13.4 Evaluate Inverse Trigonometric Functions

Name Key Date ____________

1. Use your knowledge of the unit circle to find the value(s) of θ. Pay attention to the ***units*** **(degrees or radians)** and to the ***interval*** required for each question. A unit circle is given for your reference.

(a) $\tan\theta = \sqrt{3},\ 0^\circ \le \theta \le 360^\circ$

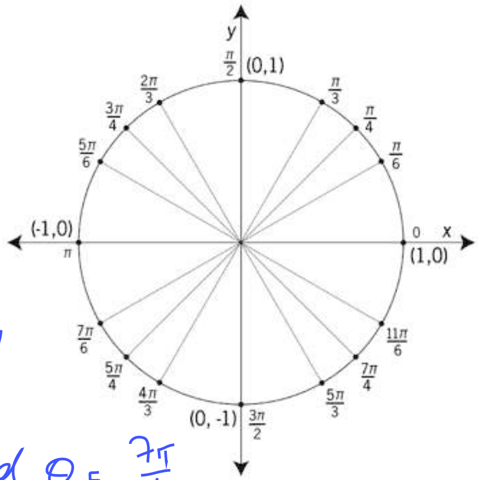
$\theta = 60^\circ$ (or $\frac{\pi}{3}$)

$\theta = 240^\circ$ (or $\frac{4\pi}{3}$)

(b) $\cos\theta = \frac{\sqrt{2}}{2},\ -180^\circ \le \theta \le 180^\circ$

$\theta = 45^\circ$

$\theta = -45^\circ$



(c) $\sin\theta = -\frac{1}{2},\ 0^\circ \le \theta \le 360^\circ$

$\theta = 210^\circ$

$\theta = 330^\circ$

(d) $\cot\theta = -1,\ 0 \le \theta \le 2\pi$

↳ same as $\tan\theta = -1$

$\theta = \frac{3\pi}{4}$ and $\theta = \frac{7\pi}{4}$

(e) $\cos\theta = -1,\ 0 \le \theta \le 2\pi$

$\theta = \pi$

(f) $\csc\theta = 2,\ -\pi \le \theta \le \pi$

$\frac{1}{\sin\theta} = 2 \Rightarrow \sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}$ (or 30°)

$\theta = \frac{5\pi}{6}$ (or 150°)

Now try the same thing using your calculator. Recall that $\sin^{-1}\left(\frac{2}{3}\right)$ gives you the angle whose sine is $\frac{2}{3}$.

2. Use your graphing calculator to evaluate:

(a) $\tan^{-1}\sqrt{3}$ [Degrees]

60°

(b) $\cos^{-1}\frac{\sqrt{2}}{2}$ [Degrees]

45°

(c) $\sin^{-1}\left(-\frac{1}{2}\right)$ [Radians]

$-\frac{\pi}{6}$ (or -30°)

What do you observe?

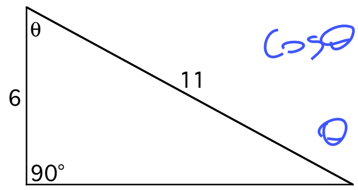
Only 1 answer, how does the calculator choose

Recall that in a function, each input value (x) is matched with one output value (y or in this case θ), i.e. one input cannot have 2 different outputs. Thus in order for $\theta = \sin^{-1} x$, $\theta = \cos^{-1} x$, and $\theta = \tan^{-1} x$ to be functions, one θ-value out of all the possible values is chosen to correspond to each x-value.

For right triangle problems, there is no problem; when x is positive, the chosen θ value is in the first quadrant, i.e. θ is an acute angle.

3. Find θ in each of the following, rounding your answers to the nearest tenth of a degree.

(a)

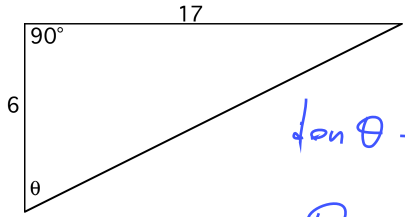


$\cos\theta = \frac{6}{11}$

$\theta = \cos^{-1}\left(\frac{6}{11}\right)$

$\theta \approx 56.94°$

(b)



$\tan\theta = \frac{17}{6}$

$\theta = \tan^{-1}\left(\frac{17}{6}\right)$

$\theta \approx 70.56°$

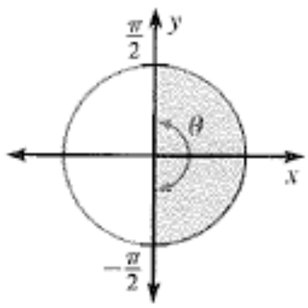
However, for negative trig values, different quadrants have been chosen for the outputs for $\theta = \sin^{-1} x$, $\theta = \cos^{-1} x$, and $\theta = \tan^{-1} x$.

