpected generation, if I don't take it into consideration and just easily deny $s_n{}' \neq a_n$ and refuse the students' idea, I will never explore the question further: is there

any other way to solve the problem by leading the students from the common problems

and contrasting the coefficient of s_n' and a_n ?

After students' exploration, we can find a surprising result that if the general term

formula of the arithmetic progression $\{a_n\}$ is $a_n = pn + q$, the sum of the first n terms is

$$s_n = \frac{n(p + q + pn + q)}{2} = \frac{n(pn + p + 2q)}{2} = \frac{pn^2}{2} + \frac{(p + 2q)n}{2} , \quad s_n' = pn + \frac{p + 2q}{2} , \quad \text{and}$$

further find the values of q through the constant term $\dfrac{p + 2q}{2}$ of s_n' and the values of p.
This finding is worth popularizing to solve such kinds of mathematical problems.
However, in the process of generative teaching, the teacher should protect the students'
desire to observe, guess, and create. For the students who have a hard time with effective
generation, the teacher could use different ways to deal with them. For example, drawing
inferences about other cases from one instance, probing into the new question under the
guidance of the teacher, referring back to the problem temporally, and evaluating
extemporaneously could be used.

INFORMATION CAPTURE IS AN EFFECTIVE COMPLEMENT TO CONSTRUCT
INTELLIGENT AND WISE CLASSES.

Sukhomlinski pointed out, "Education is not the skill of being able to foresee all the
details of a course, but to capture some valuable details in the light of prevailing
circumstances, and skillfully to make corresponding adjustments and changes to which
the students are unconscious." Information capture includes information-receiving,
analysis, and feedback. Information-receiving refers to information acquisition, which
you can gain by the students' speaking in class, role playing, group learning and other
activities. You can also acquire information from the textbooks. Regarding such
information, teachers can spend more time and wisdom to construct questions to guide
students to learn.

There is an example in the high school mathematics book II *the Positional Relationship between the Straight Line and the Circle*, which is published by People's Education Press.

If line l passes through the point $M(-3,-3)$, and it's cut off by the circle $x^2 + y^2 + 4y - 21 = 0$, the length of chord is $4\sqrt{5}$, find the equation of line l.

When preparing lessons, I captured the valuable information that the textbook doesn't include the part about the line that passes through the point M with the slope 0. Why does the textbook use a different way to solve the problem? To find the answer, I designed the following exploratory exercise. If a given line l passes through the point M that is outside the circle, and the distance between the line l and the center of the circle is d, please draw the graph and prove how many lines satisfy these conditions. After discussing and exploring and with the teacher's guidance, students find that there are only 2 lines that satisfy the conditions. However, there are 2 different results given in the textbook. In that case, I ask students not to consider the lines without slope. I feel very strongly that if there is some valuable information worth exploring, the class will be wise and brilliant.

As for the acquisition of information available in classroom activities, it is necessary for teachers to make their analysis in a short time, including validity and correlation analyses, which will directly affect the effectiveness of teaching. Some teachers, due to inadequate preparation, can't capitalize on the proper timing for students' creative generation, or they may spend unnecessary time and effort following the generation of a tangential idea or question.

For example, I have listened to two classes with the same subject: *Chord through Focus of a Parabola*. Both teachers explained the example: Draw a line with a dip angle of 60 degrees. It passes through the point F which is the focus of the parabola $y^2 = 2px(p > 0)$,

the chord joints two points A and B (point A is under the X axis), so $\dfrac{\left|AF\right|}{\left|BF\right|} = \underline{\hspace{2cm}}$.
Most of the students who solved the problem in both classes used the special case
method. That is: if p=2, using given conditions to find the coordinates of points A and B,
then find the value of the two radius and their ratio.

There were students in both of these two classes who questioned if they could solve this
problem through the general method. Given the same information, the two teachers used
different ways as following: Teacher A: "We will need a large amount of calculation If
we solve it with the general method, and obviously it's more difficult. Here we
recommend the special case method to obtain the answer." Teacher B: "This problem can
be resolved through the general method, and although we need a large amount of
calculation, I know that you are not afraid of a challenge in learning. Let's work together
to address this issue." Teacher A's analysis of this information was that it is not worth
wasting time because of the large amount of calculation. While Teacher B's feedback
was different, he encouraged the students to have the spirit of fearless determination and
at the same time he guided the students to solve it by the general method. As a result,
they obtained a series of opportunities for beautiful generation, while at the same time
they arrived at the ratio formula of focal radius, Chord Length and the formula of focal
radius.

Information capture is at the forefront of generative teaching, and is an effective
complement to construct wise classes.

FORM CONVERSION IS A POWERFUL GUARANTEE OF CONSTRUCTING
INTELLIGENT AND WISE CLASSES.

Academic form is used when compiling textbooks and publishing papers. It is formalized
with rigorous deduction and logical inference, and it shows some simple and brief
formalized content. Educational form is to take formalized content and transform it into a
format that is easy to accept and understand through teachers' efforts. It's all math

teachers' responsibility to shape and convert math academic form into educational form. Teachers should process math knowledge creatively by virtue of their wisdom in order to transfer the original "cold and static" math knowledge into "vigorous and dynamic" teaching content, making mathematics come alive for students. Only in this way can a smarter class be built.

There is an example in the high school mathematics bookⅢ *the basic algorithmic statement,* which is published by People's Education Press.

Exchange the value of two Variables A and B, and output the value before exchanging.

Procedure: INPUTA，B

PRINTA，B

X＝A

A＝B

B＝X

PRINTA，B

END

As for the exchangeable variables program, suppose A and B are two cups filled with water. If we want to put the water of A into B, and we also want to load the water of B into A, we need to find an empty cup X. First pour cup A's water into the empty cup X, then pour the water of cup B into cup A, and finally pour the water of cup X into cup B. Such comprehension methods which accord with the common sense of daily life can help students to digest the above procedures.

There is another example in the high school mathematics bookⅠ，*Finding the Approximate Solution of the Equation with Dichotomy,* which is published by People's Education Press. When the teacher explains the materials, they may make an analogy to

Fortune 52, which is a popular entertainment program on CCTV, to lead the students to guess the price of goods. As a result, the students can gain further insight into the materials. What they should do is just change the expressions in the materials, such as when they take the original area's average and divide by two, or take the game's critical phrase "more or less" and change it to a comparison between zero and the product of the function value according to the end of the line, then choose the area. This kind of game-like explanation can help us transfer mathematical form into an easily received educational form, thus students will never get bored with abstract and formalized conceptual contents, providing a strong guarantee to build Intelligent and wise classes.

