Thermo Haake
Material Characterization

Introduction to Elasticity

by

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Contents

1. Introduction

2. Theory

2.1 Definitions

2.2 Modeling of Viscoelastic Behavior

2.2.1 Basic Model For An Ideal Elastic Substance

2.2.2 Basic Model For A Purely Viscous Substance

2.2.3 Basic Model For A Viscoelastic Substance

2.3 Summary

3. Principles of Viscoelastic measurements

3.1 Viscoelastic Model Under Dynamic Conditions

3.2 Ideal Elastic Model Under Dynamic Conditions

3.3 Purely Viscous Model Under Dynamic Conditions

4. Instrument Design

5. Evaluation

6. HAAKE Measuring Instruments to Determine Viscoelastic Properties

7. HAAKE Software Oscillation

8. Examples of Measurement

9. Symbols and Units

10. Conclusion

1. INTRODUCTION

Measuring the viscosity of a substance is a standard method which characterizes the rheological behavior of substances that flow. By recording a flow curve (shear stress τ vs. shear rate D), information about flow behavior, viscosity, yield value, and thixotropy can be obtained.

In most cases, liquids and pastes do not exhibit a purely "viscous" behavior. Instead, they exhibit a "Viscoelastic" behavior. Therefore, measuring only the viscosity is often an insufficient method for the characterization of rheological behavior.

"Viscoelasticity" means that liquids have a distinctly "elastic" component which can cause difficulties in industrial processes or use of the substance. An elastic behavior can also be desirable. Following are some examples which illustrate the effect of Viscoelasticity:

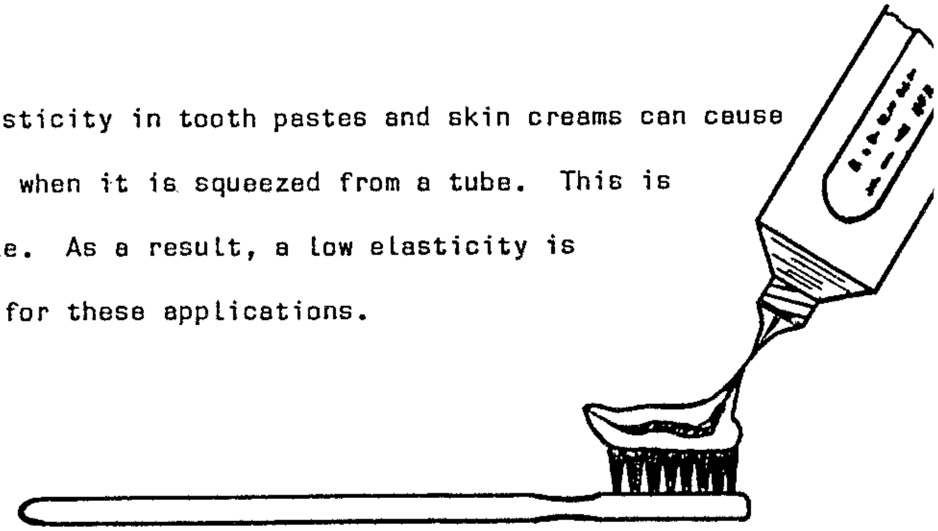
FOOD

Viscoelasticity is an important element in describing the "mouth feel" or texture of food and beverages. A natural orange juice, for example, is a purely viscous beverage. Often, a thickener is added to natural juices. The thickener can have a distinct elastic component which can change the mouth feel to a very negative feeling.

On the other hand, an elastic property can also be desired. For example, cheese slices with a little elasticity can be separated more easily than slices made from purely viscous cheese.

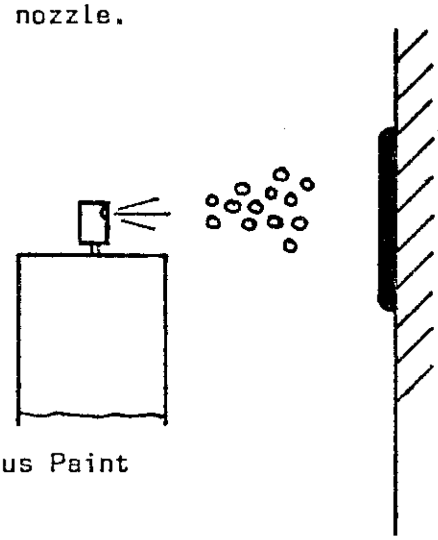
COSMETICS

A high elasticity in tooth pastes and skin creams can cause "strings" when it is squeezed from a tube. This is undesirable. As a result, a low elasticity is preferred for these applications.

