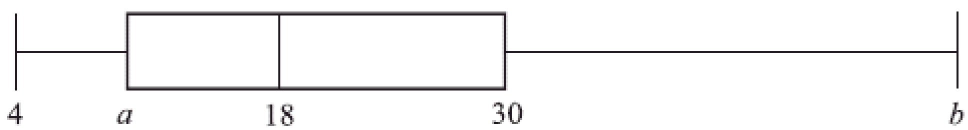
IB Practice Exam: 09 Paper 2 Zone 2 – 90 min, Calculator Allowed

Note: The syllabus for Math SL changes with first exams starting May 2014. In particular, it no longer includes matrices. As such, you may skip those problems.

Name:____ Date: _________________ Class:__________

1. The following diagram is a box and whisker plot for a set of data.



The interquartile range is 20 and the range is 40.

(a) Write down the median value.

(1)

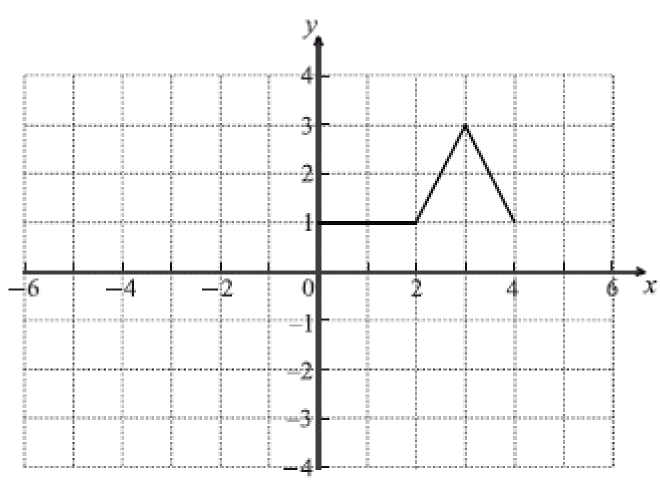
(b) Find the value of

(i) a;

(ii) b.

(4)
(Total 5 marks)

2. Consider the graph of f shown below.



(a) On the **same** grid sketch the graph of $y = f(-x)$.

(2)

The following four diagrams show **images** of f under different transformations.

Diagram A

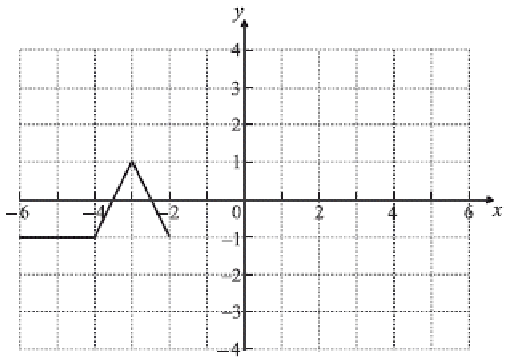


Diagram B

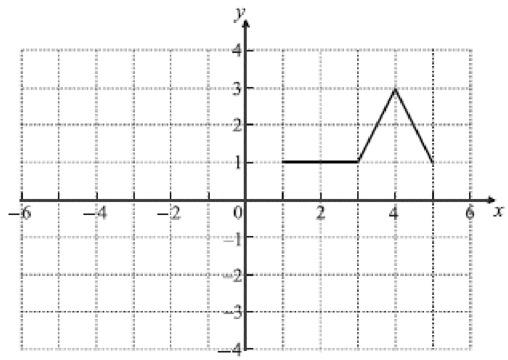


Diagram C

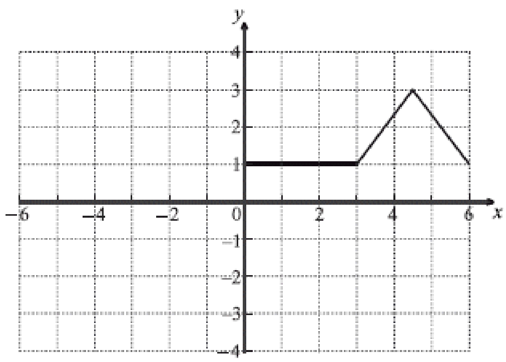
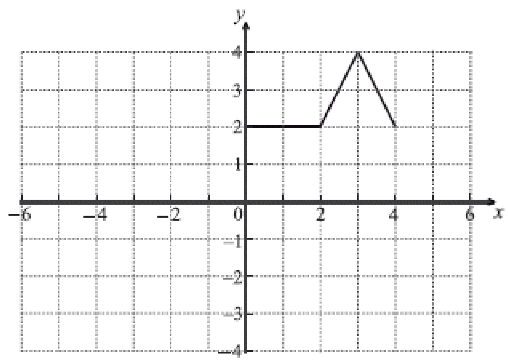


Diagram D



(b) Complete the following table.

