

Mathematician Helaman Ferguson combines science and art with his unique mathematical sculptures. Ferguson's sculptures are the concrete embodiments of mathematical concepts that incorporate ideas such as series expansions and vector fields into their creation. Some of the basic images in his work are tori and double tori, Möbius strips, and trefoil knots.

Observations

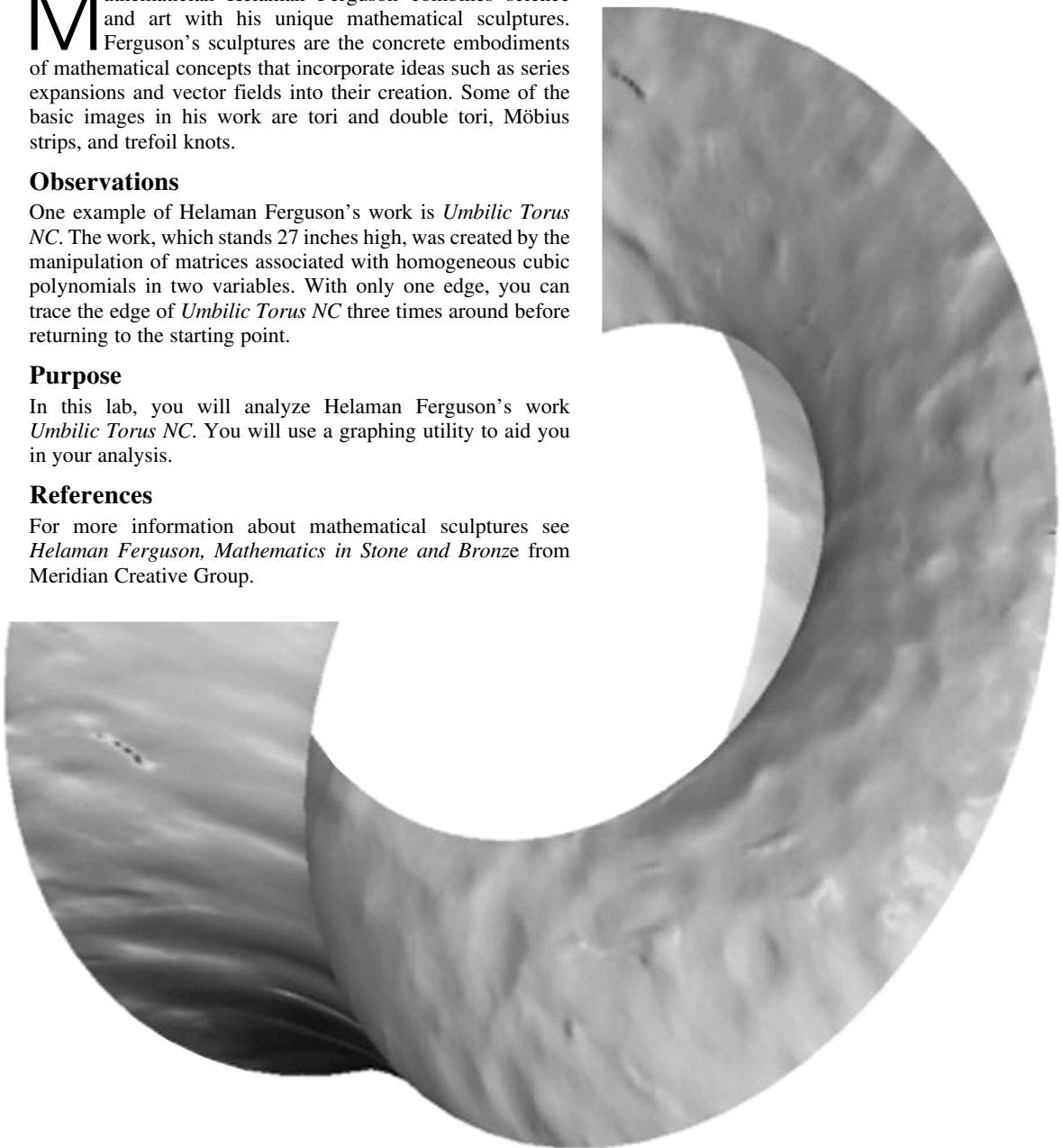
One example of Helaman Ferguson's work is *Umbilic Torus NC*. The work, which stands 27 inches high, was created by the manipulation of matrices associated with homogeneous cubic polynomials in two variables. With only one edge, you can trace the edge of *Umbilic Torus NC* three times around before returning to the starting point.

Purpose

In this lab, you will analyze Helaman Ferguson's work *Umbilic Torus NC*. You will use a graphing utility to aid you in your analysis.

References

For more information about mathematical sculptures see *Helaman Ferguson, Mathematics in Stone and Bronze* from Meridian Creative Group.