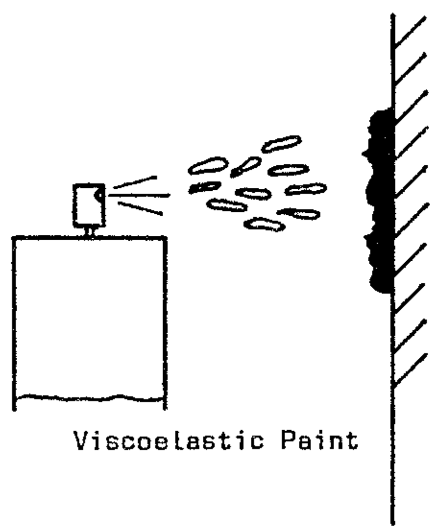


PAINTS AND VARNISH

A purely viscous fluid that is sprayed will disperse into fine spherical droplets which produces a smooth and evenly coated surface. Viscoelastic fluids disperse into elongated drops when sprayed. This effect causes an unevenly coated surface and possible clogging of the spray nozzle.



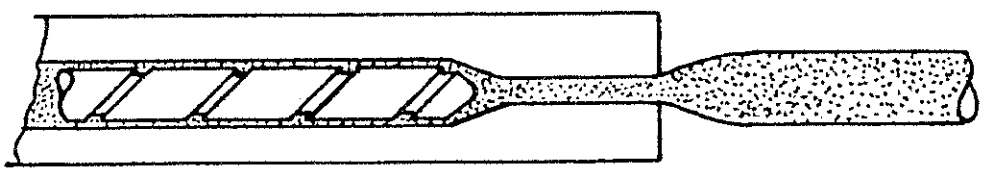
Viscous Paint



Viscoelastic Paint

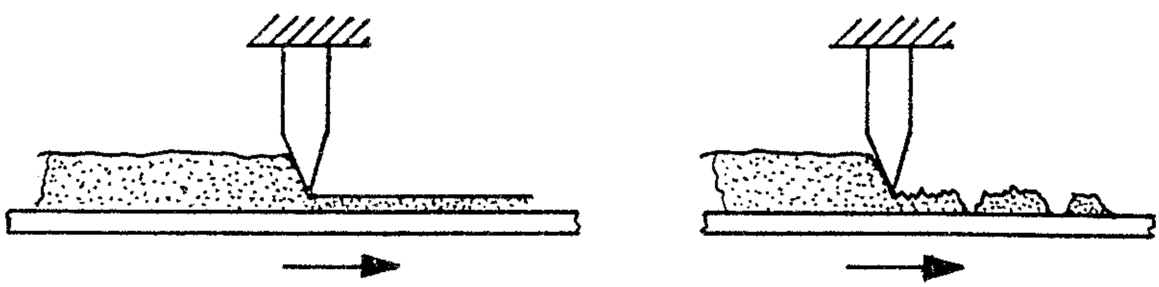
PLASTICS/RUBBER

During extrusion, plastic and rubber materials will expand after they leave the die (die swell), due to a high elastic component. As a result the dimensions of the extruded material will be larger than the die opening.



COATINGS

Coatings are often produced by applying a thin layer of material onto a moving substrate. A doctor blade controls the thickness of the coating. Highly elastic substances develop a structured surface, because they stick to the doctor blade. On the other hand, purely viscous materials pass the doctor blade without sticking and a smooth coating surface forms.



FLOW OPTIMIZER

Viscoelastic fluids flow more easily through pipes than purely viscous fluids (Tom's effect). Therefore, tensides are added to these viscous fluids to make them Viscoelastic. In this way, flow through pipes can be increased without an increase in pressure drop.