Description of transformation	**Diagram letter**
Horizontal stretch with scale factor 1.5	
Maps f to $f(x) + 1$	

(2)

(c) Give a full geometric description of the transformation that gives the image in Diagram A.

(2)
(Total 6 marks)

3. Solve the equation $e^x = 4 \sin x$, for $0 \le x \le 2\pi$.

(Total 5 marks)

4. The diagram below shows a triangle ABD with AB = 13 cm and AD = 6.5 cm.
Let C be a point on the line BD such that BC = AC = 7 cm.

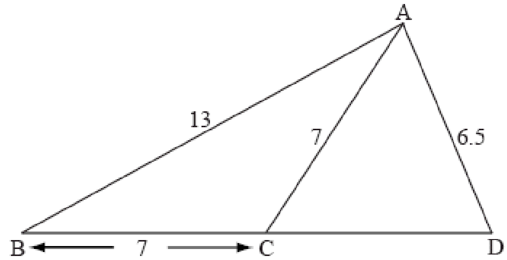


diagram not to scale

(a) Find the size of angle ACB.

(3)

(b) Find the size of angle CAD.

(5)
(Total 8 marks)

5. (a) Expand $\sum_{r=4}^{7} 2^r$ as the sum of four terms.

(1)

(b) (i) Find the value of $\sum_{r=4}^{30} 2^r$.

(ii) Explain why $\sum_{r=4}^{\infty} 2^r$ cannot be evaluated.

(6)
(Total 7 marks)

6. Consider the curve $y = \ln(3x - 1)$. Let P be the point on the curve where $x = 2$.

(a) Write down the gradient of the curve at P.

(2)

(b) The normal to the curve at P cuts the x-axis at R. Find the coordinates of R.

(5)
(Total 7 marks)

7. The quadratic equation $kx^2 + (k - 3)x + 1 = 0$ has two equal real roots.

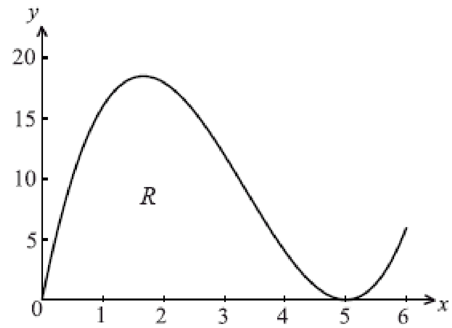
(a) Find the possible values of k.

(5)

(b) **Write down** the values of k for which $x^2 + (k - 3)x + k = 0$ has two equal real roots.

(2)
(Total 7 marks)

8. Let $f(x) = x(x - 5)^2$, for $0 \le x \le 6$. The following diagram shows the graph of f.



Let R be the region enclosed by the x-axis and the curve of f.

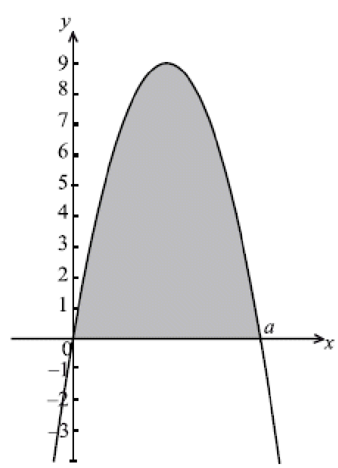
(a) Find the area of R.

(3)

(b) Find the volume of the solid formed when R is rotated through 360° about the x-axis.

(4)

(c) The diagram below shows a part of the graph of a quadratic function $g(x) = x(a - x)$. The graph of g crosses the x-axis when $x = a$.



The area of the shaded region is equal to the area of R. Find the value of a.

(7)
(Total 14 marks)

9. A van can take either Route A or Route B for a particular journey.

If Route A is taken, the journey time may be assumed to be normally distributed with mean 46 minutes and a standard deviation 10 minutes.

If Route B is taken, the journey time may be assumed to be normally distributed with mean μ minutes and standard deviation 12 minutes.

(a) For Route A, find the probability that the journey takes **more** than 60 minutes.

