# Teaching Mathematics to All Learners 

## INCLUDING English Language Learners

## Miriam A. Leiva, Founding President, TODOS: Mathematics for All



Equity without Excellence is useless. Excellence without equity is unjust. G. Mendieta 2008
All students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study - and support to learn - mathematics. Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students.

NCTM Equity Principle, p.12. Principles and Standards far Schaal Mathematics, 2005
Every student should have equitable and optimal opportunities to learn mathematics free from bias - intentional or unintentional- based on race, gender, socioeconomic status, or language. In order to close the achievement gap, all students need the opportunity to learn challenging mathematics from a well-qualified teacher who will make connections to the background, needs, and cultures of all learners.

NCTM Position Paper, Clasing the achievement Gap, 2005
Ideally, all children who are not proficient in English should receive mathematics instruction in their first language as they work to acquire English proficiency. Alternative and authentic assessment practices should be used, and the federal mandate for achievement, insofar as it is based on inadequate and inequitable standardized assessments, should be challenged.

NCTM Position Paper, Clasing the achiewement Gap, 2005

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## Our Responsibilities for ALL Students: High Expectations, Rigorous programs

DEMAND MORE: Each student has unique talents, abilities. Have high expectations and demand knowledge, and discipline. Let them know that you believe in their abilities. If you believe it you will see it.

DIFFERENCES: Provide for a variety of learning styles, prior knowledge, experiences, and all differences. HONOR: Promote pride in each student's personal history, ethnic and cultural heritage.

CLASSROOM: Develop a community of learners in a supportive non-threatening environment.
LESSONS: Design lessons to draw on students' strengths and experiences. Include language in the lesson plan, as an integral part of the lesson!

- TEACHING TOOLS: Make use of visuals, models, manipulatives, graphs, diagrams, technology, realia, and other resources Connect models to words so that students understand the math and the problems.
- COLLABORATION: Use creative cooperative teaching with teachers of other


TEACHING STRATEGIES: Accommodate differences! Use the best teaching strategies, see NCTM Professional Teaching Standards. Speak clearly and face the class. Connect spoken words to written visually. Use dialogues (teacher and student-centered) not monologues (teacher-centered). Do Wait Time.

- GROUPING: Whole class, small groups, pairs. Assign students partner/s to help, content or language.

PROBLEM SOLVING, APPLICATIONS: Do not focus on mindless worksheets but on Problem Solving. Give problems/projects that require time (hours, day, week, month).. Applications and real world problems are not solved in a minute. Life is full of complex problems not exercises.

- CONTEXT: Learning is contextual. Students learn, Mathematics and English, when they learn it in a context, applications, meaningful to them.
- REASONING, DISCOURSE: Encourage reflection, discussion, and communication. Conjecturing and justifying are essential to demonstrating knowledge, and assessing understanding.


## COMMUNICATIVE TEACHING: USE LANGUAGE 1 and LANGUAGE 2, AS NEEDED.

- LANGUAGE: Vocabulary focus: words, phrases, sentences. Use L1, L2. Use English to learn Mathematics and Mathematics and social English to learn Academic English.
- COMMUNICATION: We must communicate to learn languages and mathematics. Focus on mathematical representations, mathematics language syntax, grammar, and semantics.
- Model: Use objects, drawings, pictures, and graphs to model a situation
- Read: Help students read, interpret, understand information, and restate the problem.
- Write: Write to describe, record and explain, to create and solve problems.
- Speak: Discuss and solve in small groups, explain to the class

ASSESSMENT: Seamless approach to Curriculum, Teaching, Assessment. Assess if the barriers are academic or linguistic. When assessing, focus on both! Be flexible with grammar or spelling.

WAIT TIME, PATIENCE: In all instances, academic an otherwise, we must support students adjustments to school/classroom, culture, routines. Patience is Golden.

## Summary of key points in presentation.

All students must have an equal opportunity to be different, M. A. Leiva
What language to use: Learning is most efficient in the child's primary language. For English Language Learners try use the first language to teach the second (English) and the third (Mathematics).

