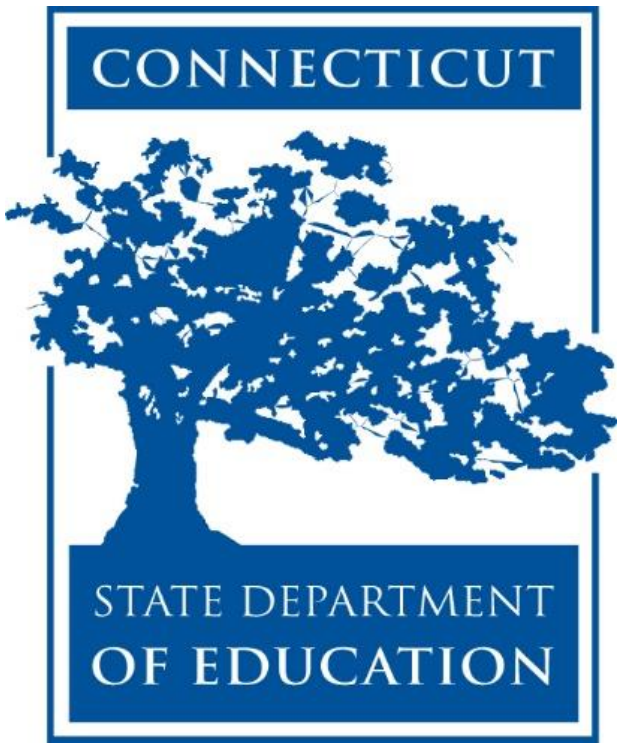


Mathematics Instructional Cycle Guide

Ratios and Proportions (6.RP.A.3)

Created by Josh Egan, 2014 Connecticut Dream Team teacher

CT CORE STANDARDS

This Instructional Cycle Guide relates to the following *Standards for Mathematical Content* in the *CT Core Standards for Mathematics*:

Insert the cluster heading and Content Standard(s) here.
6.RP.A.3 Use ratio and rate reasoning to solve real-world problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating qualities with whole-number measurements, find missing values in the tables, and plot the pairs of values in the coordinate plane. Use tables to compare ratios.

This Instructional Cycle Guide also relates to the following *Standards for Mathematical Practice* in the *CT Core Standards for Mathematics*:

MP.3 Construct viable arguments and critique the reasoning of others.

- Make conjectures and build a logical progression of statements to explore the truth of their conjectures.

MP.4 Model with mathematics.

- Apply the mathematics they know to solve problems that arise in everyday life, society and the workplace.

WHAT IS INCLUDED IN THIS DOCUMENT?

- A Mathematical Checkpoint to elicit evidence of student understanding and identify student understandings and misunderstandings **(page 2)**
- A student response guide with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint **(pages 3-7)**
- A follow-up lesson plan designed to use the evidence from the student work and address the student understandings and misunderstandings revealed **(pages 8-12)**
- Supporting lesson materials **(pages 13-15)**
- Precursory research and review of standard **6.RP.A.3** and assessment items that illustrate the standard **(pages 16-18)**

HOW TO USE THIS DOCUMENT

1) Before the lesson, administer the **(Insert Checkpoint Name)** ***Mathematical Checkpoint*** individually to students to elicit evidence of student understanding.
2) Analyze and interpret the student work using the ***Student Response Guide***
3) Use the next steps or ***follow-up lesson plan*** to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint
4) Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan

MATERIALS REQUIRED

- Response Boards, Dry Erase Markers, Large poster paper (multiple sheets)
- SmartBoard, document camera, and/or chart paper to facilitate the sharing of student work

TIME NEEDED

Ratios and Proportions administration: **15-20 minutes**
Follow-Up Lesson Plan: **1 to 2 instructional classes (45 minutes each)**

Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class.

<table>
<tr><th colspan="3">Step 1: Elicit evidence of student understanding</th></tr>
<tr><th colspan="3">Mathematical Checkpoint</th></tr>
<tr><th>Question(s)</th><th colspan="2">Purpose</th></tr>
<tr><td rowspan="2">Dayton wants to buy soda for a graduation party.
• 50-60 people will be coming to the party.
• A 2-liter bottle of soda will serve 4 people.
• Each 2-liter bottle from a local grocery store costs $2.50.