CONCLUSION

A single teaching scenario, a single creative generation, a single piece of captured information could play an important role in creating a wise class, so mathematics teachers in high school should be good at creating, capturing and leading in the process of teaching. Only in this way can we help the students be full of passion in the learning process, and let mathematics class be filled with vitality and wisdom.

5th Classroom Teaching Research for All Students Conference
22 June – 25 June, 2013, Fuzhou, Fujian, China

ANALYZING THE LITERACY

THAT EXCELLENT MATHEMATICS TEACHERS

SHOULD HAVE

ACCORDING TO SEVERAL CASES

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Excellent teachers are those who have the ideas of noble teacher ethics, student center, excellent teaching skills and lifelong learning , and those who are full of passion, mental health, and instruct students according to educational law. Based on the basic concept and content of "High School Teacher Professional Standards (for Trial Implementation)", the excellent teachers should constantly learn, practice, and accumulate as well as innovate.

The implementation of "Do people's satisfactory education" requires teachers to have good quality and emerges more excellent ones. This paper shows the required qualities from them in some cases, some of which should be possessed by all excellent teachers and some of which are only possessed by science and mathematics teachers.

1. EXCELLENT TEACHERS ARE THOSE WHO CONSIDER TEACHER ETHICS THE FIRST

Case 1 :

Many years ago, there is a very popular book named " The best for the soul" . On the first volume writes a very evocative story:

Twenty-five years ago, a sociology professor at one university, as well as his students, made a survey about 200 boys in a city slum. They did the research about these boys' families and living standard, and made predictions for their futures. The result was that they were not promising boys.

Twenty-five years later, another professor found the study, and carried on the follow-up survey, finding that except 20 boys who had moved away or died, 176 among the remaining 180 boys have achieved a lot, many of whom have become lawyers, doctors and businessmen.

The professor was very surprised, then asked the young people once surveyed: "what's the greatest reason for your success?" They all replied: "because I met a good teacher."

Then the professor found out the teacher who was still alive and asked her how she could make the slum children successful. The kind old lady smiled and replied: "actually, nothing special, I love these kids."

Comment: "love" revealed the essence of education. Of course, the teacher ethics is the soul of education and love is the core of the teacher ethics.

Deng Xiaoping said: "I am the son of the Chinese people, I love my motherland and the people affectionately!"

Ai Qing said: "The reason why I was flooded with tears often was that I love this land deeply."

Ba Jin said: "My only wish is to melt into the soil, leaving along with the warm footprints of people."

It is the love to the motherland and the people that make them successful.

Successful education and successful teachers also cannot do without the teacher's love. The teachers with love can tolerate some shortcomings or mistakes of students and also have more patience, more expectations for students. Loving the students means caring about them, helping them and regarding them as their own children to skillfully teach them on step by step , and to instruct them without weariness.

Actually, "Love, responsibility, patience and carefulness" is the requirement of High School Teacher Professional Standards (for Trial Implementation)" (Document of Ministry of Education   teachers (2012) No. 1 item 14). "Professional standard" is the basic professional requirement of qualified high school teacher; is the basic norms of middle school teachers to carry out the education teaching activity; is the basic principles, leading the middle school teachers' professional development and is an important basis for the training, admission, assessment to the teachers and so on.

2. EXCELLENT TEACHERS ARE THOSE WHO ARE STUDENT-CENTERED
ONES

Case 2

In a research learning course of Social Science, two junior kids had such different views with their teacher in the task report that conflict happened between them. As a result, she issued a report about the two kids to the school, and called for a "suspension" punishment on them.

This aroused the attention of one math teacher and he decided to intervene, but he was also caught in a dilemma. As is often the case, school teachers should be the side of the school and his colleagues. However, the math teacher listened to the two kids statement. After thinking about it , he wrote a letter to the principal and principal assistant respectively to express his view. In his view, though the two kids were naughty, they were not bad kids. Eventually, the School accepted the teacher's advice and did not punish them, which left a deep impression on all students. In these students' view, the math teacher was a real good teacher, and held justice, disregarding of his personal gains and losses.

Comment: there are many good qualities that the excellent teachers should be required, such as ability, backbone, courage, élan and vitality. But most importantly, they should have justice. Protecting the legitimate rights and interests of students is the embodiment of the "student-centered".

Many teachers with the idea of the "student-centered" and a sense of justice do not want to see that the conflict breaks out between students, let alone to say what is obviously wrong is right. In fact, the moral core of Chinese intellectuals is justice and conscience. In all kinds of offices with highly- educated people, Justice should be stressed. The sense of justice is the basis of the behavior of the teachers. " To be a model for others and to take the lead" mostly refers to the justice and principles of teachers. Upholding the justice

is to depreciate evil trend, and setting the inspiring justice can stimulate teachers' potential creation, and can experience all kinds of wisdom and inspiration. Setting the justice is to let excellent people get affirmation and praise; to let the ordinary people realize aspirations and desires; to let the backward people feel pressure and crisis; to let the evil people be punished.

The "student-centered" and the pursuit of justice show a higher level of "teacher ethics" or "teaching", which requires the teachers not only to pursue justice, but also to cultivate the students with justice.

3 EXCELLENT TEACHERS ARE THOSE WHO LAY EMPHASIS ON "TEACHING-SKILLS"

Case 3   In the 2000 Annual National Highest Scientific Award, the winner, academician Wu Wenjun, answered a reporter's question. He said emotionally: "when I was young, I did not have much interest in mathematics. Even worse, I wanted to leave school. It was a teacher, Mr. Wu, who changed my views on mathematics with his wonderful teaching. As a result, my interest in mathematics developed so much that I regarded the mathematical research as my lifelong career."

Comment: After reading it, we, as a teacher, will arouse the encouragement and delight. Actually, the teachers themselves should also be interested in their own subjects, otherwise it will cause negative influence. Interest, of course, is the starting point of education, and also is the result of education. Some teachers change interested students into non-interested ones, which is a failure in teaching.

