NAME ______________________ CLASS __________ DATE __________

Reteaching 13.6 Inverses of Trigonometric Functions

◆ Skill A Evaluating inverse trigonometric relations and functions

Recall The **domain and range** of a function become the **range and domain** respectively, of the inverse.

◆ Example 1

Find each value. Give answers in degrees and radians.

a. $\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)$ **b.** $\text{Cos}^{-1}\left(-\frac{1}{2}\right)$ **c.** $\text{Tan}^{-1}(-1)$

◆ Solution

a. Since $\sin 60° = \frac{\sqrt{3}}{2}$, then $\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60°$ or $\frac{\pi}{3}$ radians.

Notice that although other angles have a sine of $\frac{\sqrt{3}}{2}$, you must choose an angle that is between $-90°$ and $90°$ in order to have a value in the appropriate range.

b. Since $\cos 120° = -\frac{1}{2}$, then $\text{Cos}^{-1}\left(-\frac{1}{2}\right) = 120°$ or $\frac{2\pi}{3}$ radians.

c. Since $\tan(-45°) = -1$, then $\text{Tan}^{-1}(-1) = -45°$ or $-\frac{\pi}{4}$ radians.

◆ Example 2

Evaluate each expression.

a. $\sin\left(\text{Cos}^{-1}\left(\frac{1}{2}\right)\right)$ **b.** $\text{Tan}^{-1}(\sin 90°)$

◆ Solution

a. Begin inside the parentheses.

$\text{Cos}^{-1}\left(\frac{1}{2}\right) = 60°$

So, $\sin\left(\text{Cos}^{-1}\left(\frac{1}{2}\right)\right) = \sin 60° = \frac{\sqrt{3}}{2}$

b. $\sin 90° = 1$

Therefore, $\text{Tan}^{-1}(\sin 90°) = \text{Tan}^{-1}(1)$

$= 45°$ or $\frac{\pi}{4}$ radians

Find each value. Give answers in degrees and in radians. (It may be helpful to review what you learned about 30°-, 45°-, and 60°-angles.)

1. $\text{Sin}^{-1}\left(\frac{1}{2}\right)$ __________ **2.** $\text{Cos}^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ __________ **3.** $\text{Tan}^{-1}(\sqrt{3})$ __________

4. $\text{Sin}^{-1}(-1)$ __________ **5.** $\text{Cos}^{-1}\left(\frac{\sqrt{3}}{2}\right)$ __________ **6.** $\text{Tan}^{-1}(-1)$ __________

Evaluate each composite trigonometric expression.

7. $\tan\left(\text{Cos}^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$ __________ **8.** $\cos\left(\text{Sin}^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)$ __________ **9.** $\sin\left(\text{Tan}^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right)$ __________

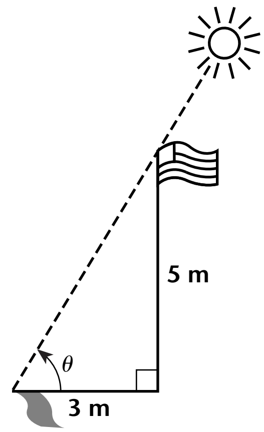
10. $\text{Sin}^{-1}(\cos 0°)$ __________ **11.** $\text{Tan}^{-1}(\sin 0°)$ __________ **12.** $\text{Sin}^{-1}(\sin 90°)$ __________

◆ **Skill B** Applying inverse trigonometric functions

Recall $\sin\theta = \dfrac{\text{opp.}}{\text{hyp.}}$ $\cos\theta = \dfrac{\text{adj.}}{\text{hyp.}}$ $\tan\theta = \dfrac{\text{opp.}}{\text{adj.}}$

◆ **Example**

At a certain time of the day, the 5 meter flagpole shown at right casts a shadow that is 3 meters long. What is the angle of elevation of the sun at this time?



◆ **Solution**

Since 3 meters is the length of the side **adjacent** to θ and 5 meters is the length of the side **opposite** θ, use the tangent function.

$\tan\theta = \dfrac{5}{3}$

$\theta = \tan^{-1}\left(\dfrac{5}{3}\right)$

This last equation states that θ is the angle that has a tangent of $\dfrac{5}{3}$.

$\theta \approx 59°$ *Use calculator in **degree** mode.*

Find the measure of each angle to the nearest whole degree.

13. Find the measure of the smallest angle in a right triangle with sides of 3, 4, and 5 centimeters. ____________

14. What is the angle between the bottom of the ladder and the ground as shown at right?

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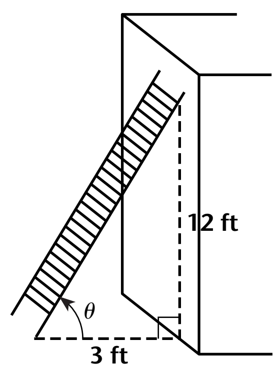

15. Find the angle at the peak of the roof as shown at right.

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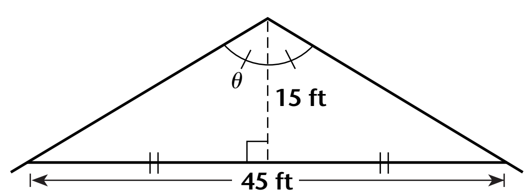

16. The hypotenuse of a right triangle is 3 times as long as the shorter leg. Find the measure of the angle between the shorter leg and the hypotenuse. ____________