Part A

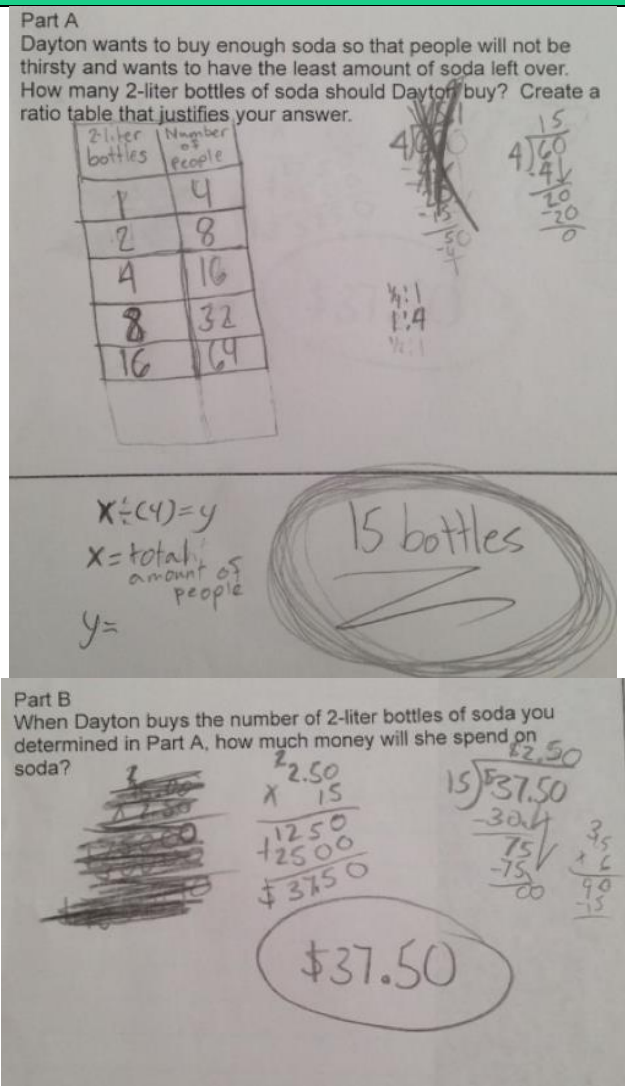
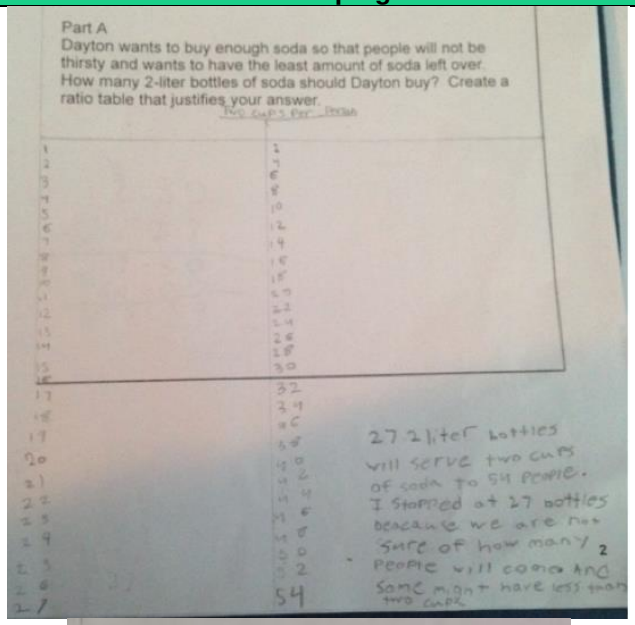
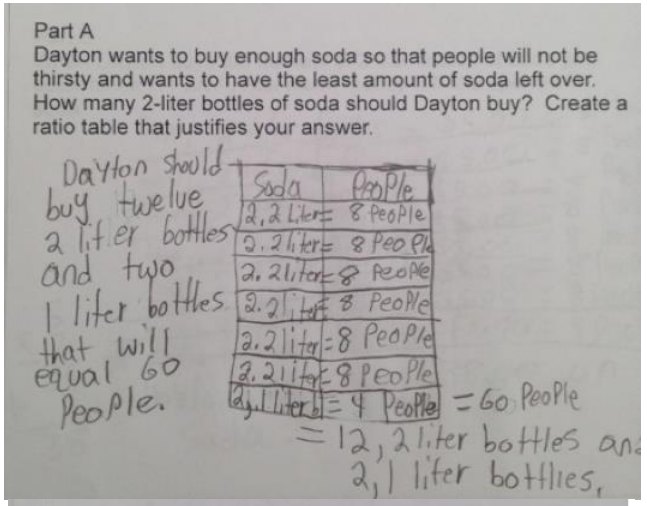
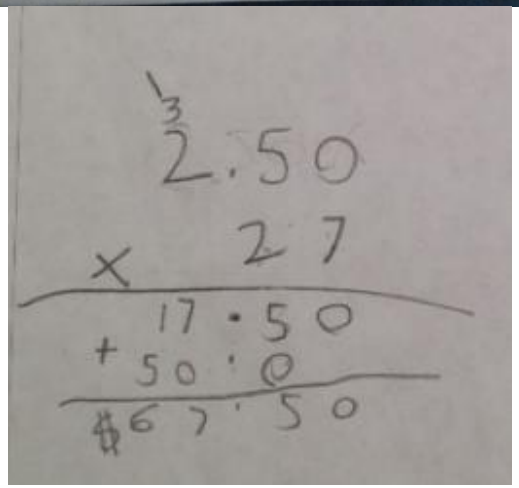
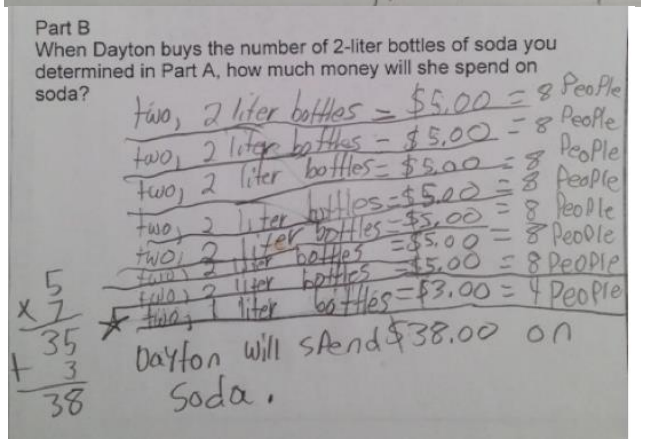
Dayton wants to buy enough soda so that people will not be thirsty and wants to have the least amount of soda left over. How many 2-liter bottles of soda should Dayton buy? Create a ratio table that justifies your answer.