2. THEORY

2.1 Definition

To define "Viscoelasticity" we consider two extreme cases:

- ideal elastic behavior
- purely viscous behavior

The behavior of real substances is somewhere between these two extremes. Therefore they are called:

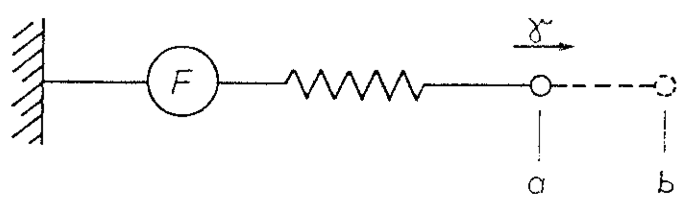
"Viscoelastic substances"

2.2 Modeling of Viscoelastic Behavior

To understand, characterize and quantify Viscoelastic behavior, models have been created for both extreme cases:

2.2.1 Basic Model For An Ideal Elastic Substance:

The model for an "ideal elastic" substance is a spring:



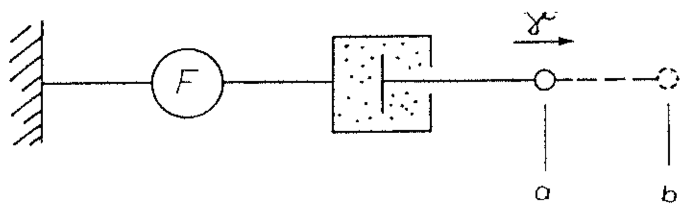
To pull the end of a spring from point a to point b, a measured force F is required. This force is directly proportional to the deformation of the spring. This deformation is called strain and is denoted by the symbol γ.

$$F \sim \gamma$$

In addition, a spring that is deformed to point b will immediately return to its original position a when it is released, because the spring (and so an ideal elastic substance) has fully stored the applied energy.

2.2.2 Basic Model For A Purely Viscous Substance

The model for a "purely viscous" substance is a dashpot:



In order to pull the dashpot from a to b, a measured force F is required. This force is independent of deformation, but proportional to deformation velocity:

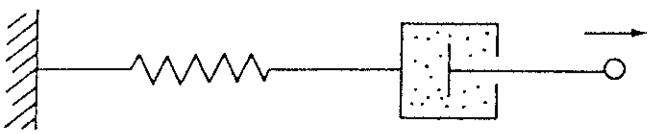
$$F \sim d\gamma / dt$$

$$F \sim v$$

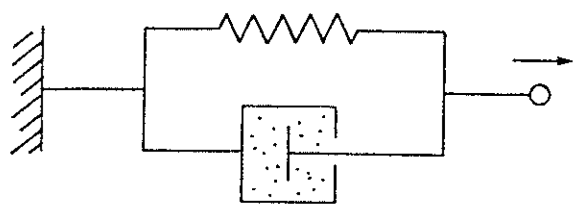
If we release the dashpot in position b it will not jump back to position a. That means that in a dashpot (and so at a "purely viscous" substance) the applied energy is fully lost.

2.2.3 Basic Model For A Viscoelastic Substance

To be able to characterize the behavior of a Viscoelastic substance, the two extreme models can be put together in every thinkable configuration. Well known configurations are the Maxwell and the Kelvin/Voigt model:



Maxwell Model



Kelvin/Voigt Model

2.3 Summary

For ideal elastic substances:

- energy invested is fully stored and recoverable
- the force required to deform the system depends on the deformation (strain).

For purely viscous substances:

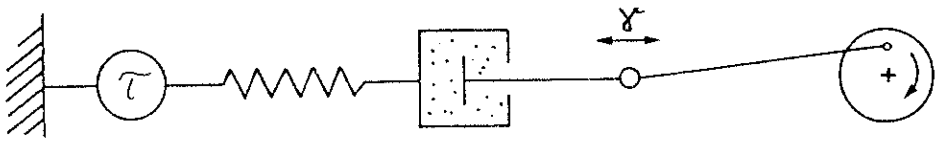
- energy invested is fully lost in the substance
- the force required depends on the deformation velocity and is independent of the degree of deformation.

For Viscoelastic substances:

- a mixture of the two pure substance behavior is observed.