"Interest is the best teacher." To stimulate students' interest in learning, teachers must inspire the students from the different angle to let the students understand. To stimulate students' interest in learning, teachers must be knowledgeable. To stimulate students' interest in learning, teachers must protect students' curiosity and childishness -- this childishness or curiosity is motive force of scientific development. It is not only a very

important scientific spirit, but also a kind of humanistic spirit. To stimulate students' interest in learning, teachers must create more spiritual resonance, widen their knowledge and improve their thought to enable students to have feeling of "suddenly-understand" , " enlightening" and "understand with ease" and to experience excitingly, imaginatively and wonderfully.

Professional ability includes teaching design ability, teaching implementation ability, classroom management and education activity ability, education teaching evaluation ability, communication and cooperation ability, and reflection and development ability and so on. "Creating a good learning environment and atmosphere, and arousing and protecting the high school students' interest in learning" is the professional teaching ability dimension requirement in item 40 in "High School Teachers' Professional Standards (for Trial Implementation)".

4 EXCELLENT TEACHERS ARE THOSE WHO ARE "LIFELONG LEARNING" TEACHERS

Case 4, Yuan Longping, the first National Science Award Winner , liked to inquire deeply into every question in the secondary school. He asked the teacher why he got a positive after multiplying two negative (two negatives make a positive.) The teacher couldn't answer it. Later on, he also encountered some difficult problems, and the teacher did not give him satisfying answer, so he came to a conclusion that "mathematics is unreasonable". As a result, he lost interest in learning mathematics.

Comment: Teachers' professional quality is the basis of engaging in education. Only a good professional quality can teach students well. The teacher must keep on learning and thinking to adapt to the environment. The reasons why the teacher couldn't answer the student's problem are that he was lack of the sense of responsibility; and that he didn't improve his knowledge and ability; and that he had no desire for lifelong learning.

In fact, constant learning and reflection is beneficial to understanding the teaching material, and also is good at answering the students' questions. A teacher not only makes the students know that 1 chicken +1 chicken =2 chickens, 1 duck +1 duck =2 ducks, but also lets students know what 1 chicken +1 duck equals to? That is to say, teachers should have a higher abstract thinking ability. What is the core competence of science teacher? I think it is the ability of solving problems. A teacher's intelligence depends on the ability of solving problem. A teacher who does a job with skill and ease in teaching often benefits from the ability of solving problem well. In order to improve a teacher's intelligence, he must keep on learning and solving problems.

Professional knowledge includes the knowledge of education, the knowledge of subject, the knowledge of subject teaching and common sense. The teacher mentioned in this case lacked both the knowledge of subject, academic ability and the attitude of lifelong learning.

5 EXCELLENT TEACHERS ARE THOSE WHO ARE FULL OF PASSION

Case 5   it was reported in Education Digest Weekly that Zhang Fangao was the most excellent teacher in one city and had been rated as outstanding teachers in the city for many times. Surprisingly, after the school was reorganized to a private one, his name was eliminated in the first list. Mr. Zhang was not convinced, determined to compete among these eliminated teachers. Because of the reasons as everyone knew, there were much more people listening to his class. After class, he got a lot of teacher's affirmation: "this class is so vigorous",

 "there are 9 ways to solve the problem, which we never think of" " The connecting are so wonderful that Mr. Zhang is great."

Soon, the conclusion of the Human Resources Department also came out: after discussing of the committee, Zhang Fangao lacks of passion, hoping that he must

improve himself in the 3 months of probation, otherwise, the school will not accept him.

Comment: obviously, Mr. Zhang should be a good teacher, but no passion has become a shortcoming of him, which is a great pity.

In fact, the passion is not only the requirements of the education, but also the requirements of all kinds of work. In the interview enrollment staff of "Microsoft", examiners have to pay special attention to the following four questions: (1) whether or not smart enough? (2) if there is a creative passion? (3) whether there is team spirit? (4) how the professional ability is?

What is education? It not only gives knowledge, but also becomes a dream maker. Lang Clark, one of the 2000 outstanding teachers in the USA, said what lacked most among the knowledge the educators gave students was passion, one that embraced the life; that showed a love of humanity; that dreamed of the future; and that pursued all unknown. Passionate teachers love life, and love the class. Also, they are full of vitality to infect themselves and students. Accordingly, that the private school strongly expect passion in teaching is understandable.

6 EXCELLENT TEACHERS ARE THOSE WHO ARE MENTAL HEALTH

Case 6   It was reported that in Shan Xi province, a 25- year -old teacher due to suffering from severe mental disorders committed suicide, leaving less than one-year- old son. The reason was that "She didn't teach students well and she was very sad."

In Yancheng city, Jiangsu province, a 30-year-old teacher also committed suicide because he couldn't answer the questions from students.

In the blue peacock kindergarten, in western blocks, Wenling City, Zhejiang Province, the teacher Yan Yanhong abused children and then uploaded photos on the Internet, which annoyed net citizens. These annoyed net citizens also found in

her QQ space that there were a lot of photos of children's mouth being kept silent with tapes; of children's face being covered; and of children's head being inserted into a garbage can. Every one was greatly shocked. "she is not the first time to abuse these children. All evidences are here!"

The investigation by Police verified that since 2010, Ms. Yan did abuse her kids in the ways mentioned above and took photos for fun. Nowadays, the Police confirmed that, at least five of ten kids appearing in Ms. Yan's QQ space were in the class where Ms Yan was teaching.

The Wenling authorities have set up a working group, specifically psychological counseling for these kids in Yan's class. What can not be ignored is that it is Yan's request to expose these photos and upload the one "she was pulling the kids by their ears" to "micro message", which was found and reported.

Yan Yanhong accepted the reporter's interview in the kindergarten. Yan Yanhong still wearing the Leopard Coat appeared panic. When closing to her, we could feel she was trembling. When asked "why to pull the kids by the ears, and let others take pictures", Yan said "for fun". Reporter asked: "don't you find that children are crying?" Yan Yanhong was silent.

These children were punished cruelly only because of some shortcomings. Some were punished to "eat flies"; some to slap each other; some to tattoo on their faces; some to cut off the fingers; some to be scalded with tongs.

