Unit 2 Ratios and Proportional Relationships: Multistep Ratios and Proportions

In this unit, students will reduce fractions to lowest terms by canceling. Students will turn ratios with fractional terms into ratios with whole-number terms through multiplication, and they will find examples of equivalent ratios and solve problems involving ratios with fractional terms. Students will solve proportions both in isolation and in context with fractions, complex fractions, decimals ratios, and unit rates. Students will write equivalent statements for proportions by keeping track of the part and the whole and solve proportions.

Students will review the balance model to solve addition and multiplication equations and then equations including negative addend and coefficient. Students will cross-multiply to write an equation for problems that involve proportions and solve the proportions by solving equations. By the end of the unit, students will recognize if two quantities are in proportional relationship or not. Finally, students will solve discount and markup word problems using diagrams.

RP7-26 Canceling Through Multiplication

Pages 35–36

Standards: 7.NS.A.2

Goals:
Students will use canceling to reduce fractions to lowest terms.

Prior Knowledge Required:
Can find factors and factor pairs
Can find the GCF of two numbers
Can divide by the GCF to reduce fractions to lowest terms
Can multiply fractions and integers

Vocabulary: canceling, common factor, factor, factor pairs, greatest common factor (GCF), lowest terms, reducing a fraction

Review factors and common factors. Have students brainstorm ways of multiplying two whole numbers to get 20. (1 × 20 or 20 × 1, 2 × 10 or 10 × 2, 4 × 5 or 5 × 4) Remind students that the numbers that appear in products that make 20—1, 2, 4, 5, 10, and 20—are called factors of 20.

Exercises: Find all the factors.
a) 12 b) 18 c) 30
Answers: a) 1, 2, 3, 4, 6, 12; b) 1, 2, 3, 6, 9, 18; c) 1, 2, 3, 5, 6, 10, 15, 30

Remind students that the factors of a number come in pairs. For example, 4 × 5 = 20, so 4 and 5 are a factor pair for 20. ASK: What is another factor pair for 20? (2 and 10, or 1 and 20) List on the board all the factor pairs for 20. (1 × 20, 2 × 10, 4 × 5)

Exercises: Write the factor pairs.
a) 12 b) 18 c) 24
Answers: a) 1 × 12, 2 × 6, 3 × 4; b) 1 × 18, 2 × 9, 3 × 6; c) 1 × 24, 2 × 12, 3 × 8, 4 × 6

Remind students that factors that two numbers have in common are called common factors. SAY: Here are the factors of 12 and 20, in order from least to greatest. Write on the board:

12: 1, 2, 3, 4, 6, 12

20: 1, 2, 4, 5, 10, 20

ASK: What factors do 12 and 20 have in common? (1, 2, and 4) What is the greatest common factor (GCF) of 12 and 20? (4)

Exercises: Find the GCF of the pair of numbers. Hint: List the factors of both numbers in order from least to greatest.

a) 15 and 20 b) 18 and 30 c) 64 and 72 d) 30 and 60

Answers: a) 5, b) 6, c) 8, d) 30

Identifying fractions in lowest terms. SAY: A fraction is reduced to lowest terms when you can't divide the numerator and denominator by a number greater than 1 and, therefore, the GCF of the numerator and the denominator is 1. Write on the board:

$$\frac{2}{6} \qquad \frac{3}{5} \qquad \frac{1}{4} \qquad \frac{6}{15}$$

Have volunteers decide if each fraction is in lowest terms. Students can do this by testing systematically if 2, 3, 5, 7, or 11 are factors of the numerator and the denominator. SAY: As soon as a number greater than 1 is identified as a common factor, we know that the fraction is not in lowest terms. (3/5 and 1/4 are in lowest terms; 2/6 (common factor of 2) and 6/15 (common factor of 3) are not)

Exercise: Copy the fractions that are in lowest terms in your notebook.

a) $\frac{3}{5}$ $\frac{4}{4}$ $\frac{4}{8}$ $\frac{4}{9}$ $\frac{4}{10}$ $\frac{3}{7}$ $\frac{2}{8}$ $\frac{2}{9}$ $\frac{3}{9}$ $\frac{5}{15}$ **Bonus:** $\frac{12}{50}$ $\frac{42}{96}$ $\frac{36}{175}$

Answers: a) 3/5, 4/9, 3/7, 2/9; Bonus: 36/175

Reducing fractions to lowest terms. Remind students that we can write a fraction in lowest terms by dividing the numerator and denominator by their GCF. Write on the board:

$$\frac{15}{20}$$

ASK: What is the GCF of 15 and 20? (5) Divide the numerator and denominator by 5, as shown below:

$$\frac{15 \div 5}{20 \div 5} =$$

ASK: What can 15/20 be reduced to? (3/4) Write the answer on the board, as shown below:

$$\frac{15 \div 5}{20 \div 5} = \frac{3}{4}$$

Write on the board:

$$\frac{-6}{21}$$

SAY: Because the original fraction is negative, we know the reduced fraction will be negative. So let's ignore the negative sign for the moment. ASK: What is the GCF of 6 and 21? (3)
SAY: We divide the numerator and the denominator by 3 to reduce the fraction. Don't forget to add in the negative sign. Reduce the fraction and write the answer on the board, as shown below:

$$\frac{-6 \; {\scriptstyle \div 3}}{21 \; {\scriptstyle \div 3}} = \frac{-2}{7}$$

Exercises: Reduce the fraction to lowest terms.

a) $\frac{12}{20}$ b) $\frac{6}{18}$ c) $\frac{-5}{25}$ d) $\frac{14}{-35}$

Bonus:

e) $\frac{700}{750}$ f) $\frac{336}{504}$ g) $\frac{-378}{-420}$ h) $\frac{147}{-231}$

Answers: a) $\frac{3}{5}$, b) $\frac{1}{3}$, c) $\frac{-1}{5}$, d) $\frac{2}{-5}$, Bonus: e) $\frac{14}{15}$, f) $\frac{2}{3}$, g) $\frac{-9}{-10}$ or $\frac{9}{10}$, h) $\frac{7}{-11}$

Introduce canceling. SAY: To multiply fractions, we multiply the numerators and the denominators. Write on the board:

$$\frac{3}{5} \times \frac{2}{3} = \frac{3 \times 2}{5 \times 3}$$

SAY: Let's switch the order of the numbers in the numerator to line up the threes. Write on the board:

$$= \frac{2 \times 3}{5 \times 3}$$

SAY: Now if we separate the fraction, we see that we are multiplying 2/5 by 3/3, which is the same as multiplying 2/5 by 1. Write the answer on the board, as shown below:

$$= \frac{2}{5} \times \frac{3}{3} = \frac{2}{5}$$

Exercises: Follow the example on the board to multiply the fractions.

a) $\frac{2}{7} \times \frac{7}{9}$ b) $\frac{3}{8} \times \frac{8}{11}$ c) $\frac{5}{12} \times \frac{12}{17}$ **Bonus:** $\frac{23}{72} \times \frac{72}{151}$

Answers: a) 2/9, b) 3/11, c) 5/17, Bonus: 23/151

SAY: Now let's actually multiply the fractions. Write on the board:

$$\frac{2}{3} \times \frac{3}{7} = \frac{6}{21}$$

ASK: Is this fraction in lowest terms? (no) How can you reduce it? (divide by 3, the GCF) What is the reduced fraction? (2/7) Write the reduced fraction on the board:

$$\frac{2}{3}\times\frac{3}{7}=\frac{6}{21}=\frac{2}{7}$$

Exercises: Follow the example on the board to multiply the fractions.

a) $\frac{1}{3}\times\frac{3}{4}$ b) $\frac{2}{5}\times\frac{5}{3}$ c) $\frac{6}{7}\times\frac{5}{6}$ **Bonus:** $\frac{9}{11}\times\frac{8}{9}$

Answers: a) 3/12 = 1/4, b) 10/15 = 2/3, c) 30/42 = 5/7, Bonus: 72/99 = 8/11

SAY: Let's find a way to make this faster. In the first example, we were able to separate the fraction 3/3, then ignore it. In the second example, we ended up dividing by 3, the number that was common to the numerator and the denominator. Let's just eliminate the number in common right away! This process is called *canceling*. Write two more examples on the board:

$$\frac{3}{5}\times\frac{2}{3}=\frac{\cancel{3}\times 2}{5\times\cancel{3}}=\frac{2}{5}$$

$$\frac{2}{7}\times\frac{7}{3}=\frac{2\times\cancel{7}}{\cancel{7}\times 3}=\frac{2}{3}$$

Exercises: Cancel first, then multiply the fractions.

a) $\frac{3}{4}\times\frac{4}{7}$ b) $\frac{5}{9}\times\frac{2}{5}$ c) $\frac{8}{11}\times\frac{3}{8}$ d) $\frac{5}{7}\times\frac{7}{8}$

Bonus:

e) $\frac{125}{4}\times\frac{3}{125}$ f) $\frac{6}{379}\times\frac{379}{7}$ g) $\frac{9}{1{,}245}\times\frac{1{,}245}{11}$ h) $\frac{104{,}525}{5}\times\frac{2}{104{,}525}$

Answers: a) 3/7, b) 2/9, c) 3/11, d) 5/8, Bonus: e) 3/4, f) 6/7, g) 9/11, h) 2/5

Multiplying positive and negative fractions by canceling first. Write on the board:

(+) × (+) = + (−) × (−) = + (+) × (−) = − (−) × (+) = −

Tell students that these facts that will help when reducing fractions with negative numbers. Write on the board:

$$\frac{5}{9}\times\left(-\frac{9}{11}\right)$$

SAY: The first step is to decide if the answer is positive or negative. In this case, we have (+) × (−), so the answer is (−). Then we cancel the fractions. Write on the board:

$$\frac{5}{9}\times\left(-\frac{9}{11}\right)=-\frac{5\times\cancel{9}}{\cancel{9}\times 11}=-\frac{5}{11}$$

Write on the board:

$$\left(-\frac{7}{8}\right)\times\left(-\frac{1}{7}\right)$$

ASK: What is the sign of the answer? (+) Why? (because (−) × (−) = (+)) Have a volunteer use canceling to find the answer, as shown below:

$$\left(-\frac{7}{8}\right)\times\left(-\frac{1}{7}\right)=\frac{\cancel{7}\times 1}{8\times\cancel{7}}=\frac{1}{8}$$

Exercises: Cancel first, then multiply. Hint: Decide the sign of the answer before you cancel.

a) $-\frac{5}{11}\times\frac{11}{8}$ b) $\frac{6}{7}\times\left(-\frac{5}{6}\right)$ c) $-\frac{1}{3}\times\left(-\frac{3}{4}\right)$ d) $-\frac{10}{11}\times\frac{9}{10}$

Answers: a) $-\frac{5}{8}$, b) $-\frac{5}{7}$, c) $\frac{1}{4}$, d) $-\frac{9}{11}$

Using canceling to reduce fractions to lowest terms when the numerator is the GCF. SAY: We can use canceling to reduce fractions to lowest terms. Rewrite the numerator and denominator as a product of their GCF, which will then be canceled out. Work through these two examples on the board as a class:

$$\frac{5}{15}=\frac{1\times\cancel{5}}{3\times\cancel{5}}=\frac{1}{3} \qquad -\frac{3}{21}=-\frac{1\times\cancel{3}}{7\times\cancel{3}}=-\frac{1}{7}$$

Exercises: Cancel to reduce to lowest terms.

a) $\frac{7}{35}$ b) $\frac{6}{42}$ c) $-\frac{8}{32}$ d) $-\frac{5}{15}$

Answers: a) $\frac{1}{5}$, b) $\frac{1}{7}$, c) $-\frac{1}{4}$, d) $-\frac{1}{3}$

Using canceling to multiply a fraction by its denominator. SAY: Let's look at what happens when you multiply a fraction by its denominator. We can also do this by canceling. Write on the board:

$$\frac{5}{6}\times 6=\frac{5}{6}\times\frac{6}{1}=\frac{5\times\cancel{6}}{\cancel{6}\times 1}=\frac{5}{1}=5 \text{ or more simply, } \frac{5}{\cancel{6}}\times\cancel{6}=5$$

Exercises: Cancel to multiply.

a) $\frac{7}{8}\times 8$ b) $6\times\frac{5}{6}$ c) $-\frac{4}{9}\times 9$ d) $\frac{6}{-13}\times(-13)$

Answers: a) 7, b) 5, c) −4, d) 6

Canceling to multiply three fractions. SAY: We can also cancel when we are multiplying three fractions. Just cancel out all the numbers that are common to the numerator and the denominator. Write on the board:

$$\frac{2}{3}\times\frac{3}{7}\times\frac{7}{5}=\frac{2\times\cancel{3}\times 7}{\cancel{3}\times 7\times 5}=\frac{2\times\cancel{7}}{\cancel{7}\times 5}=\frac{2}{5}$$

SAY: You can also do the canceling in one step. Write another example on the board:

$$\frac{3}{5}\times\frac{7}{3}\times\frac{5}{2}=\frac{\cancel{3}\times 7\times\cancel{5}}{\cancel{5}\times\cancel{3}\times 2}=\frac{7}{2}$$

Exercises: Cancel as much as you can before multiplying.

a) $\frac{5}{8}\times\frac{8}{11}\times\frac{11}{6}$ b) $\frac{3}{4}\times\frac{1}{3}\times\frac{4}{7}$ c) $\frac{6}{11}\times\frac{5}{7}\times\frac{7}{6}$ d) $\frac{8}{5}\times\frac{9}{8}\times\frac{2}{9}$

Answers: a) 5/6, b) 1/7, c) 5/11, d) 2/5

Write on the board:

$$-\frac{11}{3}\times\frac{3}{7}\times\left(-\frac{2}{11}\right)$$

SAY: You can count the negative signs to decide whether the product is positive or negative. If there is an even number of negative signs, the product is positive. If there is an odd number of negative signs, the product is negative. Here there are two negative signs, so the answer is positive. Then, we cancel to multiply. Finish solving the problem, as shown below:

$$-\frac{11}{3}\times\frac{3}{7}\times\left(-\frac{2}{11}\right)=\frac{\cancel{11}\times\cancel{3}\times 2}{\cancel{3}\times 7\times\cancel{11}}=\frac{2}{7}$$

Exercises: Cancel first, then multiply. Hint: What is the sign of the answer?

a) $-\frac{3}{4}\times\frac{5}{3}\times\frac{1}{5}$ b) $\frac{5}{6}\times\left(-\frac{3}{5}\right)\times\left(-\frac{6}{7}\right)$ c) $-\frac{2}{5}\times\left(-\frac{5}{9}\right)\times\left(-\frac{7}{2}\right)$

Answers: a) $-\frac{1}{4}$, b) $\frac{3}{7}$, c) $-\frac{7}{9}$

Canceling when the factors are not visible. SAY: Remind students that two numbers that multiply to make another number are called factor pairs of that number. Write on the board:

$$\frac{3\times 14}{5\times 21}$$

SAY: We can’t see any common factors at the moment. Let’s use factor pairs to rewrite the 14 and 21 to see if there are any common factors. ASK: What is a factor pair of 14? (1 × 14 or 2 × 7) of 21? (1 × 21 or 3 × 7) SAY: Fractor pairs that include 1 are not very useful because canceling out 1 does not make the fraction any simpler.

Rewrite the question on the board using 14 = 2 × 7 and 21 = 3 × 7, as shown below:

$$\frac{3\times 14}{5\times 21}=\frac{3\times 2\times 7}{5\times 3\times 7}$$

SAY: Now we can cancel to get an answer in lowest terms. Write the answer on the board, as shown below:

$$\frac{3\times14}{5\times21}=\frac{\cancel{3}\times2\times\cancel{7}}{5\times\cancel{3}\times\cancel{7}}=\frac{2}{5}$$

(MP.7) Exercises: Use factor pairs to rewrite the fraction. Then cancel to reduce to lowest terms.

a) $\frac{2\times6}{3\times10}$ b) $\frac{7\times10}{2\times49}$ c) $\frac{4\times27}{9\times15}$ **Bonus:** $\frac{6\times10\times21}{11\times14\times15}$

Answers: a) 2/5, b) 5/7, c) 4/5, Bonus: 6/11

Write on the board:

$$\frac{(-4)\times3}{(-6)\times(-5)}$$

ASK: What will the sign of the answer be? (negative) How do you know? (there are an odd number of negative signs) Which numbers can we rewrite as factor pairs? (4 = 2 × 2 and 6 = 2 × 3) Rewrite the fraction on the board using factor pairs, then cancel to multiply, as shown below:

$$\frac{(-4)\times3}{(-6)\times(-5)}=-\frac{\cancel{2}\times2\times\cancel{3}}{\cancel{2}\times\cancel{3}\times5}=-\frac{2}{5}$$

(MP.7) Exercises: Cancel to multiply. Hint: Decide the sign of the answer first.

a) $\frac{(-5)\times9}{3\times(-10)}$ b) $\frac{7\times(-15)}{(-3)\times(-14)}$

Bonus:

c) $\frac{(-12)\times(-10)}{7\times6\times(-5)}$ d) $\frac{2}{3}\times(-16)\times\frac{3}{8}$

Answers: a) $\frac{3}{2}$, b) $-\frac{5}{2}$, Bonus: c) $-\frac{4}{7}$, d) −4

SAY: With canceling, you can reduce the most complicated looking fraction problems in a snap!

Exercise: Cancel to multiply.

$\frac{1}{2}\times\frac{2}{-3}\times\frac{-3}{4}\times\frac{4}{5}\times\frac{5}{6}\times\frac{6}{-7}\times\frac{-7}{-8}\times\frac{-8}{9}\times\frac{9}{10}$

Answer: 1/10

Extensions

1. Explore two ways of multiplying fractions.

(MP.7) a) Complete the table.

	Multiply first, then reduce the fraction.	Cancel first, then multiply.
Example:	$\frac{7}{6}\times\frac{3}{14}=\frac{21\ {\div 21}}{84\ {\div 21}}=\frac{1}{4}$	$\frac{7}{6}\times\frac{3}{14}=\frac{\cancel{7}\times\cancel{3}}{2\times\cancel{3}\times2\times\cancel{7}}=\frac{1}{4}$
i)	$\frac{7}{10}\times\frac{20}{21}$	$\frac{7}{10}\times\frac{20}{21}$
ii)	$\frac{15}{14}\times\frac{7}{12}$	$\frac{15}{14}\times\frac{7}{12}$
iii)	$5\times\frac{8}{26}\times\frac{13}{40}$	$5\times\frac{8}{26}\times\frac{13}{40}$

(MP.1) b) Did you get the same answer both ways? If not, find your mistake.

(MP.5) c) Which method is faster?

(MP.1) d) Fill in the chart to compare your solutions to part a). In both cases, did you divide by the same number?

	What factor did you divide by to reduce the fraction?	What number did you cancel?	Are these numbers the same?
Example:	21	7 × 3 = 21	yes
i)			
ii)			
iii)			

Answers: a) i) 2/3, ii) 5/8, iii) 1/2; c) the second method (canceling) is faster; d) i) 70, ii) 21, iii) 520

2. You can use canceling to reduce any fraction to lowest terms. Canceling is useful when you don't know the GCF, but you do see a common factor. For example:

$$\frac{72}{192}=\frac{36\times2}{96\times2}=\frac{36}{96}=\frac{12\times3}{32\times3}=\frac{12}{32}=\frac{3\times4}{8\times4}=\frac{3}{8}$$

Cancel to reduce the fraction to lowest terms.

a) $\frac{90}{150}$ b) $\frac{96}{144}$ c) $\frac{66}{162}$ d) $\frac{480}{576}$

Answers: a) 3/5, b) 2/3, c) 11/27, d) 5/6

RP7-27 Ratios and Rates with Fractional Terms

Pages 37–38

Standards: 7.RP.A.1

Goals:
Students will understand that ratios with fractional terms can be expressed as equivalent ratios with whole-number terms through multiplication.

