Name ____

Adv. Alg. 2 6-2 Class Examples: Inverse Functions and Relations

Example 1: How are inverse functions related to measurement conversions?

Most scientific formulas involve measurements given in SI (International System) units. The SI units for speed are meters per second. However, the United States uses customary measurements such as miles per hour. To convert x miles per hour to an approximate equivalent in meters per second, you can evaluate

$$f(x) = \frac{x\ miles}{1\ hour} \cdot \frac{1600\ meters}{1\ mile} \cdot \frac{1\ hour}{3600\ seconds} \text{ or } f(x) = \frac{4}{9}x.$$

To evaluate x meters per second to an approximate equivalent in miles per hour, you can evaluate

$$g(x) = \frac{x\ meters}{1\ second} \cdot \frac{3600\ seconds}{1\ hour} \cdot \frac{1\ mile}{1600\ meters} \text{ or } g(x) = \frac{9}{4}x.$$

Notice that $f(x)$ multiplies a number by 4 and divides it by 9. The function $g(x)$ does the inverse operation of $f(x)$. It divides a number by 4 and multiplies it by 9. The functions $f(x) = \frac{4}{9}x$ and $g(x) = \frac{9}{4}x$ are inverses.

Finding Inverses

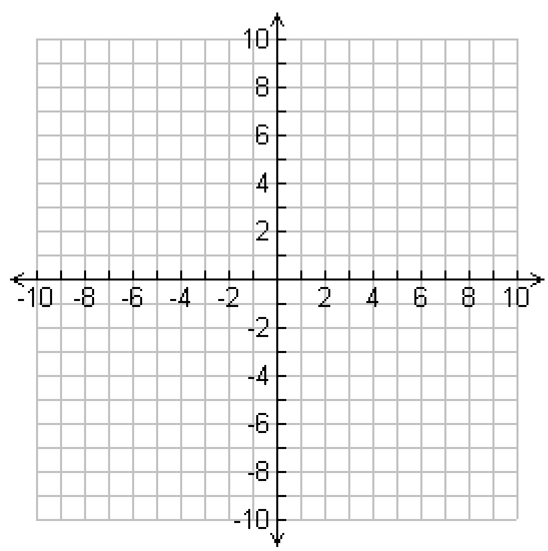
What is a relation? ____

The **inverse relation** is the **set of ordered pairs obtained by reversing the coordinates of each original ordered pair. The domain of a relation becomes the range of the inverse and the range of the relation becomes the domain of the inverse.**

Let $Q = \{(1,2),(3,4),(5,6)\}$. Let S be the inverse of Q. Write the ordered pairs for set S.

S = ____

Example 2: Finding an Inverse Relation



Graph the ordered pairs in the relation A.
A=$\{(2,1),(5,1),(2,-4)\}$

What figure do these coordinates form if you connect the points? ______________________________

Find the inverse of this relation.

B={ }

Graph them and determine the figure that these coordinates form. ____

The two graphs appear to be related how?____

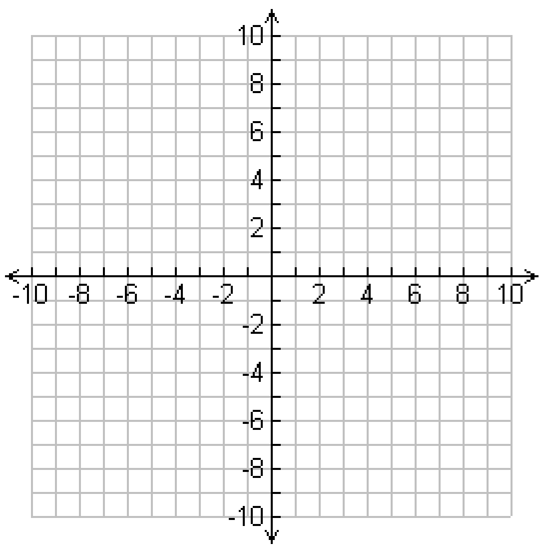
____

Recall that the points (x, y) and (y, x) are reflection images of each other over the identity function, that is, the line with equation $y = x$. The reflection over the identity function switches the coordinates of the ordered pairs. So the graphs of any relation and its inverse are reflection images of each other over the line $y = x$.

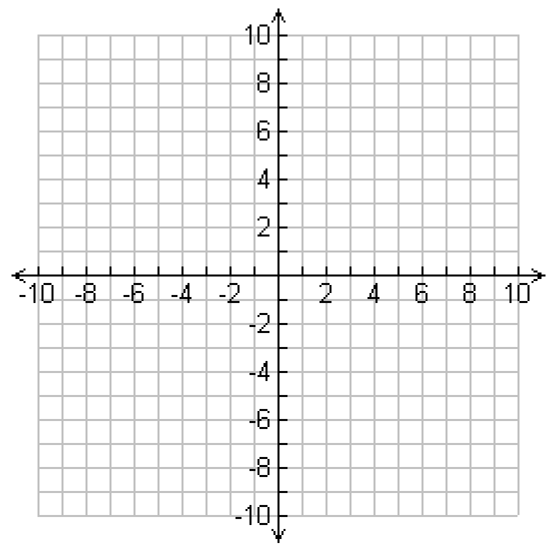
<u>Example 3: Graphing an Equation and its Inverse</u>

For problems 1-4, a) Graph the function. Write a t-chart of at least 3 ordered pairs that are on the line. b) Using the first t-chart, write a t-chart for the **<u>inverse relation</u>**. c) Plot the ordered pairs of the inverse relation and connect them. d) Using a dashed line, plot the line of reflection for the graph of the function and the inverse relation.