3. PRINCIPLES OF VISCOELASTIC MEASUREMENTS

3.1 Viscoelastic Model Under Dynamic Conditions

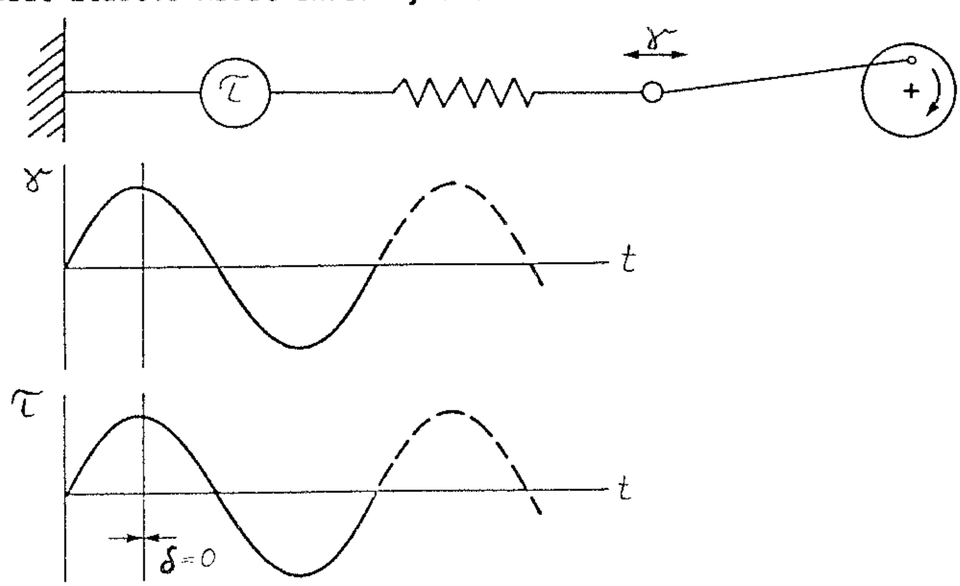


1. Pulling a spring/dashpot over an infinate distance as described in section two, is not a reasonable way to measure elasticity. A better solution is, to keep the spring/dashpot model (= Viscoelastic substance) under an oscillating deformation.

We rotate the disk. The rotation generates an oscillating deformation at the end of the spring/dashpot model.

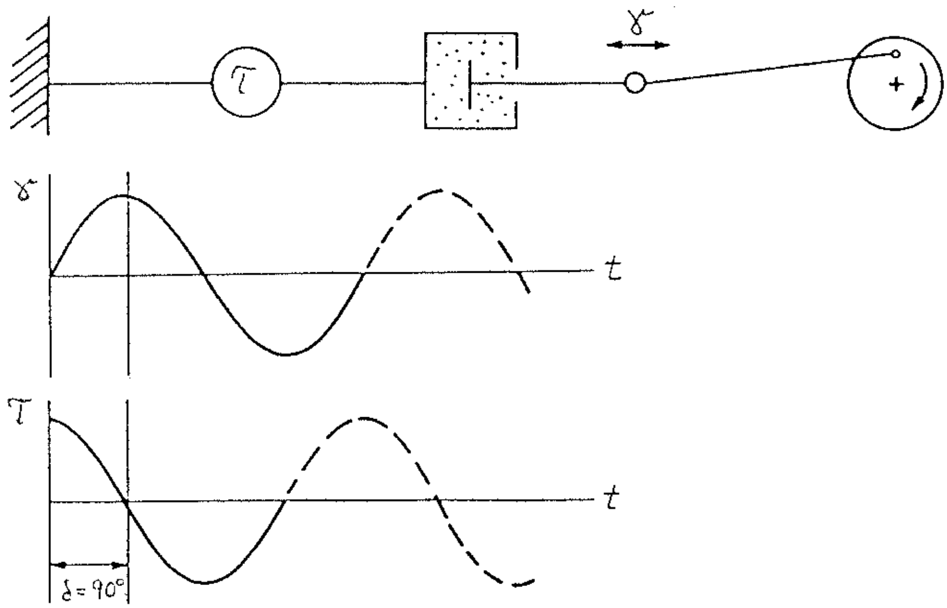
With a sensor, we measure the reacting stress on the system as τ = F/A. As we know, the two basic models react differently under deformation. The ideal elastic substance (spring) depends on the deformation, but the purely viscous material depends on the deformation velocity.

