Unit 1: Inequalities, Equations, and Graphs

Interval Notation

Interval notation is a convenient and compact way to express a set of numbers on the real number line.

Graphic Representation	Inequality Notation	Interval notation
____________________	$-2 < x < 3$	
____________________	$1 \le x \le 4$	
____________________	$-1 < x \le 2$	
____________________	$x \ge 2$	
____________________	$x < -1$	

Inequality Properties

1. If $a > b$, then $a + c > b + c$
2. If $a > b$ and $c > 0$, then $ac > bc$
3. If $a > b$ and $c < 0$, then $ac < bc$

Ex. 1 Solve each inequality (note that the degree is 1) and write the solution using interval notation:

a) $-3x + 5 > 12$ b) $-9 \le 2x + 10 < 5$ c) $-3 < \dfrac{7 - 2x}{3} \le 4$

Ex. 2 Solve each inequality and write the solution using inequality notation.

a) $0 \le 2x - \frac{\pi}{3} \le 2\pi$

b) $0 \le x - \frac{\pi}{2} \le 2\pi$

c) $0 \le 2x + \pi \le 2\pi$

Polynomial Inequalities with degree two or more and Rational Inequalities

Solve $x^2 - 4x + 7 > 4$ by making a sign chart. Write your answer using interval notation.

1. Set one side of the inequality equal to zero.
2. Temporarily convert the inequality to an equation.
3. Solve the equation for x. If the equation is a rational inequality, also determine the values of x where the expression is undefined (where the denominator equals zero). These are the partition values.
4. Plot these points on a number line, dividing the number line into intervals.
5. Choose a convenient test point in each interval. Only one test point per interval is needed.
6. Evaluate the polynomial at these test points and note whether they are positive or negative.
7. If the inequality in step 1 reads > 0, select the intervals where the test points are positive. If the inequality in step 1 reads < 0, select the intervals where the test points are negative.

Ex. 3 Solve each inequality. Show the sign chart. Draw the solution on the number line and express the answer using interval notation.

a) $x(x+4)(x-3)^2 \le 0$

b) $x^2 - 3x - 4 > 0$

c) $\dfrac{x-3}{x^2-4} < 0$

d) $\dfrac{3}{x+4} \le \dfrac{2}{x-1}$

Absolute Value

$$|x| = \begin{cases} x & if\ x \geq 0 \\ -x & if\ x < 0 \end{cases}$$

The absolute value of a real number x is the distance on the number line that x is from 0.

Absolute value equations

Ex. 4 Solve the equation (check your answers for extraneous solutions):

a) $\left|\dfrac{2x+1}{x-3}\right| = 4$

b) $|2x+3| = 1$

Absolute value inequalities

1. if $|x| < a$, then $-a < x < a$ ________________________________

2. if $|x| > a \geq 0$, then $x < -a$ or $x > a$ ________________________________

Ex. 5 Solve the inequality. Express your answers in interval notation and graph the solution:

a) $|4x-1| < .01$

b) $|2x-1| \geq 5$

c) $|x^2 + 3x - 4| < 6$

Equations and Graphs

Lines

The equation $y = mx + b$ is a linear equation where m and b are constants. This is called Slope-Intercept form where m is the slope and b is the y-intercept.

In general,

$m > 0$ $m < 0$ $m = 0$ m is undefined

The slope of a Line

Point-Slope equation of a line:

Ex. 1 Find the point-slope equation of a line passing through the points (-1, -2) and (2,5).

Ex. 2 Write the equation of a line passing through the points (4,7) and (0,3).

Parallel and Perpendicular Lines

Two non-vertical lines are parallel iff they have the same slope.

Two lines with non-zero slopes m_1 and m_2 are perpendicular iff $m_1 \cdot m_2 = -1$.

Ex. 3 Find the equation of the line passing through the point (-3,2) that is parallel to $5x - 2y = 3$.

Ex. 4 Find the equation of the line passing through (-4,3) which is perpendicular to the line passing through (-3,2) and (1,4).