Prior Knowledge Required:
Can use canceling when multiplying a fraction by a whole number
Can find the LCM of two numbers
Understands ratio tables

Vocabulary: canceling, equivalent ratios, factor pairs, lowest common multiple (LCM), ratio, ratio table

Review using canceling to multiply a fraction by its denominator. Write on the board:

$\frac{1}{7} \times 7 =$ $\qquad$ $\frac{3}{11} \times 11 =$ $\qquad$ $16 \times \frac{13}{16} =$

Ask volunteers to complete the multiplication by canceling. (1, 3, 13) ASK: What do you notice about the answers? (the numerator becomes the answer, and the answer is a whole number) SAY: When you multiply a fraction by its denominator, the denominator and the multiplier cancel and the answer will be a whole number.

Exercises: Multiply by canceling.
a) $\frac{6}{13} \times 13$ $\qquad$ b) $12 \times \frac{5}{12}$ $\qquad$ c) $21 \times \frac{19}{21}$ $\qquad$ **Bonus:** $\frac{1{,}245}{3{,}967} \times 3{,}967$
Answers: a) 6, b) 5, c) 19, Bonus: 1,245

Using canceling to multiply a fraction by a multiple of its denominator. Write on the board:

$$\frac{2}{5} \times 15$$

SAY: At the moment, there is nothing to cancel. Let's try writing 15 as a factor pair. ASK: How could we do that? (3 × 5 and 1 × 15) Which pair would be useful for canceling? (3 × 5) Cancel the 5s to find the answer, as shown below:

$$= \frac{2}{\cancel{5}} \times 3 \times \cancel{5} = 6$$

Write on the board:

$$\frac{5}{6} \times 12$$

ASK: How can we rewrite 12 so we have something to cancel? (6 × 2) Finish the example on the board, as shown below:

$$= \frac{5}{\cancel{6}} \times \cancel{6} \times 2 = 10$$

Exercises: Cancel to multiply.

a) $\frac{3}{4} \times 28$ b) $\frac{2}{3} \times 9$ c) $\frac{3}{5} \times 20$ d) $16 \times \frac{3}{8}$

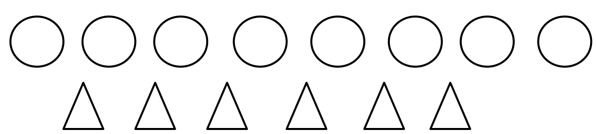
Answers: a) 21, b) 6, c) 12, d) 6

Write on the board:

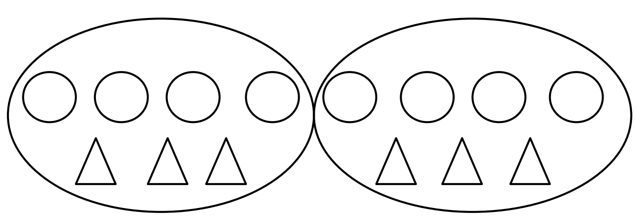
$\frac{1}{3} \times \square = 1$ $\frac{2}{5} \times \square = 2$ $\frac{3}{4} \times \square = 3$ $\frac{5}{6} \times \square = 15$

ASK: What number do you need to multiply by to have a whole-number answer to the equation? (3, 5, 4, 18)

Review ratios. Draw on the board:



SAY: There are 8 circles for every 6 triangles. Using ratios, we write “circles : triangles = 8 : 6.” We could group these objects differently. Circle the shapes on the board, as shown below:



SAY: We can also say there are 4 circles for every 3 triangles. The ratio is circles : triangles = 4 : 3. We say that 8 : 6 and 4 : 3 are equivalent ratios.

Making equivalent ratios from a ratio with one fractional term. Remind students that they can make an equivalent ratio by multiplying (or dividing) both terms in a ratio by the same number. For example, the ratios 1/2 : 3 and 1 : 6 are equivalent ratios because multiplying both terms in the first ratio by 2 gives the terms of the second ratio. Write on the board:

$$\frac{1}{3} : 4$$

SAY: I find whole numbers easier to work with. ASK: What number do I multiply by to turn the fractional term of the ratio into a whole number? PROMPT: 1/3 × ? = 1 (3) ASK: When you multiply by 3, what does the first term become? (1) What does the second term become? (4 × 3 = 12) What is the new ratio? (1 : 12) Finish answering the question on the board, as shown below:

$$\frac{1}{3} : 4 = \left(\frac{1}{3} \times 3\right) : (4 \times 3) = 1 : 12$$

Exercises: Write an equivalent ratio with whole-number terms.

a) $\frac{1}{3} : 5$ b) $3 : \frac{5}{6}$ c) $4 : \frac{7}{9}$ d) $\frac{5}{12} : 6$

Answers: a) 1 : 15, b) 18 : 5, c) 36 : 7, d) 5 : 72

Write on the board:

$\frac{1}{2}$ cup of ice for every 3 cups of juice   6 mi pedaled every $\frac{2}{5}$ hour

ASK: How can we write these ratios with both terms as whole numbers? (multiply both terms by the denominator) Write on the board:

$$\begin{array}{ccc} & \frac{1}{2} : 3 & \\ \times 2 \downarrow & & \downarrow \times 2 \\ & 1 : 6 & \end{array} \qquad\qquad \begin{array}{ccc} & 6 : \frac{2}{5} & \\ \times 5 \downarrow & & \downarrow \times 5 \\ & 30 : 2 & \end{array}$$

1 cup of ice for every 6 cups of juice   30 mi pedaled every 2 hours

Exercises: Write a ratio with whole-number terms.

a) $\frac{3}{8}$ of a pizza for every 3 people   b) $\frac{1}{3}$ cup of cheese for every 4 eggs

c) 45 heartbeats every $\frac{3}{4}$ of a minute   d) 26 miles per $\frac{2}{3}$ gallons of gas

Answers: a) 3 pizzas for every 24 people, b) 1 cup of cheese for every 12 eggs, c) 180 heartbeats every 3 minutes, d) 78 miles per 2 gallons of gas

Making equivalent ratios from a ratio with two fractional terms. Write on the board:

$$\frac{1}{3} : \frac{1}{4}$$

SAY: We want to write this ratio with whole numbers. Let's break the problem into steps. Start by multiplying both terms by the first denominator. Write on the board:

$$\frac{1}{3} : \frac{1}{4} = \frac{1}{3} \times 3 : \frac{1}{4} \times 3 = 1 : \frac{3}{4}$$

SAY: Then multiply both terms by the second denominator. Write on the board:

$$1 \times 4 : \frac{3}{4} \times 4 = 4 : 3$$

So $\frac{1}{3} : \frac{1}{4} = 4 : 3$

Repeat these steps with the example shown below:

$$\frac{1}{5} : \frac{3}{7} = 1 : \frac{15}{7} = 7 : 15$$

ASK: In this problem, could we multiply both terms by a single number instead of doing two steps? (yes, we could multiply by 35, which is the lowest common multiple (LCM) of the two denominators) SAY: The number we multiply by has to be a multiple of the two denominators, so we might as well use the lowest common multiple. Write on the board:

$$\frac{1}{5} : \frac{3}{7} = \left(\frac{1}{5} \times 35\right) : \left(\frac{3}{7} \times 35\right) = 7 : 15$$

Exercises: Multiply by the LCM of the denominators to make an equivalent ratio with whole-number terms.

a) $\frac{1}{5} : \frac{1}{4}$ b) $\frac{1}{3} : \frac{1}{7}$ c) $\frac{1}{2} : \frac{3}{5}$ d) $\frac{5}{6} : \frac{2}{5}$

Bonus:

e) $\frac{1}{3} : \frac{5}{6}$ f) $\frac{7}{15} : \frac{3}{5}$ g) $\frac{2}{9} : \frac{5}{12}$ h) $\frac{9}{20} : \frac{7}{18}$

Answers: a) 4 : 5, b) 7 : 3, c) 5 : 6, d) 25 : 12, Bonus: e) 2 : 5, f) 7 : 9, g) 8 : 15, h) 81 : 70

Using ratio tables to find equivalent ratios. Remind students that ratio tables are another way to represent ratios. SAY: In a ratio table, we can find equivalent ratios by multiplying each term in a row by the same number to get another row. A ratio table can have fractions in it. Draw on the board:

$\frac{2}{5}$	$\frac{1}{6}$

SAY: What number do we need to multiply by to get whole numbers in the next row? (30, the LCM of 5 and 6) Add arrows as shown below:

$\frac{2}{5}$	$\frac{1}{6}$

× 30 × 30

Point to the left cell and ASK: What number goes here? (2/5 × 30 = 2 × 30 ÷ 5 = 12) Write “12” in the cell. Point to the right cell and ASK: And what number goes here? (1/6 × 30 = 1 × 30 ÷ 6 = 5) Write “5” in the cell. The table should look like this:

$\frac{2}{5}$	$\frac{1}{6}$
12	5

× 30 × 30

ASK: What is the ratio equivalent to 2/5 : 1/6? (12 : 5) Write on the board:

So $\frac{2}{5}:\frac{1}{6} = 12:5$

Exercises: Complete the ratio table.

a)

$\frac{1}{7}$	$\frac{5}{6}$

× 42 × 42

So $\frac{1}{7}:\frac{5}{6} =$ ______

b)

$\frac{1}{2}$	$\frac{3}{5}$

So $\frac{1}{2}:\frac{3}{5} =$ ______

c)

$\frac{2}{3}$	$\frac{3}{4}$

So $\frac{2}{3}:\frac{3}{4} =$ ______

d)

$\frac{5}{9}$	$\frac{3}{11}$

So $\frac{5}{9}:\frac{3}{11} =$ ______

Bonus: Draw a ratio table and find equivalent ratios with whole numbers.

e) $\frac{5}{12}:\frac{2}{3}$ f) $\frac{3}{10}:\frac{7}{20}$ g) $\frac{3}{4}:\frac{5}{6}$ h) $\frac{5}{9}:\frac{4}{15}$

Answers: a) 6 : 35, b) 5 : 6, c) 8 : 9, d) 55 : 27, Bonus: e) 5 : 8, f) 6 : 7 g) 9 : 10, h) 25 : 12

Remind students that the arrows in a ratio table can point from bottom to top as well as from top to bottom.

Exercises: Complete the ratio table.

a)

$\frac{5}{11}$	$\frac{3}{7}$

× 77 × 77

So ______ $= \frac{5}{11}:\frac{3}{7}$

b)

$\frac{2}{3}$	$\frac{3}{5}$

So ______ $= \frac{2}{3}:\frac{3}{5}$

c)

$\frac{2}{5}$	$\frac{3}{10}$

So ______ $= \frac{2}{5}:\frac{3}{10}$

d)

$\frac{3}{4}$	$\frac{7}{10}$

So ______ $= \frac{3}{4}:\frac{7}{10}$

Answers: a) 35 : 33, b) 10 : 9, c) 4 : 3, d) 15 : 14

Extensions

1. Have students convert mixed numbers in a ratio into improper fractions, then find an equivalent ratio in which both terms are whole numbers. For example:

$$3\frac{2}{3}:1\frac{2}{5}=\frac{11}{3}:\frac{7}{5}=\frac{11}{3}\times 15:\frac{7}{5}\times 15=55:21$$

a) $2\frac{3}{4}:3\frac{1}{2}$ b) $4\frac{2}{5}:3\frac{2}{3}$

Bonus:

c) $6\frac{2}{7}:3\frac{1}{3}$ d) $31\frac{3}{4}:21\frac{2}{3}$

Answers: a) 11 : 14, b) 66 : 55, Bonus: c) 132 : 70, d) 381 : 260

2. Find equivalent ratios in which all terms are whole numbers. Multiply all terms by the LCM of all the denominators.

a) $\frac{1}{3}:\frac{1}{4}:\frac{1}{5}$ b) $\frac{1}{2}:\frac{2}{3}:\frac{5}{6}$ c) $\frac{2}{5}:\frac{1}{2}:\frac{3}{4}$ **Bonus:** $\frac{1}{2}:\frac{2}{3}:\frac{3}{4}:\frac{4}{5}$

Answers: a) 20 : 15 : 12, b) 3 : 4 : 5, c) 8 : 10 : 15, Bonus: 30 : 40 : 45 : 48

3. It's always convenient to use the smallest whole numbers possible. Fractions can be reduced to lowest terms. For example, 4/6 = 2/3. Similarly, ratios can be reduced to lowest whole-number terms. For example, 4 : 6 = 2 : 3. To write a ratio in its lowest terms, divide both terms in the ratio by the GCF (see example below).

5 : 20 = 5 ÷ 5 : 20 ÷ 5 = 1 : 4 or

	5	20	
÷5	1	4	÷5

Write the ratio in lowest terms.

a) 12 : 48 b) 16 : 40 c) 21 : 49 d) 24 : 28

Bonus:

e) 18 : 24 : 33 f) 20 : 8 : 28

g) Change both terms to whole numbers, then write the ratio in lowest terms: $\frac{6}{7}:\frac{2}{3}$

Answers: a) 1 : 4, b) 2 : 5, c) 3 : 7, d) 6 : 7, Bonus: e) 6 : 8 : 11, f) 5 : 2 : 7, g) 18 : 14 = 9 : 7

RP7-28 Ratio Word Problems with Fractional Terms

Pages 39–40

Standards: 7.RP.A.1, 7.RP.A.2b

Goals:
Students will solve problems involving ratios with fractional terms.

Prior Knowledge Required:
Can divide fractions using the invert-and-multiply rule
Can multiply fractions by whole numbers
Can multiply whole numbers by fractions
Can find equivalent ratios
Understands ratio tables

Vocabulary: canceling, equivalent ratios, fractional multiplier, invert-and-multiply rule, rate, ratio, ratio table

Review dividing whole numbers by fractions. Write on the board:

$5 \div \frac{1}{3} =$ $\qquad$ $3 \div \frac{4}{7} =$ $\qquad$ $6 \div \frac{2}{5} =$ $\qquad$ $6 \div \frac{4}{5} =$

Remind students that we can use the invert-and-multiply rule when dividing by a fraction. Ask volunteers to show how this rule is applied to evaluate each division. (5 × 3/1 = 15, 3 × 7/4 = 21/4, 6 × 5/2 = 30/2 = 15, 6 × 5/4 = 30/4 = 7 1/2)

Write on the board:

$$\frac{1}{2} \times \square = 6$$

ASK: What number makes the equation true? (12) How do you know? (because 6 ÷ 1/2 = 12)
Write on the board:

$$\frac{2}{3} \times \square = 4$$

ASK: How can we find the missing number? (4 ÷ 2/3 = 4 × 3/2 = 12/2 = 6)

Exercises: Write a division sentence to find the number that makes the equation true.

a) $\frac{1}{5} \times \square = 2$ $\qquad$ b) $\frac{1}{8} \times \square = 3$ $\qquad$ c) $\frac{3}{4} \times \square = 6$ $\qquad$ d) $\frac{4}{9} \times \square = 12$

Answers: a) 2 ÷ 1/5 = 10, b) 3 ÷ 1/8 = 24, c) 6 ÷ 3/4 = 8, d) 12 ÷ 4/9 = 27

Using equivalent ratios to solve problems. Tell students that Jenny can walk 1/2 a mile for every 1/6 of an hour. ASK: What two quantities are being compared? (distance in miles and time in hours) SAY: Let's put this information into a ratio table. When solving word problems, you should label the columns with the two quantities that are being compared. Draw on the board:

Distance (miles)	Time (hours)

ASK: What quantities can we add to the table? (1/2 in the distance column and 1/6 in the time column) Add this information to the table, as shown below:

Distance (miles)	Time (hours)
$\frac{1}{2}$	$\frac{1}{6}$

SAY: Jenny wants to know how long it will take her to walk 3 miles at this rate. ASK: Where do we put 3 in the table? (in the miles column) What number are we missing? (the hours) Write "3" and "?" in the table, as shown below:

Distance (miles)	Time (hours)
$\frac{1}{2}$	$\frac{1}{6}$
3	?

(MP.6) Emphasize the importance of labeling the columns. SAY: If you don't label them correctly, you could mistakenly put "3" with the hours column, which would give you a different answer.

Remind students that the rows are equivalent in ratio tables. SAY: To find the missing number, you need to find the number that you multiply (or divide) each term by in one ratio to get the other ratio. Point to the table and ASK: What number do I need to multiply 1/2 by to get 3? PROMPT: 1/2 × ? = 3 (multiply by 6) How do you know? (because 3 ÷ 1/2 = 6) Draw an arrow and write "× 6", as shown below:

	Distance (miles)	Time (hours)
	$\frac{1}{2}$	$\frac{1}{6}$
× 6	3	?

SAY: Multiply the other term in the ratio by 6. ASK: What number is missing? (1) Draw another arrow beside the table and write “× 6”. Erase the question mark, and write “1” instead. The table should look like this:

	Distance (miles)	Time (hours)	
	$\frac{1}{2}$	$\frac{1}{6}$	
× 6	3	1	× 6

ASK: What ratio is equivalent to 1/2 : 1/6? (3 : 1) So how long will it take Jenny to walk 3 miles at this rate? (1 hour)

Exercises: Solve by making a ratio table and finding the equivalent ratio. Remember to label the columns.

a) A recipe for salad dressing uses $\frac{1}{3}$ cup of oil for every $\frac{1}{9}$ cup of vinegar. How many cups of vinegar are needed for 1 cup of oil?

b) On average, your hair grows $\frac{1}{8}$ inch every $\frac{1}{4}$ of a month. How much will it grow in 2 months?

c) On a map, every $\frac{1}{2}$ inch represents $\frac{3}{4}$ of a mile. How many inches on the map represent 6 miles?

d) A trail mix recipe uses $\frac{1}{4}$ bag of chocolate chips for every $1\frac{1}{3}$ cups of mixed nuts. How many cups of mixed nuts are needed for 3 bags of chocolate chips?

Answers:

a)

Cups of oil	Cups of vinegar
1/3	1/9
1	1/3

b)

Inches	Months
1/8	1/4
1	2

c)

Inches	Miles
1/2	3/4
4	6

d)

Cups of chocolate chips	Cups of nuts
1/4	1 1/3
3	16

Review canceling when multiplying by fractions. Remind students that we can cancel out a common factor when multiplying by fractions. Work through the following examples on the board:

$$\frac{3}{\cancel{4}}\times\cancel{4}=3 \qquad \frac{7}{8}\times 16=\frac{7}{\cancel{8}}\times\cancel{8}\times 2=14 \qquad 14\times\frac{1}{7}=\cancel{7}\times 2\times\frac{1}{\cancel{7}}=2$$

Exercises:

1. Find the product.

a) $5\times\frac{7}{5}$ b) $20\times\frac{11}{4}$ c) $32\times\frac{3}{8}$ d) $12\times\frac{2}{3}$

Answers: a) 7, b) 55, c) 12, d) 8

2. Find the numbers that make the equation true.

a) $3 \times \frac{\square}{\square} = 8$ b) $5 \times \frac{\square}{\square} = 9$ c) $6 \times \frac{\square}{\square} = 13$ d) $12 \times \frac{\square}{\square} = 20$

Answers: a) 8/3, b) 9/5, c) 13/6, d) 20/12 or 5/3

Using ratio tables with a fractional multiplier. Draw on the board:

5	6
7	?

Point to the table and ASK: What number do I need to multiply 5 by to get 7?

PROMPT: $5 \times \frac{\square}{\square} = 7$? (7/5) Draw an arrow and write "$\times\frac{7}{5}$" as shown below:

	5	6
$\times\frac{7}{5}$ ↓	7	?

SAY: Multiply the other term in the ratio by 7/5. ASK: What number is missing? (6 × 7/5 = 42/5) Draw another arrow and erase the question mark. Write "42/5" in the table instead, as shown below:

	5	6	
$\times\frac{7}{5}$ ↓	7	$\frac{42}{5}$	↓ $\times\frac{7}{5}$

Exercises: Multiply by a fraction to find the missing term.

a)

	3	5	
$\times\frac{8}{3}$ ↓	8		↓ $\times\frac{8}{3}$

b)

7	4
9	

c)

5	3
7	

d)

6	5
11	

Answers: a) 5 × 8/3 = 40/3, b) 4 × 9/7 = 36/7, c) 3 × 7/5 = 21/5, d) 5 × 11/6 = 55/6

Emphasize that in ratio tables, the direction of the arrow is always toward the missing number. Therefore, the arrow sometimes points from bottom to top.