Part B

When Dayton buys the number of 2-liter bottles of soda you determined in Part A, how much money will she spend on soda?</td><td>CT Core Standard:</td><td><u>6.RP.A.3</u> Use ratio and rate reasoning to solve real-world problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
a. Make tables of equivalent ratios relating qualities with whole-number measurements, find missing values in the tables, and plot the pairs of values in the coordinate plane. Use tables to compare ratios.</td></tr>
<tr><td>Target question addressed by this checkpoint:</td><td>Do students understand ratios? To what extent do they…
• Create a ratio table that compares different quantities.
• Create a multiplication equation to represent a real-world problem.</td></tr>
</table>

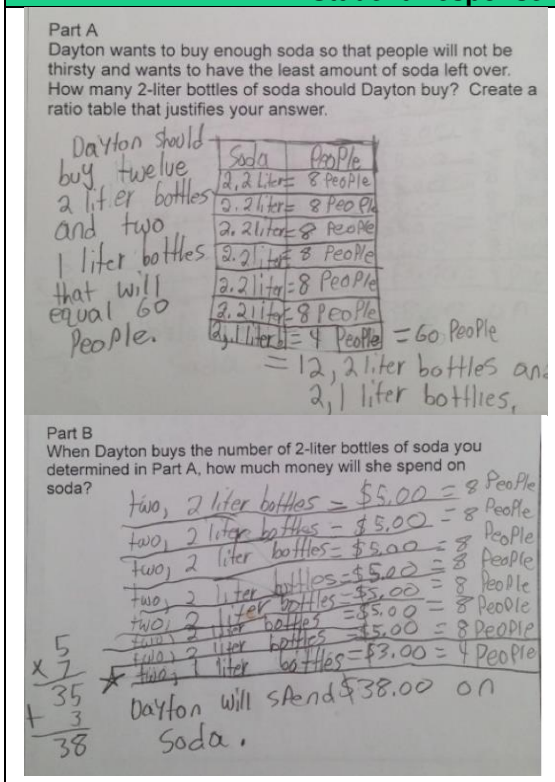
Step 2: Analyze and Interpret Student Work