Comment: it can be said that psychology of those teachers is not healthy, and the soul of YanYanhong is distorted. The harm of mental health is enormous. It hurts themselves and others. According to the sampling survey in Fujian, among 265 primary and secondary school teachers, 29% of them have slight mental barriers; 10% have moderate barriers; 4% even have mental diseases. Nearly half of the teachers are not mental health. Moreover, woman teacher, the head teacher and the

teachers whose students will graduate soon have more serious mental health problems. Similar survey has been made in other places. As a result, there are similarities.

It doesn't mean that excellent teachers have no mental problems. Most importantly, they can realize problems and solve them. For example, they will ask others for help, or heal themselves. The life is cherishing. Whether they are students or teachers, they can't teach a kid healthily without mental health. A student said in QQ signature: because of the darkness, I was lost. The author suggested that: because it is dark, I pursue light. Mentality changes, so the world will change. Mentality change, so dark can become bright. In this sense, we teachers can change ourselves, change the world.

In the field of "professional idea and ethics" dimension "individual tutelage and behavior, "High School Teacher Professional Standards (for Trial Implementation)" puts forward five requirements: love, responsibility, patience and carefulness; optimistic, enthusiastic, affinity; self- adjusting mood, a calm state of mind; diligently learning, constantly enterprising; and neatly dressed, appropriate language, good behavior. The "optimistic, enthusiastic, affinity; self adjusting mood, a calm state of mind; diligently learning, constantly enterprising." is to emphasize the teacher's mental health

7 EXCELLENT TEACHERS ARE THOSE WHO CAN INSTRUCT STUDENTS ACCORDING TO EDUCATIONAL LAW.

Case 7   One teacher having taught for many years always liked to teach more in class. Every time the bell rang, he always wanted to delay a few minutes. He always repeated the same problem in fear that the students didn't understand it. He asked students question frequently and the way of questioning problems was simple. He took it for granted that the teacher should teach more. He thought even if the student forgot half, the students could master another half. If the teacher taught less, the students would master

little knowledge, after forgetting half. However, his teaching was not popular with the students and students' marks were poor. So, he felt frustrated and confused.

Comment: the students are disgusted about not dismissing class on time. The students thought the teacher deprived of their rest time. It was a pity if the teacher didn't realize this.

The purpose of teachers is not bad and also an" active action", but he forgot a basic law of teaching: teaching is the external conditions to learn and learning is the intrinsic motivation to teach. Teaching is hard to succeed without characteristics of cognition and learning motivation. The class is only 45 minutes, during which the task of teaching should be accomplished. And accomplishing the task of teaching is not only achieved by teachers, and it should be dealt with according to the characteristics of the teaching content. When necessary, the students can learn by themselves or between peers. Sometimes the teachers needn't to instruct. It is wrong that teaching more will get more and teaching less will get less.

" In order to provide appropriate education to every student, we should follow the laws of education and the physical and mental development of students ", which is the requirement of item 11 in "High School Teacher Professional Standards (for Trial Implementation)" professional competence dimension and the field of teaching implementation. Teaching overtime and extending learning time are contrary to the laws of education and it is not a better choice.

In short, excellent teachers should follow the basic ideas in High School Teacher Professional Standards "(for Trial Implementation)" (the four basic ideas: teacher ethics being the first place, student centered, excellent teaching skills and lifelong learning); the basic content (including three levels: "dimension", "field" and "basic requirements" , i.e. three dimensions, fourteen fields, sixty-three basic requirements". "The three dimensions" are a "professional idea and ethics", "professional knowledge" and "professional ability"; in each dimension are four to six different fields; in each field are three to six basic

requirements). On the basis of these ideas and contents, the teachers can constantly learn, practice, and accumulate as well as innovate.

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How Big is Humongous?

Mathematics Conversations

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Abstract

Questioning strategies, while reading selected storybooks, promote young children to express measurement concepts using descriptive language.

How big is humongous? "As big as a giant's teeth." Why, in real-life, do you need to measure accurately? "Because the giant needs the right size of glasses." Using questioning strategies during an interactive reading of *Jim and the Beanstalk* (Briggs, 1997), a pre-kindergarten child describes the size of a giant's teeth using rich vocabulary and makes authentic connections to the purpose of measuring precisely. Implementing questioning strategies, during interactive storybook reading, encourages young children to express mathematics concepts in their conversations.

Benefits of Storybooks to Promote Mathematics Conversations. Children's literature enhances mathematical learning in classrooms by providing an alternative for communicating about mathematics. Through purposeful book selection, interactive reading and hands-on activities, students become engaged in "math-talk" to further develop an understanding of measurement concepts. Math-talk is an instructional conversation directed by the teacher, teacher-aide, or parent and includes student-to-student talk (Fuson, Atler, Roedel, & Zaccariello, 2009). Reading aloud and engaging

children interactively with the book fits naturally with the established routines in the classroom and encourages math-talk. Stories that focus on concept development can bring life to seemingly isolated and abstract mathematical ideas for many students. Questioning strategies, which can be implemented quickly and easily, promote conversations to express measurement concepts.

A standards-based mathematics classroom exists when teachers *and* students contribute to the mathematics conversations. Math-talk challenges students to express and justify their mathematical thinking, thereby constructing and exchanging knowledge. Talking about mathematics encourages children to increase their vocabulary and compels them to really consider what words mean and how they are used. Storybooks are the springboard to enhance mathematical concepts by introducing precise vocabulary. In language-rich classrooms, the meaning of mathematical terms and their connections to the real-world objects or pictures become the emphasis. When language objectives are integrated in teaching mathematics concepts, English language learners can concurrently develop their English language skills while they are learning mathematics (Furner, Yahya & Duffy, 2005).

Research Support. The NCTM *Standards* advocates the use of children's books as a vehicle for communicating mathematical ideas (2000). Shared storybook reading has proven to be an effective context in which to embed specific strategies to increase the language and early literacy skills of young children (Mol, Bus, de Jong, & Smeets, 2008; Whitehurst et al., 1988). Other work suggests how teachers can integrate children's

literature to enhance mathematics learning in classrooms (Whitin & Wilde, 1992; Columba, Kim, & Moe, 2009). To implement the Core Curriculum State Standards in Mathematics (CCSSM) (2010) in primary classrooms: "The classroom library would have mathematics-concept literature books and manipulatives alongside toys and games to challenge growth and learning" (White & Dauksas, 2012, p. 443).