3.2 Ideal Elastic Model Under Dynamic Conditions



The stress of an ideal elastic material depends on the degree of deformation. Therefore we find that stress and strain are "in-phase". There is no phase shift ($\delta = 0^\circ$).

3.3 Purely Viscous Model Under Dynamic Conditions:

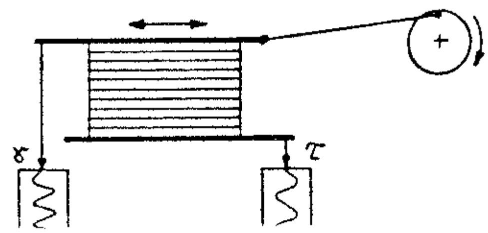


The stress of a purely viscous material depends on the deformation velocity. At the two maximum strain values, the strain velocity is zero, because the system changes direction. Therefore we find that stress and strain are 90° out of the phase ($\delta = 90^\circ$)

All "Viscoelastic substances" having a viscous as well as an elastic part show a phase shift between 0° and 90°. Therefore the phase shift δ is a measure of the Viscoelasticity of a substance. The closer the phase shift is to 0°, the more elastic a given material is. The closer the phase shift is to 90°, the more viscous a given material is.

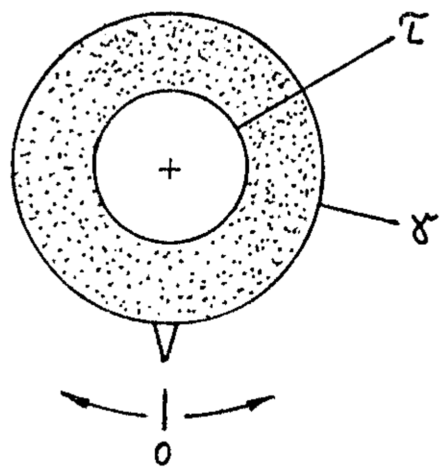
4. INSTRUMENT DESIGN

Modifying the kinematic parallel-plate model from Isaac Newton to a dynamic model, we have the following:



The strain γ as well as the shear stress τ can be recorded on an x-t recorder and the phase shift determined from the curves.

If the parallel plates are bent into a coaxial cylinder system, then a more reasonable instrument would look like the following:



With this coaxial cylinder system we generate an oscillating deformation with the outer cylinder and measure the torque response with the inner cylinder.

The strain is a function of the amplitude of deformation, and the size of the gap.

The stress depends on the measured torque, and the surface area of the inner cylinder. Both strain γ and stress τ can be calculated from these parameters.

5. EVALUATION

With the above mentioned type of instrument, it is possible to make a defined strain and measure a defined stress. We know that these two signals can be in-phase or out-of-phase depending on the Viscoelastic behavior of the measured material.

$\gamma = \gamma_0 \sin(\omega t)$ γ_0 = max. amplitude of strain

$\tau = \tau_0 \sin(\omega t + \delta)$ τ_0 = max. amplitude of stress

Now we have to find a way to describe these Viscoelastic properties.

The relationship between stress and strain is defined as:

$$\tau = G^* \cdot \gamma$$

using the complex modulus G*.

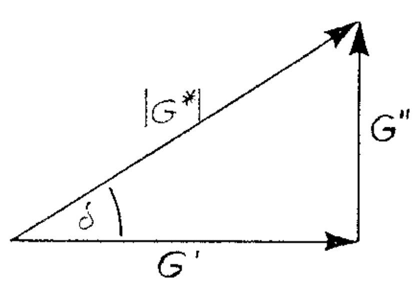
The complex modulus G* includes the complete information of the Viscoelastic properties; the elastic component, the viscous component as well as the phase shift between stress and strain.

$$G^* = G' + iG''$$

where G' is the storage modulus and G" is the loss modulus.

The storage modulus G' is a measurement for the energy stored and recovered in the material, while the loss modulus is a measurement for the energy lost as heat in the material.