Ex. 5 A new car costs $29,000. Its useful lifetime is approximately 12 years, at which time it will be worth an estimated $2000.00.

a) Find the linear equation that expresses the value of the car in terms of time.
b) How much will the car be worth after 6.5 years?

Ex. 6 The manager of a furniture factory finds that it costs \$2220 to manufacture 100 chairs and \$4800 to manufacture 300 chairs.

a) Assuming that the relationship between cost and the number of chairs produced is linear, find an equation that expresses the cost of the chairs in terms of the number of chairs produced.
b) Using this equation, find the factory's fixed cost (i.e. the cost incurred when the number of chairs produced is 0).

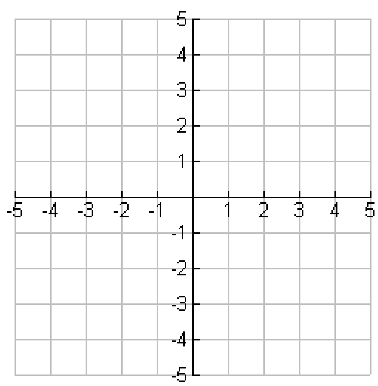
Ex. 7 Find the slope-intercept equation of the line that has an x-intercept of 3 and a y-intercept of 4.

Circles

Recall the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

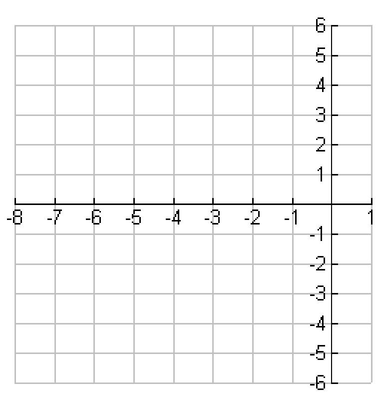
The Standard form for the equation of a circle is:

Ex.1 Write the equation of a circle with center (-1,2) and radius 3. Sketch this circle.



Ex.2 Write the equation of a circle with center at the origin and radius 1.

Ex.3 Find the equation of the circle with center (-4,1) that is tangent to the line x = -1.



Ex. 4 Find the equation of the circle with center (4,3) and passing through the point (1,4).

Ex. 5 Express the following equations of a circle in standard form. Identify the center and radius:

a) $x^2 + y^2 + 4x - 6y = 3$

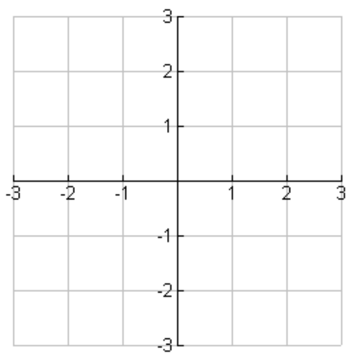
b) $x^2 - 2x + y^2 + 4y = 4$

The intercepts of a graph

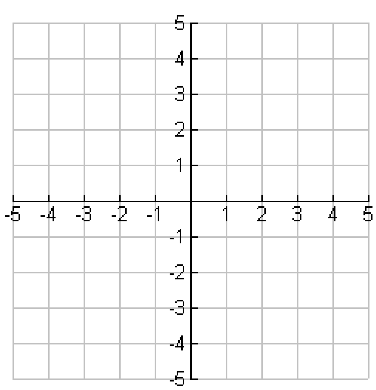
The x-coordinates of the x- intercepts of the graph of an equation can be found by setting $y=0$ and solving for x.

The y-coordinates of the y- intercepts of the graph of an equation can be found by setting $x=0$ and solving for y.