Exercises: Multiply by a fraction to find the missing term.

a)

$\times\frac{5}{4}$ ↑	5		↑ $\times\frac{5}{4}$
	4	9	

b)

7	
4	11

c)

9	
5	8

d)

13	
6	7

Answers: a) 9 × 5/4 = 45/4, b) 11 × 7/4 = 77/4, c) 8 × 9/5 = 72/5, d) 7 × 13/6 = 91/6

Using ratio tables to solve problems. SAY: To make orange paint, you need to mix 2 pints of red paint with 3 pints of yellow paint. We want to know how many pints of yellow paint are needed for 5 pints of red paint. ASK: How can we show this information in a ratio table? Draw on the board:

Pints of red paint	Pints of yellow paint
2	3
5	

ASK: What number do we need to multiply 2 by to get 5? (5/2) ASK: How do you get the missing number? (3 × 5/2) What is the missing number? (15/2 or 7 1/2) Write "7 1/2" in the empty cell and SAY: So we need 7 1/2 pints of yellow paint. Point out that it is more convenient to write the answer as a mixed number because it gives a better picture of how much is needed. SAY: You can see right away that the answer is between 7 and 8 pints.

Provide several problems such as these for extra practice. Keep the same context, just change the numbers and the position of the words "pints red" and "pints yellow" in the problems.

(MP.4) Exercises: Write the answer as a mixed number.
a) We need 4 pints of red paint for every 5 pints of yellow paint. If we have 8 pints of yellow, how many pints of red do we need?
b) We need 2 pints of yellow for every 5 pints of red. How many pints of red do we need for 7 pints of yellow?
Answers: a) 6 2/5 pints, b) 17 1/2 pints

Solving problems that provide a total. SAY: To make a cup of hot chocolate, mix 1/8 cup of chocolate syrup with 3/4 cup of hot milk. We want to know how much of each ingredient is needed to make 21 cups of hot chocolate. ASK: What ratio with whole-number terms is equivalent to 1/8 : 3/4? (1 : 6) SAY: Let's work with whole numbers instead of fractions because it's easier. ASK: How can we show this information in a ratio table? Draw on the board:

Cups of chocolate syrup	Cups of milk
1	6

ASK: How many parts in total does this recipe give? (1 + 6 = 7) How many parts in total do we want? (21) Add a "total" column to the table and fill the empty cells, as shown below:

Cups of chocolate syrup	Cups of milk	Total
1	6	7
?	?	21

ASK: What number do we multiply 7 by to get 21? (3) SAY: Multiply the first row by 3. Erase the two question marks and write “3” and “18” in their places. The final table should look like this:

Cups of chocolate syrup	**Cups of milk**	**Total**
1	6	7
3	18	21

ASK: How much syrup do we need? (3 cups) How much milk do we need? (18 cups)

(MP.4) Exercises: Use a ratio table to solve.

a) A punch recipe calls for orange juice and ginger ale in the ratio $\frac{2}{3}:\frac{1}{4}$. If Kyle made 22 cups of punch, how many cups of each ingredient did he use?

b) Rani makes purple paint by mixing blue paint and red paint in the ratio $\frac{2}{3}:\frac{2}{5}$. If she makes 80 pints of paint, how many pints of each color did she use?

Bonus: If Rani makes 20 pints of purple paint, how many pints of each color did she use?

Solutions:

a) 2/3 : 1/4 = 8 : 3

Cups of orange juice	**Cups of ginger ale**	**Total**
8	3	11
?	?	22

× 2

16 cups of orange juice and 6 cups of ginger ale

b) 2/3 : 2/5 = 10 : 6 = 5 : 3

Pints of blue paint	**Pints of red paint**	**Total**
5	3	8
?	?	80

× 10

50 pints of blue paint and 30 pints of red paint

Bonus:

Pints of blue paint	**Pints of red paint**	**Total**
5	3	8
?	?	20

× 20/8 or × 5/2

12 1/2 pints of blue paint and 7 1/2 pints of red paint

Extensions

(MP.4) 1. When solving ratio problems that involve money, you can convert fractional answers to decimals through long division or equivalent fractions. For example, if 4 mangoes cost \$5, then 6 mangoes cost \$30/4. We want to write \$30/4 as a decimal.

Using long division:

$$\begin{array}{r} 7.5 \\ 4\overline{)30.0} \\ \underline{28} \\ 2\,0 \\ \underline{2\,0} \\ 0 \end{array}$$

So $\$\dfrac{30}{4} = \$7.50$

Using equivalent fractions:

$$\frac{30}{4} = 7\frac{2}{4} = 7\frac{1}{2} = 7\frac{5}{10} = 7.5$$

So $\$\dfrac{30}{4} = \$7.50$

(MP.2) It doesn't always make sense to convert fractions to decimals. For example, we commonly measure in fractions of a cup when baking. Solve the problem and express your answer to the problem as a decimal or a fraction, as appropriate.

a) If every 2 necklaces cost \$11, how much do 7 necklaces cost?

b) In a recipe, 4 cups of cranberry juice are mixed with 5 cups of ginger ale. How many cups of ginger ale should be mixed with 9 cups of cranberry juice?

c) In a recipe, 5 cups of raisins are mixed with 7 cups of nuts. How many cups of nuts should be mixed with 8 cups of raisins?

d) If every 5 pizzas cost \$16, how much do 8 pizzas cost?

Answers: a) \$38.50, b) 11 1/4 cups, c) 11 1/5 cups, d) \$25.60

2. When expressing minutes as a fraction of an hour, the denominator is 60, because there are 60 minutes in an hour. For example, 10 minutes is $\frac{10}{60}$ of an hour, which is reduced to $\frac{1}{6}$ of an hour.

a) Express the time as a fraction of an hour.

i) 12 minutes ii) 15 minutes iii) 20 minutes iv) 30 minutes

b) John skates $\frac{7}{2}$ km in 10 minutes and bikes $\frac{19}{4}$ km in 12 minutes.

i) Convert the minutes to hours.

ii) Find how far John can skate every hour and how far he can bike every hour.

iii) Does he skate or bike faster?

Answers: a) i) 1/5 of an hour, ii) 1/4 of an hour, iii) 1/3 of an hour, iv) 1/2 of an hour;
b) i) 10 minutes = 1/6, 12 minutes = 1/5; ii) he skates 21 km per hour, he bikes 23 3/4 km per hour; iii) he bikes faster

RP7-29 Unit Rates

Pages 41–43

Standards: 7.RP.A.1, 7.RP.A.2b

Goals:
Students will use unit rates and ratios involving fractions and decimals to solve problems in isolation and in context.

Prior Knowledge Required:
Knows that a unit rate is a rate with one term equal to one
Can use division to find equivalent ratios
Can convert improper fractions to mixed numbers
Can convert fractions to decimal fractions and write decimal fractions as decimals
Knows that a number divided by itself is one

Vocabulary: decimal fraction, equivalent ratios, rate, ratio, ratio table, unit rate

Review unit rates. Remind students that a rate is called a unit rate if one of the quantities is equal to 1. SAY: Unit rates are easy to work with because 1 is easy to multiply and divide by. Write on the board:

4 pens cost $8
1 pen costs _____

Ask a volunteer to fill in the blank. ($2) Then SAY: Because I know that I have to divide 4 by 4 to get 1, I know that I have to divide $8 by 4 to get how much one pen costs. Draw arrows as shown below:

4 pens cost $8
÷ 4
1 pen costs _____

SAY: "One pen costs $2" is a unit rate because the quantity of pens is 1, and we know the cost per pen.

Exercises: Use division to find the unit rate.

a) 3 pounds of rice for 12 cups of water
1 pound of rice for _____ cups of water

b) $128 for 8 hours
_____ for 1 hour

c) 385 miles in 7 hours
_____ miles in 1 hour

d) 5 T-shirts for $70
1 T-shirt for _____

Answers: a) 4, b) $16, c) 55, d) $14

Using equivalent ratios to find unit rates. SAY: Ethan paid $60 for 5 tickets. ASK: What ratio is this? (60 : 5) SAY: He wants to know how much he paid for 1 ticket. ASK: How do we write this information as equivalent ratios? Write on the board:

60 : 5 = ? : 1

ASK: How do we find the unit rate? (5 ÷ 5 = 1, so divide 60 ÷ 5) Draw arrows as shown below:

÷ 5
60 : 5 = ? : 1
÷ 5

ASK: How much does one ticket cost? ($12)

(MP.4) Exercises: Write the ratio from the description, find the unit rate, then answer the question in a full sentence.
a) 4 books cost $48. How much does each book cost?
b) A car travels 95 miles on 5 gallons of gas. How many miles does it travel on 1 gallon of gas?
c) A 420 g box of cereal has 15 servings. How many grams are in each serving?
d) 8 pears cost $8. How much does one pear cost?
Answers: a) 4 : 48 = 1 : 12, each book costs $12; b) 95 : 5 = 19 : 1, a car travels 19 miles per gallon of gas; c) 402 : 15 = 28 : 1, there are 28 g in one serving; d) 8 : 8 = 1 : 1, 1 pear costs $1

Comparing rates. SAY: Unit rates are also useful because they allow two rates to be easily compared. Suppose you have the choice of three different jobs with the following hourly pay rates. Write on the board:

A. $19 for 2 hours **B.** $31 for 3 hours **C.** $43 for 4 hours

Break the class into three groups and assign each group one of the jobs. Have students in each group individually calculate the hourly rate of that job as a mixed number. Have one member from each group write their unit rate on the board. (A: $9 1/2 : 1, $10 1/3 : 1, $10 3/4 : 1)

ASK: Which job has the lowest hourly rate? (job A) Cross out job A on the board. ASK: Jobs B and C both pay more than $10 per hour, but which one pays more? (job C) Why? (3/4 > 1/2 and 1/3 <1/2, so 3/4 > 1/3, or 1/3 = 4/12 and 3/4 = 9/12 and 9/12 > 4/12)

Exercises: Calculate the unit rate to compare the fees of the landscaping companies.
a) Which is cheaper, A: $144 for 3 hours or B: $250 for 5 hours?
b) Which is more expensive, C: $139 for 3 hours or D: $185 for 4 hours?
c) Which prices are equivalent: E: $105 for 2 hours, F: $260.25 for 5 hours, G: $420 for 8 hours?
Answers: a) A's rate is $48 : 1, B's rate is $50 : 1, so A is cheaper; b) C's rate is $46 1/3 : 1, D's rate is $46 1/4 : 1, so C is more expensive; c) E's rate is $52.50 : 1, F's rate is $52.05 : 1, G's rate is $52.50 : 1, so E and G are equivalent

Unit rates with fractional and decimal terms. Tell students that ratios and rates do not have to have whole-number terms. SAY: We often represent quantities and prices using fractions or decimals. For example, 1 drink costs $1.25, 22.3 miles per gallon, or 1/2 cup of butter for every cup of sugar. Write on the board:

4 pens cost $9
1 pen costs _____

Ask a volunteer to find the unit rate expressed as a fraction. (1 pen costs $9/4) ASK: What is $9/4 as a mixed number? ($2 1/4) SAY: Let's write this answer as a decimal, which is how we commonly write money. Review the steps for converting simple fractions to decimals. Write on the board:

$$\$2\frac{1}{4} = \$2\frac{25}{100} = \$2.25$$

Exercises: Divide to find the missing information. Write your answer as a mixed number, then convert the fraction to a decimal fraction and write your answer as a decimal.

a) 4 sandwiches cost $10
1 sandwich costs _____

b) 2 cinema tickets cost $19
1 cinema ticket costs _____

c) 5 pens cost $6
1 pen costs _____

d) 10 cards cost $29
1 card costs _____

Answers: a) $2.50, b) $9.50, c) $1.20, d) $2.90

Using ratio tables to find fractional unit rates. Draw on the board:

2	7
1	?

SAY: To find the missing number, first I find what the number is being divided by in the first column. ASK: What is that number? (2) SAY: Now we divide by that number (2) in the second column to find the missing number. Use the table to find the missing number. (7/2). The final picture should look like this:

	2	7	
÷2	1	$\frac{7}{2}$	÷2

ASK: What is the unit rate? (2 : 7 = 1 : 7/2)

SAY: The 1 can be in the first or second position of the unit ratio. When working with ratio tables, identify the position of the unit rate and find what the number is being divided by in that column.

Exercises: Use division to find the missing number in the unit rate.

a)

3	8
1	

So 3 : 8 = 1 : $\square$

b)

7	5
	1

So 7 : 5 = $\square$: 1

c)

4	9
	1

So 4 : 9 = $\square$: 1

d)

6	11
1	

So 6 : 11 = 1 : $\square$

Answers: a) 8/3, b) 7/5, c) 4/9, d) 11/6

Remind students that the arrows can go from bottom to top as well. Draw on the board:

÷ 4	1	?	÷ 4
	4	7	

ASK: What is the missing number? (7/4)

Exercises: Use division to find the missing number in the unit rate.

a)

	1
2	7

So $\square$: 1 = 2 : 7

b)

1	
5	3

So 1 : $\square$ = 5 : 3

c)

	1
6	7

So $\square$: 1 = 6 : 7

d)

1	
3	14

So 1 : $\square$ = 3 : 14

Answers: a) 2/7, b) 3/5, c) 6/7, d) 14/3

Finding fractional and decimal unit rates using equivalent ratios. Write on the board:

4 lemons for $5

ASK: What is the ratio of lemons : cost? (4 : 5) SAY: It would be useful to know how much one lemon costs. ASK: What is the equivalent unit rate for one lemon? (1 : 5/4) SAY: 4/5 : 1 is also an equivalent unit rate. ASK: What does this ratio mean? (4/5 lemons cost $1) Why is this unit rate not useful? (you can't buy 4/5 of a lemon) SAY: For this reason, we usually write unit rates for costs as per 1 item (so $5/4 per lemon), not per 1 dollar (or 4/5 lemons per $1). Write on the board:

$$4:5=1:\frac{5}{4}$$

ASK: How do we usually write money? (as a decimal) Remind students that ratios and rates can have decimal terms. Have a volunteer convert the fraction in the sample problem (5/4) to a decimal. (5/4 = 1 1/4 = 1 25/100 = 1.25) ASK: What is the unit rate as a decimal? (1 : 1.25) What is the cost of one lemon? ($1.25)

Exercises: Write the fraction to complete the unit rate. Convert the fraction to a decimal.

a) 4 apples cost \$1
   4 : 1 = 1 : $\square$

b) 12 minutes to answer 5 questions
   12 : 5 = $\square$: 1

c) 10 coconuts for \$15
   10 : 15 = 1 : $\square$

d) 3 cups of ice for 5 cups of juice
   3 : 5 = $\square$: 1

Answers: a) 1 : 0.25, b) 2.4 : 1, c) 1 : 1.5, d) 0.6 : 1

Unit rates with units of measurement. Write on the board:

3 miles pedaled in $\frac{1}{6}$ of an hour

ASK: What is the ratio? (3 mi : 1/6 hr) What would you multiply the rate by to make the fraction equal to 1? (6) Why? (6 is the denominator) What is the unit rate? (18 mi : 1 hr) Write on the board:

$$3 \text{ mi} \times 6 \ : \ \frac{1}{6} \text{hr} \times 6 = 18 \text{ mi} : 1 \text{ hr}$$

Exercises: Find the unit rate. Include the units in your work.

a) 6 mi every $\frac{1}{2}$ hr

b) 3 messages every $\frac{1}{4}$ minute

c) $\frac{1}{3}$ cup of oil for every 4 eggs

d) $\frac{1}{5}$ of a pizza for every 2 people

Answers:
a) 6 mi × 2 : 1/2 hr × 2 = 12 mi : 1 hr,
b) 3 messages × 4 : 1/4 min × 4 = 12 messages : 1 min,
c) 1/3 cup × 3 : 4 eggs × 3 = 1 cup : 12 eggs,
d) 1/5 pizza × 5 : 2 people × 5 = 1 pizza : 10 people

Extensions

(MP.4) 1. Calculate the unit rate to make comparisons. Write your answer as a mixed number or as a decimal, as needed.

a) Which is the higher paying job?
   Job A: \$75 for 6 hr
   Job B: \$186 for 15 hr

b) Which is the slower speed?
   Car A: 9 mi in $\frac{1}{5}$ hr
   Car B: 186 mi in 4 hr

c) Which is the cheaper price?
   Pencil A: \$17 for 20 pencils
   Pencil B: \$12 for 15 pencils

d) Which is the most fuel efficient car?
   Car A: 70 mi per $3\frac{1}{2}$ gallons
   Car B: 99 mi per 5 gallons
   Car C: 105 mi per $5\frac{1}{4}$ gallons

e) Which is the least expensive juice?
   Juice A: \$47 for 50 boxes
   Juice B: \$21 for 20 boxes
   Juice C: \$5 for 5 boxes

f) Which is the most powerful waterfall?

Iguazú Falls: 1,164 cubic meters of water in $\frac{2}{3}$ of a second

Victoria Falls: 816 cubic meters of water in $\frac{3}{4}$ of a second

Niagara Falls: 1,505 cubic meters of water in $\frac{5}{8}$ of a second

Answers:
a) Job A: \$12.50 per hr, Job B: \$12.40 per hr, so Job A pays higher;
b) Car A: 45 mph, Car B: 46.5 mph, so Car A is slower;
c) Pencil A: 0.85¢ per pencil, Pencil B: 0.80¢ per pencil, so Pencil B is cheaper;
d) Car A: 20 mi per gal, Car B: 19.8 mi per gal, Car C: 20 mi per gal, so Car B is the most fuel efficient;
e) Juice A: 0.94¢ per box, Juice B: \$1.05 per box, Juice C: \$1 per box, so Juice A is the least expensive;
f) Iguazú Falls: 1,746 cubic meters per second, Victoria Falls: 1,088 cubic meters per second, Niagara Falls: 2,408 cubic meters per second, so Niagara Falls is the most powerful

2. You can write unit rates with the unit in the first or second position. Show that these are equivalent ratios. Hint: Multiply by the denominator to write the ratios with whole numbers.

a) First position 1 : $\frac{4}{7}$ Second position $\frac{7}{4}$: 1

b) First position 1 : $\frac{6}{11}$ Second position $\frac{11}{6}$: 1

c) First position 1 : $\frac{3}{8}$ Second position $\frac{8}{3}$: 1

Answers: a) both ratios are equivalent to 7 : 4, b) both ratios are equivalent to 11 : 6, c) both ratios are equivalent to 8 : 3 **NOTE:** Another method is to multiply both terms of the ratio by the inverse of the fraction: a) $1\times\frac{7}{4}:\frac{4}{7}\times\frac{7}{4}=\frac{7}{4}:1$, b) $1\times\frac{11}{6}:\frac{6}{11}\times\frac{11}{6}=\frac{11}{6}:1$, c) $1\times\frac{8}{3}:\frac{3}{8}\times\frac{8}{3}=\frac{8}{3}:1$

RP7-30 Unit Rates and Complex Fractions

Pages 44–45

Standards: 7.RP.A.1, 7.RP.A.2b

Goals:
Students will use unit rates and ratios involving complex fractions to solve problems in isolation and in context.