We are dealing with several languages: Social English, Academic English, and the Language of Mathematics:

- Mathematics and language: Students must learn the language of Mathematics (symbols, syntax, grammar) and become proficient in translating to and from various representations: symbols, graph, data, pictures, models, the Mathematical Language "in English" to the English Language and back.
- Problems with words are a challenge to ALL students.
- Being fluent in social situations in English is far from being fluent in the math classroom.

Mathematics and Problem Solving. For success in problem solving we must promote:

- Students translating problems to-from the "languages" used. They must be multilingual.
- Students reasoning, making connections from "exercises" to more complex, realistic problems
- Students solving problems, reflecting on their solution, and communicating results effectively.
- Students solving problems with applications/connections with mathematics and other disciplines

Language and assessment: Measure mathematics knowledge and understanding and language proficiency.

## Language and Teaching Strategies:

- Seamless Approach to Instruction, Curriculum, and Assessment.
- High Expectations. Problem Solving Approach.
- Flexible groupings.
- Use of visuals, models and manipulatives.

Instruction that ...diminishes educational inequities by considering the linguistic factors ... must be designed and implemented for Latino children to have an opportunity to maximize their learning from instruction. N. Hernandez, p. 57, Perspectives on Latinos, NCTM (1999)

Language and concept development - Adapted from Garrison/Mora
Reference: L. Garrison, J.K.Mora, Adapting Mathematics Instruction for ELL, Changing Faces of Mathematics: Perspectives on Latinos, NCTM, (1999).

| Language of <br> instruction | Mathematics <br> concept | Learning | Teaching |
| :--- | :--- | :--- | :--- |
| Unknown | Unknown | Limited Learning <br> opportunities; modify <br> instruction | Use students' language to teach <br> concept or develop concepts <br> through other known concepts, <br> pictures, manipulatives, acting out. |
| Known | Unknown | Concept development | Use known language to teach <br> concepts. Develop English <br> through visuals, etc. |
| Unknown | Known | Language development | Use known concept to develop <br> language. |
| Known | Known | Advance to next conceptual <br> or linguistic development | Review, Assessment, reevaluate <br> language and mathematics <br> concept. |

As a member of TODOS you will receive a copy of the Research Monograph, on the Achievement Gap \& The first two issues of our Research Journal: Teaching for Excellence and Equity in Mathematics Education


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TODOS: Mathematics for ALL

## An Affiliate of the National Council of Teachers of Mathematics

The mission of TODOS: Mathematics for All is to advocate for an equitable and high quality mathematics education for all students, in particular Latino/Hispanic students, by increasing educators' equity awareness and their ability to foster students' proficiency in rigorous and coherent mathematics.

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## World Wide Web Resources

Center for Applied Linguistics http://www.cal.org/

Center for Bilingual Education and Research (CBER) http://www.asu.edu/educ/cber

Center for Research on Education, Diversity \& Excellence http://www.crede.ucsc.edu/

## Directory of ESL Programs in the United States

http://www.globalstudy.com/embark/us/
Eisenhower National Clearinghouse
http://www.enc.org
EQUALS, Lawrence Hall of Science, University of California, Berkeley http://equals.lhs.berkeley.edu/

ESL Magazine
http://www.eslmag.com

Fundacion Cientec<br>http://www.cientec.or.cr.matematica.html

The K-12 Mathematics Curriculum Center http://www2.edc.org/mce/

Mathematical Association of America
http://www.maa.org/
Math Forum
http://mathforum.org/dr.math/
Mathematics Projects
http://www.eduplace.com/projects/mathproj.html
National Academy of Sciences
http://www.nas.edu/

National Center for Education Statistics
http://nces.ed.gov
The National Coalition for Equity in Education, University of California, Santa Barbara http://ncee.education.ucsb.edu/

National Council of Teachers of Mathematics
http://www.nctm.org
National Science Foundation
http://www.nsf.gov/
Pew Hispanic Center
http://pewhispanic.org
Society for Advancement of Chicanos and Native Americans in Science http://www.SACNAS.org/
U.S. Department of Education
http://www.ed.gov/index.jhtml
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