This relationship can be visualized in the following graph:



The absolute magnitude of the complex modulus G* can now be calculated as the peak stress divided by the peak strain:

$$|G^*| = \frac{\tau_0}{\gamma_0}$$

Out of this value, the storage and the loss modulus can be calculated using trigonometric identities:

$$G' = |G^*| \cos \delta$$

$$G'' = |G^*| \sin \delta$$

A useful parameter which is a measurement of the ratio of energy lost to energy stored is the loss tangent:

$$\tan \delta = G'' / G'$$

As an alternative to G*, the phase relationship can also be described by a complex viscosity:

$$\eta^* = \eta' - i\eta''$$

with $\eta' = G'' / \omega$ and $\eta'' = G' / \omega$.

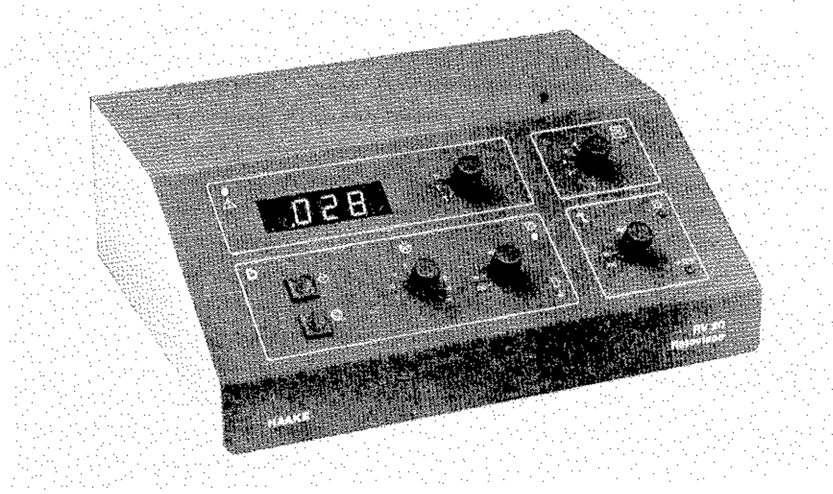
When both the amplitude of strain and the frequency becomes very small, a Viscoelastic fluid will behave more like a newtonian fluid. In this case, the dynamic viscosity approaches the steady-shear viscosity :

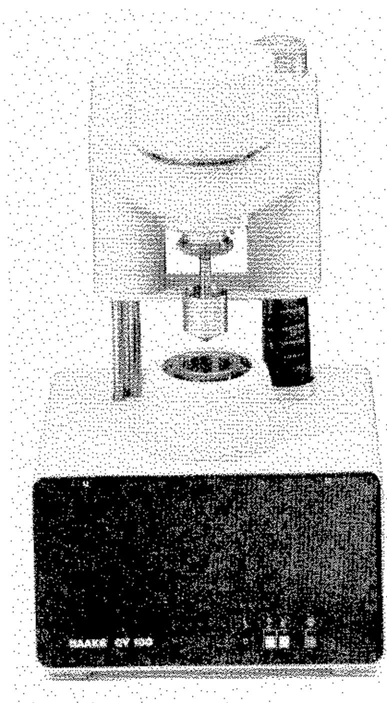
$$\lim_{\omega \to 0} \eta' = \eta$$

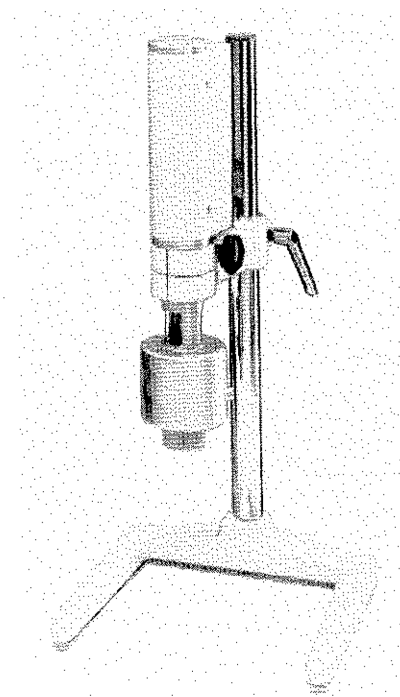
6. HAAKE MEASURING INSTRUMENTS TO DETERMINE VISCOELASTIC PROPERTIES

Dynamic measurements can be done with the HAAKE Rotational Viscometer, RV20. Both, a Searle System (M5-Osc) as well as a Couette System (CV 100) is available.