Prior Knowledge Required:
Knows that a number divided by itself is one
Knows that a unit rate is a rate with one term equal to one
Can use division to find equivalent ratios
Can use the invert-and-multiply rule to divide by fractions

Vocabulary: complex fraction, equivalent ratios, invert-and-multiply rule, unit rate

Review finding unit rate by division. Write on the board:

$5 \div ? = 1$ $\qquad$ $7 \div ? = 1$ $\qquad$ $14 \div ? = 1$ $\qquad$ $2{,}954 \div ? = 1$

ASK: What numbers make these equations true? (5, 7, 14, 2,954) Why? (a number divided by itself is 1) Write on the board:

$$\begin{array}{l} 3 : 7 \\ 1 : \square \end{array}$$

SAY: These ratios are equivalent. ASK: How do we find the missing number? (3 ÷ 3 = 1, so you divide 7 by 3) Draw arrows as shown below:

$$\begin{array}{c} 3 : 7 \\ \xrightarrow{\div 3} 1 : \square \xleftarrow{\div 3} \end{array}$$

ASK: What is the missing number? (7/3) What is the unit rate? (1 : 7/3) Write on the board:

So $3 : 7 = 1 : \frac{7}{3}$

SAY: In the next example, we will find the unit rate with 1 in the second position. Write on the board:

$$\begin{array}{l} 3 : 7 \\ \square : 1 \end{array}$$

ASK: Now that 1 (the unit) is in the second position, how will this change the way you find the answer? (this time, you have to divide by 7) Draw on the board:

$$\begin{array}{c} 3 : 7 \\ \div 7 \quad \square : 1 \quad \div 7 \end{array}$$

ASK: What is the missing number? (3/7) What is the unit rate? (3/7 : 1) Write on the board:

So 3 : 7 = $\frac{3}{7}$: 1

SAY: The unit rate can have the 1 in the first or second position. Look carefully at the position of the 1 when you are deciding what to divide by to get the unit rate.

Exercises: Find the number to divide by to make 1 in each position. Write the equivalent ratio.

a) First position

$$\begin{array}{c} 5 : 6 \\ 1 : \square \end{array}$$

So 5 : 6 = 1 : ___

Second position

$$\begin{array}{c} 5 : 6 \\ \square : 1 \end{array}$$

So 5 : 6 = ___ : 1

b) First position

$$\begin{array}{c} 7 : 9 \\ 1 : \square \end{array}$$

So 7 : 9 = 1 : ___

Second position

$$\begin{array}{c} 7 : 9 \\ \square : 1 \end{array}$$

So 7 : 9 = ___ : 1

Answers: a) 6/5 and 5/6, b) 9/7 and 7/9

Using division to find the unit rate in ratios with fractional terms. Write on the board:

$$\frac{5}{5} = \qquad \frac{-4}{-4} = \qquad \frac{1.7}{1.7} = \qquad \frac{\frac{2}{3}}{\frac{2}{3}} =$$

ASK: What do these numbers equal? (1, 1, 1, 1) Why? (if you divide any number (except 0) by itself, the result is 1) SAY: This is true for all numbers except 0: negative numbers, decimal numbers, and fractions. The last example is what we call a *complex fraction*. A complex fraction is a fraction in which the numerator, denominator, or both, contain a fraction.

Write on the board the table below, but leave the numerator and denominator columns blank. Ask students to identify the numerator and the denominator. (answers are given in italics)

Complex fraction	Numerator	Denominator
$\frac{5}{\frac{2}{3}}$	*5*	$\frac{2}{3}$
$\frac{\frac{1}{4}}{3}$	$\frac{1}{4}$	*3*
$\frac{\frac{2}{5}}{\frac{3}{8}}$	$\frac{2}{5}$	$\frac{3}{8}$

Write on the board:

$$\frac{5}{6} : 7$$

$$1 : \square$$

ASK: What do we divide 5/6 by to make 1? (5/6) SAY: To make this an equivalent ratio, we also have to divide 7 by 5/6. This will give us a complex fraction. Complete the ratio on the board, as shown below:

$$\frac{5}{6} : 7 \quad \xrightarrow{\div \frac{5}{6}} \quad 1 : \frac{7}{\frac{5}{6}}$$

Exercises: Find the fraction you divide by to make 1 in the given position. Then complete the equivalent ratio using a complex fraction.

a) $\frac{2}{5} : 9 \xrightarrow{\div \frac{2}{5}} 1 : \square$ b) $4 : \frac{3}{7} \rightarrow \square : 1$ c) $\frac{7}{8} : 5 \rightarrow 1 : \square$ d) $8 : \frac{2}{9} \rightarrow \square : 1$

Answers: a) $\frac{9}{\frac{2}{5}}$, b) $\frac{4}{\frac{3}{7}}$, c) $\frac{5}{\frac{7}{8}}$, d) $\frac{8}{\frac{2}{9}}$

Write on the board:

$$\frac{7}{\frac{5}{6}} \qquad \frac{\frac{2}{9}}{5} \qquad \frac{\frac{3}{4}}{\frac{1}{7}} \qquad \frac{\frac{2}{5}}{\frac{3}{4}}$$

SAY: Complex fractions may look scary, but have no fear! We can simplify them by dividing, because a fraction is just another way of showing division. As a class, write the division statements associated with each example on the board. (see answers below)

$$\frac{7}{\frac{5}{6}} = 7 \div \frac{5}{6} \qquad \frac{\frac{2}{9}}{5} = \frac{2}{9} \div 5 \qquad \frac{\frac{3}{4}}{\frac{1}{7}} = \frac{3}{4} \div \frac{1}{7} \qquad \frac{\frac{2}{5}}{\frac{3}{4}} = \frac{2}{5} \div \frac{3}{4}$$

SAY: That looks better already, doesn't it? And you know how to use the invert-and-multiply rule to divide by fractions. As a class, finish evaluating the examples on the board, as shown below:

$$= 7 \times \frac{6}{5} = \frac{42}{5} \qquad = \frac{2}{9} \times \frac{1}{5} = \frac{2}{45} \qquad = \frac{3}{4} \times \frac{7}{1} = \frac{21}{4} \qquad = \frac{2}{5} \times \frac{4}{3} = \frac{8}{15}$$

Exercises: Write the division statement associated with the complex fraction. Then, evaluate the division statement.

a) $\dfrac{5}{\frac{2}{3}}$ b) $\dfrac{\frac{3}{5}}{9}$ c) $\dfrac{\frac{6}{11}}{\frac{1}{5}}$ d) $\dfrac{\frac{5}{7}}{\frac{6}{13}}$

Answers: a) 5 ÷ 2/3 = 5 × 3/2 = 15/2, b) 3/5 ÷ 9 = 3/5 × 1/9 = 3/45 or 1/15, c) 6/11 ÷ 1/5 = 6/11 × 5/1 = 30/11, d) 5/7 ÷ 6/13 = 5/7 × 13/6 = 65/42

Word problems with complex fractions. SAY: It takes me 1/3 of an hour to run 2 miles. ASK: How do we write this as a ratio? (1/3 : 2) How do we write this as a unit rate with the 1 in the first position? (divide by 1/3) Write on the board:

$$\frac{1}{3}:2=1:\frac{2}{\frac{1}{3}}$$

ASK: What division statement do we write for the complex fraction? (2 ÷ 1/3) How do we evaluate it? (2 × 3 = 6) Finish the example on the board, as shown below:

$$\frac{2}{\frac{1}{3}}=2\div\frac{1}{3}=2\times 3=6$$

ASK: What is the unit rate? (1 : 6) Write on the board:

$$\text{So } \frac{1}{3}:2=1:6$$

SAY: This unit rate shows how many miles I run at this pace in 1 hour. If we were to write the unit rate with the 1 in the second position, we would know how much time it would take me to run 1 mile.

Exercises:

1. Write a complex fraction to find the unit rate with the 1 in the given position.

a) $\frac{1}{4}$ hour to sing 5 songs

$\frac{1}{4}$: 5 = 1 : $\square$

b) $\frac{3}{4}$ cup of fruit sauce for 3 cheesecakes

$\frac{3}{4}$: 3 = $\square$: 1

c) $\frac{1}{3}$ hour to bike 6 miles

$\frac{1}{3}$: 6 = $\square$: 1

d) $\frac{3}{4}$ of a pizza for 6 people

$\frac{3}{4}$: 6 = 1 : $\square$

Answers: a) $\dfrac{5}{\frac{1}{4}}$, b) $\dfrac{\frac{3}{4}}{3}$, c) $\dfrac{\frac{1}{3}}{6}$, d) $\dfrac{6}{\frac{3}{4}}$

2. Evaluate the complex fractions in your answers to Exercise 1 above, and find the unit rate in whole numbers.
Answers: a) 5 ÷ 1/4 = 20, so 1 : 20, or 1 hour to sing 20 songs; b) 3/4 ÷ 3 = 1/4, so 1/4 : 1, or 1/4 cup of fruit sauce for 1 cheesecake; c) 1/3 ÷ 6 = 1/18, so 1/18 : 1, or 1/18 of an hour to bike 1 mile; d) 6 ÷ 3/4 = 8, so 1 : 8, or 1 pizza for 8 people

(MP.4) 3. Find the unit rate of grams of sodium per serving of food.
Hint: Write the unit rate with a complex fraction, then evaluate the complex fraction.
a) Bacon contains 2 g of sodium in $\frac{8}{9}$ of a serving.

b) Sweet potatoes contain $\frac{1}{8}$ g of sodium in 2 servings.

c) Potato chips contain $\frac{1}{4}$ g of sodium in $\frac{1}{2}$ a serving.

Bonus: There is $\frac{3}{4}$ g of sodium in $\frac{2}{3}$ of a serving of soy sauce. What food has more sodium than soy sauce?
Answers: a) 9/4 g per serving, b) 1/16 g per serving, c) 1/2 g per serving, Bonus: soy sauce has 9/8 g per serving; bacon has more sodium than this

(MP.1) Show students that complex fractions are another way of finding the unit rate when the ratio has fractional terms. Give this example: Nancy walks 3/4 of a mile in 1/3 of an hour.
SAY: To find how far she walks per hour, we solve this equivalent ratio. Write on the board:

$$\frac{3}{4}:\frac{1}{3}=\square:1$$

Method 1: SAY: You can multiply by the LCM to find the unit rate. Write on the board:

$$\frac{3}{4}\times 12:\frac{1}{3}\times 12=9:4$$

The unit rate is $\frac{9}{4}:1$.

Method 2: SAY: Or, you can use complex fractions to find the unit rate. Write on the board:

$$\frac{3}{4}:\frac{1}{3}=\frac{\frac{3}{4}}{\frac{1}{3}}:1$$

$$\frac{\frac{3}{4}}{\frac{1}{3}}=\frac{3}{4}\div\frac{1}{3}=\frac{3}{4}\times\frac{3}{1}=\frac{9}{4}$$

The unit rate is $\frac{9}{4}:1$

SAY: In both cases, we get the same answer: Nancy walks 9/4, or 2 1/4 miles per hour.

Extensions

1. Evaluate the complex fraction. Hint: Change the mixed numbers to improper fractions first.

a) $\dfrac{6\frac{1}{3}}{9}$ b) $\dfrac{5}{3\frac{2}{7}}$ c) $\dfrac{3\frac{2}{5}}{5\frac{3}{4}}$ d) $\dfrac{2\frac{3}{5}}{7\frac{1}{3}}$ **Bonus:** $\dfrac{\frac{\frac{1}{2}}{\frac{1}{3}}}{\frac{\frac{1}{5}}{\frac{1}{4}}}$

Answers: a) 19/27, b) 35/23, c) 68/115, d) 39/110, Bonus: 15/8

(MP.4) 2. A unit rate can be expressed as a fraction with a denominator of 1 unit. For example, we can write "400 lbs of grass seed per acre" as $\dfrac{\text{400 lb}}{\text{acre}}$.

a) Express the unit rate as a fraction.
 i) 15.8 miles per gallon
 ii) \$10 per hour
 iii) 40 words per minute
 iv) 55 miles per hour

b) Write the unit rate in fraction form.
 i) Bamboo grows 4 feet in 24 hours.
 ii) 125 square feet can be covered by $\frac{1}{2}$ gallon of paint
 iii) A small worm travels $6\frac{1}{2}$ feet in $\frac{1}{4}$ of an hour
 iv) A shower uses $10\frac{1}{2}$ gallons of water in 5 minutes.
 Bonus: 10 pounds of potatoes make $3\frac{1}{2}$ pounds of potato chips

c) Energy is measured in kilowatt hours (kWh). Electricity costs $\frac{1}{2}$¢ for $\frac{1}{24}$ kWh in the US and 5¢ for $\frac{1}{4}$ kWh in the UK. In which country is electricity more expensive?

Answers: a) i) 15.8 miles/gallon, ii) \$10/hour, iii) 40 words/minute, iv) 55 miles/hour;
b) i) $\dfrac{\frac{1}{6}\text{ foot}}{\text{hour}}$, ii) 250 sq feet/gallon, iii) 26 feet/hour, iv) $\dfrac{2\frac{1}{10}\text{ gallons}}{\text{minute}}$, Bonus: $\dfrac{2\frac{6}{7}\text{ lbs potatoes}}{\text{lb potato chips}}$;
c) US: $\dfrac{12¢}{\text{kWh}}$ and UK: $\dfrac{20¢}{\text{kWh}}$, so the UK has more expensive electricity

RP7-31 Using Proportions to Solve Percent Problems I

Pages 46–48

Standards: 7.RP.A.3

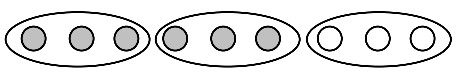
Goals:
Students will write equivalent statements for proportions by keeping track of the part and the whole, and by solving easy proportions.

Prior Knowledge Required:
Can write equivalent ratios
Can name a ratio from a picture

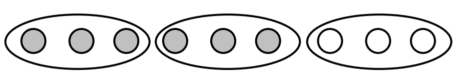
Vocabulary: comparison fraction, equivalent ratios, multiplier, part-to-whole ratio, percent, proportion, ratio

Materials:
BLM Three Types of Percent Problems (p. N-79)

Using pictures to review equivalent ratios. Draw on the board:



Have students brainstorm ways of interpreting this picture. SAY: The picture shows four equivalent statements. Write on the board:



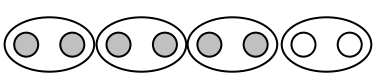
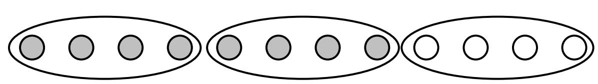
$\frac{6}{9}$ of the circles are shaded.

$\frac{2}{3}$ of the circles are shaded.

6 is $\frac{2}{3}$ of 9

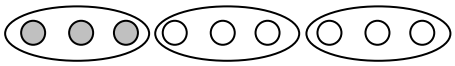
6 : 9 = 2 : 3

Exercises: Write four equivalent statements for the picture.

a)  b) 

Answers: a) 6/8 are shaded, 3/4 are shaded, 6 is 3/4 of 8, 6 : 8 = 3 : 4;
b) 8/12 are shaded, 2/3 are shaded, 8 is 2/3 of 12, 8 : 12 = 2 : 3

Writing part-to-whole as a ratio. Tell students that they can write different part-to-whole ratios from the same picture. Draw on the board:



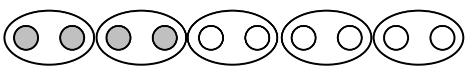
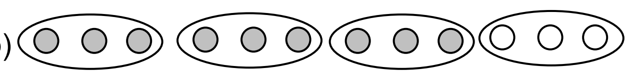
ASK: How many circles are there? (9) How many are shaded? (3) Write on the board:

part : whole = 3 : 9

ASK: How many groups are there? (3) How many groups are shaded? (1) Write on the board:

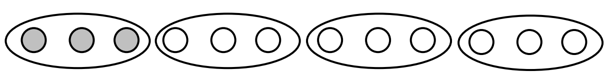
part : whole = 1 : 3

Exercises: Write a pair of equivalent ratios for the picture.

a)  b) 

Answers: a) part : whole = 4 : 10 = 2 : 5, b) part : whole = 9 : 12 = 3 : 4.

Writing part-to-whole as a fraction. Tell students that they can write different fractions of the form $\frac{\text{part}}{\text{whole}}$ from the same picture. Draw on the board:



ASK: How many circles are there? (12) How many are shaded? (3) Write on the board:

$$\frac{\text{part}}{\text{whole}} = \frac{3}{12}$$

Point to the fraction 3/12 and SAY: The comparison fraction is 3/12. ASK: How many groups are there? (4) How many groups are shaded? (1) Write on the board:

$$\frac{\text{part}}{\text{whole}} = \frac{1}{4}$$

Point to the fraction 1/4 and SAY: The comparison fraction is 1/4.

Exercises:
1. Write a pair of equivalent fractions for each picture from the previous set of exercises.
Answers: a) part/whole = 4/10 = 2/5, b) part/whole = 9/12 = 3/4

2. Determine the part, the whole, and the comparison fraction. Write an equivalent fraction.

a) 3 is $\frac{1}{4}$ of 12 b) 4 is $\frac{2}{3}$ of 6 c) 6 is $\frac{3}{5}$ of 10

Answers: a) part = 3, whole = 12, comparison fraction = 1/4, part/whole = 3/12 = 1/4; b) part = 4, whole = 6, comparison fraction = 2/3, part/whole = 4/6 = 2/3; c) part = 6, whole = 10, comparison fraction = 3/5, part/whole = 6/10 = 3/5

Writing ratios with missing parts. Tell students that in the previous exercises, all four numbers in the equivalent ratios or fractions were given. Usually in questions like those, one number (out of four) is missing. Explain to students that to write a proportion, they have to determine the part, the whole, and what fraction or ratio of the whole the part is, then write them in the correct places. Write on the board:

8 is $\frac{2}{3}$ of what number?

SAY: In this question, the part is 8 and the whole is missing. ASK: What is the comparison fraction? (2/3) What is the ratio of part-to-whole? (2 : 3) Write on the board:

8 : ? = 2 : 3, $\frac{\text{part}}{\text{whole}} = \frac{8}{?} = \frac{2}{3}$

Emphasize that writing each number in the correct place is very important, because writing a number in the wrong place leads to a wrong answer.

Exercises: Determine the part, the whole, and the comparison fraction. Write the proportion, but replace the missing number with a question mark.

a) 3 is $\frac{1}{2}$ of what number? b) 4 is $\frac{1}{3}$ of what number?

c) 6 is $\frac{2}{5}$ of what number? d) What number is $\frac{3}{4}$ of 20?

e) What number is $\frac{4}{5}$ of 20? f) What number is $\frac{2}{7}$ of 21?

Answers: a) part = 3, whole = ?, comparison fraction = 1/2, so 1/2 = 3/?; b) part = 4, whole = ?, comparison fraction = 1/3, so 1/3 = 4/?; c) part = 6, whole = ?, comparison fraction = 2/5, so 2/5 = 6/?; d) part = ?, whole = 20, comparison fraction = 3/4, so 3/4 = ?/20; e) part = ?, whole = 20, comparison fraction = 4/5, so 4/5 = ?/20; f) part = ?, whole = 21, comparison fraction = 2/7, so 2/7 = ?/21

Changing a verbal proportion problem into a known problem. Write on the board:

12 is how many fifths of 30?

Underline "how many fifths" and point out that this is the same as "?/5." SAY: The denominator tells you that the size of the parts is a fifth, and the numerator—the unknown—tells you the number of fifths. So “12 is how many fifths of 30” is another way of saying “12 is ?/5 of 30.” This is now easy to change to an equivalent ratio. Write on the board:

? : 5 = 12 : 30

Exercises: Write an equivalent ratio for the question. Then write the fraction form.
a) 8 is how many thirds of 12?
b) 21 is how many quarters of 28?
c) 18 is how many tenths of 30?
Answers: a) ? : 3 = 8 : 12, ?/3 = 8/12; b) ? : 4 = 21 : 28, ?/4 = 21/28; c) ? : 10 = 18 : 30, ?/10 = 18/30

Writing percent statements in terms of ratios. Remind students that asking how many hundredths is like asking for “?/100.” ASK: What is another name for a fraction with denominator 100? PROMPT: What do we use to compare numbers to 100? (a percent) Since students can write fraction statements as equivalent ratios, and a percent is just a fraction with denominator 100, students can now write percent statements as equivalent ratios.

Exercises: Write the proportion (without solving it). Then write the proportion in terms of fractions. Replace the missing number with a question mark.
a) 19 is how many hundredths of 20?
b) 13 is how many hundredths of 50?
c) 36 is how many hundredths of 60?
Answers: a) ? : 100 = 19 : 20, ?/100 = 19/20; b) ? : 100 = 13 : 50, ?/100 = 13/50; c) ? : 100 = 36 : 60, ?/100 = 36/60

Remind students that a percent is a hundredth, so asking what is 15% of 40 is asking what is 15 hundredths of 40. If they know how to find a fraction of a whole number, then they know how to find a percent of a whole number. In questions in which the percent is unknown, students can write a comparison fraction with denominator 100 and a question mark in the numerator.

