MATH 442 / OR 442 Midterm Examination

120 Minutes

Rules:

1. Close book except one-page note
2. No calculator
3. No cheating

Name:_____ ID: _____

1. (10) Before burning out, a light bulb gives X hours of light, where X is normally distributed with mean 100, and standard deviation 100. Namely $X \sim N(100, 100^2)$. If we have 4 bulbs, what is the probability that they will give a total of at least 560 hours?

(Critical point for standard normal distribution: $z_{0.5} = 0.0$, $z_{0.55} = 0.1$, $z_{0.6} = 0.2$, $z_{0.65} = 0.3$, $z_{0.7} = 0.5$, $z_{0.75} = 0.6$, $z_{0.8} = 0.8$, $z_{0.85} = 1.0$, $z_{0.9} = 1.2$, $z_{0.95} = 1.6$, $z_{0.99} = 2.3$, $z_{0.999} = 3.0$. If you can not find the exact number, please use the closest number as an approximation.)

2. Consider an inventory system in which the sequence of events during each period is as follows. (1) We observe the inventory level (call it i) at the beginning of the period. (2) If $i \leq 1$, 3-i units are ordered. Otherwise, 0 units are ordered. Delivery of all ordered units is immediate. (3) With probability 1/3, 0 unit is demanded during the period; with probability 1/3, 1 unit is demanded during the period; and with probability 1/3, 2 units are demanded during the period. (4) We observe the inventory at the beginning of the next period. Define a period's state to be the period's beginning inventory level.

(a) (8)   Determine the transition matrix.

(b) (7)   Is this Markov chain ergodic? Why?

3. There are two restaurants in a small town: M and B. A survey shows that 20% of M customers in this week will switch to B restaurant next week, and 30% of B customers will go to M next week.

(a) (7)   If the market share for M restaurant is currently 50% this week, what is the M restaurant's market share next week?

(b) (7)   What is the steady-state market share for these two restaurants?

(c) (7)   What is the mean first passage time from M to B?

(d) (9)   Suppose the total annual restaurant profit in this small town is $1,000,000. The owner of M restaurant is considering to add a new restaurant N to gain more market share and hopefully can make more money. The owner has done a study and found out the following if the restaurant N is added:

   • For M customers, 10% of them will go to B and 20% will go to N next week,

   • For B customers, 20% of them will go to M and 20% will go to N next week,

   • For N customers, 20% of them will go to B and 20% will go to M next week,

   • There is an additional cost of $100,000 for operation of the N restaurant.

   Should the owner open the new restaurant?

4. Nancy wants to sell some special material called "CC" at the price of $35 per pound at this Sunday Festival. The order price from the manufacturer is $20 per pound. She can return the unsold material to the manufacturer at $10 per pound. Suppose the probability distribution for demand is given in the flowing table.

Demand (pounds)	Probability
10	0.2
15	0.3
20	0.5

(a) (8)   How many pounds of CC should Nancy order? You can use any method.

(b) (7)   If Nancy decides to order 20 pounds, what is her expected profit?

5. This is the same as Question (4) except the probability distribution. Again, Nancy wants to sell some special material called "CC" at the price of $35 per pound at this Sunday Festival. The order price is $20 per pound. She can return the unsold materials at $10 per pound. Suppose the probability distribution for demand is uniformly distributed between 10 and 20 pounds.

(a) (7)   How many pounds of CC should Nancy order? Please use marginal analysis.

(b) (8)   Please use the generic method to do part (a).

6. Each week, a computer store sells an average of 100 sets of external hard drive. Weekly demand is normally distributed with a standard deviation of 4 sets. The store orders hard drives from a regional distributor. Each order is filled in one week. The cost of placing each order is $50, and the weekly cost of holding one set of hard drive in inventory is $1. The per-unit stockout cost is assumed to be $5. The store is willing to assume that all demand is back-logged.

(a) (7)   Determine a good size of order.

(b) (8)   Determine a good reorder point.

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