Depending on the application, you have a choice of a coaxial cylinder, plate and cone, or plate and plate sensor systems.







Control of the RV20 and data evaluation is done with a computer using HAAKE Software "Oscillation". Computer systems such as a Hewlett Packard HP 86B, an IBM PC/XT/AT, or an IBM PS/2 can be used.
The interface between the RV20 and the computer is the HAAKE Rheocontroller RC20.

7. HAAKE-SOFTWARE OSCILLATION

The HAAKE Software Oscillation controls the Rotovisco RV20 and reads and evaluates the measured data.
The software is designed so that the following parameters can be calculated:

- Complex modulus G^*
- Storage modulus G'
- Loss modulus G''
- Loss tangent $\tan\delta$
- Dynamic viscosity η'
- Complex viscosity η^*

All parameters can be determined as a function of:

- strain γ
- angular velocity ω
- time t
- temperature T

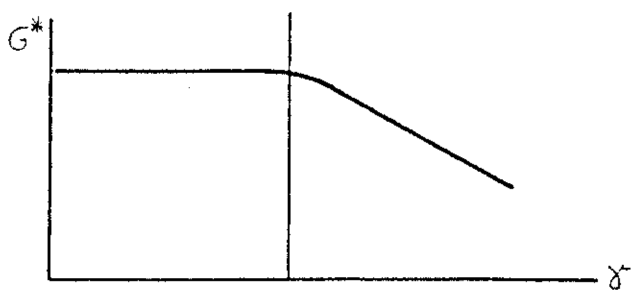
If the deformation amplitude φ and the frequency f is selected as test parameters, the values can be varied in the following ranges:

	$0.017 \leqslant \varphi \leqslant 0.67$	deformation amplitude
or	$1 \leqslant \varphi \leqslant 38$ deg	
	$0.05 \leqslant \gamma \leqslant 120$ rad	strain
	$0.05 \leqslant f \leqslant 5$ Hz	frequency
or	$0.3 \leqslant f \leqslant 35$ rad/s	

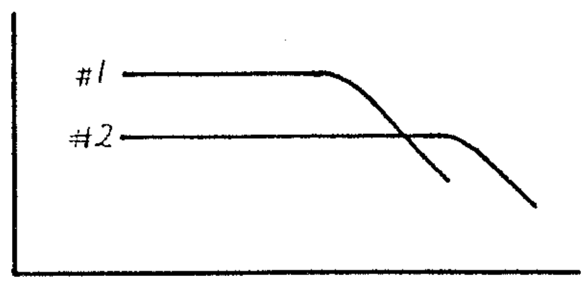
8. EXAMPLES OF MEASUREMENTS

Complex modulus = f (strain) ω=const

This measurement is used to find the linear Viscoelastic range or to measure the stability of a material:



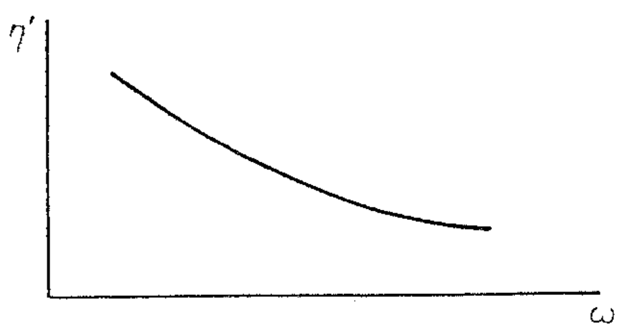
If the strain is outside of the range, the structure of the material will be destroyed.



Material #2 is more stable than #1.