Exercises: Write the question as a proportion, in ratio form and in fraction form.
a) What is 15% of 40?
b) What is 32% of 50?
c) What is 75% of 48?
d) 24 is 80% of what number?
e) 62 is 25% of what number?
f) 12 is 30% of what number?
g) What percent of 20 is 19?
h) What percent of 24 is 6?
Answers: a) ? : 40 = 15 : 100, or ?/40 = 15/100; b) ? : 50 = 32 : 100, or ?/50 = 32/100; c)? : 48 = 75 : 100, or ?/48 = 75/100; d) 24 : ? = 80 : 100, or 24/? = 80/100; e) 62 : ? = 25 : 100, or 62/? = 25/100; f) 12 : ? = 30 : 100, or 12/? = 30/100; g) 19 : 20 = ? : 100, or 19/20 = ?/100; h) 6 : 24 = ? : 100, or 6/24 = ?/100

Distribute **BLM Three Types of Percent Problems**. All three types of questions from the exercises above are summarized on the BLM. Students can use BLM Three Types of Percent Problems as a reference to help them solve the remaining exercises in this lesson.

Solving proportions. Show students how to solve proportions using equivalent ratios. Use this problem: If 4 bus tickets cost $9, how much would 12 tickets cost?

Step 1: Make the proportion. Write a fraction on the board, the top of which is the unknown quantity (in this example, Dollars) and the bottom of which is the other quantity (in this example, Tickets). Write on the board the complete ratio of dollars to bus tickets (9 : 4) in fraction form, then write the incomplete ratio of dollars to bus tickets (? : 12) in fraction form, as shown below:

$$\frac{\text{Dollars}}{\text{Tickets}} = \frac{9}{4} = \frac{?}{12}$$

Step 2: Find the multiplier. Find the number the first denominator is being multiplied by to get the second denominator (in this example, 3). Write on the board:

$$\begin{array}{ccc} 9 & \longrightarrow & ? \\ \hline & = & \\ 4 & \xrightarrow[\times 3]{} & 12 \end{array}$$

Step 3: Find the missing number. Multiply the numerator by that multiplier to find the missing number, as shown below:

$$\begin{array}{ccc} 9 & \xrightarrow{\times 3} & 27 \\ \hline & = & \\ 4 & \xrightarrow[\times 3]{} & 12 \end{array}$$

SAY: Since 9/4 = 27/12, 12 tickets cost $27.

Have volunteers complete the first few exercises below, then have students answer the rest on their own.

Exercises:
a) If 3 bus tickets cost $4, how much will 15 bus tickets cost?
b) Five bus tickets cost $6. How many can you buy with $30?
c) On a map, 3 cm represents 10 km. How many kilometers do 15 cm represent?
d) Milly gets paid $25 for 3 hours of work. How much would she get paid for working 6 hours?
e) Three centimeters on a map represents 20 km in real life. If a lake is 6 cm long on the map, what is its actual length?
f) There are 2 apples in a bowl for every 3 oranges. If there are 12 oranges, how many apples are there?
Bonus: A goalie stopped 18 out of every 19 shots. There were 38 shots. How many goals were scored? Hint: How many did she not stop?
Answers: a) $20, b) 25, c) 50 km, d) $50, e) 40 km, f) 8, Bonus: 2

Extensions
1. Determine decimals as the value of a percent.
a) What percent of 30 is 16.5? b) What percent of 18 is 2.7? c) What percent of 14 is 2.8?
Answers: a) 55%, b) 15%, c) 20%

2. Give word problems involving decimals as the value of a percent.
a) A book that costs $18 came to $20.70 after taxes.
   i) How much were the taxes?
   ii) What percent is the tax?
b) The regular price of a book is $18. The sale price is $12.60.
   i) How much was taken off the regular price?
   ii) What percent was taken off the regular price?
Answers: a) i) $2.70, ii) 15%; b) i) $5.40, ii) 30%

RP7-32 Using Proportions to Solve Percent Problems II

Pages 49–50

Standards: 7.RP.A.3

Goals:
Students will write equivalent statements for proportions by keeping track of the part and the whole, and by solving the proportions.

Prior Knowledge Required:
Can write equivalent ratios
Can solve proportions

Vocabulary: equivalent ratios, lowest terms, multiplier, percent, proportion, ratio

Materials:
calculators
BLM Three Types of Percent Problems (p. N-79)

NOTE: Students can use **BLM Three Types of Percent Problems** as a reference to help them solve the exercises in this lesson.

Review percent proportions in terms of fractions. Remind students that to write a proportion, they have to determine the part, the whole, and what fraction of the whole is the part.
SAY: Suppose that we want to find what percent of 25 is 7. ASK: What is the whole in this question? (25) What is the part? (7) ASK: What is the part-to-whole fraction? (7/25) SAY: There is another way of writing the part-to-whole, which is the missing percent/100 or ?/100, so we can equate the two part-to-whole ratios. Write on the board:

$$\frac{\text{part}}{\text{whole}} = \frac{7}{25} = \frac{?}{100}$$

Emphasize that writing each number in the correct place is very important, because writing a number in the wrong place leads to a wrong answer.

Remind students that writing part-to-whole ratios is the first step of solving proportions. In the second step, they have to find the relation between two given numerators or denominators. In this example, students have to find the number the first denominator is being multiplied by to get the second denominator. Write on the board:

$$\frac{7}{25} \overset{\longrightarrow}{\underset{\xrightarrow[\times 4]{}}{=}} \frac{?}{100}$$

In the third step, students have to multiply the numerator by that multiplier to find the missing number, as shown below:

$$\frac{7}{25} \underset{\times 4}{\overset{\times 4}{=}} \frac{28}{100}$$

Exercises: Write the proportion in fraction form, then solve the proportion.

a) What percent of 20 is 9?

b) What is 50% of 50?

c) 9 is what percent of 25?

d) 13 is 26% of what number?

Answers: a) 9/20 = ?/100, so 9 is 45% of 20; b) ?/50 = 50/100, so 25 is 50% of 50; c) 9/25 = ?/100, so 9 is 36% of 25; d) 13/? = 26/100, so 13 is 26% of 50

(MP.1) Solving proportions that need simplifying. Write on the board:

$$\frac{9}{20} = \frac{?}{100}$$

Explain to students this proportion is easy to solve because the relationship between the two denominators is obvious. SAY: You can find the second denominator by multiplying the first denominator by 5. You find the multiplier by dividing 100 by 20. Write on the board:

$$\frac{7}{35} = \frac{?}{100}$$

SAY: In this proportion, the relation between two denominators is not clear. Ask students to use their calculators to divide 100 by 35 to find the multiplier. ASK: Is the answer a well-known decimal? (no) SAY: Don't give up! Try to reduce 7/35 to lowest terms. Write on the board:

$$\frac{7}{35} = \frac{1}{5}$$

SAY: Replace 7/35 by 1/5. Write on the board:

$$\frac{1}{5} = \frac{?}{100}$$

ASK: Is this proportion easy to solve? (yes) Ask a volunteer to solve the proportion as shown below:

$$\frac{1}{5} \underset{\times 20}{\overset{\times 20}{=}} \frac{20}{100}, \text{ so } \frac{7}{35} = \frac{20}{100}$$

Exercises: Find an equivalent ratio to rewrite the proportion. Solve the new proportion.

a) $\frac{?}{100}=\frac{11}{22}$ b) $\frac{6}{24}=\frac{?}{100}$ c) $\frac{12}{?}=\frac{40}{100}$

d) $\frac{?}{100}=\frac{24}{32}$ e) $\frac{?}{48}=\frac{75}{100}$ f) $\frac{12}{60}=\frac{?}{100}$

Answers: a) ?/100 = 1/2, so ? = 50; b) 1/4 = ?/100, so ? = 25; c) 12/? = 2/5, so ? = 30; d) ?/100 = 3/4, so ? = 75; e) ?/48 = 3/4, so ? = 36; f) 1/5 = ?/100, so ? = 20

Word problem practice.

Exercise: Use your answer to each problem to obtain the answer to the next problem. Discuss the similarities and differences between the problems.

a) 12 is how many fifths of 30?
b) How many fifths of 30 is 12?
c) 12 is what percent of 30?
d) What percent of 30 is 12?
e) A shirt costs $30, and $12 was taken off. What percent was taken off?

Answers: a) 2, b) 2, c) 40, d) 40, e) 40

Selected solutions: a) 12/30 = ?/5, so ? = 2; b) ?/5 = 12/30, so ? = 2; c) 12/30 = ?/100, so ? = 40. The difference between a) and b) is just the order of fractions, but in c) the question asks for percentage so the denominator is 100.

Finding the whole from the part. Write on the board:

$\frac{2}{3}$ of a number is 100. What is the number?

ASK: Is 100 the part or the whole? (the part) What is the whole? (the number that we don't know) Tell students that this is a part-to-whole ratio. Write on the board:

$$\frac{2}{3}=\frac{100}{?}=\frac{\text{part}}{\text{whole}}$$

Have students solve the proportion. (? = 150)

Exercises: Write the proportion, then find the number.

a) $\frac{3}{4}$ of a number is 9

b) $\frac{4}{9}$ of a number is 24

c) $\frac{7}{13}$ of a number is 21

Answers: a) 3/4 = 9/?, so the number is 12; b) 4/9 = 24/?, so the number is 54; c) 7/13 = 21/?, so the number is 39

More word problem practice. Write on the board:

$\frac{2}{3}$ of the beads in a box are red.
100 beads are red.
How many beads are in the box?

NOTE: This is the same problem as before, but written a little differently. Use the following sentences and prompts to demonstrate this.

Write on the board:

$\frac{2}{3}$ of the number of beads in the box is <u>the number of red beads in the box</u>.

ASK: What is the number of red beads in the box? (100) Continue writing on the board:

$\frac{2}{3}$ of the number of beads in the box is 100.

Now underline part of that sentence, as shown below:

$\frac{2}{3}$ of <u>the number of beads in the box</u> is 100.

Tell students this underlined part is what we want to know. Continue writing on the board:

$\frac{2}{3}$ of what number is 100?

This is exactly the problem we solved earlier, so the number is 150.

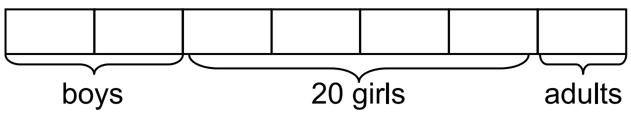
Exercises: A box holds red and blue beads. Find the total number of beads in the box.
a) $\frac{2}{3}$ of the beads are red. 8 beads are red.
b) $\frac{4}{9}$ of the beads are red. 24 beads are red.
c) $\frac{7}{13}$ of the beads are red. 21 beads are red.
Answers: a) 2/3 = 8/?, so the total number of beads is 12; b) 4/9 = 24/?, so the total number of beads is 54; c) 7/13 = 21/?, so the total number of beads is 39

Have students compare parts b) and c) of the exercises above with parts b) and c) of the previous set of exercises.

Solving problems with three groups instead of two. Write on the board:

> $\frac{2}{7}$ of the people (boys, girls, and adults) at the park are boys, and $\frac{1}{7}$ are adults.
> There are 20 girls at the park. How many people are at the park?

ASK: What fraction of the people are boys or adult? (2/7 + 1/7 = 3/7) ASK: What fraction of the people are girls? (4/7) Challenge students to find a model to help solve the problem. One possible model could be to draw a bar divided into sevenths, then add the given information to it, as shown below:



Have students use this model, or their own, to solve the problem. SAY: From the picture, it is clear that 4/7 of the total number of people is 20. ASK: 4/7 of what number is 20? (35) Ask a volunteer to write a proportion and solve it to find the number of people at the park, as shown below.

$\frac{4}{7} = \frac{20}{?}$, so ? = 35

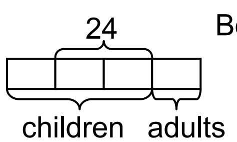
Exercises: Draw a model, then solve the problem.

a) 75% of the people at a park are children. There are 24 more children than adults at the park. How many people are at the park? Hint: There are 50% more children than adults at the park.

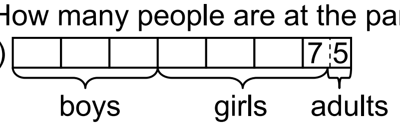
Bonus:

b) There are 5 adults at a park. There are 7 more girls than boys at a park. $\frac{3}{7}$ of the people at the park are boys. How many people are at the park?

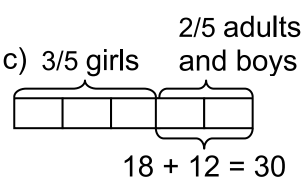
c) There are 18 boys at a park. $\frac{3}{5}$ of the people (boys, girls, and adults) are girls. There are 6 fewer adults than boys. How many people are at the park?

Solutions: a)


48 people at the park

Bonus: b)


84 people at the park

c)


75 people at the park

Solving problems with other contexts.

Exercises: Draw a model, then solve the problem.

a) Ken and Liz pay for a meal. Ken pays \$12. Liz pays $\frac{3}{5}$ of the total cost. How much did they pay altogether?

b) In a parking lot, $\frac{3}{4}$ of the vehicles are cars, $\frac{1}{5}$ are trucks, and the rest are buses. There are 4 buses. How many vehicles are in the parking lot?

c) Jake has a rock collection. He found $\frac{2}{5}$ of his rocks in Arizona, $\frac{1}{3}$ in California, and the rest in Alaska. He found 8 more rocks in California than he did in Alaska.

i) What fraction of his rocks did he find in Alaska?

ii) What fraction of the total number of rocks do the 8 rocks represent? Hint: 8 is the difference between the number found in California and the number found in Alaska.

iii) How many rocks does Jake have altogether?

d) At a school, $\frac{3}{7}$ of the people are boys, $\frac{2}{5}$ are girls, and the rest are adults. There are 6 more students of one gender than the other.

i) Are there more boys or more girls?

ii) What fraction of the total number of people do the 6 extra students of one gender represent?

iii) How many people are there in the school altogether?

Answers: a) \$30; b) 80; c) i) 4/15, ii) 1/15, iii) 15 × 8 = 120; d) i) 3/7 = 15/35 and 2/5 = 14/35, so there are more boys, ii) 3/7 − 2/5 = 15/35 − 14/35 = 1/35, iii) 6 × 35 = 210

Extensions

(MP.1, MP.7) 1. Find the number.

a) $\frac{3}{5}$ of $\frac{4}{5}$ of a number is 60. What is the number?

b) $\frac{2}{3}$ of $\frac{3}{4}$ of a number is 36. What is the number?

c) $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of a number is 18. What is the number?

NOTE: Some students might find a shortcut way to solve these problems. For example, 2/3 of 3/4 is 2/4 or 1/2, so the question is really asking 1/2 of a number is 36. What is the number?

Solutions: a) Solve in two steps: if 3/5 of something is 60, then the something is 100, so 4/5 of a number is 100, this means the number is 125; b) 72; c) Solve in three steps: if 2/3 of (3/4 of 4/5 of a number) is 18, then the part in brackets is 27; 3/4 of 4/5 of a number is 27, so in the bracket is 36, so 4/5 of a number is 36 and the number is 45

2. a) Two thirds of Helen's age is half of David's age. David is 10 years older than Helen. How old is Helen?

b) Tim's age is two thirds of Sara's age. Sara's age is three fifths of Mark's age. Mark is 9 years older than Tim. How old is Sara?

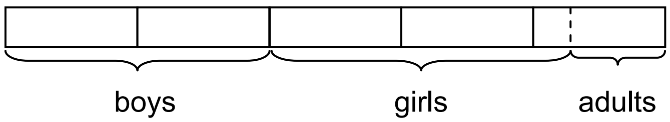
Answers: a) 30, b) 9

3. Hanna emptied her piggy bank, which only contained pennies. The contents weigh about 1 kg 300 g in total. If each penny weighs $2\frac{1}{3}$ g, about how much money was in the piggy bank? Round your answer to the nearest half dollar.

Answer: 1,300 ÷ 2 1/3 = 1,300 ÷ 7/3 = 1,300 × 3 ÷ 7 ≈ 557 pennies = \$5.57, so there is about \$5.50 **NOTE:** To divide by a mixed number, it is essential to change the mixed number to an improper fraction. We cannot write 1,300 ÷ (2 1/3) = 1,300 ÷ (2 + 1/3) = 1,300 ÷ 2 + 1,300 ÷ 1/3, because the distributive law does not apply here. If you try to use the distribute law, you should see right away that doing so gives an answer that can't be correct: 1,300 ÷ 2 is larger than what you started with, 1,300 ÷ (2 1/3), because 2 is less than 2 1/3 and dividing by a smaller number gives a larger result.

4. $\frac{2}{5}$ of the people (boys, girls, adults) at the park are boys. There are 3 more girls than boys. There are 7 adults. How many people are at the park?

Solution:



From the model, it is clear that 1/5 of the total number of people is 10. 1/5 of 50 is 10, so there are 50 people at the park.

RP7-33 Solving Equations (Introduction)

Pages 51–53

Standards: preparation for 7.EE.B.4

Goals:
Students will use the balance model to solve addition and multiplication equations including negative addends and coefficients.

Prior Knowledge Required:
Is familiar with balances
Can solve a simple equation to find an unknown value
Can substitute numbers for unknowns in an expression
Can check whether a number solves an equation

Vocabulary: balance, equation, expression, integer, pan balance, quotient, **sides** (of an equation), variable

Materials:
pan balance
apples, cubes, or other small objects for demonstration and for each pair of students to use with a pan balance
paper bags, one per pair of students and some for display
masking tape
40 connecting cubes for demonstrations

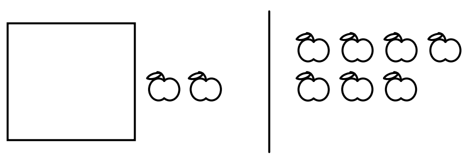
NOTE: You will need a number of identical objects for demonstrations throughout this lesson. The objects you use should be significantly heavier than a paper bag, so that the presence of a paper bag on one of the pans of the balance does not skew the pans. Apples are used in the lesson plan below (to match the pictures in the AP Book), but other objects, like small fruit of equal size, metal spoons, golf balls, tennis balls, or cereal bars, will work well. If a pan balance is not available, refer to a concrete model, such as a seesaw, to explain how a pan balance works, and use pictures or other concrete models during the lesson.

Review pan balances. Show students a pan balance. Place the same number of identical (or nearly identical) apples on both pans, and show that the pans balance. Remind students that when the pans, or scales, are balanced, this means there is the same number of apples on each pan.

Removing the same number of apples from both pans keeps them balanced. Place some apples in a paper bag and place it on one pan, then add some apples beside the bag. Place the same total number of apples on the other pan. ASK: Are the pans balanced? (yes) What does this mean? (the same number of apples are on each pan) Take one apple off each pan. ASK: Are the pans still balanced? Repeat with two apples. Remove the same number of apples from each pan until one pan has only the bag with apples on it. ASK: Are the pans balanced? Can you tell how many apples are in the bag? Show students the contents of the bag to check their answer. Repeat the exercise with a different number of apples in the bag.

Solving addition equations given by a balance model. Make a line on a desk with a piece of masking tape and explain that the parts on either side of the line will be the pans. Ask students to imagine that the pans are balanced. As you did before, place a paper bag with apples in it along with some other apples on one side of the line, and place the same number of apples (altogether) on the other side of the line. Ask students how many apples need to be removed from both sides of the balance to find out how many apples are in the bag. Students can signal their answer. Remove the apples, then ask students to tell how many apples are in the bag. Show the contents of the bag to check the answer. Repeat with a different number of apples.