Student Response Guide

Got It	Developing	Getting Started
		
		

Getting Started

Student Response Example



Indicators

- Student may not understand what a ratio table is.
- Student response may indicate a basic understanding of a ratio but does not understand equivalent ratios or how they are created.
- Student may not reach a reasonable solution.
- Student response may indicate the ratio table is not attempted or does not represent the context or given data.
- Student may not determine the correct dollar amount for the total bottles of soda purchased.

In the Moment Questions/Prompts

Q: What is a ratio/ratio table? Is it similar to a fraction?

P: Excellent job on your ratio table, tell me a little more about it.

Q: What is the ratio of soda bottles to people?

Q: If 1 bottle of soda is enough for 4 people, how many bottles of soda will be needed for 8 people?

P: How did you come up with your answer?

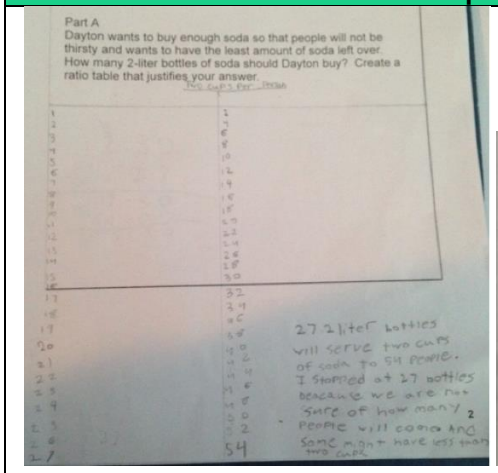
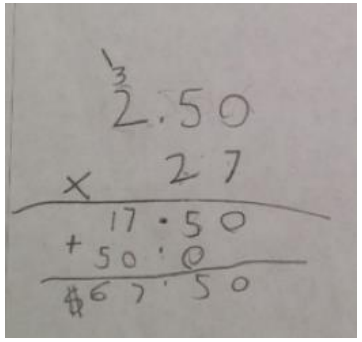
Closing the Loop (Interventions/Extensions)

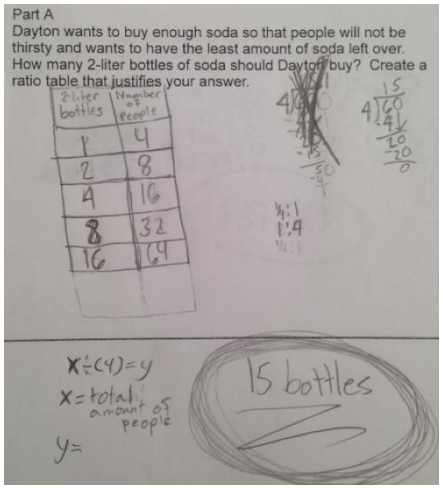
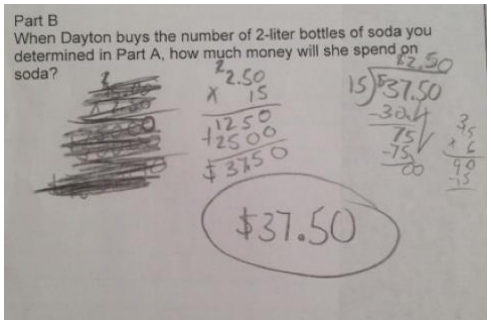
Provide students with counters, cups and beans, or pattern blocks. Students may benefit from modeling a simple ratio and how the two different items relate.