(2)

(b) For Route B, the probability that the journey takes **less** than 60 minutes is 0.85. Find the value of μ.

(3)

(c) The van sets out at 06:00 and needs to arrive before 07:00.

(i) Which route should it take?

(ii) Justify your answer.

(3)

(d) On five consecutive days the van sets out at 06:00 and takes Route B. Find the probability that

(i) it arrives before 07:00 on all five days;

(ii) it arrives before 07:00 on at least three days.

(5)
(Total 13 marks)

10. Let $f(x) = 3\sin x + 4\cos x$, for $-2\pi \le x \le 2\pi$.

(a) Sketch the graph of f.

(3)

(b) Write down

(i) the amplitude;

(ii) the period;

(iii) the x-intercept that lies between $-\frac{\pi}{2}$ and 0.

(3)

(c) Hence write $f(x)$ in the form $p \sin(qx + r)$.

(3)

(d) Write down one value of x such that $f'(x) = 0$.

(2)

(e) Write down the two values of k for which the equation $f(x) = k$ has exactly two solutions.

(2)

(f) Let $g(x) = \ln(x + 1)$, for $0 \le x \le \pi$. There is a value of x, between 0 and 1, for which the gradient of f is equal to the gradient of g. Find this value of x.

(5)

(Total 18 marks)

IB Practice Exam: 09 Paper 2 Zone 2 – MarkScheme

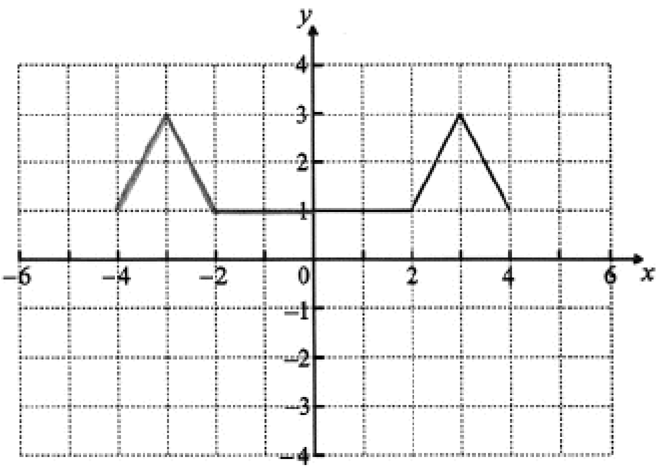
A1 N1

(b) (i) 10 A2 N2

(ii) 44 A2 N2

[5]

2. (a)



A2 N2

(b)

Description of transformation	Diagram letter
Horizontal stretch with scale factor 1.5	**C**
Maps *f* to *f*(*x*) + 1	**D**

A1A1 N2

(c) translation (accept move/shift/slide *etc.*) with vector A1A1 N2

[6]

3. evidence of appropriate approach M1

e.g. a sketch, writing $e^x - 4\sin x = 0$

$x = 0.371$, $x = 1.36$ A2A2 N2N2

[5]

4. (a) **METHOD 1**

evidence of choosing the cosine formula (M1)

correct substitution A1

e.g. $\cos A\hat{C}B = \frac{7^2+7^2-13^2}{2\times 7\times 7}$

$A\hat{C}B = 2.38$ radians (= 136°) A1 N2

METHOD 2

evidence of **appropriate** approach involving right-angled triangles (M1)

correct substitution A1

e.g. $\sin\left(\frac{1}{2}A\hat{C}B\right) = \frac{6.5}{7}$

$A\hat{C}B = 2.38$ radians (= 136°) A1 N2

(b) **METHOD 1**

$A\hat{C}C = \pi - 2.381$ (180 – 136.4) (A1)

evidence of choosing the sine rule in triangle ACD (M1)
correct substitution A1

e.g. $\frac{6.5}{\sin(0.760)} = \frac{7}{\sin A\hat{D}C}$

$A\hat{D}C = 0.836...$ (= 47.9...°) A1

$C\hat{A}D = \pi - (0.760... + 0.836...)$ (180 – (43.5... + 47.9...))
= 1.54 (= 88.5°) A1
N3

METHOD 2

[illegible] (A1)

evidence of choosing the sine rule in triangle ABD (M1)
correct substitution A1

e.g. [illegible]

$A\hat{D}C = 0.836...$ (= 47.9...°) A1

$C\hat{A}D = \pi - 0.836... - (\pi - 2.381...)$ (= 180 – 47.9... – (180 – 136.4))
= 1.54 (= 88.5°) A1
N3