Writing equations from scales. Remind students that we often use variables to represent numbers we do not know. Place a paper bag containing 5 apples and 2 more apples on one side of the line, and place 7 apples on the other side of the line. Ask students to write an expression for the number of apples on the side with the paper bag, as shown below. Explain that an equation is like a pair of balanced pans (or scales), and the equal sign shows that the number of apples in each pan is the same. Remind students that the parts of the equation on either side of the equal sign are called the *sides* of the equation. Each pan of the balance becomes a side in the equation and the “balance” on the desk becomes $x + 2 = 7$. Draw the on the board:



$\longrightarrow$ $x + 2 = 7$

The “pans” balance each other. $\longrightarrow$ The numbers on both sides are equal.

Create more models and have students write the equation for each one. After you have done a few models that follow this pattern, start placing the bag on different sides of the line, so that students have to write the expressions with unknown numbers on different sides of the equation.

Solving addition equations using the balance model. Return to the model that corresponds to the equation $x + 2 = 7$. ASK: What do you need to do to find out how many apples are in the bag? (remove two apples from each side) Invite a volunteer to remove the apples, then have students write the old equation and the new equation ($x = 5$), one below the other. Repeat with a few different examples.

ASK: What mathematical operation describes taking the apples away? (subtraction) Write the subtraction vertically for the equation above, as shown below.

$$\begin{array}{rcr} x + 2 & = & 7 \\ \underline{\quad -2} & & \underline{-2} \end{array}$$

ASK: How many apples are left on the right side of the equation? (5) What letter did we use to represent how many apples are in the bag? (x) Remind students that we write this as "$x = 5$."

Solving addition equations without using the balance model. Present a few equations without a corresponding model. Have students signal how many apples need to be subtracted from both sides of the equation, then write the vertical subtraction for both sides.

Exercises: Solve the equation.

a) $x + 5 = 9$ b) $n + 17 = 23$ c) $14 + n = 17$ d) $p + 15 = 21$

Answers: a) $x = 4$, b) $n = 6$, c) $n = 3$, d) $p = 6$

Students who have trouble deciding how many apples to subtract without the model can complete the following problems.

Exercises: Write the missing number. Part a) has been done for you.

a) $\begin{array}{r} x + 15 \\ -\boxed{15} \\ \hline x \end{array}$ b) $\begin{array}{r} x + 55 \\ -\boxed{} \\ \hline x \end{array}$ c) $\begin{array}{r} x + 91 \\ -\boxed{} \\ \hline x \end{array}$ **Bonus:** $\begin{array}{r} x + 38 \\ -\boxed{} \\ \hline 38 \end{array}$

Answers: b) 55, c) 91, Bonus: x

Finally, give students a few more equations and have them work through the whole process of subtracting the same number from both sides to find the unknown number.

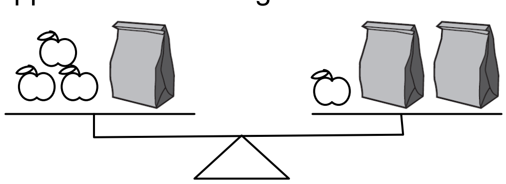
Exercises: Solve the equation by subtracting the same number from both sides to find the unknown number.

a) $x + 5 = 14$ b) $x + 9 = 21$ c) $2 + x = 35$ d) $x + 28 = 54$

Sample solution:

a) $\begin{array}{rcl} x + 5 & = & 14 \\ \underline{-5} & & \underline{-5} \\ x & = & 9 \end{array}$

Bonus: The scale below is balanced. Each bag has the same number of apples in it. How many apples are in the bag? Hint: You can cross out whole bags too!



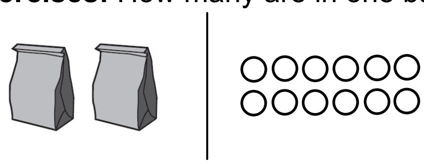
Answers: b) 12, c) 33, d) 26, Bonus: 2

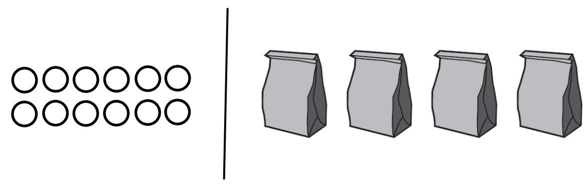
Solving multiplication equations given by a model. Divide a desk into two parts using masking tape and place three bags (with 4 cubes in each) on one side of the line, and 12 separate cubes on the other side of the line. Tell students that the "pans" are balanced. ASK: What does this say about the number of cubes in both pans? (they are equal) How many cubes are on the pan without the bags? (12) How many cubes are in the bags in total? (12) How many cubes are in each bag? (4) How do you know? (divide 12 into 3 equal groups, 12 ÷ 3 = 4) Invite a volunteer to group the 12 cubes into 3 equal groups to check the answer. Show students the contents of the bags to confirm the answer.

Repeat the exercise with 4 bags and 20 cubes, 5 bags and 10 cubes, 2 bags and 6 cubes. Students can hold up the right number of fingers to signal the number of cubes in one bag each time.

Writing equations from models. Remind students that the pans of the balance become the sides of an equation, and that the equal sign in the equation shows that the pans are balanced. When they have, say, 3 bags with the same number of cubes in each, they write the total number of cubes in the bags as 3 × *b*. Present a few equations in the form of a model, and have students write the corresponding equations using the letter *b* for the unknown number.

Exercises: How many are in one bag?

a)


b)


Answers: a) 6, b) 3

Now have students solve the equations by looking at the model. Show them how to write the solution below the equation and have students record their solutions. Write on the board:

$$2 \times b = 12$$
$$b = 6$$

Drawing a model to solve the equations. Tell students that the next task will be the opposite of what they have been doing: now they will start with an equation and draw a model for it. Remind students that when we draw pictures in math class, it is important to draw the correct numbers of objects. Shading, color, and other artistic features or details are not important. Our drawings in math should be simple and we shouldn't spend too much time on them. Demonstrate making a simple drawing of a pan balance, and remind students that they can use circles or squares or big dots for cubes and boxes for paper bags.

Exercises: Draw a model and use it to solve the equation.

a) 3 × *b* = 15 b) 4 × *b* = 8 c) 9 × *b* = 18 d) 8 × *b* = 24

Answers: a) 5, b) 2, c) 2, d) 3

Using division to find the missing factor. ASK: Which mathematical operation did you use to write an equation for each balance? (multiplication) Which mathematical operation did you use to find the number of apples in each bag? (division) Have students show the division in the models they have drawn by circling equal groups of dots. For example, in Exercise a) above, they should circle three equal groups of dots. ASK: What number do you divide by? (the number of bags)

Multiplying and dividing by the same number does not change the starting number. Write on the board:

$(5 \times 2) \div 2$ $(3 \times 2) \div 2$ $(8 \times 2) \div 2$
$(5 \times 4) \div 4$ $(9 \times 3) \div 3$ $(10 \times 6) \div 6$

Have students solve each question. (5, 3, 8, 5, 9, 10)

SAY: Look at the questions you solved. ASK: How are they all the same? (they start with a number, multiply by another number, then divide by the same number) Did you get back to the same number you started with? (yes) Does it matter what number you started with? Does it matter what number you multiplied and divided by as long as it was the same number? (no) Have students write their own question of the same type, and check that they get back to the same number they started with.

Explain that you can do the same with unknown numbers. Show one paper bag with some cubes and SAY: I want to multiply this by 3. ASK: What will the answer look like? (3 bags) SAY: I want to divide the result by 3. What will you get? (1 bag again) Write on the board:

$(\square \times 3) \div 3 =$ ___

ASK: What will we get when we perform the multiplication and the division? (the box) Write on the board:

$(b \times 3) \div 3$ $(b \times 5) \div 5$ $(b \times 6) \div 6$ $(b \times 10) \div 10$

Have students solve each question. (3, 5, 6, 10)

Solving equations by dividing both sides by the same number. Write on the board:

$(b \times 7) \div$ ____ $= b$ $(b \times 2) \div$ ____ $= b$ $(b \times 4) \div$ ____ $= b$
$(b \times 8) \div$ ____ $= b$ $(b \times 12) \div$ ____ $= b$ $(b \times 9) \div$ ____ $= b$

For each equation, have students hold up the correct number of fingers to signal the number they would divide the product by to get back to *b*. (7, 2, 4, 8, 12, 9)

If a pan balance is available, show students the balance with 3 bags of 5 apples (other objects will work equally well—cereal bars, metal spoons, etc.) on one pan, and 15 apples on the other pan. Invite a volunteer to write the equation for the balance on the board, as shown below:

$3 \times 5 = 15$

ASK: How many apples are in one bag? (5) Have a volunteer make three groups of five apples on the side without the bags. Point out that there are three equal groups of apples on both sides of the balance. Remove two of the bags from one side, and two of the groups from the other side. SAY: I have replaced three equal groups on each side with only one of these groups.

ASK: What operation have I performed? (division by 3) Are the scales still balanced? (yes) Point out that when you perform the same operation on both sides of the balance, the scales remain balanced. ASK: What does that mean in terms of the equation? Write on the board:

$$b \times 3 \div 3 = 15 \div 3$$

Have students calculate the result on both sides. (b on the left side, 5 on the right side) Write on the board (align the equal signs vertically):

$$b = 5$$

Demonstrate that the bags indeed contain 5 apples. Repeat with a few more examples. Finally, have students solve equations by dividing both sides of the equation by the same number.

Exercises: Solve the equation.

a) $b \times 7 = 21$　　b) $b \times 2 = 12$　　c) $b \times 4 = 20$

d) $b \times 6 = 42$　　e) $b \times 3 = 27$　　f) $b \times 9 = 72$

Selected solution: a) $b \times 7 = 21$, $b \times 7 \div 7 = 21 \div 7$, $b = 3$

Answers: b) 6, c) 5, d) 7, e) 9, f) 8

Review adding and subtracting negative numbers. Remind students that they can subtract a negative number by adding its opposite. Write on the board:

$$5 - (-2) = 5 + 2 = 7$$

SAY: For adding negative numbers, you can subtract the opposite. Write on the board:

$$5 + (-2) = 5 - 2 = 3$$

Solving equations with integer addends. Write on the board:

$$x + (-3) = 5$$

ASK: If there was no negative sign, how would you solve the equation? (by subtracting both sides by the same number to isolate x) SAY: Let's do the same for the equation above and see the result. Write on the board:

$$\begin{aligned} x + (-3) - (-3) &= 5 - (-3) \\ x &= 5 + 3 \\ x &= 8 \end{aligned}$$

SAY: Let's check the answer by replacing x with 8 in the equation. Ask a volunteer to check the answer, as shown below:

$$x + (-3) = 8 + (-3) = 8 - 3 = 5 \checkmark$$

SAY: You can solve equations with negative numbers the same way you solve equations with positive numbers.

Exercises: Solve the equation by doing the same thing to both sides of the equation.

a) $x + (-5) = 7$	b) $x + 5 = 7$	c) $x - 5 = 7$	d) $x - (-5) = 7$
e) $x + 5 = -7$	f) $x - 5 = -7$	g) $x - (-5) = -7$	h) $x + (-5) = -7$

Answers: a) 12, b) 2, c) 12, d) 2, e) −12, f) −2, g) −12, h) −2

Review dividing integers. Remind students that they can divide integers the same way they divide whole numbers. SAY: To divide integers, first determine the sign of the quotient, then divide. Write on the board:

$(+) \div (+) = +$ $\quad$ $(+) \div (-) = -$ $\quad$ $(-) \div (+) = -$ $\quad$ $(-) \div (-) = +$

Exercises: Divide the integers. Determine the sign of the quotient first, then divide as though they are whole numbers.

a) $-32 \div 4$ $\quad$ b) $25 \div -5$ $\quad$ c) $-24 \div (-3)$

Answers: a) −8, b) −5, c) 8

Solving equations with integer coefficients. Write on the board:

$$3x = -15$$

ASK: If there was no negative sign, how would you solve the equation? (by dividing both sides by the same number, 3) SAY: Let’s do the same for the equation above and see the result. Write on the board:

$$3x \div 3 = -15 \div 3$$
$$x = -5$$

SAY: Let’s check the answer by replacing *x* with −5 in the equation. ASK: Is 3 times −5 equal to −15? (yes) Students can signal their answer with thumbs up or thumbs down. SAY: You can solve equations with negative coefficients the same way you solve equations with positive coefficients. Write on the board:

$$-3x = 15$$

Ask a volunteer to divide both sides by −3 to find the answer. Ask the volunteer to check the answer by replacing *x* with −5 in the equation.

Exercises: Solve by dividing both sides of the equation by the same number.

a) $6x = -18$	b) $-6x = 18$	c) $x \times (-6) = 18$	d) $x \times (-6) = -18$
e) $-12x = -36$	f) $\frac{1}{2}x = -5$	g) $-\frac{1}{2}x = 5$	h) $-\frac{1}{2}x = -5$

Answers: a) −3, b) −3, c) −3, d) 3, e) 3, f) −10, g) −10, h) 10

Extensions

(MP.5) a) Does the model work to solve the equation?

i) $x + 3 = 8$ ii) $x - 3 = 8$ iii) $3x = 12$ iv) $-3x = 12$ v) $0.6x = 1.8$

b) Does doing the same thing to both sides work?

Answers: a) i) yes, ii) no, iii) yes, iv) no, v) no; b) i) yes, ii) yes, iii) yes, iv) yes, v) yes

RP7-34 Cross-Multiplication (Introduction)

Pages 54–55

Standards: 7.RP.A.3

Goals:
Students will cross-multiply to write an equation for problems involving proportions.

Prior Knowledge Required:
Can convert a fraction *a*/*b* to a decimal by dividing *a* ÷ *b*
Can find equivalent fractions by multiplying the numerator and denominator by the same number
Can write an equivalent multiplication statement for a given division statement

Vocabulary: canceling, commutative property, complex fraction, **cross-multiply**, equivalent fractions

Materials:
calculators

Review writing a fraction as a division statement. Remind students that we can calculate the value of a fraction such as 3/4 by dividing 3 ÷ 4. For a quick reminder of why this is true,
SAY: To find 1/4 of something, I would divide it into four equal groups. So to find 1/4 of something, divide it by 4. You can think of 1/4 as 1/4 of 1, so that is 1 ÷ 4. But 3/4 is three times as much as 1/4, so 3/4 is 3 × 1 ÷ 4 = 3 ÷ 4.

Exercises: Write as a division statement and use a calculator to find the answer.
a) $\frac{3}{5}$ b) $\frac{5}{8}$ c) $\frac{7}{20}$ d) $\frac{3}{10}$ e) $\frac{8}{25}$
Answers: a) 3 ÷ 5 = 0.6, b) 5 ÷ 8 = 0.625, c) 7 ÷ 20 = 0.35, d) 3 ÷ 10 = 0.3, e) 8 ÷ 25 = 0.32

Writing fraction statements as equivalent multiplication statements. Remind students that a division statement can be written as a multiplication statement. For example, 12 ÷ 3 = 4 can be rewritten as 12 = 3 × 4.

Exercises: Change the division statements in the previous set of exercises to multiplication statements.
Answers: a) 3 = 5 × 0.6, b) 5 = 8 × 0.625, c) 7 = 20 × 0.35, d) 3 = 10 × 0.3, e) 8 = 25 × 0.32

To guide students in the following exercises, write this template on the board:

_____ ÷ _____ = _____

so _____ = _____ × _____

Exercises: Change the fraction statement to a division statement, then to a multiplication statement.

a) $\frac{7}{8} = 0.875$ b) $\frac{1}{4} = 0.25$ c) $\frac{17}{20} = 0.85$ d) $\frac{4}{5} = 0.8$

Answers: a) 7 ÷ 8 = 0.875, so 7 = 8 × 0.875; b) 1 ÷ 4 = 0.25, so 1 = 4 × 0.25; c) 17 ÷ 20 = 0.85, so 17 = 20 × 0.85; d) 4 ÷ 5 = 0.8; so 4 = 5 × 0.8

Finding a pattern. Now have students look at their answers to the questions in the previous set of exercises. ASK: If we know the value of a fraction as a decimal, how can we use that to write a multiplication statement? Write on the board:

____ = ____ × ____

ASK: In which blank does the numerator—the top number—of the fraction go? (the first blank) What number goes in the second blank, the denominator or the value? (it doesn't matter, because multiplication follows the commutative property) Explain that when you know the decimal value of a fraction, the numerator of the fraction can be written as the product of the denominator and the decimal value.

Exercises:

1. Write the fraction as a product.

a) $\frac{7}{4} = 1.75$ b) $\frac{9}{20} = 0.45$ c) $\frac{7}{35} = 0.2$ d) $\frac{9}{25} = 0.36$

Answers: a) 7 = 1.75 × 4, b) 9 = 0.45 × 20, c) 7 = 0.2 × 35, d) 9 = 0.36 × 25

2. Calculate the value of the fraction, then write a multiplication statement.

a) $\frac{2}{5}$ b) $\frac{9}{10}$ c) $\frac{21}{25}$ d) $\frac{19}{20}$

Bonus:

e) $\frac{23}{5}$ f) $\frac{192}{25}$

Answers: a) 2 = 0.4 × 5, b) 9 = 0.9 × 10, c) 21 = 0.84 × 25, d) 19 = 0.95 × 20,
Bonus: e) 23 = 4.6 × 5, f) 192 = 7.68 × 25

Writing fraction statements that involve variables as a product. Write on the board:

$$\frac{10}{x} = 2$$

SAY: I don't know what number *x* is, but I know that whatever it is, 2 times *x* is equal to 10. Write on the board:

$10 \div x = 2$, so $2x = 10$

Exercises: Rewrite the equation so that it uses multiplication instead of division.

a) $\frac{24}{x}=2$ b) $\frac{24}{x}=3$ c) $\frac{24}{x}=4$ d) $\frac{x}{3}=5$ e) $\frac{x}{6}=5$ f) $\frac{x}{7}=8$

g) $\frac{8}{2}=x$ h) $\frac{15}{3}=x$ i) $\frac{18}{2}=x$ j) $\frac{18}{x}=3$ k) $\frac{33}{3}=x$ l) $\frac{x}{2}=15$

Answers: a) 24 = 2x, b) 24 = 3x, c) 24 = 4x, d) x = 5 × 3, e) x = 5 × 6, f) x = 8 × 7, g) 8 = 2x, h) 15 = 3x, i) 18 = 2x, j) 18 = 3x, k) 33 = 3x, l) x = 15 × 2

Changing an equation of equivalent fractions to an equation of multiplication statements. Write on the board:

$$\frac{3}{5}=\frac{12}{20}$$

SAY: I can write each fraction as a division statement. Write on the board:

3 ÷ 5 = 12 ÷ 20

Have students verify this equation by doing long division. (3 ÷ 5 = 0.6 and 12 ÷ 20 = 0.6) Tell students that you find it easier to work with multiplication than with division. SAY: I would like to be able to verify this equality by using multiplication instead of division, and I know a trick that lets me change the equation so I can do that. Work through the steps below as a class. Write on the board:

$$\frac{3}{5}=\frac{12}{20}$$

SAY: Start by multiplying both sides by 5 × 20 (the product of the denominators). Write on the board:

$$\frac{3}{5}\times 5\times 20=\frac{12}{20}\times 5\times 20$$

SAY: Then, cancel common factors and rewrite the equation. The equations should look like this:

$$\frac{3}{\cancel{5}}\times \cancel{5}\times 20=\frac{12}{\cancel{20}}\times 5\times \cancel{20}$$
$$3\times 20=12\times 5$$

Point out that we have now created an equation of multiplication statements instead of fractions. Have students use multiplication to verify the equation. ASK: Was it easier to use multiplication to verify the equation or was it easier to use division? (multiplication)

Have students use this method to complete the following exercises.