Data

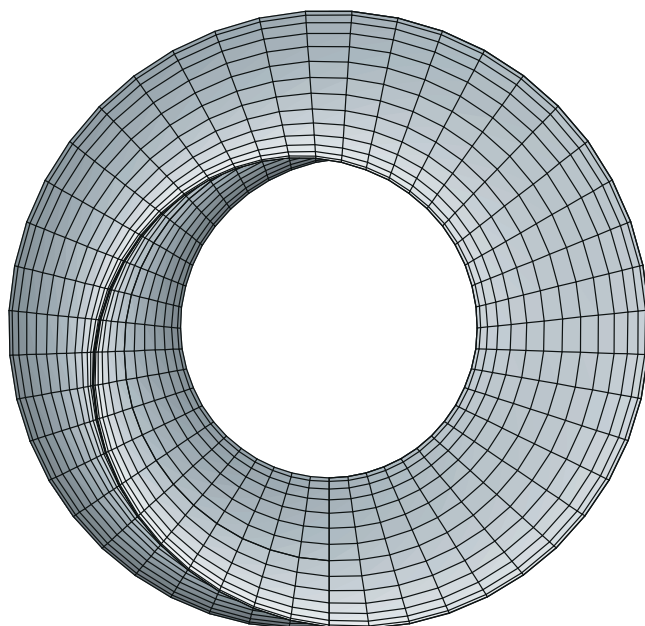
Helaman Ferguson's work *Umbilic Torus NC* is shown below. This form can be written as a parametric surface using the following set of parametric equations.

$$x = \sin u \left[7 + \cos\left(\frac{u}{3} - 2v\right) + 2 \cos\left(\frac{u}{3} + v\right) \right]$$

$$y = \cos u \left[7 + \cos\left(\frac{u}{3} - 2v\right) + 2 \cos\left(\frac{u}{3} + v\right) \right]$$

$$z = \sin\left(\frac{u}{3} - 2v\right) + 2 \sin\left(\frac{u}{3} + v\right)$$

$$-\pi \leq u \leq \pi, -\pi \leq v \leq \pi$$



A graph of the parametric equations is stored in the graphing utility file called LAB19.92T.

Exercises

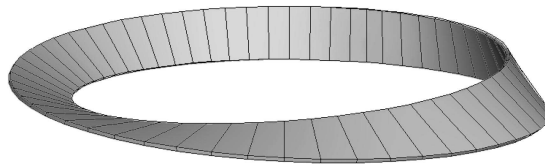
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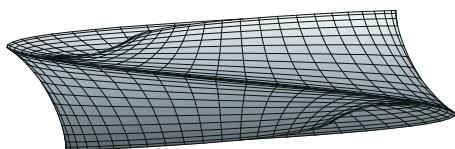
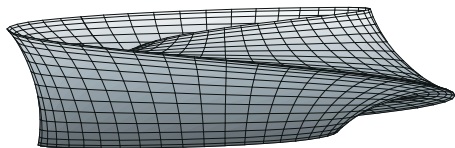
Instructor _____

1. **An “Elliptical” Torus.** When viewed along the z -axis, the torus appears nearly circular. Is it possible to alter this appearance so that the shape of the torus is more like that of an elongated ellipse? If so, explain how this could be done.

2. **Comparing Graphs.** Explain how the umbilic torus shown in this lab’s Data is similar to the Möbius strip shown below.

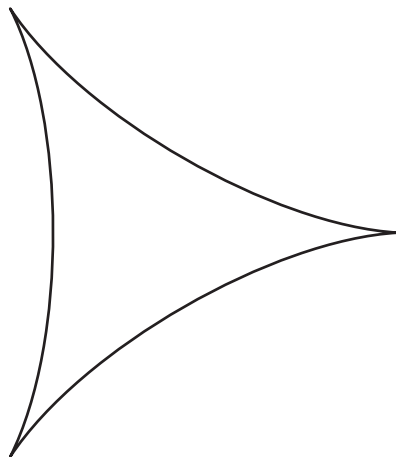


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3. **Viewing the Torus From Different Points in Space.** In this lab's Data, the point in space from which the umbilic torus is viewed is $(0, 0, 10)$. Determine the point in space from which each graph of the umbilic torus below is viewed. Explain your strategy in determining the point of view.





4. **Cross Section.** An example of a cross-section of *Umbilic Torus NC* is shown below.



The cross-section is a hypocycloid with three arches. A hypocycloid is a curve generated by a point on the circumference of a circle rolling within the circumference of a larger circle. The parametric equations of a hypocycloid in the xy -plane are

$$x = (a - b) \cos \theta + b \cos \left[\frac{(a - b)\theta}{b} \right]$$


$$y = (a - b) \sin \theta - b \sin \left[\frac{(a - b)\theta}{b} \right]$$

where $0 \leq \theta \leq 2\pi$. One possible set of parametric equations of the hypocycloid shown above is as follows.

$$x = 2 \cos \theta + \cos 2\theta$$

$$y = 2 \cos \theta + \cos 2\theta$$

Use a graphing utility to graph a hypocycloid with four arches and a hypocycloid with five arches. What values of a and b did you use to create your graphs? Determine the relationship between a and b for each of the following hypocycloids: one with three arches, one with four arches, and one with five arches. Determine the relationship between a and b for a hypocycloid with n arches.

-  **5. Complete the Table.** Use the relationship from Exercise 4 and the values of a and b in the table to predict the number of arches each hypocycloid has. Then use a graphing utility and the values of a and b to graph each hypocycloid. Does the number of arches in the graph match the number of arches predicted in the table? If not, explain why and determine the correct relationship between a and b .

a	b	Number of Arches
8	2	
21	7	
18	3	
20	4	

- 6. Graphing the Torus Using Different Ranges.** The graph of the umbilic torus below was created using the same set of parametric equations given in this lab's Data, except that one of the parameter ranges used was altered, as was the point in space from which the umbilic torus was viewed. Determine which parameter range was changed and what the parameter's new range is. Then determine the new point in space from which the umbilic torus is viewed.