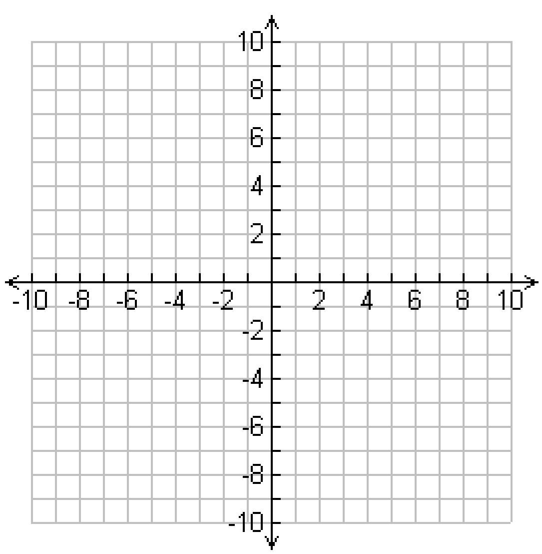
1. $y = 2x - 5$



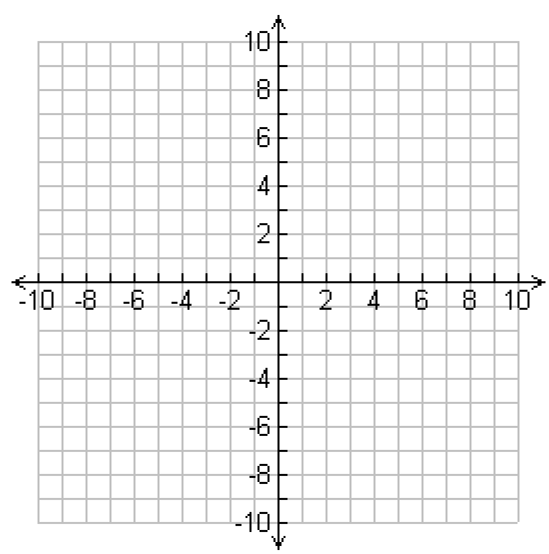
2. $6y = 24$



3. $y = x + 5$



4. $y = -x$



Example 4: Determining Whether the Inverse of a Function is a Function

The **inverse** of a **relation** is **always a relation**. But the **inverse** of a **function** is **not always a function**.

How do we determine whether a graph represents a function? ________________________

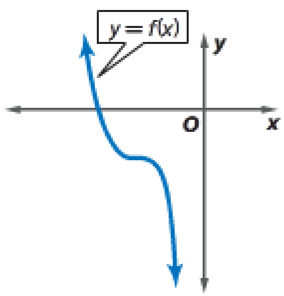
To determine whether the inverse of a function will be a function, we use the **Horizontal Line Test**.

Theorem (Horizontal-Line Test for Inverses)
The inverse of a function f is itself a function if and only if no horizontal line intersects the graph of f in more than one point.

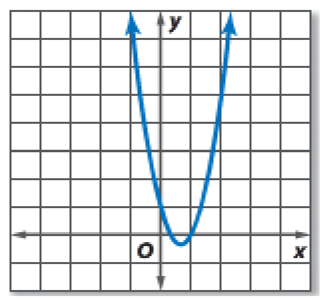
When the inverse of a function is also a function, then the original function is said to be one-to-one.

Tell whether the inverse of each function is a function. Explain your reasoning.

1.



2.



NOW: Go back to Example 3 and examine the GIVEN functions. Which ones pass the HLT? _____________

Which ones have inverses that are functions? ________________ Relations? ___________________

Example 5: Finding the equation of the inverse function, $f^{-1}(x)$

To find the equation of the inverse function, $f^{-1}(x)$, follow these steps:

1. Graph or draw a sketch of the given problem. Then, ask yourself this question: Does the graph pass the HLT?
   - If the graph DOES NOT pass the HLT, then write this: The graph fails the HLT, therefore, there is an inverse, but the inverse is not a function.
   - If the graph DOES pass the HLT, then write down the Domain and Range, and complete steps 2-5.
2. Replace $f(x)$ with y.
3. Switch x and y.
4. Resolve for y.
5. Replace y with $f^{-1}(x)$. Write down the new Domain and Range (switch those found in step 1.)

Find the inverse of each function, if it exists.

1. $f(x) = 2x - 5$

2. $f(x) = x^3 + x^2 - 3x + 1$

3. $f(x) = \frac{x-3}{5}$

4. $f(x) = 3x^2$

5. $f(x) = \frac{4x+3}{2}$

6. $f(x) = \frac{5x-2}{10}$

Example 6: Verifying Two Functions are Inverses

To determine whether two functions are inverses of each other, look at their compositions, one with the other. Check to see if $f(g(x)) = g(f(x)) = x$. If so, the two functions are inverses.

In 1 and 2 below determine whether or not the two functions are inverses of each other.

1. $f(x) = x - 9$ and $g(x) = x + 9$

2. $f(x) = 3x + 5$ and $g(x) = 3x - 5$