Provide students with the opportunity to look at ratios as fractions. Create equivalent fractions and allow students to look into equivalent ratios.

http://learnzillion.com/lessons/610-solve-missing-values-in-ratio-problems-using-multiplicative-reasoning

Q: What is the smallest number of people that may come to the party? Most? Q: How would you make sure that there are enough bottles of soda for all of the guests (if all 60 came)?	**http://learnzillion.com/lessons/583-convert-between-parttopart-and-parttototal-ratios-by-drawing-a-picture** **http://learnzillion.com/lessons/581-visualize-parttototal-ratios-using-pictures**

Developing	
Student Response Example	**Indicators**
 	• Student model may show an appropriate representation, but the representation may be inaccurate or incomplete. • Student model may show an understanding of ratios and equivalent ratios but the solution may be inaccurate or incomplete. • Student response may include some confusion about what the question is asking and how to relate the material that is included to solve the problem. • Student may not complete the correct ratio in order to solve the given problem. • Student may determine the correct dollar amount needed to purchase the determined bottles of soda in Part A.
In the Moment Questions/Prompts	**Closing the Loop (Interventions/Extensions)**
P: Tell me about your ratio table. Q: How did you begin your table? What does the ratio of 1:2 mean? Q: In the problem, how many people can be served by a 2-liter bottle of soda? Q: Let's say that 12 people came to the party, how many bottles of soda should Dayton buy? Q: Let's look back at your work, what do you mean by "cups"? Q: If you multiplied 27 by 4, what product would you get?	Provide student with various ratio tables and allow them to find the missing numbers using multiplication. Provide students with a different ratio of bottles to guests. Keep adding guests while concentrating on developing equivalent ratios. **http://learnzillion.com/lessons/608-solve-missing-values-in-ratio-problems-using-a-table**

Got it	
Student Response Example	**Indicators**
 	• Student may show that 15 bottles of soda would satisfy all of the possible guests at the party. • Student response may create a complete ratio table. • Student may use an understanding of equivalent fractions. • Student may determine the amount of money needed to purchase at 15 bottles of soda.
In the Moment Questions/Prompts	**Closing the Loop (Interventions/Extensions)**
P: Explain your thinking. Tell me about your ratio table. Q: Can you explain your equation to me (x/(4)=y)? Q: Why do you believe that Dayton should buy 15 bottles of soda? Q: How did you check your solution? Q: How many bottles of soda would you buy if only 50 people came to the party?	Have students analyze other approaches for solving the problem and explain the thinking behind those approaches. Have students analyze other approaches for solving the problem when the number of guest's change or the number of people served from one bottle of soda varies. Create an algebraic expression where "n" is the number of guests and serves "k" people. **http://ctdreamteam.learnzillion.com/lessons/610-solve-missing-values-in-ratio-problems-using-multiplicative-reasoning**

Steps 3 and 4: Act on Evidence from Student Work and Adjust Instruction	
Lesson Objective:	Use ratio and rate reasoning to solve real-world problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
Content Standard(s):	**6.RP.A.3 Use** ratio and rate reasoning to solve real-world problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating qualities with whole-number measurements, find missing values in the tables, and plot the pairs of values in the coordinate plane. Use tables to compare ratios.
Targeted Practice Standard :	**MP.3 Construct viable arguments and critique the reasoning of others.** • Make conjectures and build a logical progression of statements to explore the truth of their conjectures. **MP.4 Model with mathematics.** • Apply the mathematics they know to solve problems that arise in everyday life, society and the workplace.

Mathematical Goals	Success Criteria
• Understand that ratios must be equal to each other and remain constant. • Understand the concept of a ratio and how to describe the relationship between two quantities. • Understand that equivalent ratios are always multiplied or divided by the same number.	• Create a ratio table that compares different quantities to solve a real-life problem. • Create a multiplication equation to represent a real-world problem.

Launch (Probe and Build Background Knowledge)

Purpose: Assess and activate background knowledge about ratios and ratio tables.

Provide students with response boards and dry erase markers. Project the following questions/statement on the board one at a time. Instruct students to answer each of the following as best they can. They should leave no question blank.