Note: *Two triangles are possible with the given information. If candidate finds $A\hat{D}C$ = 2.31 (132°) leading to $C\hat{A}D$ = 0.076 (4.35°), award marks as per markscheme.*

[8]

5. (a) $\sum_{r=4}^{7} 2^r = 2^4 + 2^5 + 2^6 + 2^7$ (accept 16 + 32 + 64 + 128) A1 N1

(b) (i) **METHOD 1**

recognizing a GP (M1)

$u_1 = 2^4$, $r = 2$, $n = 27$ (A1)

correct substitution into formula for sum (A1)

e.g. $S_{27} = \frac{2^4(2^{27}-1)}{2-1}$

$S_{27} = 2147483632$ A1
N4

METHOD 2

recognizing $\sum_{r=4}^{30} = \sum_{r=1}^{30} - \sum_{r=1}^{3}$ (M1)

recognizing GP with $u_1 = 2$, $r = 2$, $n = 30$ (A1)

correct substitution into formula for sum

$S_{30} = \frac{2(2^{30}-1)}{2-1}$ (A1)

= 2147483646

$\sum_{r=4}^{30} 2^r = 2147483646 - (2 + 4 + 8)$

= 2147483632 A1
N4

(ii) valid reason (*e.g.* **infinite** GP, diverging series), **and** $r \geq 1$ (accept $r > 1$) R1R1
N2

[7]

6. (a) gradient is 0.6 A2 N2

(b) at R, $y = 0$ (seen anywhere) A1

at $x = 2$, $y = \ln 5$ (= 1.609...) (A1)

gradient of normal = –1.6666... (A1)

evidence of finding correct equation of normal A1

e.g. $y - \ln 5 = -\frac{5}{3}(x - 2)$, $y = -1.67x + c$

$x = 2.97$ (accept 2.96) A1

coordinates of R are (2.97, 0) N3

[7]

7. (a) attempt to use discriminant (M1)

correct substitution, $(k - 3)^2 - 4 \times k \times 1$ (A1)

setting **their** discriminant equal to zero M1

e.g. $(k - 3)^2 - 4 \times k \times 1 = 0$, $k^2 - 10k + 9 = 0$

$k = 1$, $k = 9$ A1A1 N3

(b) $k = 1$, $k = 9$ A2 N2

[7]

8. (a) finding the limits $x = 0$, $x = 5$ (A1)

integral expression A1

e.g. $\int_0^5 f(x)\,dx$

area = 52.1 A1 N2

(b) evidence of using formula $v = \int \pi y^2\,dx$ (M1)

correct expression A1

e.g. volume = $\pi \int_0^5 x^2(x-5)^4\,dx$

volume = 2340 A2 N2

(c) area is $\int_0^a x(a - x)\,dx$ A1

$= \left[\frac{ax^2}{2} - \frac{x^3}{3}\right]_0^a$ A1A1

substituting limits (M1)

e.g. $\frac{a^3}{2} - \frac{a^3}{3}$

setting expression equal to area of R (M1)

correct equation A1

e.g. $\frac{a^2}{2} - \frac{a^3}{3} = 52.1$, $a^3 = 6 \times 52.1$,

$a = 6.79$ A1 N3

[14]

9. $A \sim N(46, 10^2)$ $B \sim N(\mu, 12^2)$

(a) $P(A > 60) = 0.0808$ A2 N2

(b) correct approach (A1)

e.g. $P\left(Z < \frac{60-\mu}{12}\right) = 0.85$, sketch

$\frac{60-\mu}{12} = 1.036...$ (A1)

$\mu = 47.6$ A1

N2

(c) (i) route A A1

N1

(ii) **METHOD 1**

$P(A < 60) = 1 - 0.0808 = 0.9192$ A1

valid reason R1

e.g. probability of A getting there on time is greater than probability of B

$0.9192 > 0.85$

N2

METHOD 2

$P(B > 60) = 1 - 0.85 = 0.15$ A1

valid reason R1

e.g. probability of A getting there late is less than probability of B

$0.0808 < 0.15$

N2

(d) (i) let X be the number of days when the van arrives before 07:00

$P(X = 5) = (0.85)^5$ (A1)

$= 0.444$ A1

N2

(ii) **METHOD 1**

evidence of adding correct probabilities (M1)

e.g. $P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5)$

correct values $0.1382 + 0.3915 + 0.4437$ (A1)