Seeley (2009) recommends developing imaginative thinking and creativity to stimulate students' interest in mathematics. Storybooks can provide a unique alternative to formulaic approaches and facilitate math-talk. The National Institute for Literacy (2009) concludes that the more teachers intentionally make time for talking and sharing experiences, such as reading books with rich concepts, the more support there is for children's language development and later reading comprehension success.

Research Design. Over a three-year period, my co-authors and I provided professional development for parents, teachers and teacher-aides to increase math-talk with pre-kindergarten children. We adapted and modified strategies to promote mathematical conversations, which are used for successful reading and language development and are presented in this manuscript. We provided storybooks selected to provide specific mathematics content, such as, measurement concepts and hands-on materials. In addition, we provided reader's guides (see Figure 2), which have been developed by the authors, to provide suggestions and structure when implementing an interactive math-talk approach in the classroom. A reader's guide, similar to a lesson plan, includes a summary of the book, objectives, key concepts and recommended

questions to focus on the mathematics content and to engage students in the mathematics process. Recommended questions to encourage dialog was in bold font on the reader's guide to facilitate meaningful mathematics discussions of pre-kindergarten children.

In the earliest study, we examined systematically the effect of instructing parents to focus on mathematical concepts and vocabulary during shared storybook reading. Specific research questions were as follows (a) did parents increase their use of math talk during shared storybook reading following training? (b) did parents generalize intervention strategies? and (c) did children increase their use of math-talk during shared storybook reading? Parent and child utterances were recorded and transcribed and then coded for mathematics content and process. Results from a yoked multiple baseline design with six dyads, indicated variability across the dyads with two general patterns. For three of the dyads, there was an increase in math-talk following training whereas verbal mathematical behavior did not show consistent change for the other three dyads. The mean frequency of mathematics utterances during reading sessions increased from baseline to intervention for all child participants.

During the next phase of our research, for ten weeks, five preschool teachers in the intervention condition were instructed in interactive reading strategies with the emphasis on using storybook reading as a means of engaging in "math-talk." One of the authors served as a literacy coach. Professional development involved modeling, rehearsal, feedback, and weekly consultation. Five preschool teachers in the control condition were instructed in using interactive reading strategies with no emphasis placed

on the use of "math-talk." Professional development involved modeling, rehearsal, and feedback, as well as, weekly consultation. Teachers were provided with storybooks with reader's guides that focus solely on interactive reading procedures. The children's mathematics growth was measured through pre- and post-test data. Initial analyses indicate growth across both groups of students.

Questioning Strategies to Promote Mathematics Conversations. How we read to young learners is as important as how often we read to them. Reading children's books provides nourishment for language-rich classrooms and provides a real-life context for the development of mathematical terms and measurement concepts. Children learn most from books when they are actively involved in the story.

The first two strategies were developed by Whitehurst et al. (1998). Presented as an acronym each letter may not be used on every page of the book, in this order, or in every conversation with the child(ren). They are a guide that is easily implemented to engage the child(ren) in talking, measuring, comparing, estimating, representing, and using descriptive language.

The first strategy to encourage "math-talk" is PEER.

Prompts the student to say something about the book;

Evaluates the child's response;

Expands the student's response by rephrasing and adding information to it; and,

Repeats the prompt to make sure the student has learned from the expansion.

An example from our research demonstrates math-talk between a parent (A) and a child (C) using the PEER strategy while reading *How Big is a Foot?* (Myller, 1991). The child is prompted to point to the length and width of the bed and to count the number of feet for each in the drawing. Instantly, the child is evaluated and guided to repeat the counting for accuracy and receives confirmation.

Text: *He counted that the bed must be three feet wide and six feet long to be big enough to fit the queen. (Including the crown which the queen sometimes liked to wear to sleep).*

A: **How many feet this way?** **(Prompt)**

C: Three!

A: **Three, three feet wide. And how many feet this way? (Evaluate)**

C: One, two, three, four, five, six.

Text: *One, two, three, four, five, six.* (beside drawings of feet.)

C: So, one, two, three, so three equals six, it makes, one, two, three, four, five, six, seven, eight.

A: **Count them again. (Expand)**

C: One, two, three, four, five, six, seven, eight, nine.
A: **Nine. (Repeat)**

A second strategy to encourage mathematics conversations is CROWD.

Completion prompts—Fill in the blank. Provides child with information on the structure of language;

Recall prompts—Can you tell me? Helps child to identify plot and sequence;

Open-ended prompts—Tell me what's happening. Helps the child to increase expressive fluency;

Wh-prompts—What, where, when, why, how? Encourages the child to develop new vocabulary; and,

Distancing prompts—Relate pictures or words outside the book. Allows the child to make connections between the book and the real world.

An example of a conversation about measurement from our research using CROWD, while reading *Inch by Inch* (Lionni, 1960), illustrates how to develop measurement concepts recommended in the *Measurement and Data* domain in the CCSSM (2010). The standard states "to directly compare two objects with a measurable attribute in common, to which object has "more of"/"less of" the attribute, and describe the difference" (p. 12). Pictures in the storybook present a learning opportunity for young mathematicians to compare and describe the difference between two objects. Additionally, opportunities are presented for the adult (A) to probe further to discover how the child (C) worked out the answer.

Text: *He measured the toucan's beak…*

A: **Do you think the toucan's beck is longer than the flamingo's neck?**

C: Um, longer than it, the flamingo's neck?

A: **You think the flamingo's neck is longer?** **(Recall)**

C: Yeah.

A: **So, how many inches do you think the flamingo's neck is?** **(Completion)**

C: One, two, three, four, five, six, seven.

A: **Only seven?**

C: Yeah.

A: **Okay, and so if the flamingo's neck is longer, then how long do you think the toucan's beak is?** **(Open-ended)**

C: One two, three, four, five, six, seven, eight, nine, ten, eleven.

A: **Is eleven longer than seven?**

C: Yeah.

A: **It is?** **Well how can the toucan's beak be 11 and the flamingo's neck be 7 if the flamingo's neck is longer?** **(Distancing-connecting to real-life; guiding to immediately correct a misconception)**

C: Oh, I know what it is.

A: **What?** **(Wh-Prompts)**

C: It's 20!

A: 20?   **(Wh-Prompts-How? An opportunity to probe deeper)**

C: Yeah!