- What is a ratio?
- Explain how to use a ratio table.
- A store sells oranges at the price of 4 oranges for $1.50, how much would eight oranges cost? Explain your reasoning.

After all questions/statements are answered, students discuss the answers in their small groups. They should come to a consensus about the answers to each of the three problems and share with the entire class.

Instructional Task

Purpose: Introduce the trail mix task and provide students time to brainstorm, reason and problem-solve.

Engage (Setting Up the Task)

1. Introduce the task by showing students a bag of trail mix. Use the following prompts/questions:
   a. Ask students, “Have you ever had trail mix as a snack?”
   b. Ask students, “What are some of the ingredients that make up a bag of trail mix?”

c. Trail mix ingredients are packaged using ratios. What does the ratio of 2:3 mean if we we're talking about raisins and nuts? Explain.

2. Project the problem on the board. Instruct students to take 1-2 minutes to think about how they may approach this problem. Tell students to write down ideas they come up with on their dry erase boards.

Trail Mix Task (Ratios and Proportions)

Giovanny was mixing raisins and mixed nuts in the ratio of 2:3 to make trail mix. He wants to make 45 ounces of trail mix for his class's field trip. He decided to begin the following table to help him think about the problem, but is not sure what he should do next.

Ounces of Raisins	Ounces of Mixed Nuts	Ounces of Trail Mix
2	3	5
4	6	10

a. Explain in words how to continue to add numbers into the table.

b. Write an explanation to Giovanny about how he can use his table to find out how many ounces of raisins and how many ounces of mixed nuts he will need to make 45 ounces of trail mix.

3. Students should, after taking some time to brainstorm, share their thinking with a partner. Each partner will have 2-minutes to share their approach.

4. Facilitate a class discussion about the problem by using the following prompts and questions. As the students respond, take notes on a large sheet of paper.
   - Tell me what is going on in the problem in your own words.
   - What information does the chart already provide to us?
   - How can those given ratios help us to solve the problem?
   - What do we have to find out? What's the problem asking us to do?
   - What mathematical questions could we ask about this story?
   - How might we go about finding solutions to our questions?

5. Explain to students that they will now be working in their small groups to determine a solution to the problem. Provide copies of the task to each student. Students may use any tools that are available around the room to assist them in solving this problem (counters, charts, pattern blocks, fraction bars, etc.). Explain to the class that they will have a specific amount of time to work on the task.

Explore (Solving the Task)

6. Provide students time to work on the task in small groups. Observe, question and take notes.

   Focusing Questions
   - What is some of the information that you already know to assist you in solving the problem?
   - What are the given ratios we already know from the ratio table?
   - How can we use those given ratios to create other equivalent ratios involving raisins and mixed nuts?
   - What does the third column in our chart tell us?
   - How do we arrive at the numbers in the third column?
   - How do we create equivalent ratios?
   - Can we use fractions to better understand ratios?
   - Besides the ratio needed to make 45 ounces of trail mix, what else do we have to do to fully solve this task?

   Probing Questions
   - Are all of your ratios equivalent to the given ratios? How can you tell?
   - How do you know that you have the correct ratio for 45 ounces of trail mix?
   - Can you show/explain more about how you added numbers to the ratio table?
   - How does your ratio table show your solution?
   - It's still not clear how you came up with your ratio of __:__, can you further explain it?
   - How can you check your solution to make sure that it makes sense?

   Advanced Questions
   - How might you prove your solution in a different way?
   - Why did you decide on your ratio of 18 ounces of raisins and 27 ounces of mixed nuts would solve this problem?
   - How could you solve this problem quickly if I asked the ratio of raisins to mixed nuts needed to make 50 ounces of trail mix? 95 ounces of trail mix?

Elaborate (Discuss Task and Related Mathematical Concepts)

7. Call the class back together to begin a discussion of the trail mix task. After carefully observing the class, allow groups with different solutions or approaches to share. Project the following questions for students to consider as they actively listen to others' solutions during the discussion.
   - What did you like about their approach to solving the problem?
   - What, in their approach, was different from what we did as a group?
   - Does this approach make sense to me? Do you have any questions for the group presenting?
   - Does the solution make sense? Why or why not?