Ex. 1 Find the x and y intercepts of the line and sketch its graph: $-x+2y=1$



Ex. 2 Find the x and y intercepts of the circle and sketch its graph: $x^2+y^2=9$

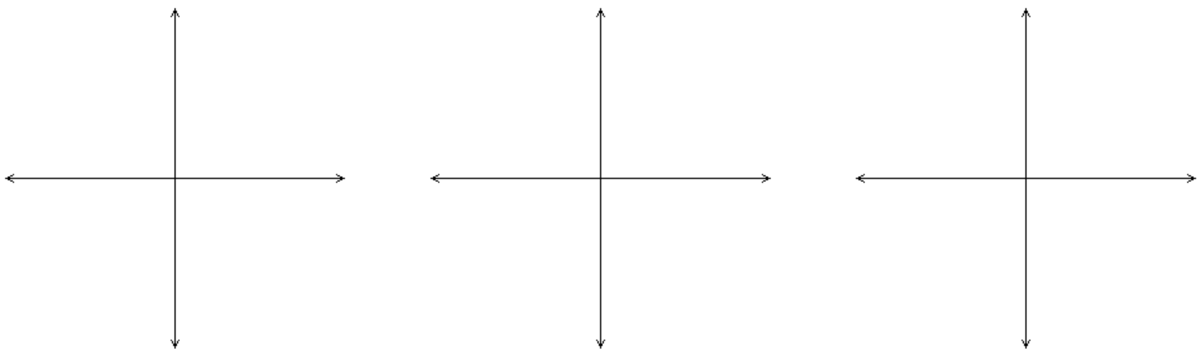


Ex. 3 Find the intercepts of the graphs of the equations.

a) $x^2-y^2=9$

b) $y = 2x^2 + 5x - 12$

Symmetry



In general :

- A graph is symmetric with respect to the $y-$axis if whenever (x, y) is on a graph $(-x, y)$ is also a point on the graph.
- A graph is symmetric with respect to the $x-$ axis if whenever (x, y) is on a graph $(x, -y)$ is also a point on the graph.
- A graph is symmetric with respect to the origin if whenever (x, y) is on a graph $(-x, -y)$ is also a point on the graph.

Tests for Symmetry:

The graph of an equation is symmetric with respect to:

a) the $y-axis$ is replacing x by $-x$ results in an equivalent equation.

b) the $x-axis$ is replacing y by $-y$ results in an equivalent equation.

c) the origin if replacing x and y by $-x$ and $-y$ results in an equivalent equation.

Ex. 1 Show that the equation $y = x^2 - 3$ has $y-$axis symmetry.

Ex. 2 Show that the equation $x + y^2 = 10$ has $x -$ axis symmetry.

Ex. 3 Show that the equation $x^2 + y^2 = 9$ has symmetry with respect to the origin.

Ex. 4 Find any intercepts of the graph of the given equation. Determine whether the graph of the equation possesses symmetry with respect to the $x -$ axis, $y -$ axis, or origin.

a) $x = y^2$

b) $y = x^2 - 4$

c) $y = x^2 - 2x - 2$

d) $y = |x - 9|$

Algebra and Limits

Difference of two squares: $a^2 - b^2 = (a-b)(a+b)$

Difference of two cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Sum of two cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Binomial Expansion $n = 2$: $(a \pm b)^2 = a^2 \pm 2ab + b^2$

Binomial Expansion $n = 3$: $(a \pm b)^3 = a^3 \pm 3a^2 b + 3ab^2 \pm b^3$

Limits

Ex. 1 Estimate $\lim_{x \to 2} \frac{x-2}{x^2-4}$ numerically by completing the following chart:

x	y	x	y
1.9		2.1	
1.99		2.01	
1.999		2.001	

Conclusion: $\lim_{x \to 2} \frac{x-2}{x^2-4} =$

Properties of Limits

If a and c are real numbers, then $\lim_{x \to a} c = c, \lim_{x \to a} x = a, \lim_{x \to a} x^n = a^n$

Ex. 2 Find the limit:

a) $\lim_{x \to 2} (x^3 - x + 4)$

b) $\lim_{x \to -1} (2x + 7)$

Ex. 3 Find the given limit by simplifying the expression

a) $\lim\limits_{x\to 2}\frac{x^2+x-6}{x^2-5x+6}$

b) $\lim\limits_{x\to -2}\frac{x^2-4}{x^3+8}$

c) $\lim\limits_{x\to 1}\frac{2-\sqrt{x+3}}{x-1}$

d) $\lim\limits_{x\to 2}\frac{x^2-7x+10}{x-2}$

e) $\lim\limits_{x\to -2}\frac{x+2}{\sqrt{x^2+5}-3}$