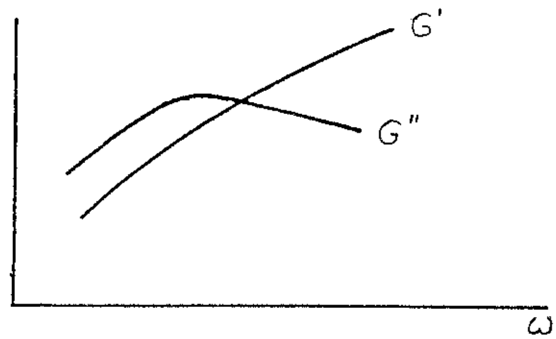
Dynamic Viscosity = f (angular velocity) γ=const

This measurement gives a qualitative information about the flow behavior of the material.



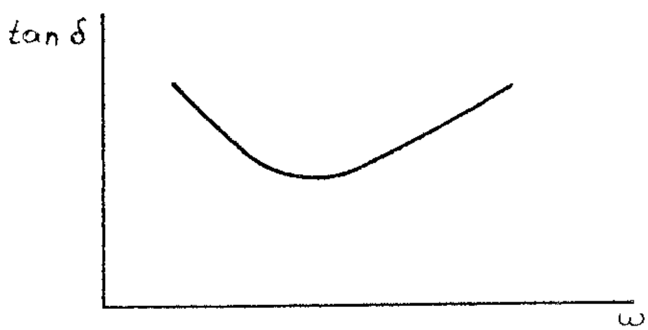
Storage/Loss modulus = f (angular velocity)

This test gives information about the elastic (storage modulus G') and the viscous (loss modulus G") components of the material.



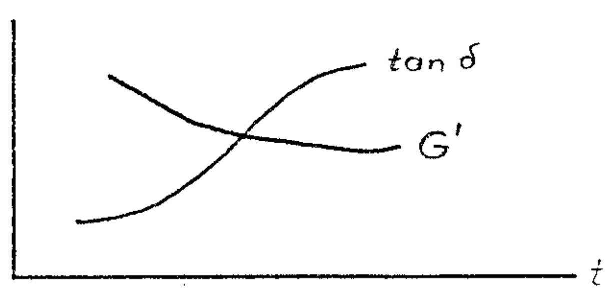
Loss tangent = f (angular velocity) γ=const

This test gives information about the elastic/viscous relation independent of the complex modulus G*



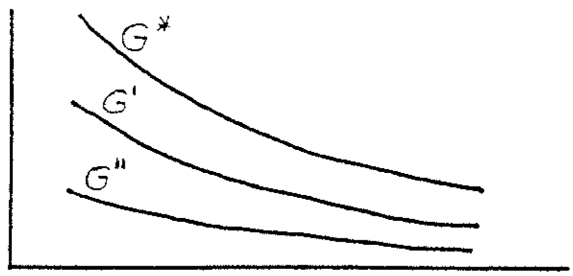
Storage modulus / Loss tangent = f (time) γ=const ω=const

This test is a good way to describe the elasticity during tireing effects, curing processes or chemical reactions.



G-moduli = f (temperature) γ=const ω =const

This plot shows a way to describe the Viscoelastic behavior as a function of the temperature; eg. during polymerization.



9. Symbols and Units

Description	Symbol	Unit
Strain	γ	rad
Shear Stress	τ	Pascal=Pa
Complex modulus	G*	Pascal=Pa
Storage modulus	G'	Pascal=Pa
Loss modulus	G"	Pascal=Pa
Phase shift	δ	Dimensionless
Loss tangent	tan δ	Dimensionless
Deformation	γ	degrees or radians
Frequency	f	Hz
Angular velocity	ω	radians/sec
Complex viscosity	η^*	Pa.s
Dynamic viscosity	η'	Pa.s
Steady shear viscosity	η	Pa.s

10. CONCLUSION

Measuring the viscoelasticity via an oscillating strain is a useful method to describe viscous and elastic behaviors of a test material in absolute numbers.

With the HAAKE Rotovisco RV20, an instrument is available to measure both viscosity and elasticity with the same system configuration.

Several measuring systems and many sensor systems may be used to cover nearly every application.

The measurements performed are fully computerized and the results can be shown on the computer screen, printer, or plotter. Therefore, time consuming calculations are no longer necessary.

Elasticity measurements are another tool to characterize your material as completely as possible and measuring elasticity can now be performed easily and routinely with the HAAKE RV20 system.

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