8. Once all of the groups have shared their information, allow students some time to share their answers to the questions presented above as an entire class.

Checking for Understanding

Purpose: Provide students with the opportunity to answer the following question to elicit evidence of students' understanding of completing ratio tables with equivalent ratios using multiplication to develop equivalent ratios.

Complete the following ratio table.

Red	Green
2	6
3	9
5	15
6	18
9	
	30
15	

What did you do in order to solve for the missing numbers? Explain.

Common Misunderstanding

Purpose: Address a common misunderstanding in ratio problems where students often use addition or subtraction where multiplication or division is necessary.

Remind students that when we were solving the trail mix problem, we needed to use equivalent ratios throughout the problem. Project the following problem on the board and provide students time to brainstorm.

Fish Food (Ratios and Proportions)

The smaller fish is 10 cm long and the larger fish is 15 cm long. If the smaller fish gets 12 grams of fish food, how much food does the larger fish get? Use ratios to solve this problem.

Josh says that since the smaller fish is 10 cm long and gets 12 grams of food, the larger fish, being 15 cm long, must get 17 grams of food. Is Josh correct? Explain your reasoning.

- Instruct students brainstorm individually. If it is helpful, they can use their response boards to work out the problem.
- In their small groups from the mixed nuts task, explain to your partners if you believe that Josh is correct or incorrect. Use evidence to back up your response.
- *Who thinks that Josh is correct? Incorrect? Uncertain?*
- Facilitate a discussion and have students who represent each group (Correct, incorrect and uncertain) share their responses with the class.
- *How did you arrive at your answer?*
- *In order for a ratio to be equal, which operation do we need to use?*
- *Let's look at each fish. Are the ratios that Josh gave as an answer equivalent? Explain.*
- *How can we help Josh to understand that his ratios are not equal?*
- *What would have been a better way for Josh to solve this problem?*
- *What are some similarities between this problem and the problem involving the trail mix?*

Checking for Understanding

Purpose: To elicit evidence of students' understanding when working with equivalent ratios and ratio tables, pose the following question to students as an exit ticket.

A local store sells candy bars at a price of 6 for $9.00.

a. Enrique wants to buy 24 candy bars for his classmates. How much will Enrique spend if he buys the 24 candy bars at the local store? Explain your reasoning in a few sentences.
b. Enrique decides he does not have enough money to buy 24 candy bars. Instead he can only buy 15. How much will Enrique spend on the 15 candy bars? Explain your reasoning.

Closure

Purpose: Provide students with an opportunity to reflect on their own learning through journal writing. Project the following questions on the board and allow students ample time to write their responses in their journals. Collect and evaluate for future lessons.

1. What can you tell me about when to use ratios and ratio tables as tools when helping solve real-world problems?
2. How do you know that two ratios are equivalent? Explain your answer.
3. Complete the following statement, "I still feel like I need more time learning…."

Extension Task

Purpose: The following is an extension task for students who want to deepen their understanding of ratio tables used to solve real world problems.

Leah rode her bike 20 miles in 120 minutes.

a. How far did she ride in 12 minutes?
b. How long did it take her to ride 5 miles?
c. What was her pace in minutes per mile?
d. How fast did she ride in miles per hour?

Name:_____Date:_____

Trail Mix Task (Ratios and Proportions)

Giovanny was mixing raisins and mixed nuts in the ratio of 2:3 to make trail mix. He wants to make 45 ounces of trail mix for his class's field trip. He decided to begin the following table to help him think about the problem, but is not sure what he should do next.

Ounces of Raisins	Ounces of Mixed Nuts	Ounces of Trail Mix
2	3	5
4	6	10

c. Explain in words how to continue to add numbers into the table.

_____
_____
_____
_____
_____
_____
_____
_____
_____
_____
_____

d. Write an explanation to Giovanny about how he can use his table to find out how many ounces of raisins and how many ounces of mixed nuts he will need to make 45 ounces of trail mix.

_____
_____
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Name:_____Date:_____

Fish Food (Ratios and Proportions)

The smaller fish is 10 cm long and the larger fish is 15 cm long. If the smaller fish gets 12 grams of fish food, how much food does the larger fish get? Use ratios to solve this problem.