The third is strategically asking open-ended questions. Asking questions, such as "Tell me about this" and "What's this?" at key points in the book encourages oral responses and active participation from the children.   Explaining "why" requires reflective thinking and diminishes guesses or responses based on rote memorization. Teacher questioning promotes thinking and understanding of ideas and allows the teacher to probe deeper after an answer is given.

An example from our research, while reading *Inch by Inch* (Lionni, 1960), illustrates how an adult (A) guides a child (C) to use the nonstandard measurements, an inch-worm cutout and fingers, to measure the toucan's beak. The child enthusiastically measures the toucan's beak.

Text: *He measured the toucan's beak…*

A: That's a long beak, isn't it? **(Tell me about this…)**

C: Let's count. One, two… (using an inchworm cutout).

A: Oh you know what, I'll show you something. See these two fingers here? (modeling first two fingers) (Actively engaging the child)

C: Mm-hmm.

> **A: These together make about an inch! So you can go like this. One, two three, four, five six, seven, eight, nine, ten eleven, twelve! You can pretend you're like the inch worm! What do you think about that?** (**What is this? Guiding the child to clarify the meaning**)
>
> C: Yeah!

Selecting Measurement Storybooks. Children's literature presents mathematics content in different ways. Mathematics concept books can provide *explicit* content, such as measurement concepts. In *Measuring Penny* (Leedy, 1997), Lisa, the main character, learns about measurement by measuring her dog, using standard and nonstandard units.

In some storybooks, mathematical concepts are *implicit* because they present mathematical concepts subtly. In *How Big is a Foot?* (Myller, 1991), students discover the usefulness of standardized measuring tools when the king decides to have a bed built for the queen and everyone has difficulty determining the size of a bed. Measurement is indirectly part of the plot. For lesson ideas related to *How Big is a Foot?* see *illuminations.nctm.org/LessonDetail.aspx?id=L635*.

When mathematics content is *invisible,* creative teachers construct meaningful connections to the mathematics concepts. Or the book can serve as an invitation to introduce other books and resources about the topic in order to construct a fuller, more complete understanding of the concepts. Teachers accomplish these types of connections

by organizing learning experiences around topics of high interest to students, allowing connections to occur across curriculum areas. *The Mitten* (Brett, 1989) illustrates a content invisible book. Shared reading integrates measurement concepts by engaging the students in estimating the length of real mittens.

However, all of these features—purpose, audience, genre, text structure, and content—are necessary to judge the appropriateness of a book's selection. Other selection criteria for quality children's books to guide teachers' selections are provided by a set of questions, which are:

1. Is the book engaging to the reader?

2. Is the book age appropriate?

3. Does the book contribute to the balance of the classroom collection?

4. Does the book include meaningful and relevant mathematics concepts? (Columba, Kim, & Moe, 2009, p. 31) These overarching principles provide a justification for each storybook selection. See Figure 1 for other storybook selections at the primary level that foster the teaching of measurement concepts.

So whether students are measuring humongous teeth or long beaks, these math-talk examples demonstrate how incorporating questioning strategies into interactive storybook reading promotes mathematical conversations and increase the awareness of measurement concepts. For young children, language plays an essential role in learning how to express themselves, which often results in new understandings of measurement concepts.

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THE ROLE OF ATTENTION IN ASSESSMENT

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Abstract

Attention to details is one of the most important specific features of a mathematical way of thinking. When evaluating mathematical questions it is important to teach students to pay attention globally and locally, consider conditions, properties and relationships, since all of these aspects are as important to develop as mathematical techniques. To develop such skills in their students school teachers should have those skills themselves. In this study, we investigate the ability of school mathematics teachers to pay attention to details and use their mathematical knowledge. We are confident that the vast majority of teachers have excellent knowledge of mathematical techniques. Hence the question is whether this kind of knowledge might structure their attention in such a way that the emphasis on procedures deviates their attention from the essential details. Two groups of teachers from New Zealand and Hong Kong were given a mini-test containing seven simple mathematics questions. All questions in the test were provocative in the sense that they looked like routine questions but in fact they had some catch. The results from the test were startling – the vast majority of the teachers did not notice any catch and gave incorrect answers to most questions in the test. After the test the teachers were given a short questionnaire to reflect on their performance on the test. The teachers' responses to the questionnaire are presented and analysed in the paper using theories on attention. Implementations of the results of the study in assessment and professional development are discussed.

Keywords: attention, assessment, knowledge, professional development

INTRODUCTION AND THEORETICAL FRAMEWORKS

In recent years there has been substantial research on teachers' mathematical knowledge and assessment of teachers' mathematical knowledge (Rowland & Ruthven, 2011; Ball, Thames & Phelps, 2008; Hill, Ball & Schilling, 2008; Hill et al, 2007;). In this paper, an attempt is made to investigate the role of attention in *using* existing mathematical knowledge by school mathematics teachers while doing simple but not routine mathematical tasks. The following theories on attention are employed as theoretical frameworks: the late selection theory of selective attention (Deutsche & Deutsch, 1963) based on the idea that all information is routinely processed and selection of response depends on the level of alertness; Kahneman's (1973) model of divided attention based on the idea of mental efforts and the level of arousal or state of alertness; and the feature-integration theory of attention (Treisman & Gelade, 1980) based on the idea that putting different features into a coherent object demands focused attention. Other theoretical considerations are based on research by Mason (2000, 2002, 2004). Mason has proposed that when we look at a mathematics question the focus of our attention may vary depending on whether we are *looking at* the symbols or *looking through* them. The idea is that we need to structure our attention, to know what we are aware of, and Mason describes a number of elements that we may focus attention on, including: the whole, the details, the relationships between the parts, the properties of the whole or the parts and deductions (2004).

We are confident that the vast majority of teachers have excellent mathematics knowledge of *knowing-that* (factual) and *knowing-how* (techniques and skills) as described by Mason and Spence (1999). Most of the formulas, rules and theorems however are not always applicable but have certain conditions and constraints. Often assessment questions focused on techniques are selected in such a way that the conditions/constraints of the relevant formula or rule are met. So the students might develop a habit of applying formulas or rules without checking the conditions/constraints.