$P(X \geq 3) = 0.973$ A1

N3

METHOD 2

evidence of using the complement (M1)

e.g. $P(X \geq 3) = 1 - P(X \leq 2)$, $1 - p$

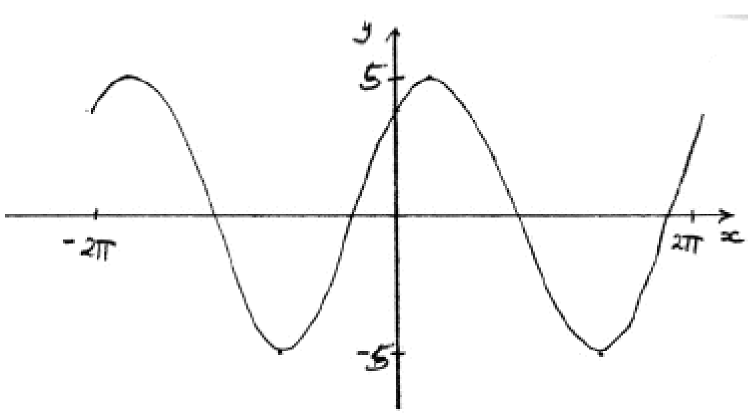
correct values $1 - 0.02661$ (A1)

$P(X \geq 3) = 0.973$ A1

N3

[13]

10. (a)



A1A1A1 N3

Note: *Award A1 for approximately sinusoidal shape,*
A1 for end points approximately correct, (–2π, 4),

(2π, 4) A1 for approximately correct position of graph, (y-intercept (0, 4) maximum to right of y-axis).

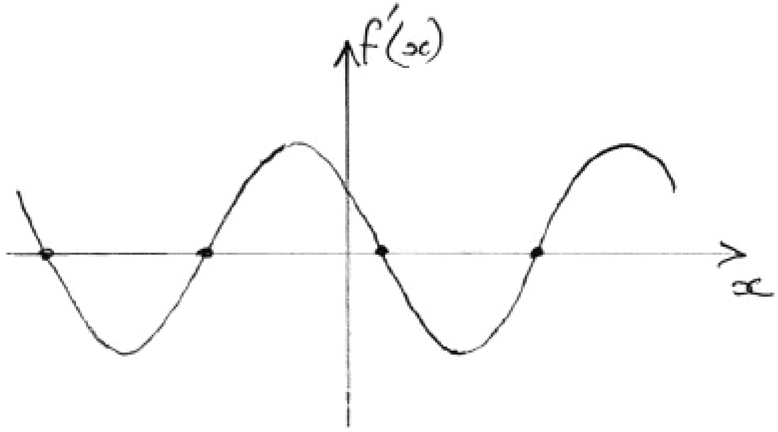
(b) (i) 5 A1 N1

(ii) 2π (6.28) A1 N1

(iii) –0.927 A1 N1

(c) $f(x) = 5\sin(x + 0.927)$ (accept $p = 5$, $q = 1$, $r = 0.927$) A1A1A1 N3

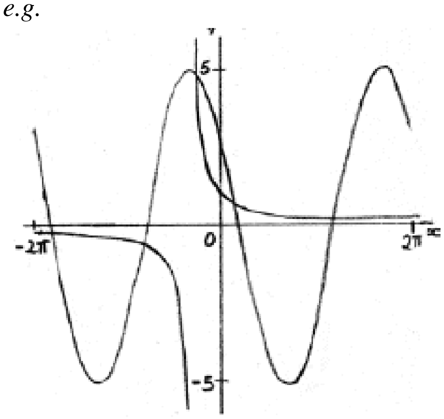
(d) evidence of correct approach (M1)
e.g. max/min, sketch of $f'(x)$ indicating roots



one 3 s.f. value which rounds to one of –5.6, –2.5, 0.64, 3.8 A1 N2

(e) $k = -5$, $k = 5$ A1A1 N2

(f) **METHOD 1**
graphical approach (but must involve derivative functions) M1
e.g.



each curve A1A1
$x = 0.511$ A2 N2

METHOD 2

$g'(x) = \frac{1}{x+1}$ A1

$f'(x) = 3\cos x - 4\sin x$ $(5\cos(x + 0.927))$ A1
evidence of attempt to solve $g'(x) = f'(x)$ M1
$x = 0.511$ A2 N2

[18]