Exercises: Change the equivalent fractions to equivalent multiplication statements.

a) $\frac{2}{3}=\frac{8}{12}$ b) $\frac{2}{5}=\frac{6}{15}$ c) $\frac{5}{9}=\frac{10}{18}$ d) $\frac{3}{8}=\frac{9}{24}$

Answers: a) 2 × 12 = 8 × 3, b) 2 × 15 = 6 × 5, c) 5 × 18 = 10 × 9, d) 3 × 24 = 9 × 8

Multiplying to verify equivalent fractions. Point out that the fractions 3/5 and 12/20 are equivalent fractions. ASK: How do I know? PROMPT: What number can we multiply both the numerator and the denominator by in 3/5 to get 12/20? (multiply 3 by 4 to get 12 and 5 by 4 to get 20) Write on the board:

$$\frac{3}{5}=\frac{12}{20}$$

ASK: How can we change this to an equation with multiplication instead? (we did it above—it was 3 × 20 = 12 × 5)

Exercises: Change the equivalent fractions to equivalent multiplication statements.

a) $\frac{3}{4}=\frac{15}{20}$ b) $\frac{3}{5}=\frac{9}{15}$ c) $\frac{7}{9}=\frac{21}{27}$ d) $\frac{3}{8}=\frac{15}{40}$

Sample solution:

a) 3/4 = 15/20

$3/\cancel{4} \times \cancel{4} \times 20 = 15/\cancel{20} \times 4 \times \cancel{20}$

3 × 20 = 15 × 4

Answers: b) 3 × 15 = 9 × 5, c) 7 × 27 = 21 × 9, d) 3 × 40 = 15 × 8

(MP.8) Finding a pattern (cross-multiplying). Have students look at their answers to the previous set of exercises. ASK: How can you find which numbers to multiply together from the fractions? PROMPT: Do you multiply both numerators together? (no) What do you multiply together? (the numerator of one fraction with the denominator of the other fraction) Go through each one, point to the answer, and verify that this is indeed what students did for each question—join the numerator of each fraction with the denominator of the other fraction to emphasize this point. Tell students that because the products from equivalent fractions can be found by drawing an X, we call this process *cross-multiplying*. Write on the board:

$$\frac{3}{4}=\frac{15}{20}$$

Exercises:

1. Verify that each pair of fractions in the previous two sets of exercises are in fact equivalent by verifying that the products you found are equal.

Answers:

a) 2 × 12 = 8 × 3	b) 2 × 15 = 6 × 5	c) 5 × 18 = 10 × 9	d) 3 × 24 = 9 × 8
24 = 24	30 = 30	90 = 90	72 = 72
a) 3 × 20 = 15 × 4	b) 3 × 15 = 9 × 5	c) 7 × 27 = 21 × 9	d) 3 × 40 = 15 × 8
60 = 60	45 = 45	189 = 189	120 = 120

(MP.7) 2. Use cross-multiplication to verify that the fractions are equivalent.

a) $\frac{3}{7}=\frac{6}{14}$ b) $\frac{4}{5}=\frac{12}{15}$ c) $\frac{3}{8}=\frac{15}{40}$ **Bonus:** $\frac{19}{24}=\frac{133}{168}$

Answers: a) 3 × 14 = 42 and 6 × 7 = 42; b) 4 × 15 = 60 and 12 × 5 = 60; c) 3 × 40 = 120 and 15 × 8 = 120; Bonus: 19 × 168 = 3,192 and 133 × 24 = 3,192

(MP.7) Cross-multiply to identify equivalent fractions. For the following exercises, have students decide whether the two fractions are equivalent by multiplying the numerator of each fraction with the denominator of the other fraction and checking whether the two products are equal.

Exercises: Cross-multiply to check if the fractions are equivalent.

a) $\frac{8}{10}$ and $\frac{64}{100}$ b) $\frac{7}{10}$ and $\frac{49}{70}$ c) $\frac{3}{5}$ and $\frac{36}{65}$ d) $\frac{4}{9}$ and $\frac{36}{90}$

e) $\frac{5}{12}$ and $\frac{45}{96}$ f) $\frac{8}{17}$ and $\frac{40}{85}$ g) $\frac{7}{9}$ and $\frac{63}{81}$ **Bonus:** $\frac{531}{792}$ and $\frac{59}{98}$

Answers: a) not equivalent, b) equivalent, c) not equivalent, d) not equivalent, e) not equivalent, f) equivalent, g) equivalent, Bonus: not equivalent

Cross-multiplying for complex fractions. SAY: You can cross-multiply complex fractions, too. Explain that complex fractions are like other fractions—they just contain fractions in the numerator, or the denominator, or both. Write on the board:

$$\frac{\frac{2}{3}}{\frac{4}{5}} \text{ and } \frac{\frac{5}{12}}{\frac{1}{2}}$$

SAY: To verify that they are equivalent, I have to multiply the numerator of the first complex fraction by the denominator of the second complex fraction, then the numerator of the second complex fraction by the denominator of the first complex fraction. Write on the board:

$$\frac{2}{3}\times\frac{1}{2}=\frac{\cancel{2}\times 1}{3\times\cancel{2}}=\frac{1}{3} \quad \text{and} \quad \frac{5}{12}\times\frac{4}{5}=\frac{\cancel{5}\times 4}{12\times\cancel{5}}=\frac{4}{12}$$

ASK: Are they equal? (yes) Students can answer by signaling thumbs up. ASK: How do you know? (because 4/12 and 1/3 are equivalent fractions) SAY: So the two complex fractions are equivalent.

Exercises: Cross-multiply to check if the complex fractions are equivalent.

a) $\frac{\frac{3}{4}}{\frac{1}{3}}$ and $\frac{\frac{6}{4}}{\frac{2}{3}}$ b) $\frac{\frac{1}{5}}{\frac{2}{7}}$ and $\frac{\frac{2}{3}}{\frac{3}{4}}$

Answers: a) equivalent, b) not equivalent

Explain that when the relation between two numerators or two denominators is clear and easy to find, they can find the missing number mentally. Otherwise, it is better to use cross-multiplication.

Exercises: How would you solve the proportion: mentally or using cross-multiplication? Circle the questions you would solve mentally.

a) $\frac{3}{5} = \frac{6}{?}$ b) $\frac{5}{6} = \frac{?}{15}$ c) $\frac{80}{100} = \frac{18}{?}$

d) $\frac{55}{100} = \frac{?}{20}$ e) $\frac{10}{11} = \frac{7}{?}$ f) $\frac{50}{65} = \frac{25}{?}$

Answers: a) mentally, ? = 10; b) cross-multiplying, ? = 12.5; c) cross-multiplying, ? = 22.5; d) mentally, ? = 11; e) mentally, ? = 7.7; f) cross-multiplying, ? = 32.5

Extensions

1. a) Have students investigate this question: If fractions $\frac{3}{5}$ and $\frac{6}{10}$ are equivalent, what fraction is $\frac{3}{6}$ equivalent to? Write on the board:

$\frac{3}{5} = \frac{6}{10}$ Cross-multiply to get 3 × 10 = 6 × 5

Have students decide what fractions they can cross-multiply to get 3 × 10 = 5 × 6. Suggest that students look for where the parts of each fraction go in the equation and compare how the equations are different. PROMPT: Which numbers switched positions, and which numbers stayed in the same position?

b) Have students cross-multiply to make a new pair of equivalent fractions, then use the commutative property of multiplication for one of the products (not both!) to make a new pair of equivalent fractions.

i) $\frac{2}{3} = \frac{6}{9}$, so ______ = ______ ii) $\frac{1}{4} = \frac{5}{20}$, so ______ = ______ iii) $\frac{3}{5} = \frac{9}{15}$, so ______ = ______

Answers: a) 3 and 10 are in the same position, but 5 and 6 get switched. So the corresponding fractions are $\frac{3}{6} = \frac{5}{10}$; b) i) 2/6 = 3/9, ii) 1/5 = 4/20, iii) 3/9 = 5/15. Emphasize to students that to find the second pair of equivalent fractions, they can read the numbers from the first pair across, from left to right.

2. **Mental math and estimation.** Tell students that you know someone who changed the fractions in Extension 1, part b.ii) to 1/20 = 5/4. ASK: How can you tell immediately that this is wrong?
Answer: 1/20 is less than 1, but 5/4 is more than 1

3. **Cross-multiplying with decimal numbers.** Cross-multiply to verify whether the fractions are equivalent.

a) $\frac{0.2}{0.3}$ and $\frac{0.4}{0.6}$ b) $\frac{0.03}{0.05}$ and $\frac{0.4}{0.7}$ c) $\frac{0.02}{0.5}$ and $\frac{0.1}{0.25}$

Sample solution: a) 0.2 × 0.6 = 0.3 × 0.4, 0.12 = 0.12 ✓
Answers: b) no, c) no

RP7-35 Using Equations to Solve Proportions

Pages 56–57

Standards: 7.RP.A.3

Goals:
Students will cross-multiply to solve problems that involve proportions.

Prior Knowledge Required:
Can convert a fraction a/b to a decimal by dividing $a \div b$
Can find equivalent fractions by multiplying the numerator and denominator by the same number
Can write a proportion to solve ratio and percent problems
Can solve multiplicative equations
Can write an equivalent multiplication statement for a given division statement

Vocabulary: cross-multiply, equation, equivalent fractions, equivalent ratios, percent, proportion, variable

Materials:
calculators

Using cross-multiplying to write equations. Show students how to cross-multiply to write an equation when there is a variable in one of the fractions. Write on the board:

$$\frac{10}{x} = \frac{2}{3}$$

SAY: I don't know what number x is, but I know that no matter what, 2 times x is equal to 10 times 3. Write on the board:

$$\frac{10}{x} = \frac{2}{3} \longrightarrow \quad 10 \times 3 = 2 \times x, \text{ so } 30 = 2x$$

Exercises: Cross-multiply to write an equation for x.

a) $\frac{24}{x} = \frac{2}{5}$ b) $\frac{24}{x} = \frac{3}{5}$ c) $\frac{24}{x} = \frac{4}{5}$ d) $\frac{x}{3} = \frac{12}{9}$

e) $\frac{x}{6} = \frac{5}{3}$ f) $\frac{x}{7} = \frac{8}{28}$ g) $\frac{8}{2} = \frac{x}{5}$ h) $\frac{15}{3} = \frac{x}{4}$

Answers: a) $24 \times 5 = 2x$, so $120 = 2x$; b) $120 = 3x$, c) $120 = 4x$, d) $9x = 36$, e) $3x = 30$, f) $28x = 56$, g) $40 = 2x$, h) $60 = 3x$

Using cross-multiplying to solve equations. Review multiplicative equations like $2 \times b = 12$. Remind students that to solve this type of equation, they have to divide both sides of the equation by the coefficient of the unknown. For example, in the equation $2b = 12$, the answer is $b = 12 \div 2$, so $b = 6$.

Exercises:
1. Have students solve the equations in their answers to the previous set of exercises.
Sample solution: d) $9x = 12 \times 3$, $9x = 36$, $x = 36 \div 9$, $x = 4$
Answers: a) $x = 60$, b) $x = 40$, c) $x = 30$, e) $x = 10$, f) $x = 2$, g) $x = 20$, h) $x = 20$

2. Rewrite the equation so that it involves multiplication, then solve for x. Check your answer by substitution.
a) $\frac{20}{x} = 5$ b) $\frac{x}{6} = 7$ c) $\frac{26}{2} = x$ d) $\frac{60}{x} = 15$ e) $\frac{x}{7} = 9$ f) $\frac{48}{8} = x$
Answers: a) $20 = 5x$, so $x = 20 \div 5 = 4$; b) $x = 6 \times 7 = 42$; c) $2x = 26$, so $x = 26 \div 2 = 13$; d) $60 = 15x$, so $x = 60 \div 15 = 4$; e) $x = 7 \times 9 = 63$; f) $48 = 8x$, so $x = 48 \div 8 = 6$

Point out that parts c) and f) do not even need to be rewritten, as they can be solved in one step. For example, c) says directly that $26 \div 2 = x$, so we don't need to first write that $2x = 26$.

Cross-multiplying when the answers are decimal numbers. Tell students to again cross-multiply to solve for x, but this time their answers will be decimal numbers. This means that they are comparing equivalent ratios rather than equivalent fractions. Remind students that we can write ratios in fraction form even when both terms are not whole numbers. Review writing fractions as decimal fractions, then as decimals. Write on the board:

$$\frac{1}{2} = \frac{5}{10} = 0.5$$
$$\frac{1}{4} = \frac{25}{100} = 0.25,\ \frac{2}{4} = \frac{1}{2} = 0.5,\ \frac{3}{4} = \frac{75}{100} = 0.75$$
$$\frac{1}{5} = \frac{2}{10} = 0.2,\ \frac{2}{5} = \frac{4}{10} = 0.4,\ \frac{3}{5} = \frac{6}{10} = 0.6,\ \frac{4}{5} = \frac{8}{10} = 0.8$$

Exercises: Solve for x.
a) $\frac{10}{3} = \frac{3}{x}$ b) $\frac{x}{3} = \frac{5}{6}$ c) $\frac{7}{5} = \frac{x}{6}$ d) $\frac{7}{x} = \frac{4}{5}$ e) $\frac{9}{x} = \frac{6}{5}$ f) $\frac{5}{3} = \frac{11}{x}$
Sample solution: c) $5x = 42$, so $x = 42 \div 5 = 42/5 = 8\ 2/5$, so $x = 8.4$
Answers: a) $10x = 9$, so $x = 0.9$; b) $6x = 15$, so $x = 2.5$; d) $4x = 35$, so $x = 8.75$; e) $6x = 45$, so $x = 7.5$; f) $5x = 33$, so $x = 6.6$

Using proportions to solve percent problems. Review writing a proportion to solve a percent problem, then demonstrate how using cross-multiplication makes the problem easy. Write on the board:

What is 30% of 8?

SAY: Suppose the answer is x and we're going to find x. If 30% of 8 is equal to x, the ratio of x to 8 is the same as the ratio of 30 to 100. Write on the board:

$x : 8 = 30 : 100$

SAY: To solve this proportion, you can write it as two equivalent fractions. Write on the board:

$$x : 8 = 30 : 100 \longrightarrow \frac{x}{8} = \frac{30}{100}$$

Remind students that they already used cross-multiplying to solve this type of equation. Ask a volunteer to solve the equation, as shown below.

$$\frac{x}{8} = \frac{30}{100}, \quad \text{so} \quad 100x = 30 \times 8$$
$$100x = 240$$
$$x = \frac{240}{100}$$
$$x = 2.4$$

Explain to students that writing the proportion is the most important part of the solving process. Write on the board:

12 is 3% of what number?

SAY: Suppose the answer is *x* and we're going to find *x*. Have students propose different ways of writing the proportion for this problem. SAY: Because there is 3% in the question, one fraction in the proportion is 3/100. Write on the board:

$$\frac{x}{12} = \frac{3}{100} \qquad\qquad \frac{12}{x} = \frac{3}{100}$$

ASK: Which proportion gives me the answer, the first or the second? (the second) Students can answer by signaling thumbs up or thumbs down as you point to each proportion. ASK: How do you know? (because 12 is a part of the question and is in the numerator of the second proportion) Ask a volunteer to use cross-multiplying to solve the proportion, as shown below.

$$\frac{12}{x} = \frac{3}{100}, \quad \text{so} \quad 3x = 12 \times 100$$
$$3x = 1{,}200$$
$$x = \frac{1{,}200}{3}$$
$$x = 400$$

Exercises: Write a proportion in fraction form, then cross-multiply and solve.

a) What is 15% of 40?
b) What is 32% of 50?
c) What is 75% of 48?
d) 24 is 80% of what number?
e) 62 is 25% of what number?
f) 12 is 30% of what number?

Answers: a) *x*/40 = 15/100, *x* = 6; b) *x*/50 = 32/100, *x* = 16; c) *x*/48 = 75/100, *x* = 36; d) 24/*x* = 80/100, *x* = 30; e) 62/*x* = 25/100, *x* = 248; f) 12/*x* = 30/100, *x* = 40

Solving percent problems mentally. Write on the board:

What is 15% of 20?

Ask a volunteer to write the proportion for the problem in fraction form and use cross-multiplication to solve it, as shown below:

$$\frac{x}{20} = \frac{15}{100}, \quad \text{so} \quad \begin{aligned} 100x &= 15 \times 20 \\ 100x &= 3{,}000 \\ x &= 3 \end{aligned}$$

Point to the fractions and SAY: Look at the denominators of these equivalent fractions. ASK: What number do I have to multiply 20 by to get 100? (5) What number do I have to multiply x by to get 15? (the same number: 5) Write on the board:

$$\frac{x}{20} \ \overset{\longrightarrow}{\underset{\longrightarrow}{=}} \ \frac{15}{100}$$

ASK: What is the value of x? (3)

Exercises: Write the question as a proportion in fraction form, then solve the equation mentally.

a) What is 20% of 25?
b) What is 18% of 50?
c) What is 75% of 10?
d) 24 is 48% of what number?
e) 42 is 21% of what number?

Answers: a) $x/25 = 20/100$, $x = 5$; b) $x/50 = 18/100$, $x = 9$; c) $x/10 = 75/100$, $x = 7.5$; d) $24/x = 48/100$, $x = 50$; e) $42/x = 21/100$, $x = 200$

Ask students to use cross-multiplication to solve the questions in the previous set of exercises. Ask them to find their mistake if they did not get the same answer both ways.

(MP.4) Approximating percents. Tell students that in some real-world questions, it is not necessary to find the exact percent. Write on the board:

90 students out of 298 Grade 7 students wear eyeglasses to read.
What percent of the Grade 7 students wear eyeglasses?

Ask a volunteer to write the proportion for the question, then use cross-multiplication to write the equation, as shown below:

$$\frac{90}{298} = \frac{x}{100}, \text{ so } 298x = 90 \times 100$$

Ask students to use a calculator to solve the equation. (30.20134228) ASK: Did you get an exact number for the question? (no) SAY: Look at the first fraction in the proportion. The denominator 298 is very close to 300. Write on the board:

$$\frac{90}{300} = \frac{x}{100}$$

Ask a volunteer to use cross-multiplication to write the equation, then to solve the new proportion, as shown below:

$\frac{90}{300} = \frac{x}{100}$, so $300x = 90 \times 100$, so $x = \frac{9{,}000}{300} = 30$

Write on the board:

About 30% of students in Grade 7 wear eyeglasses.

Exercises: Find the amount.
a) About 10% of students are left-handed. In a school with 731 students, about how many are left-handed?
b) 28 is 9% of a number.
c) 17 is 23% of a number.
Answers: a) about 73, b) 311, c) 74

(MP.4) More word problem practice.

Exercises: Solve the word problem.
a) A shirt costs \$25. After taxes, it costs \$30. What percent of the original price are the taxes?
b) A shirt costs \$40. After taxes, it costs \$46. At what rate was the shirt taxed?
d) A shirt costs \$40. It goes on sale for \$28. What percent was taken off?
Bonus: A shirt costs \$20. It goes on sale at 15% off. A 15% tax is then added. What is the final price?
Answers: a) 5/25 = ?/100, so ? = 20; b) 6/40 = ?/100, so ? = 15; c) 12/40 = ?/100, so ? = 30; Bonus: The sale price is \$17, and 15% tax is \$2.55, so the final price is \$19.55

Extension
(MP.1) NOTE: This extension emphasizes the importance of checking the answer by substitution.
a) Can $\frac{x}{3} = \frac{x}{5}$ be solved? Explain. b) Can $\frac{3}{x} = \frac{5}{x}$ be solved? Explain.
Answers: a) Cross-multiplying gives us $5x = 3x$, $2x = 0$, so $x = 0$, which can be substituted into the original equation (0/3 = 0/5, 0 = 0), so this equation can be solved;
b) Cross-multiplying gives us the same equation and the same result as in part a), $x = 0$, but substituting this value into the original equation results in 3/0 = 5/0. These fractions do not make sense! This equation cannot be solved.

RP7-36 Recognizing Proportional Relationships

Pages 58–59

Standards: 7.RP.A.2

Goals:
Students will recognize when two quantities are in a proportional relationship.

Prior Knowledge Required:
Is familiar with ratio tables
Can solve multiplicative equations
Can write a proportion for a word problem
Can solve proportions

Vocabulary: as many as, as much as, cross-multiply, equivalent fractions, for each, for every, per, proportion, ratio, ratio table

Words and phrases to indicate a relationship between quantities ("for every," "for each," and "per.") Remind students that a ratio associates two or more quantities. Explain to students that ratios can indicate a relationship between two or more things through words and phrases such as "to," "for every," "out of every," "for each," and "per." For example, "the score was 5 to 3," "5 cups of lemonade for every 2 cups of berries," "2 out of every 3 birds," "2 apples for each child," "5 miles per gallon of gas," and "5 parts red paint to 3 parts blue paint."