But in real-life problems not all functions and equations behave so nicely and ignoring conditions and constraints might lead to significant and costly errors. One of the goals in teaching mathematics is developing and enhancing students' mathematical way of thinking while helping them to learn a variety of concepts, techniques and procedures. In particular, the mathematical way of thinking is concerned to a large extent with the analytical thinking so that an individual analyses any situation, doesn't take anything for granted and always looks for evidence, proof and justification which are the essence of mathematics. We should encourage students to pay attention to every detail, for example - conditions, constraints, locality, properties and relationships. The ability to pay attention or 'discipline of noticing' as described by Mason (2002) is equally important to develop as mathematical techniques. It needs to be a natural part of their mathematical culture. Students can see that the ability to analyse carefully a mathematics question enhances their skills to analyse critically other situations outside mathematics. To develop such skills in their students teachers should possess those skills themselves. In this paper, we are not testing teachers' knowledge of mathematical techniques, procedures and algorithms but their skills of paying attention and analyzing the question before applying a relevant formula or technique, that is the ability to 'question the question'. We argue that attention plays a crucial role in doing non-routine mathematical tasks. As Mason and Spenser (1999) propose "*knowing-to* act in the moment depends on the structure of attention in the moment, depends on what one is aware of" (p.135).

THE STUDY

The study was conducted in 2011 with two groups of teachers - one in New Zealand and the other in Hong Kong. The New Zealand (the first) group consisted of 14 experienced upper secondary school mathematics teachers who attended a workshop during a one day conference as part of their voluntary professional development. The Hong Kong (the second) group consisted of 26 secondary school mathematics teachers who attended a 5-week full-time compulsory training course. It was not a comparative study but a parallel case study. A combination of two non-probability sampling methods – convenience and judgment – was used to

select the participants. Namely, it was convenient for the authors to invite the participants for the study from the groups of teachers they had easy access to. In the authors' judgement the selected participants were good representatives of the populations of mathematics teachers' in both countries. Both groups were given the mini-test containing 7 simple mathematical questions. All questions in the test were provocative in the sense that they looked like routine questions but in fact they had some catch. In some cases it was an extraneous root of an equation because of the restricted domain, in others the rule was inapplicable because the conditions were not met. The teachers had 15 minutes for the test and they were informed before the test a hint that some questions are a bit provocative. The questions from the test are below.

The mini-test

1. Find the area of the right-angled triangle if its hypotenuse is 10cm and the height dropped on the hypotenuse is 6cm.

2. Find the domain of the function $y = f(g(x))$ if $f(x) = x^2 + 1$, $g(x) = \sqrt{x-2}$.

3. Solve the equation $\ln(x^2 + 17x - 18) - \ln(x^2 + 5x - 6) = 0$.

4. Prove the identity $\sin x = \sqrt{(1 - \cos^2 x)}$.

5. Show that the equation $\dfrac{x^2 + \sqrt{x} + 1}{x - 1} = 0$ has a solution on the interval [0, 2].

6. Find the derivative of the function $y = \ln(2\sin(3x) - 4)$.

7. Find the integral $\int_{-1}^{1} \frac{1}{x} dx$.

The results and discussion of the mini-test

The results of the test are presented in Table 1.

Table 1: Correct answers to the test questions

	Q1.	Q2.	Q3.	Q4.	Q5.	Q6.	Q7.
Correct answers group 1	0%	7%	21%	7%	0%	8%	0%
Correct answers group 2	23%	12%	27%	19%	12%	15%	12%

After the test there was a detailed discussion of solutions of every question from the test.

Question 1. The correct answer is: there is no area as the triangle doesn't exist. By the Thales' theorem the hypotenuse in a right-angled triangle is a diameter of its semicircle so in this case the height cannot be bigger than 5cm.

In the first group there were no correct answers out of 14 with 12 teachers giving either 30cm^2 or 24cm^2. In the second group there were 6 correct answers out of 26 with some teachers arriving to the correct answer after checking their initial incorrect answer of 30cm^2 or 24cm^2 and rejecting it. Most teachers applied the familiar formula $A = \dfrac{ah}{2}$ ignoring the important piece of information about the right angle.

Question 2. The correct answer is: $x \geq 2$. The composite function $y = f(g(x))$ is defined whenever both $g(x)$ and $f(g(x))$ are defined.

In the first group there was just one correct answer out of 14. In the second group there were 3 correct answers out of 26. Most teachers did not pay attention to the definition of the domain of a composite function.

Question 3. The correct answer is: no solutions ($x = 1$ is outside of the domain of both log functions which can be easily checked by substitution).

In the first group there were 3 correct answers out of 14. In the second group there were 7 correct answers out of 26. Most teachers ignored the restricted domain.

Question 4. The correct answer is: the 'identity' is not true. Squaring both sides doesn't prove it because this operation is irreversible. It is not an identity but an equation with infinitely many solutions $x \in \left[2\pi n, \pi(2n+1)\right]$.

In the first group there was just one correct answer out of 14. In the second group there were 5 correct answers out of 26. Most teachers assumed that it was provable because of the wording of the question and did not pay attention that squaring is an irreversible operation.

Question 5. The correct answer is: there are no solutions. The equation $x^2 + \sqrt{x} + 1 = 0$ has no solutions (the left-hand side is always positive). If one can try to apply the Intermediate Value Theorem it is not applicable because the function $f(x) = \dfrac{x^2 + \sqrt{x} + 1}{x - 1}$

is not continuous on [0, 2].

In the first group there were no correct answers out of 14. In the second group there were 3 correct answers out of 26. Most teachers misused the Intermediate Value Theorem – they checked only that $f(0) < 0$ and $f(2) > 0$ and did not check the continuity condition.

Question 6. The correct answer is: the derivative doesn't exist because the function doesn't exist as the argument of the log function is always negative.

In the first group there was just one correct answer out of 13 (one teacher did not attempt the question). In the second group there were 4 correct answers out of 26. Most teachers failed to check the domain of the function and applied the familiar Chain Rule.

Question 7. The expected answer is: it is not a definite integral because the function $y = \dfrac{1}{x}$ is not continuous on [-1, 1]. For this reason the Newton-Leibniz formula is not applicable. It is beyond the secondary school curriculum (it is an improper integral and in this particular case it is not defined).

In the first group there were no correct answers out of 14. In the second group there were 3 correct answers out of 26. Many teachers failed to check the continuity condition of the Newton-Leibnitz formula and applied it. Some used graphs to produce incorrect solutions.