Inverse Trigonometric Functions

If $-1 \le a \le 1$, then the **inverse sine** of a is an angle θ, written $\theta = \sin^{-1} a$, where:

(1) $\sin\theta = a$

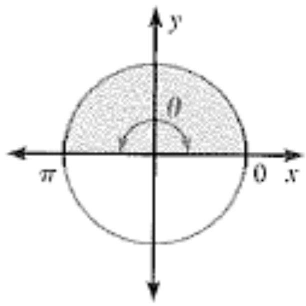
(2) $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ (or $-90° \le \theta \le 90°$)



If $-1 \le a \le 1$, then the **inverse cosine** of a is an angle θ, written $\theta = \cos^{-1} a$, where:

(1) $\cos\theta = a$

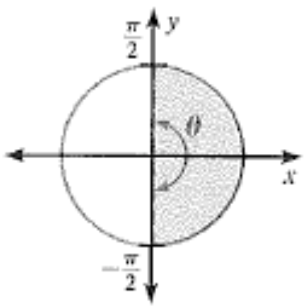
(2) $0 \le \theta \le \pi$ (or $0° \le \theta \le 180°$)



If a is any real number, then the **inverse tangent** of a is an angle θ, written $\theta = \tan^{-1} a$, where:

(1) $\tan\theta = a$

(2) $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ (or $-90° < \theta < 90°$)



You may be wondering how/why these choices were made, but unfortunately, that's something that we are saving for next year in Precalculus.

4. Evaluate in both radians and degrees. Remember, you should have only one value for each of these (expressed in radians and in degrees). **[No calculator]**

(a) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ $30°$ or $\frac{\pi}{6}$

(b) $\tan^{-1}(1)$ $45°$ or $\frac{\pi}{4}$

(c) $\sin^{-1}\left(-\frac{1}{2}\right)$ $-30°$ or $-\frac{\pi}{6}$

(d) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ $135°$ or $\frac{3\pi}{4}$

(e) $\tan^{-1}\left(-\sqrt{3}\right)$

-60 or $-\frac{\pi}{3}$

(f) $\sin^{-1} 0$

0 or 0

Sometimes we have to combine these rules for inverse trig functions with the formulas for reference angles.

5. Solve the equation for θ: $\cos\theta = 0.4$ where $270^\circ < \theta < 360^\circ$.

Note that 0.4 is not a value that we know on the unit circle, so we need to use a calculator. Here are the steps:

(a) Find $\cos^{-1} 0.4$. $\approx 66.42°$

(b) Note that the answer to (a) is in Quadrant I, while the desired answer is in Quadrant IV. Use the reference angle formula for Quadrant IV to find the desired angle.

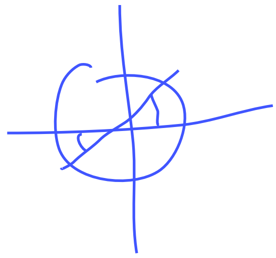
$360 - 66.42 \approx 293.58°$

6. Solve the equation for θ: $\sin\theta = -\frac{5}{8}$ where $180^\circ < \theta < 270^\circ$.

In this case, even though we want $\sin\theta = -\frac{5}{8}$, it is easier to start out with $\sin^{-1}\left(\frac{5}{8}\right)$, since this will give an angle in Quadrant I. Then we can just use the reference angle formulas that we already know. Otherwise, you have to think about the quadrant you are in, and the quadrant you want to move to, and come up with a different rule for each possible combination. It's possible, but easy to make mistakes.

$R = \sin^{-1}\left(\frac{5}{8}\right) = 38.7$

$\theta = R + 180^\circ = 218.7$



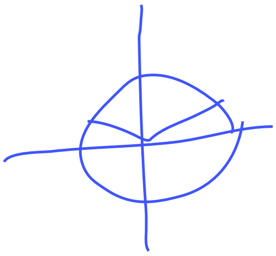
$\sin^{-1}\left(-\frac{5}{8}\right) = -38.7$

7. Solve the equation for θ.

(a) $\sin\theta = \frac{2}{7}$ where $90^\circ < \theta < 180^\circ$

$R = \sin^{-1}\left(\frac{2}{7}\right) = 16.6^\circ$

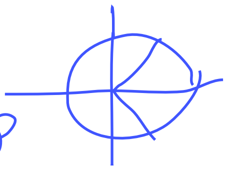
$\theta = 180 - R = 163.4^\circ$



(b) $\tan\theta = -\frac{13}{5}$ where $\frac{3\pi}{2} < \theta < 2\pi$

$R = \tan^{-1}\left(\frac{13}{5}\right) = 1.2$

or $\tan^{-1}\left(-\frac{13}{5}\right) + 2\pi$



$\Rightarrow \theta = 2\pi - R = 5.08$

(c) $\cos\theta = -\frac{3}{4}$ where $\pi < \theta < \frac{3\pi}{2}$

$R = \cos^{-1}\left(\frac{3}{4}\right) = 0.72$

$R = \cos^{-1}\left(-\frac{3}{4}\right) = 2.42$

$\theta = 2\pi - R$

$\theta = R + \pi = 3.86$