Josh says that since the smaller fish is 10 cm long and gets 12 grams of food, the larger fish, being 15 cm long, must get 17 grams of food. Is Josh correct? Explain your reasoning.

Name:_____Class:_____

Ratios and Proportions

Complete the following ratio table.

Red	Green
2	6
3	9
5	15
6	18
9	
	30
15	

What did you do in order to solve for the missing numbers? Explain.

_____

_____

_____

_____

_____

<table>
<tr><th colspan="2">Research and review of standard</th></tr>
<tr><th>Content Standard(s):</th><th>Standard(s) for Mathematical Practice:</th></tr>
<tr><td>Understand ratio concepts and use ratio reasoning to solve problems.

Standard: <u>6.RP.A.3 Use</u> ratio and rate reasoning to solve real-world problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
a. Make tables of equivalent ratios relating qualities with whole-number measurements, find missing values in the tables, and plot the pairs of values in the coordinate plane. Use tables to compare ratios.</td><td>MP.3 Construct viable arguments and critique the reasoning of others.
• Make conjectures and build a logical progression of statements to explore the truth of their conjectures.

MP.4 Model with mathematics.
• Apply the mathematics they know to solve problems that arise in everyday life, society and the workplace.</td></tr>
<tr><th>Smarter Balanced Claim</th><th>Smarter Balanced Item</th></tr>
<tr><td>Claim 2: Problem Solving
Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.</td><td>Alia wants to buy pizza for a party.
• 40 to 50 people will be coming to the party.
• A large pizza from Paolo's Pizza Place serves 3 to 4 people.
• Each large pizza from Paolo's Pizza Place costs $11.50.

<i>Part A</i>
Alia wants to buy enough pizza so that people will not be hungry, and wants to have the least amount of pizza left over. How many large pizzas should Alia buy?
[ ] pizzas

<i>Part B</i>
If Alia buys the number of large pizzas that you determined in <i>Part A</i>, how much money will she spend on pizza?
$[ ]</td></tr>
<tr><td></td><td>Conceptual Understanding and Knowledge
• Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.
• Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship.
• Understand that a ratio says how much of one item there is compared to another item.
• Understand the concept of a ratio and how to describe the relationship between two quantities.
• Understand that equivalent rations are always multiplied or divided by the same number.
• Understand that a proportion statement is the equality of two ratios.
• Understand that ratios must be equal to each other and remain constant.</td></tr>
</table>

	• Understand that there are several different ways to correctly solve ratio problems. **Procedural Skills** • Apply estimation strategies • Manipulate multi-digit decimals • Multiplication with whole numbers • Division of whole numbers • Find common factors and multiples • Compute fluently with multi-digit numbers **Representational** • Apply mathematics and develop a proportional relationship to solve a real-world problem. • Use equivalent ratios and ratio tables to solve real-world mathematical problems • Understand equivalent ratios and ratio tables **Conventions of Social knowledge** • Know that a colon separates the two numbers that make up a ratio. • Know that two numbers are included in a ratio (no zero). • Know that different ratios can be equivalent.

Standards Progression **Look at LearnZillion lessons and expert tutorials, the Progressions documents, learning trajectories, and the "Wiring Document" to help you with this section*		
Grade(s) below	**Target grade**	**Grade(s) above**
Standard: 5.G.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	**Standard: 6.RP.1 Understand** the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **Standard: 6.RP.B.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were the lawns being mowed?*	**Standard: 7.RP.2** Recognize and represent proportional relationships between quantities. **Standard: 7.RP.3** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Common Misconceptions/Roadblocks
What characteristics of this problem may confuse students? • Students unfamiliar with range problems may not realize that there is more than one possible solution to the same problem. **What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?** • Students struggle with the fact that ratios must be equal to each other and remain constant. • Students may use subtraction or addition to solve ratio problems where multiplication or division is necessary. • Students reason additively instead of reasoning multiplicatively (proportional reasoning) • Students may be uncertain on how to write ratios. • Students may be uncertain on how to complete a ratio table. **What overgeneralizations may students make from previous learning leading them to make false connections or conclusions?** • Students may believe that there is only one answer to a given problem.