The questionnaire and teachers' responses

After the discussion of the solutions the teachers were given a short questionnaire to reflect on their performance in the test. The response rate in both groups was 100%.
The questionnaire is below.

Question 1. What are your feelings after you have learnt about the correct solutions to the test questions?

As most of the teachers gave incorrect answers to the vast majority of the questions it was interesting to notice that their feelings were polarized. In the first group 50% of the teachers expressed self-criticism – "disappointed', "without thinking", "embarrassing", "didn't apply critical at all", "felt stupid –

oversimplified the questions" and the other 50% were happy to learn lessons from the test – "enlightened, a very good test and ensure reflection upon teaching practice", "attention to the words and wider picture", "feel OK", "like it – I should have known…', "fun, I love anything that knocks me out of academic boredom", "a bit more enlightened", "glad that I have the opportunity to see and think through these problems".

In the second group the majority of the teachers were embarrassed and uncomfortable about their performance on the test.

Question 2. What are the reasons for not solving all test questions correctly?

In the first group all 14 teachers gave comments on the lack of attention and careful thinking. The common responses are as follows:

Not thinking carefully about whether my solution method was appropriate to that particular problem; I did not think critically; not paying attention, impulsive reaction; I did not rely on my understanding but jumped straight to applying the rule; lack of knowledge and 'testing' things and being programmed to look for 'set' answers'; possibly that's how I was taught back home (South Korea), learnt lots of techniques (some difficult) but not to question the questions; not looking at all conditions; not thinking carefully and not reading the questions carefully; applying skills but not applying knowledge; not thinking about the structure of the expressions, considering its conditions; I knew there was something more to check but did not check thoroughly enough.

In the second group the teachers reported that the main reasons for making mistakes were carelessness and the expectation that each test questions had an answer (often a certain number).

There were no comments in both groups about lack of time to finish the test so we assume it was not a reason for poor performance. From our observation the vast majority of the teachers in both groups finished the test within 10 minutes.

Question 3. Would you make any changes in your teaching practice after doing the mini-test? If so – which changes? If not – why?

In the first group all 14 teachers reported that they would make changes in their teaching practice after doing the test. The common responses are as follows:

Introduce tricks like this to class to make them think; keep encouraging and creating environment where a deep conceptual knowledge is cultivated; encourage and reward checking of answers; more emphasis on the validity of solutions; teach them to examine the question thoroughly; give students more questions that will force them to think about the conditions surrounding the questions; I would encourage students to think through questions carefully; students need to understand, observe and consider answer to ensure they make sense and think before you solve; give students questions to challenge their knowledge; I try to make my students think more about restricted domains, check solutions and not trust graphical calculators; encourage kids to think about their solutions in light of the original question; give them problems occasionally that will 'trip' them up if they have not gone back and re-assessed their solutions; more emphasis on the nature of problem solving; stop answering impulsively, think before respond; I will expose students to such questions to get them to think more deeply about the conditions.

One teacher however, along with his/her positive response, made the following comment regarding the changes: "unless it is an element of the assessment I might not have time".

In the second group 13 out of 26 teachers reflected that they would change their teaching practice. Some of them reported that they would take the test back to their school and use it as a teaching material. The other 13 teachers reported that they would not change their teaching practice as such test questions are not common.

Some differences between the two groups in answering the questionnaire were probably due to cultural differences between New Zealand and Hong Kong.

However, addressing the effect of the cultural differences was not an intention of this paper.

ANALYSIS AND CONCLUSIONS

Teachers' performance on the test and their responses to the questionnaire demonstrated that the majority of the teachers had serious lack of attention and careful thinking that led them to fail most of the questions in the test. According to Mason and Spence (1999) those teachers, in spite of good *knowing-that* and *knowing-how* should enhance their *knowing-to* act skills that help them to perform better: "active, practical knowledge, knowledge that enables people to act creatively rather than merely react to stimuli with trained or habituated behaviour involves *knowing-to* act, in the moment" (p.136). Knowing-to act in many cases is a multistep activity and each step needs attention. We are absolutely confident that the participants of the study had the relevant knowledge (e.g. they knew the domain of the log function, the range of the sine function, the conditions of the Intermediate Value Theorem, the definition of a definite integral, and so on). So the question was about their ability to *use* their knowledge on the test. Theories of attention developed by psychologists might be helpful in analysing the relationship between knowledge and attention. Deutsch and Deutsch (1963) argue that "however alert or responsive we may be, there is a limit to the number of things to which we can attend at any one time" (p.80). Kahneman's (1973) model of divided attention (when attention is divided between two or more concurrent tasks) suggests that attention can be flexibly allocated between tasks based on processing priority. Treisman & Gelade (1980) went further claiming that "without focused attention, features cannot be related to each other" (p.98). In solving mathematical questions attention is required to each step and often the priority of allocation of attention to different steps is very important. In many cases attention is needed to the 'analysis of the question' step (e.g. checking conditions of the rule, domain of the functions, type of the equation, locality of the statement, and so on) before switching attention to the next steps – procedure, verification, etc. Ignoring the 'analysis of the question' or 'question the question' step can lead to incorrect solutions

especially in non-routine questions as the study shows. In some cases, however, the order of steps can be changed. For example, to solve Question 3 of the test one way is to find the common domain of both log functions first by solving a system of two quadratic inequalities, perform calculations using the log rules and then check whether the solution belongs to the common domain. An easier way however, is just notice that we are dealing with the restricted domain without finding it (can be time consuming), perform calculations using the log rules and then verify the resulting solution by substitution into the original equation. Feedback of the participants of the study show that one of the main reasons for poor performance on the test was not the priority of the steps but ignoring some of the crucial steps, in most cases the 'analysis of the question' step. The majority of the participants reported that they would definitely make changes in their teaching practice after the test by putting more emphasis on the analysis of the question before applying a certain formula or theorem. Those participants, who reported that they would not change their teaching practice because the test questions were uncommon, probably tend to 'teach to the test'. Including the type of questions from the mini-test into the assessment would encourage those teachers to pay more attention to details and analysis and enhance such skills in their students. After all, many situations in real life don't have a single 'correct' answer like routine questions from the traditional assessment in mathematics. We believe that solving non-routine, non-standard questions would prepare students for the real world better. Enhancing their own and their students' 'discipline of noticing' (Mason, 2002) by paying attention to details can also be a useful addition to teachers' professional development.

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