SAY: Whenever you see words or phrases meaning "for every," such as "for each" or "per," you can recognize the proportions. Write on the board:

> Four out of every nine students in a class are girls.
> There are 12 girls in the class. How many students are in the class?

ASK: Can you see a word or phrase that tells you to solve a proportion? (yes) Students can signal with thumbs up. ASK: What is the word or phrase? ("out of every") Underline the phrase "out of every." SAY: This is a part-to-whole ratio. ASK: What is part in the question? (4) What is whole? (9) Write part-to-whole fractions on the board:

$$\frac{\text{part}}{\text{whole}} = \frac{4}{9} = \frac{12}{x}$$

Remind students that they can use equivalent fractions or cross-multiplication to solve this proportion, but this question is easy to solve mentally. Ask a volunteer to solve it on the board, as shown below:

$\frac{4}{9} = \frac{12}{x}$, so x = 27. There are 27 students in the class.

Exercises: Underline the word or phrase that tells you to solve the proportion. Then solve the problem.
a) There are 3 cups of flour in each cake. How many cups of flour are in 4 cakes?
b) Bev bikes 12 km per hour. How far can she bike in 3.5 hours?
c) Cam paid $30 for every 4 shirts. How much did he pay for 1 shirt?
Answers: a) <u>in each</u>, 12 cups of flour; b) <u>per</u>, 42 km; c) <u>for every</u>, $7.50

Other ways to show "for every." Point to the last question in the previous set of exercises and SAY: I would like to write this question in a different way. Write on the board:

4 shirts cost $30. How much does 1 shirt cost?

ASK: Do you see the words or phrases "for every", "for each," or "per" in the question? (no) Students can signal with thumbs down. ASK: Even though the question doesn't use these exact words, does it still mean "for every"? (yes) Students can signal with thumbs up. ASK: How do you know? (because the shirts are all the same, the shirts have the same price) SAY: The statement "4 shirts cost $30" means "$30 for every 4 shirts that are the same." Explain to students that, in some questions, they won't see the words or phrases "for every", "for each", or "per," but the question contains the meaning of "for every." Emphasize that students have to focus on the meaning of the phrases "for every," "for each," and "per" because different questions will use different ways of stating ratios and rates.

Exercises: Rewrite the statement using the phrase "for every."
a) One ticket costs $3.25.
b) Emma can bike 25 km in 2 hours.
c) Ben shapes cookies twice as fast as Kate.
Answers: a) You pay $3.25 for every ticket, b) Emma can bike 25 km for every 2 hours, c) For every cookie that Kate shapes, Ben shapes two cookies.

Decreasing relationship. Write on the board:

4 workers can paint 3 walls a day.
How many walls can 8 workers paint a day?

SAY: Assume that everyone works at the same steady rate. Draw on the board:

Workers	Walls painted
4	3
8	?

Point to the table and ASK: Can we solve this problem with a proportion? (yes) Students can signal their answers with thumbs up or down. ASK: Why? (because “a day” means for every one day) Ask a volunteer to find the missing number in the ratio table, as shown below:

Workers	Walls painted
4	3
8	6

Write on the board:

4 workers can paint an apartment in 3 hours.
How long will it take 8 workers to paint the same apartment?

SAY: Assume that everyone works at the same steady rate. Draw on the board:

Workers	Hours
4	3
8	?

Point to the table and ASK: If more people are working, are more hours needed or are fewer hours needed to complete the job? (fewer) SAY: So this can’t be proportional. With this proportion, if there were twice as many workers, they would need twice as many hours. But they actually need half as many hours, because twice as many workers get the job done twice as fast, so it will take only 1.5 hours for 8 people to paint the apartment.

Exercises: Is the proportion twice as much, or half as much?

a) Workers are painting fences.

i)

Workers	Fences painted
2	6
4	?

ii)

Workers	Hours needed to paint
2	6
4	?

b) A car rental is $12 per hour

i)

Hours	Total cost ($)
1	12
2	?

ii)

Cars rented	Cost per hour ($)
1	12
2	?

iii)

People sharing a car rental	Cost per person
1	12
2	?

Answers: a) i) 12 fences, twice as much; ii) 3 hours, half as much; b) i) $24, twice as much; ii) $24, twice as much; iii) $6, half as much

SAY: If the input (left column) increases and the output (right column) decreases, it’s not proportional, so the table is not a ratio table. Ask students to identify which table(s) in the previous exercises are not ratio tables. (in part a) table ii), and in part b) table iii) are not ratio tables)

Exercises: Solve the problem.
a) It takes 4 workers 3 hours to fill a hole in the road. How long does it take for 6 workers to fill the same hole? Hint: How long would 1 worker take?
b) Three people need 1 hour to clean a house. How long does it take for 5 people to clean the house? Hint: How long would 1 person take?
Answers: a) 2 hours, b) 3/5 hour or 36 minutes

Constant increase relationship may not be proportional. Start with an example.
SAY: Jayden is 2 years old and Yu is 5 years old. Draw on the board:

Jayden	Yu
2	5
4	10

Jayden	Yu
2	5
4	7

ASK: Can I say that for every 2 years that Jayden gets older, Yu gets 5 years older? (no) PROMPT: If Jayden gets 2 years older, how many years older would Yu get? (2) ASK: Do the words "for every" make sense in this situation? (no) ASK: How do you know? (because after 2 years, Jayden is 4 and Yu is 7 years old)

Point to the tables and ASK: Which table is a ratio table? (the first table) Which table represents "Jayden is 2 years old and Yu is 5 years old"? (the second table) Cross out the first table and SAY: In this situation, there is no a proportional relationship between Jayden's age and Yu's age, so using a ratio table doesn't make sense. Write on the board:

A canoe rental is $8 for the first hour and $5 for every hour after that.

SAY: Let's draw a table to show how much it costs to rent a canoe for the first 4 hours. Draw on the board:

Hours	Cost ($)
1	8
2	13
3	18
4	23

Explain to students that this is not a ratio table, so there is no proportional relationship between rental hours and cost. SAY: To check if a relationship is proportional, double one quantity. If the other quantity doubles, too, then the relationship is proportional.

Exercises: Is there a proportional relationship between quantities?
a) The temperature increases 1.5°C for every week in the spring.
Hint: If the temperature increases 1.5°C one week, could it increase 3°C in two weeks?
b) For every 2 cups of flour, use 1 teaspoon of oil.
Hint: Would twice as much flour use twice as much oil?
c) Mike gets 3 years older for every 2 years that Vicky gets older.
Hint: If Mike gets 3 years older, how many years older would Vicky get?
d) For every 3 people cleaning the house, 1 hour is needed.
Hint: Would twice as many people helping take twice as long?
Answers: a) yes, b) yes, c) no, d) no

Extensions
1. A computer engineer writes a program in 6 days. Another computer engineer can write the same program in 3 days. How long would it take if they worked together to write the program?
Hint: How much do they each get done in one day? So how much do they get done in total in one day?
Solution: The first engineer can write the program in 6 days. That means he can write 1/6 of the program per day. The second engineer can write the program in 3 days. That means she can write 1/3 of the program per day. Together they can write 1/6 + 1/3 = 1/2 of the program per day, so it would take 2 days to complete the program.

2. In 2 hours, 3 people can paint a hall with an area of 360 ft^2. How long would it take for 5 people to paint a hall with an area of 450 ft^2 at the same rate of speed?
Solution: 3 people can paint 360 ft^2 in 2 hours, so each person can paint 120 ft^2 in 2 hours or 60 ft^2 in 1 hour. 5 people can paint 60 × 5 = 300 ft^2 in 1 hour, so it would take 1.5 hours to paint 450 ft^2.

RP7-37 Ratio and Percent Problems—Tape Diagrams, Discounts, and Markups

Pages 60–61

Standards: 7.RP.A.3

Goals:
Students will use diagrams to solve discount and markup word problems.

Prior Knowledge Required:
Is familiar with percent
Is familiar with number lines
Is familiar with tape diagrams

Vocabulary: discount, discounted price, markup, original price, percent, ratio

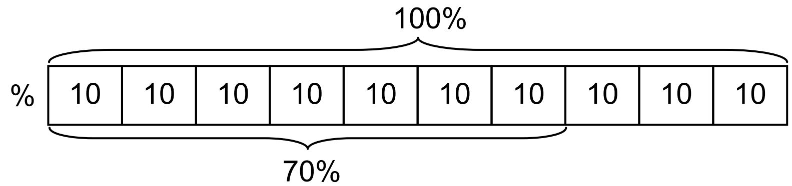
Review finding percents. ASK: If you know 10% of a number, how can you find 30% of that number? (multiply 10% of the number by 3) Tell students that you would like to find 70% of 12. ASK: What is 10% of 12? (1.2) If I know that 10% of 12 is 1.2, how can I find 70% of 12? (1.2 × 7). You can review multiplying a decimal number by a whole number with an example. (12 × 7 = 84, so 1.2 × 7 = 8.4)

Exercises: Used the method above to find the percent.

a) 60% of 15	b) 40% of 40	c) 60% of 4	d) 20% of 1.5
e) 90% of 8.2	f) 70% of 4.3	g) 80% of 5.5	h) 30% of 3.1

Answers: a) 6 × 1.5 = 9, b) 4 × 4 = 16, c) 6 × 0.4 = 2.4, d) 2 × 0.15 = 0.30, e) 9 × 0.82 = 7.38, f) 7 × 0.43 = 3.01, g) 8 × 0.55 = 4.4, h) 3 × 0.31 = 0.93

Using tape diagrams to find percents. Remind students that a percent is a ratio that compares a number to 100. Remind them that tape diagrams are useful when two quantities in a ratio have the same units, so they can use tape diagrams in ratio and percent problems. ASK: How many 10% are in 100%? (10) How many 20% are in 100%? (5) How many 25% are in 100%? (4) SAY: To find 70% of 60, I have to divide 100% into 10 parts. Draw on the board:



ASK: What is 10% of 60? (6) If 10% of a number is 6, how can you find 70% of that number? (multiply 10% of the number by 7, so 6 × 7 = 42) Draw on the board:

100%

%	10	10	10	10	10	10	10	10	10	10
Amount	6	6	6	6	6	6	6	6	6	6

70%

SAY: So 70% of 60 is 42. Emphasize that 100% is divided into 10 parts because the information about 10% of the number was available and 100% ÷ 10% = 10. If there is information about 25% of a number, the 100% is being divide into four parts because 100% ÷ 25% = 4. SAY: You can use tape diagrams to find the *discount*. For instance, the example above could be considered for the following real-world problem. Write on the board:

A T-shirt costs $60. How much of a discount would you get if the T-shirt is 70% off?

SAY: You would get a $42 discount.

Have students find the amount of the discount in different percentages.

Exercises: The *original price* is given. Find the discount.

a) 100% = $45

%	10	10	10	10	10	10	10	10	10	10

60% discount

b) 100% = $35

%	20	20	20	20	20

40% discount

c) 100% = $40

%	25	25	25	25

25% discount

d) The price of a sweater is $55. What is the discount if the sweater is 20% off?

Selected solutions:

b) 100% = $35

%	20	20	20	20	20
	7	7	7	7	7

40% discount = $14

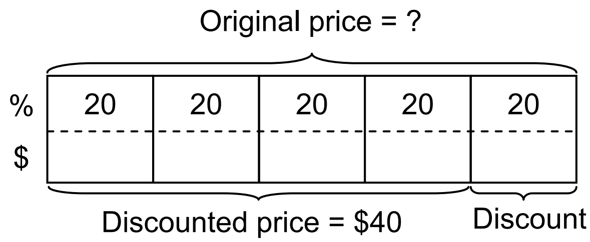
d) 100% = $55

%	20	20	20	20	20
	11	11	11	11	11

20% discount = $11

Answers: a) $27, c) $10

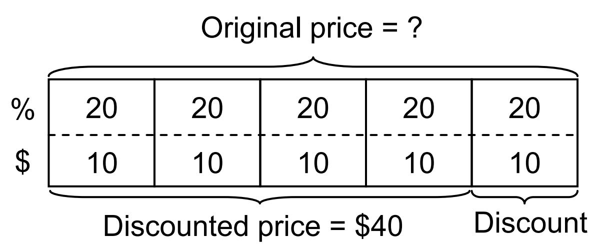
Finding the original price from the discounted price. SAY: In the previous questions, we found the part (discount) from the whole (original). Explain to students that they can also use a tape diagram to find the whole (original price) from the part (discounted price). Start with an example. SAY: After a 20% discount, I paid $40 for pants. I would like to know the original price. Because I have the information about 20%, I will divide 100% into five parts. Draw on the board:



Explain that when you get a 20% discount, you actually pay 80% of the original price, which is called the *discounted price*. Write on the board:

discounted price + discount = original price

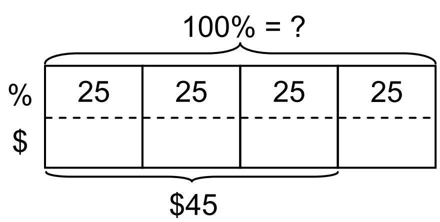
Point to the diagram and SAY: The discounted price is $40 and it's divided into four equal parts. ASK: What is each part? ($10) How do you know? (40 ÷ 4 = 10) Write $10 in each part of the diagram, as shown below:



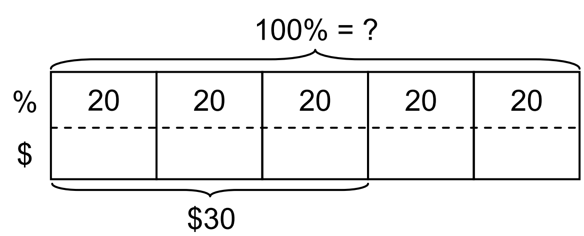
SAY: The discount is $10, so the original price was $50.

Exercises: Find the original price.

a) After a 25% discount, the price is $45. What was the original price?



b) After a 40% discount, the price is $30. What was the original price?



c) After a 20% discount, the price is $40. What was the original price?

Selected solutions:

b)

100% = $50

%	20	20	20	20	20
$	10	10	10	10	10

$30

c)

100% = $50

%	20	20	20	20	20
$	10	10	10	10	10

$40

Answer: a) $60

If some students divided 100% into 10 parts in part b), SAY: It is true, but it's easier to take the parts of 20% instead of 10%.

100% = $50

%	10	10	10	10	10	10	10	10	10	10
$	5	5	5	5	5	5	5	5	5	5

$30

SAY: Notice that both ways give the same answer.

Finding the markup from the original price. Introduce the term "*markup*." SAY: If a store buys a shirt for $10 and sells it for $15, then the markup is $5. $5 is 50% of $10, so the store marks the price up 50%. Write on the board:

A shirt costs $12. What would the price be after a 25% markup?

Explain to students that because there is information about 25%, they have to divide 100% into four parts. Draw on the board:

100% = $12

%	25	25	25	25

SAY: Markup means an extra amount, so I have to add another 25% to the whole. Then add another box of 25%, as shown below:

100% = $12

%	25	25	25	25	25
$					

Price after markup = $?

Point to the diagram and SAY: The original price is $12 and it divided into four equal parts. ASK: How much is each part? ($3) How do you know? (12 ÷ 4 = 3) Write $3 in each part of the diagram, as shown below.

100% = $12

%	25	25	25	25	25
$	3	3	3	3	3

Price after markup = $15

SAY: The markup is $3, so the price after markup is $15.

Exercises: Find the price after the given markup.
a) The original price is $30. What would the price be after a 50% markup?

100% = $30

%	50	50	50
$			

Price after markup = ?

b) The original price is $30. What would the price be after a 60% markup?

100% = $30

%	20	20	20	20	20	20	20	20
$								

Price after markup = ?

c) The original price is $40. What would the price be after a 25% markup?
Selected solution:
b)

100% = $30

%	20	20	20	20	20	20	20	20
$	6	6	6	6	6	6	6	6

Price after markup = $48

Answers: a) $45; c) $50, see model below:

100% = $40

%	25	25	25	25	25
$					

Price after markup = ?

Finding the original price before the markup. SAY: In the previous questions, we found the markup from the original price. We can also use a tape diagram to find the original price from the marked-up price.

Start with an example. SAY: After a 20% markup, I paid $30 for pants. I would like to know the original price before the markup. Because I have the information about 20%, I divide 100% into five parts. Draw on the board:

Original price = ?

%	20	20	20	20	20
$					

Explain that when the markup is 20%, you have to add another 20%. Then add another box of 20%, as shown below:

Original price = ?

%	20	20	20	20	20	20
$						

Price after markup = $30

Point to the diagram and SAY: The price after the markup is $30 and it divided into six equal parts. ASK: How much is each part? ($5) How do you know? (30 ÷ 6 = 5) Write $5 in each part of the diagram, as shown below.

Original price = ?

%	20	20	20	20	20	20
$	5	5	5	5	5	5

Price after markup = $30

SAY: The markup was $5, so the original price was $25.

Exercises: Find the original price.

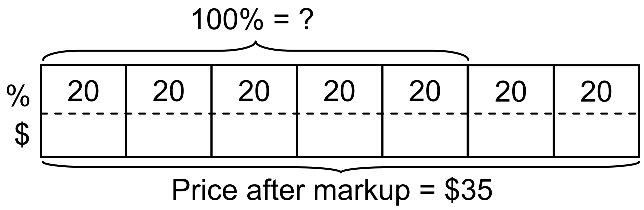
a) After a 25% markup, the price is $45. What was the original price before the markup?

100% = ?

%	25	25	25	25	25
$					

Price after markup = $45

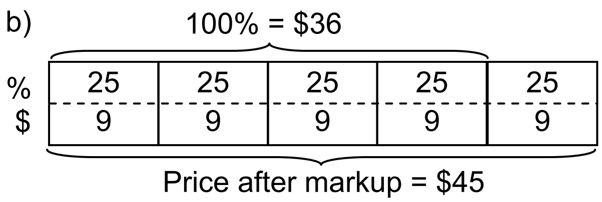
b) After a 40% markup, the price is \$35. What was the original price?



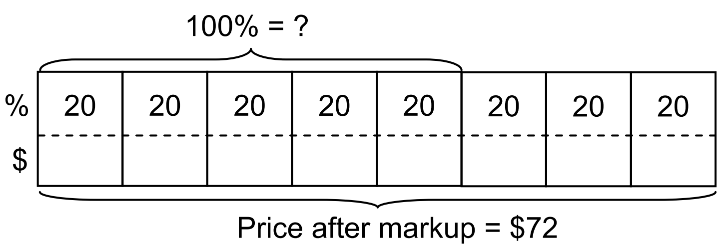
c) After a 60% markup, the price is \$72. What was the original price?

Selected solution:

b)



Answers: a) \$36; c) \$45, see model below:



Extensions

1. You can use a proportion to solve discount problems. For example: after a 20% discount, the price is \$60. What was the original price? The discounted price of \$60 is 80% of the original price, so the question is equivalent with this question: 80% of what number is 60? $\frac{60}{?} = \frac{80}{100}$, so $\frac{60}{?} = \frac{4}{5}$ and ? = 75. The original price was \$75.

Use the method above to solve the problem.

a) After a 25% discount, the price is \$45. What was the original price?

b) After a 30% discount, the price is \$28. What was the original price?

c) After a 55% discount, the price is \$36. What was the original price?

Answers: a) \$60, b) \$40, c) \$80

(MP.1) 2. You can use an equation to solve discount problems. For example: after a 20% discount, the price is \$60. What was the original price? The discounted price of \$60 is 80% of the original price, so 80% × x = 60 where x is the original price. The equation is 0.8x = 60 because 80% is 0.8. 0.8x = 60, so $x = \frac{60}{0.8} = \frac{600}{8}$ and x = 75. The original price was \$75.

Use an equation to solve each problem in the previous Extension and compare the answers.

Answers: a) \$60, b) \$40, c) \$80