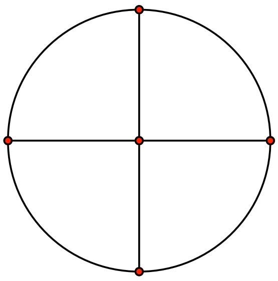
IB Math HL Y1 **Lesson 14 Radian Measure and Unit Circle**
Mr. Jauk

Name ________________________ Date:_____________

Part 1 – Review of unit circle trig

1. Suppose the diagram shows a unit circle, that is, a circle of radius 1 and center at the origin. Use the diagram to show how $\sin\theta, \cos\theta$, and $\tan\theta$, are defined as circular functions, i.e. as functions on the unit circle.



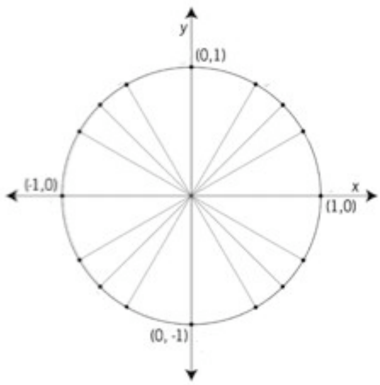
$\sin\theta =$

$\cos\theta =$

$\tan\theta =$

Show how these definitions are compatible with SOHCAHTOA.

2. Write the exact value. Use the unit circle diagram to help you.



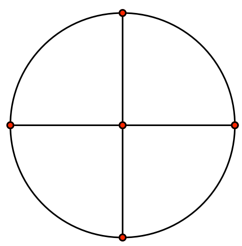
(a) $\cos 30^\circ$ (b) $\tan 315^\circ$

(c) $\sin\frac{7\pi}{6}$ (d) $\cos\frac{3\pi}{4}$

(e) $\tan\frac{\pi}{6}$ (f) $\cos\frac{5\pi}{3}$ (g) $\sin\left(-\frac{\pi}{4}\right)$ (h) $\cos\frac{19\pi}{6}$

You need to know how to find the unit circle coordinates and associated trig function values for problems like these ***QUICKLY*** and ***ACCURATELY***.

3. Indicate which of the basic trig functions is positive in each quadrant, and the reference angle formula associated with each quadrant.



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Part 2 – Review??

4. Use reference angles to answer the following, without a calculator.
   (a) Given that $\sin 28^{\circ} \approx 0.2709$, what is $\sin 332^{\circ}$?

   (b) Given that $\tan 110 \approx -2{,}747$, what is $\tan 250^{\circ}$?

5. Use reference angles and your calculator to find all solutions without graphing. Give answers correct to 3 s.f. using appropriate units.
   (a) $\cos\theta = 0.9$, on the interval $0^{\circ} \le \theta < 360^{\circ}$

   (b) $\sin\theta = -\dfrac{1}{4}$, on the interval $0 \le \theta < 2\pi$

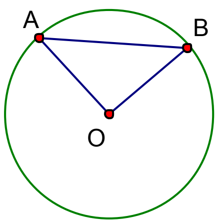
6. Use the symmetries of the unit circle to complete each of the following:
   (a) $\sin(\pi - \theta) =$

   (b) $\tan(\pi + \theta) =$

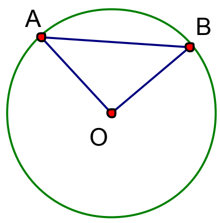
   (c) $\cos\left(\dfrac{\pi}{2} - \theta\right) =$

7. In the space below are three copies of the same diagram. Suppose $OA = 5$ cm, and $m\angle AOB = 95^{\circ}$. For each of the following, first shade the indicated part of the circle. Then determine the indicated quantity.

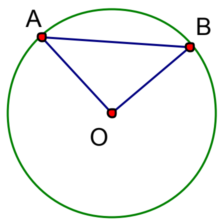
   (a) length of arc AB



   (b) area of sector AOB



   (c) ***area*** of segment AB



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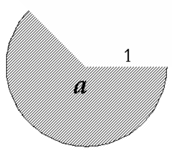
Suppose a central angle θ is measured in radians rather than in degrees. Can you figure out the new formulas for arc length and the area of a sector? [Hint: If $\theta = 2\pi$, then you should get the familiar formulas for circumference and area.

In a circle of radius r, and with a sector determined by a central angle of θ radians: (a) arc length $l =$ and (b) area of sector $A =$

8. A sector has radius 8.2 cm and arc length 13.3 cm. Find the area of the sector.

9. The sector shown has radius 1 meter, and central angle $a = \dfrac{4\pi}{3}$ radians. The figure is cut out and the two straight edges are brought together to form a cone. Find the volume of this cone.



Part 3 – Extension – Linear and Angular Velocity

Suppose an object is on the outside of a rotating circle of radius r, and that the circle makes n complete rotations in T units of time. Then

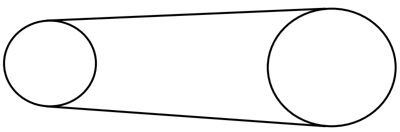
Linear velocity $v =$
and angular velocity $\omega =$ radians/unit time

10. A carousel has three concentric rings of animals. Selena is riding in the middle ring, about 5 meters from the center of the carousel. Roxanne is in the outermost ring, about 7 meters from the center of the carousel. The carousel makes one complete turn every 15 seconds.

(a) Which is the same for both girls, the linear velocity or the angular velocity?

(b) How much faster is Roxanne going than Selena?

11. Two wheels of radius 8 cm and 11 cm respectively are connected by a belt, as shown in the diagram. The smaller wheel makes 3 complete turns per second.



(a) Which is the same for both wheels, the linear velocity or the angular velocity?

(b) How fast is the larger wheel turning?